First Results from the Muon $g-2$ Experiment at Fermilab

Alessandra Lucà on behalf of the Muon $g-2$ E989 Collaboration

LIP Seminar

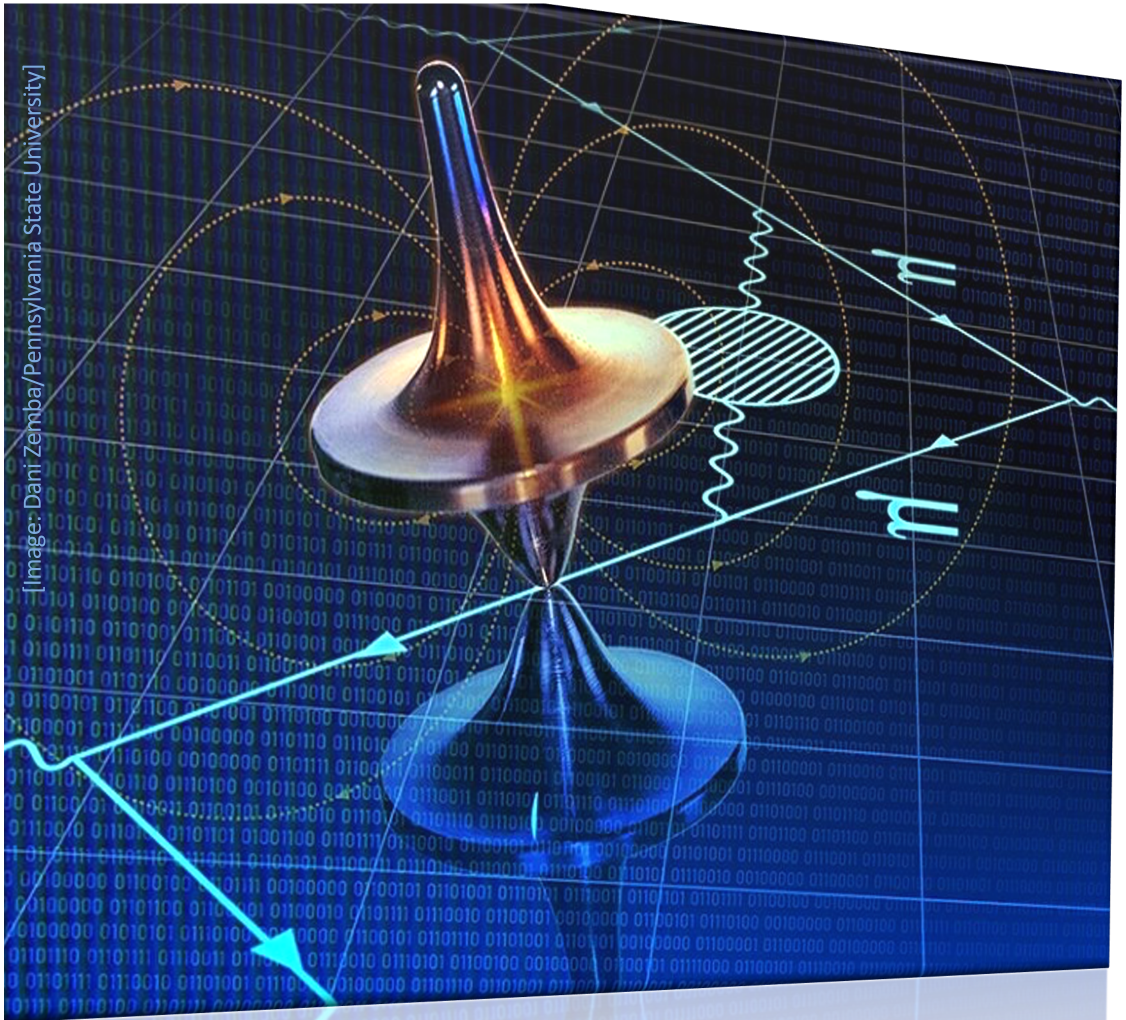
15 July 2021

Content

- **Introduction**
  - The magnetic anomaly and the g factor
  - Standard Model Prediction of the muon Anomalous Magnetic Moment

- **Experimental technique**
  - The Fermilab Muon $g - 2$ Experiment
  - Anomalous Precession
  - Magnetic Field

- **Run-1 Results**



Introduction:

Muon $g-2$ in the Standard Model

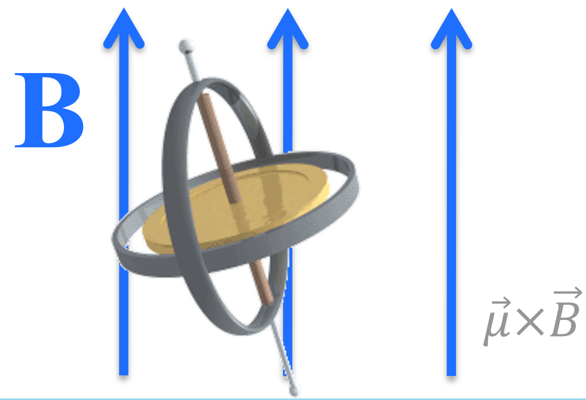
Magnetic anomaly a_e

- **The magnetic moment of a charged lepton ($\ell = e, \mu, \tau$)** is related to its intrinsic spin by the gyromagnetic ratio g_ℓ
  - $\vec{\mu}_\ell = \boldsymbol{g_\ell} \left(\frac{q}{2m_\ell}\right) \vec{s}$

- **Electron magnetic moment**
  - 1928: Dirac's equation predicted for the electron that
    - $g_e = 2$
  - 1947: Kusch and Foley's atomic spectroscopy experiment measured for the electron
    - $g_e = 2 \times (1.001\,19 + 0.000\,05)$

    $\boxed{a_e \equiv \frac{g_e - 2}{2}} = \mathbf{0.001\,19\,(5)}$
  - 1948: Schwinger proposed an additional contribution from a radiative correction, predicting the anomaly in agreement with experiment
    - $a_e = \frac{\alpha}{2\pi} \cong \mathbf{0.001\,16}$
  - Present: current theoretical prediction
    - $a_e = \mathbf{0.001\,159\,652\,181.61(23)}$

Image Credit: by Lucas Vieira from WP:PDI



The quantum theory of electron. Part II,
P. A. M. Dirac,
Proc. R. Soc. A 118, 351 (1928).

*Precision Measurement of the Ratio of the Atomic 'g Values' in the $^2P_{3/2}$ and $^2P_{1/2}$ States of Gallium**,
P. Kusch and H. M. Foley,
Phys. Rev. **72**, 1256 – Published 3 November 1947

On Quantum-Electrodynamics and the Magnetic Moment of the Electron,
Julian Schwinger
Phys. Rev. **73**, 416 – Published 15 February 1948

The Magnetic Moment of the Electron,
P. Kusch and H. M. Foley
Phys. Rev. **74**, 250 – Published 1 August 1948

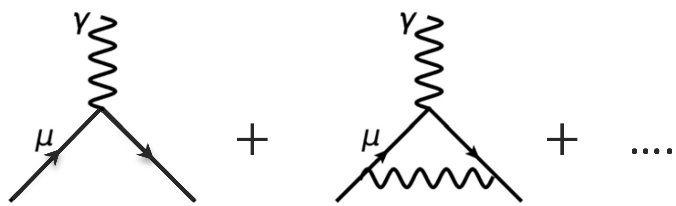
Theory of the Anomalous Magnetic Moment of the Electron,
Aoyama, T.; Kinoshita, T.; Nio, M.
Atoms 2019, **7**, 28

The scalar quantity a_ℓ is the magnetic anomaly, but is also commonly referred to as the "anomaly" or the "anomalous magnetic moment".

Anomalous Muon Magnetic Moment: a_μ^{SM}

- The lepton interacts with the magnetic field **B**, directly or *via* virtual particles

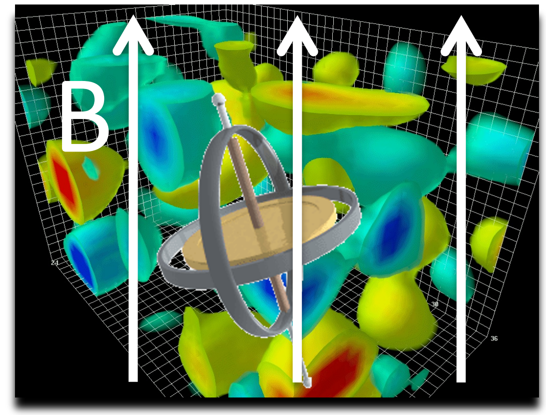
Schwinger O(α)
one-loop effect of QED



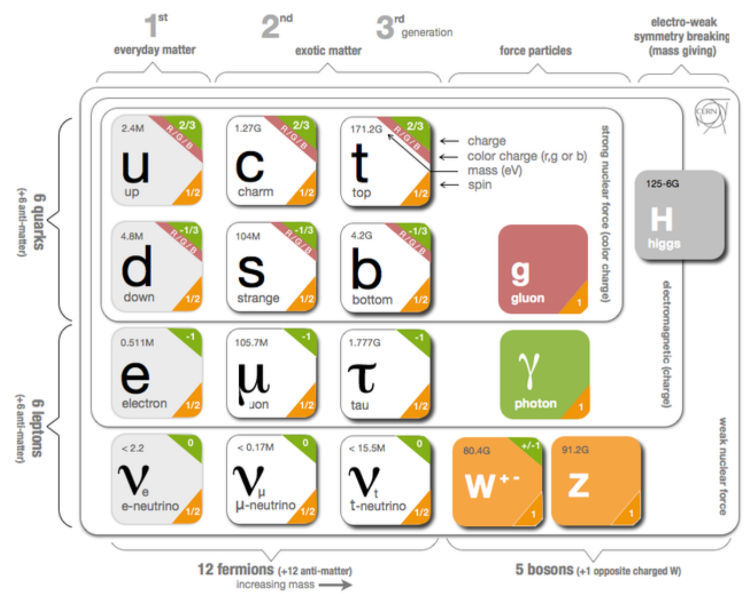
$$g_\mu = 2\times\Big(\; 1 \;+\; \underbrace{\frac{\alpha}{2\pi} \;+\; \dots}\;\Big)$$

$$a_\mu^{SM} = a_\mu^{QED} + a_\mu^{weak} + a_\mu^{hadronic}$$

Image Credit: [Derek Leinweber]



- The Standard Model (SM) contributions to the anomaly include electromagnetic, weak, and strong interactions.

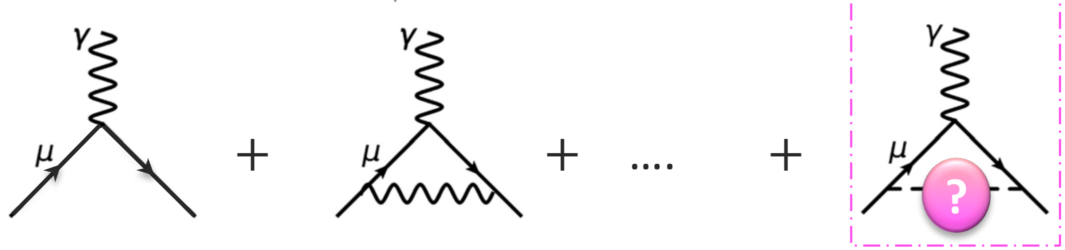


Anomalous Muon Magnetic Moment: a_μ

- The lepton interacts with the magnetic field **B**, directly or *via* virtual particles

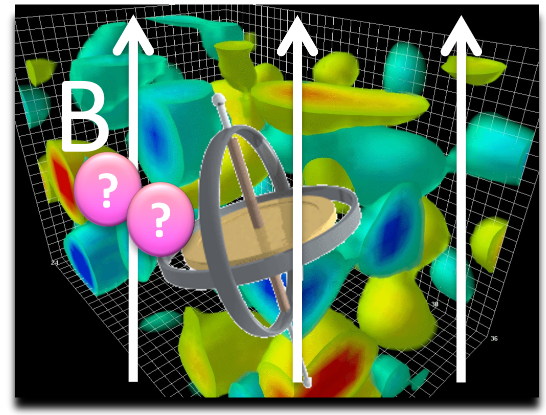
Schwinger O(α)
one-loop effect of QED

Effects of hypothetical new particles
contributing through virtual loops



$g_\mu = 2\times(\quad 1 \quad + \quad \frac{}{2\pi} \quad + \quad \dots \quad + \quad \dots \quad)$

Image Credit: Derek Leinweber



$$a_\mu = a_\mu^{QED} + a_\mu^{weak} + a_\mu^{hadronic} + \boldsymbol{a_\mu^{NP?}}$$

- ○ The Standard Model (SM) contributions to the anomaly include electromagnetic, weak, and strong interactions.

WHY MUONS?

The relative contribution of virtual heavy particles in many cases scales as $\left(\frac{m_\mu}{m_e}\right)^2 \approx 43{,}000$

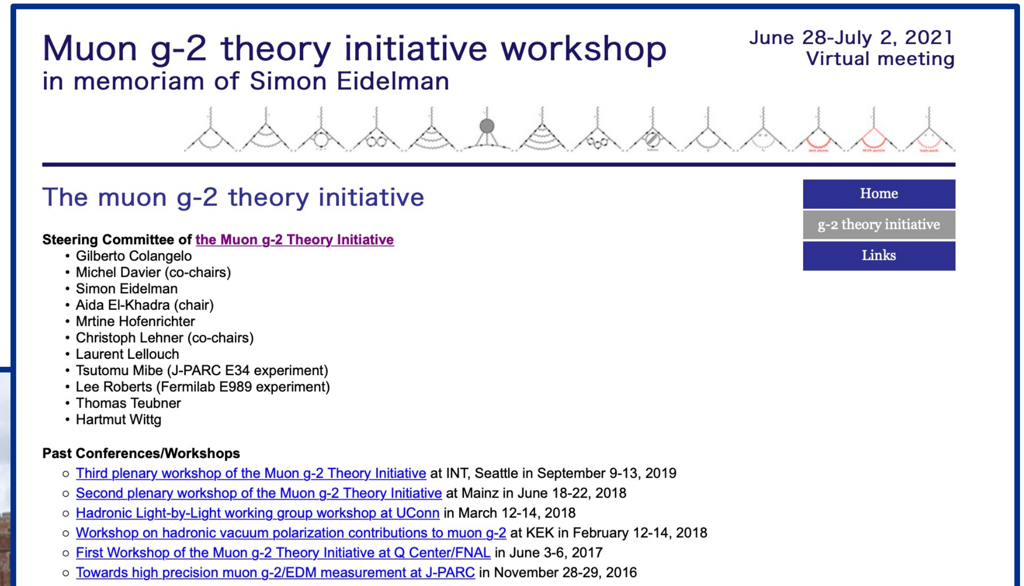
- **Electrons:** Weak and hadronic interactions have miniscule impact on a_e
- **Muons:**
  - Weak and hadronic processes have bigger effect
  - **a_μ is a very sensitive probe of New Physics (NP)**

The Muon $g-2$ Theory Initiative, White Paper 2020

- The Muon g – 2 Theory Initiative is a group formed to evaluate all aspects of the SM and to recommend a consolidated value against which new experimental results should be compared.

- This White Paper (WP) is the first product of the Initiative:
  - ``The anomalous magnetic moment of the muon in the Standard Model" T. Aoyama et al, arXiv:2006.04822, Phys. Repts. 887 (2020) 1-166]
  - WP posted 10 June 2020 (132 authors, from 82 institutions, in 21 countries)

- Our collaboration uses the WP value for comparison
  - $a_\mu^{SM} = 116\,591\,810\ (43)\times 10^{-11}$



Muon g-2 theory initiative workshop
in memoriam of Simon Eidelman

June 28-July 2, 2021
Virtual meeting

The muon g-2 theory initiative

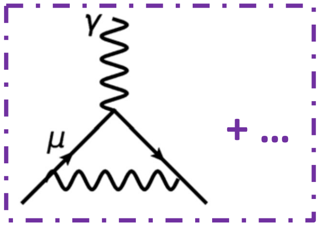
Home
g-2 theory initiative
Links

Steering Committee of the Muon g-2 Theory Initiative
- Gilberto Colangelo
- Michel Davier (co-chairs)
- Simon Eidelman
- Aida El-Khadra (chair)
- Mrtine Hofenrichter
- Christoph Lehner (co-chairs)
- Laurent Lellouch
- Tsutomu Mibe (J-PARC E34 experiment)
- Lee Roberts (Fermilab E989 experiment)
- Thomas Teubner
- Hartmut Wittg

Past Conferences/Workshops
- Third plenary workshop of the Muon g-2 Theory Initiative at INT, Seattle in September 9-13, 2019
- Second plenary workshop of the Muon g-2 Theory Initiative at Mainz in June 18-22, 2018
- Hadronic Light-by-Light working group workshop at UConn in March 12-14, 2018
- Workshop on hadronic vacuum polarization contributions to muon g-2 at KEK in February 12-14, 2018
- First Workshop of the Muon g-2 Theory Initiative at Q Center/FNAL in June 3-6, 2017
- Towards high precision muon g-2/EDM measurement at J-PARC in November 28-29, 2016

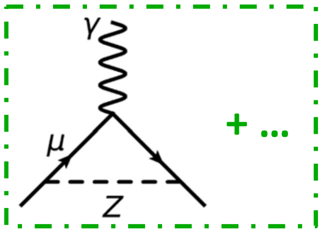
https://muon-gm2-theory.illinois.edu

The SM contributions to the muon anomaly



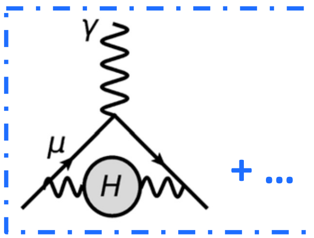
$$a_\mu^{QED} = 116\,584\,718.9\ (1)\times10^{-11} \qquad (0.001\ ppm)$$

Calculated up to tenth order in the perturbative expansion, i.e.,O(α⁵):
-> Subset of 12672 5-loop diagrams
Gives the largest contribution
[Ref. qed1, qed2]



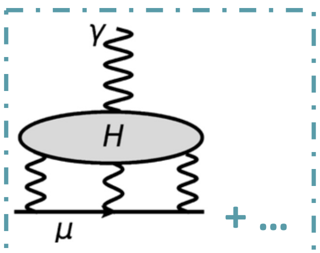
$$a_\mu^{EW} = 153.6(1.0)\times10^{-11} \qquad (0.01\ ppm)$$

Known to 2-loop. Small contribution; uncertainty 10x larger than the QED one, but still negligible with respect to the hadronic uncertainties
[Ref. ew]



$$a_\mu^{HVP} = 6\,845\ \ (40)\times10^{-11} \qquad (0.34\ ppm)$$

HVP (e+e-, LO + NLO + NNLO)
Hadronic Vacuum Polarization
Data-driven evaluations using dispersion relations to connect *e+ e-* absolute cross section data to a_μ
Largest uncertainty
[Ref. hvp]

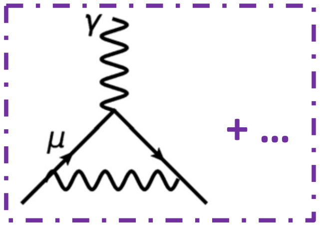


$$a_\mu^{HLbL} = 92\ \ (18)\times10^{-11} \qquad (0.15\ ppm)$$

HLbL (phenomenology + lattice + NLO)
Hadronic Light-by-Light Scattering
Essentially based on a dispersive approach
[Ref. hlbl]

$$\boldsymbol{a_\mu^{SM} = 116\,591\,810\ \ (43)\times10^{-11} \qquad (0.37\ ppm)}$$

The SM contributions to the muon anomaly



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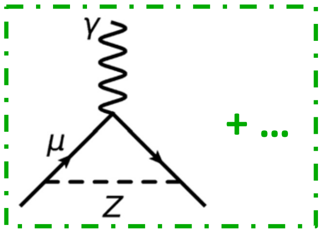

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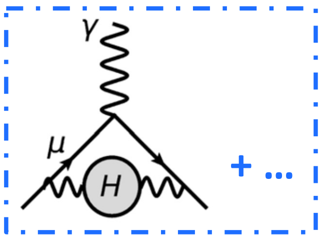

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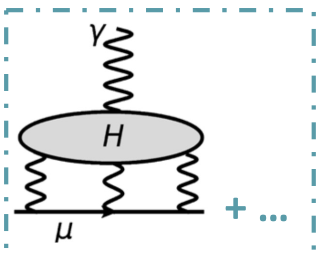

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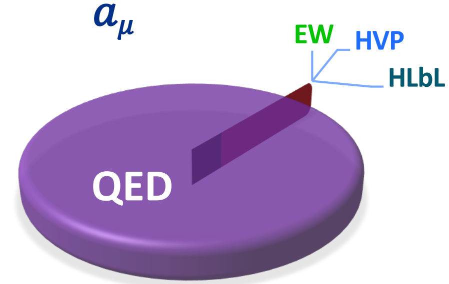
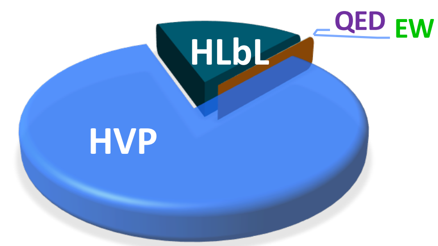
a_μ



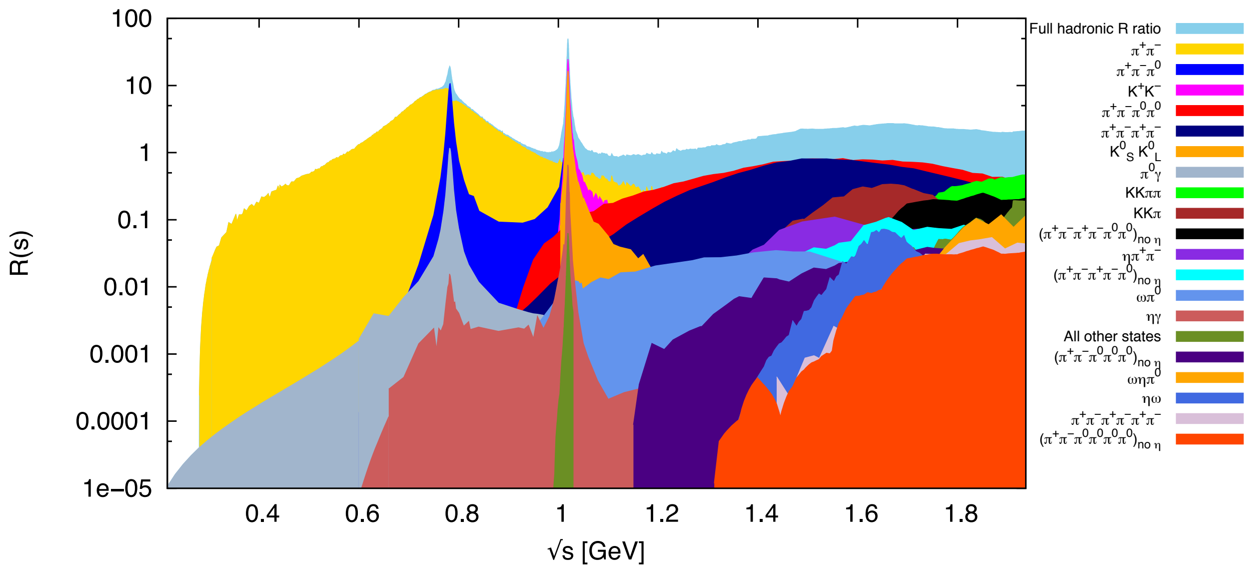
ERROR²

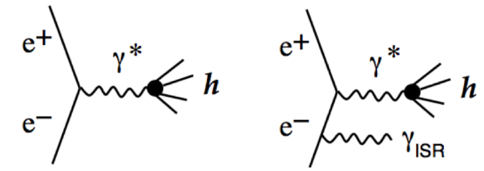


The uncertainty on the theoretical prediction of a_μ^{SM} is dominated by the strong interactions.

The a_μ^{HVP} contributions to the muon anomaly

SM uncertainty dominated by Hadronic Vacuum Polarization [KNT18, PRD97, 114025]





$$a_\mu^{\text{had;LO}} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_{m_\pi^2}^{\infty} \frac{ds}{s^2} K(s) R(s)$$

$$R \equiv \frac{\sigma_{\text{tot}}(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} \qquad K(s)/s = \mathcal{O}(1)/s$$

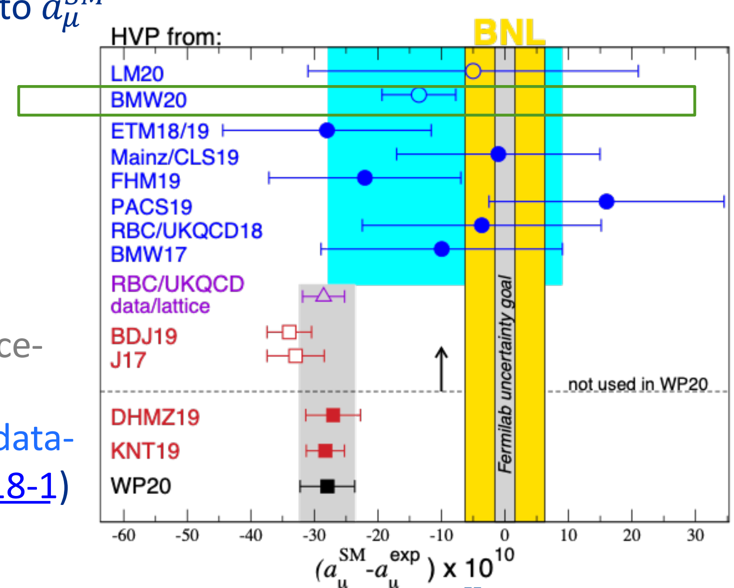
→ Major contribution from Lower energies:

→ 73% of total $a_\mu^{had;LO}$ from $\pi^+\pi^-$ channel

- Dispersive data-driven approach using e^+e^- cross section data to connect to a_μ^{SM}
  Uncertainties include:
  - Differences in data treatment and combination
  - Conservative systematic uncertainties for tensions in data

WP20 value relies on these determinations

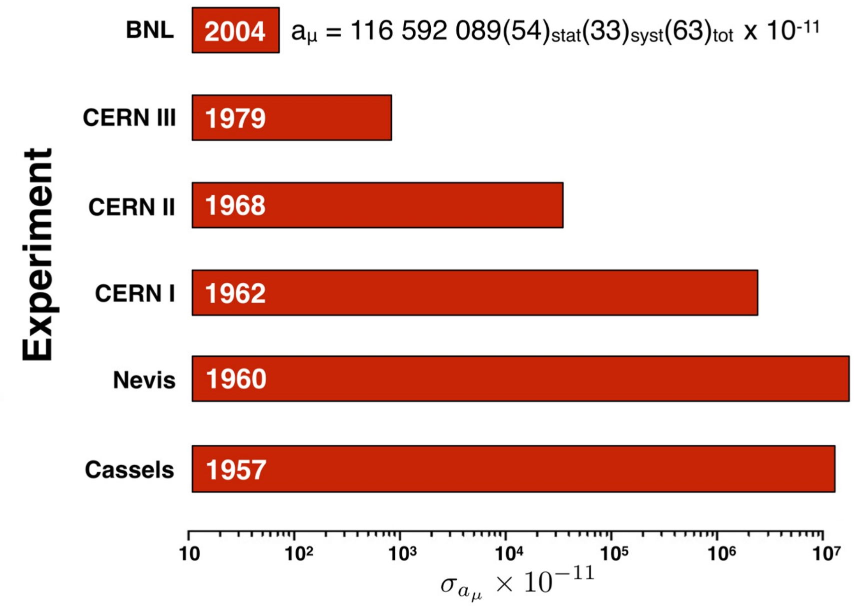
- Lattice calculations
  - Simulate the strong interaction (+photons) w. discretized Euclidean space-time - independent of scattering data stat. and syst. uncertainties
  - BMW20 result first to achieve sub-percent precision, competitive with data-driven calculations (https://www.nature.com/articles/s41586-021-03418-1)
  - BMW20 in tension with the WP20 result
  - Further development is expected

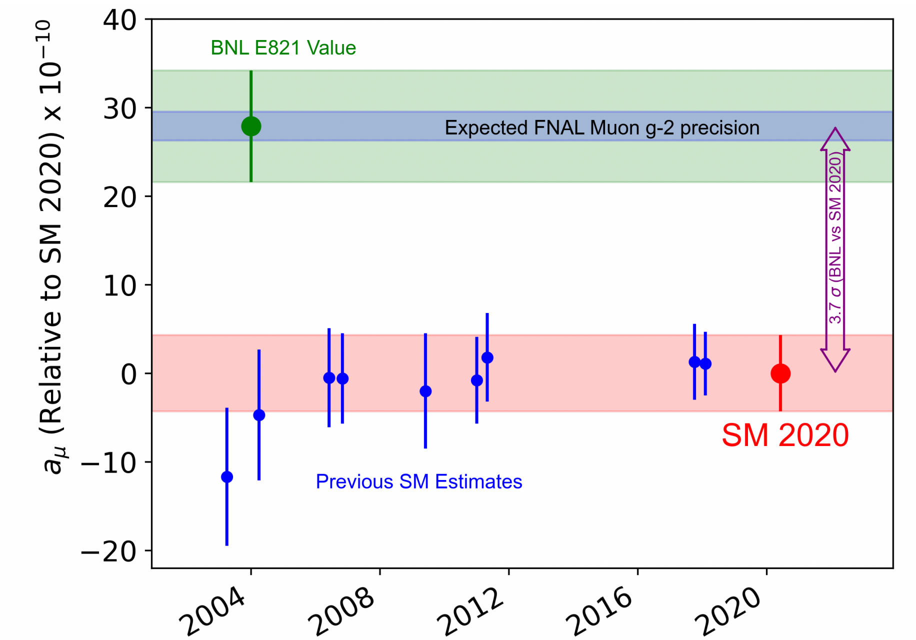


The measured a_μ

- The measurement of a_μ has become increasingly precise through a series of innovations at CERN and more recently at Brookhaven (BNL E821).
- The BNL result has a **540 ppb** uncertainty.

[BNL Result: Phys. Rev. D **73**, 072003 (2006)]





HIGH PRECISION TEST OF THE STANDARD MODEL

- E821 experiment at BNL

  a_μ(BNL) = 116,592,089(<u>63</u>) × 10^{-11} (**0.54 ppm**)

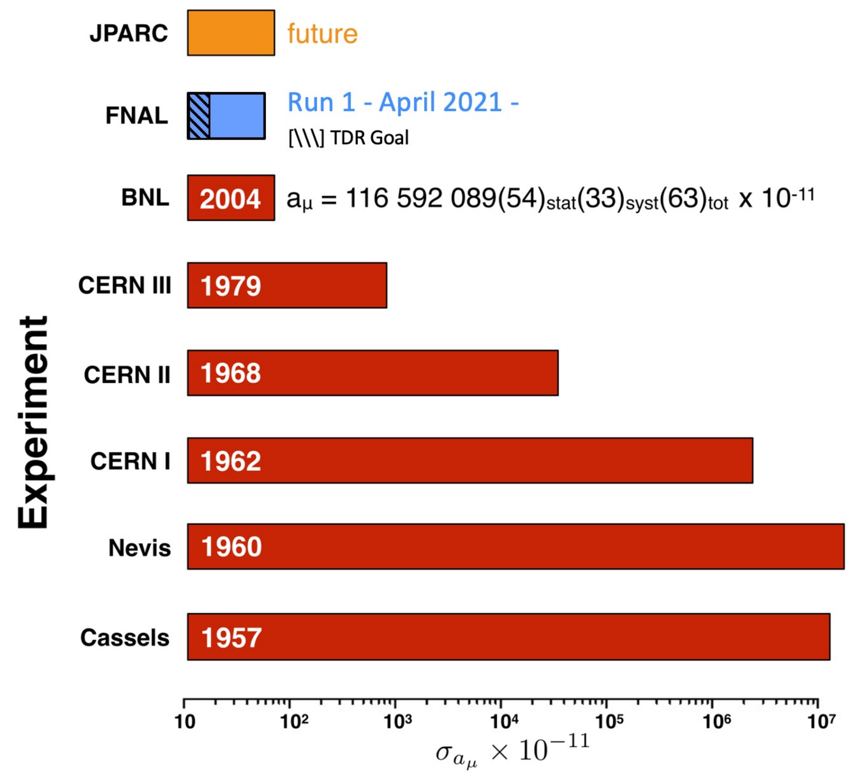
- Standard Model Prediction

  a_μ(SM) = 116,591,810(<u>43</u>) × 10^{-11} (**0.37 ppm**)

- Difference

  $\Delta a_\mu := a_\mu(\text{BNL}) - a_\mu(\text{SM}) = 279(\underline{76}) \times 10^{-11}$ **3.7 σ**

The measured a_μ



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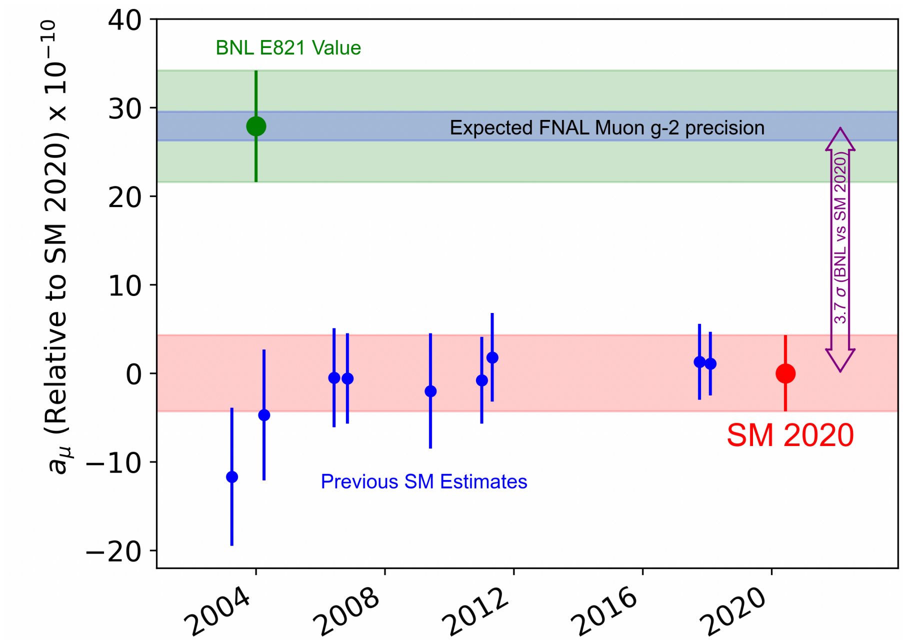
- **THE EXPERIMENT AT FNAL AIMS FOR A 4x-IMPROVEMENT** in the uncertainty: 100 ppb stat. ⊕ 100 ppb syst. Result presented in this talk is based on Run-1 dataset (similar size to BNL ~ 10 billion μ^+)



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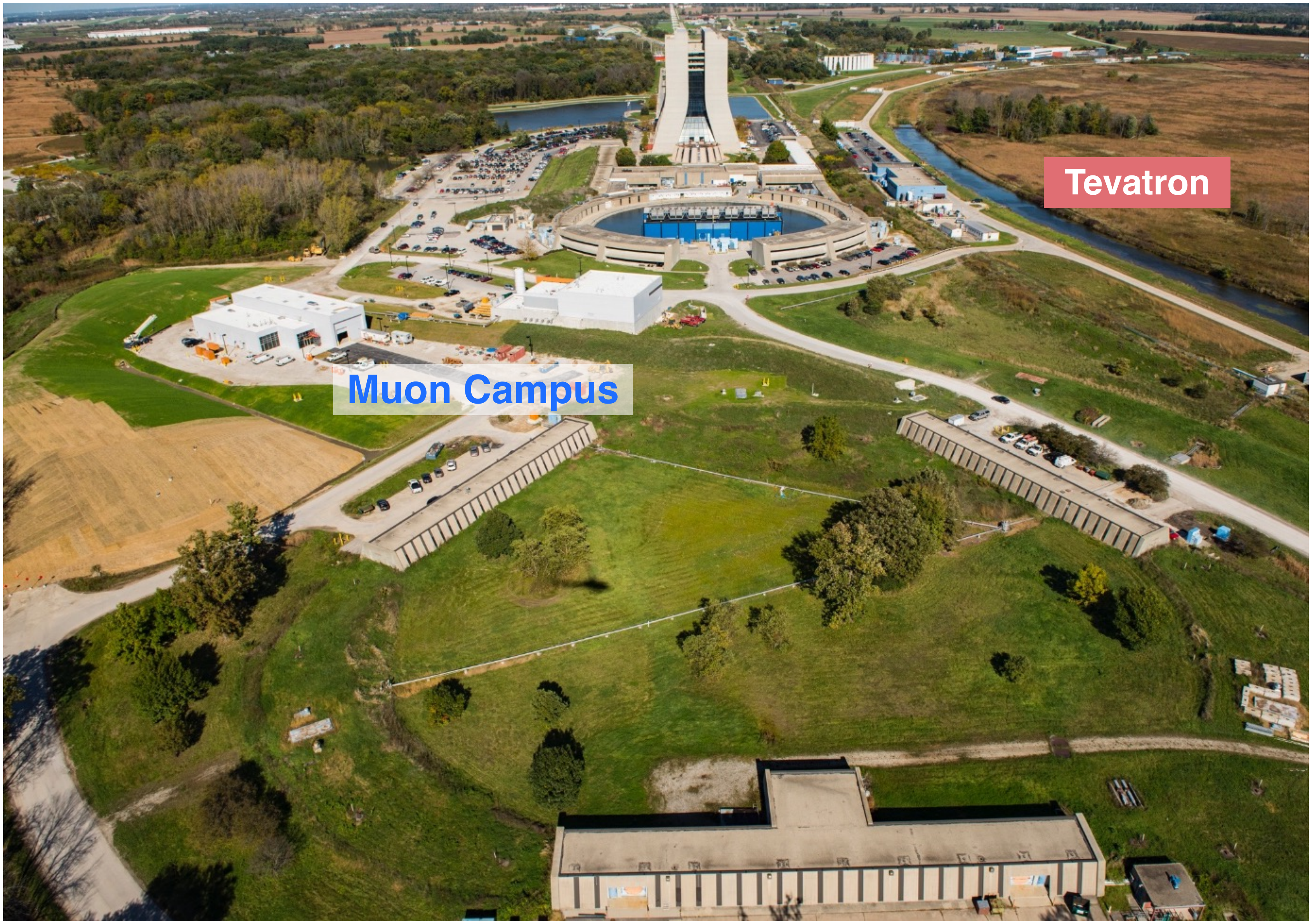
Experimental method:

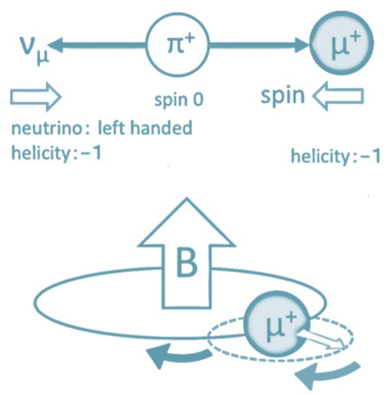
The $g_\mu - 2$ measurement at Fermilab

Fermilab

CHICAGO



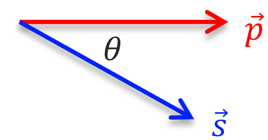
Principles of the experiment (1/2)



The experiment follows the BNL concept:

- *longitudinally polarized muon beam injected into a magnetic storage ring*

- *measurement of difference between spin precession and cyclotron frequencies*



$$\omega_a = \dot{\theta}$$

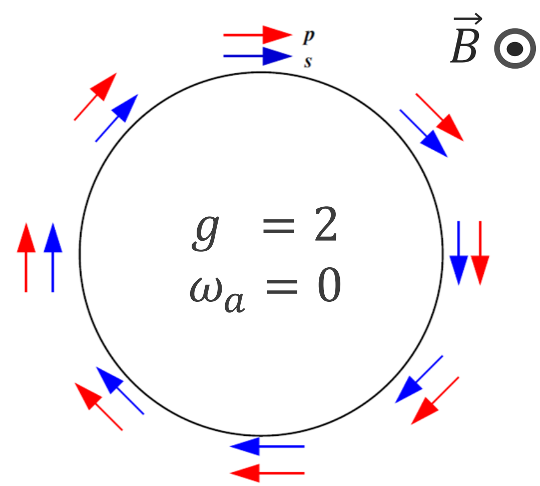
spin precession frequency: $$\vec{\omega}_s = -\frac{q\vec{B}}{\gamma m_\mu}(1 + \gamma a_\mu)$$

cyclotron frequency: $$\vec{\omega}_c = -\frac{q\vec{B}}{\gamma m_\mu}$$

anomalous spin-precession frequency:

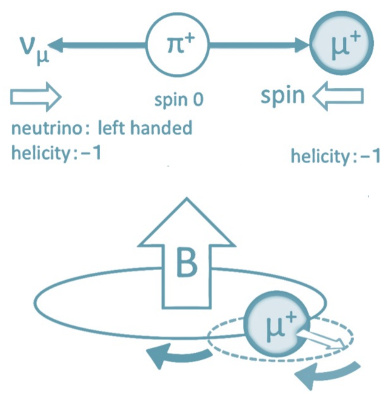
$$\vec{\omega}_a \equiv \vec{\omega}_s - \vec{\omega}_c = -a_\mu \frac{q\vec{B}}{m_\mu}$$

N.B. This equation is for a uniform vertical magnetic field and an ideal horizontal orbit anomalous spin-precession



Spin always pointing tangentially

Principles of the experiment (1/2)



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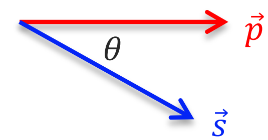

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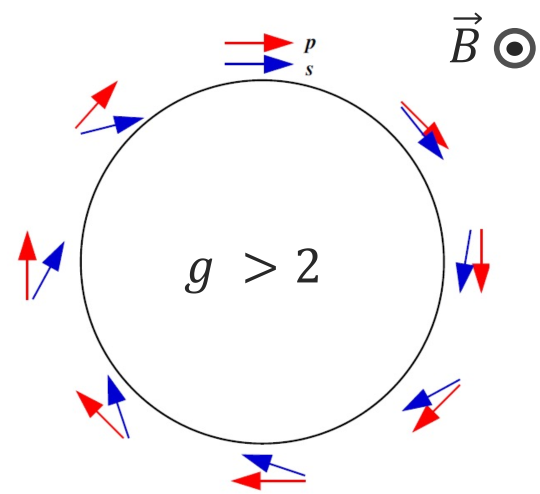


- muon spins precess in the magnetic field at a rate greater than the cyclotron frequency → $\boldsymbol{\omega_a \propto a_\mu}$

The E989 beamline

The E989 beamline delivers a muon beam with momentum p_0 = 3094 MeV/c to the storage ring.

8 GeV protons from the LINAC and the Booster are injected into the **Recycler**

- protons are combined into batches to form a more intense beam (each batch has 4 bunches of 10^{12} p's)

A set of **beamlines** connects the Recycler to the Muon Campus.

- The proton beam hits a **target**, converting the beam to a mixture of mainly π's, p's and μ's.
- Positive particles within ±10% of 3.115 GeV/c are selected and steered into a beamline
- The particles circle the **Muon Delivery Ring** several times, where p's are then removed and the remaining π's decay into μ's.
- The muon beam travels to the **MC-1 Building** and is injected into the storage ring:
  - 95% polarized muon beam injected
  - 11.4 Hz average injection (fill) rate
  - ~10,000 stored muons per fill
  - Muons observed for > 10 lifetimes

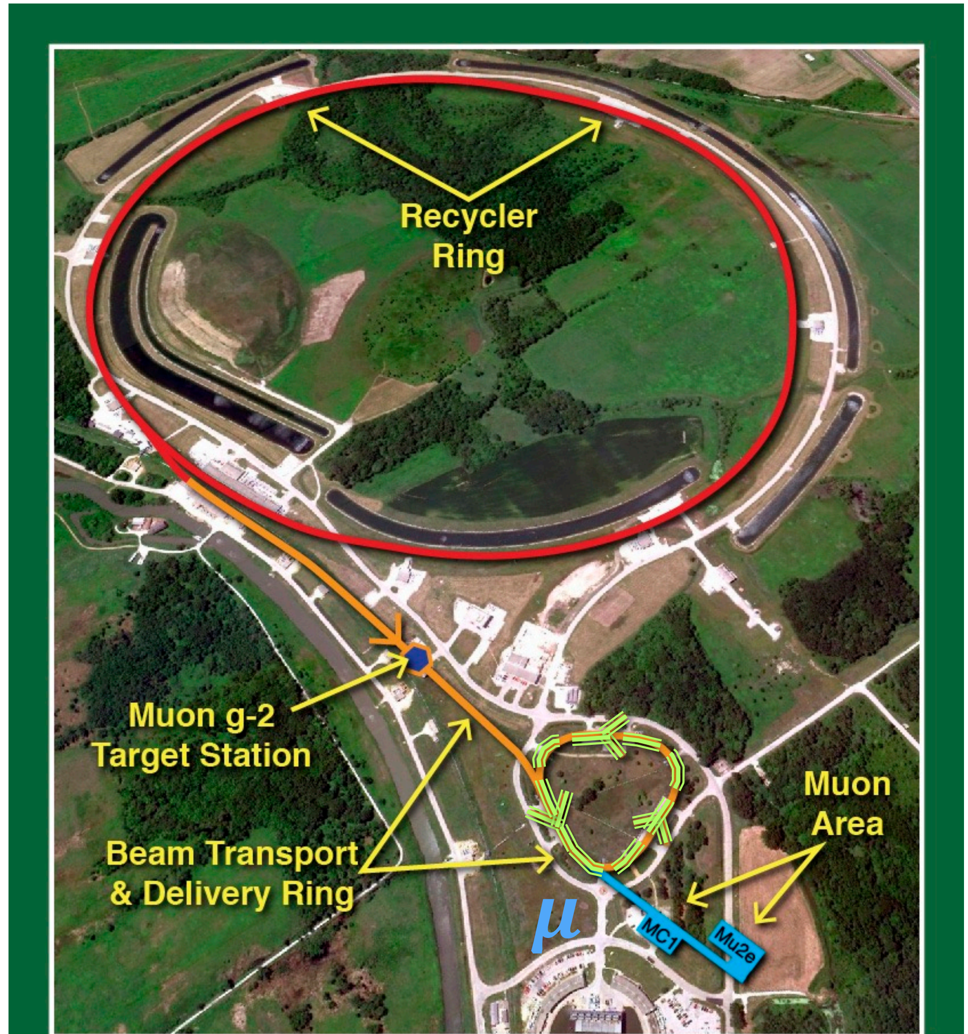


Image Credit: Fermilab

The Big Move

Image Credit: Symmetry Magazine

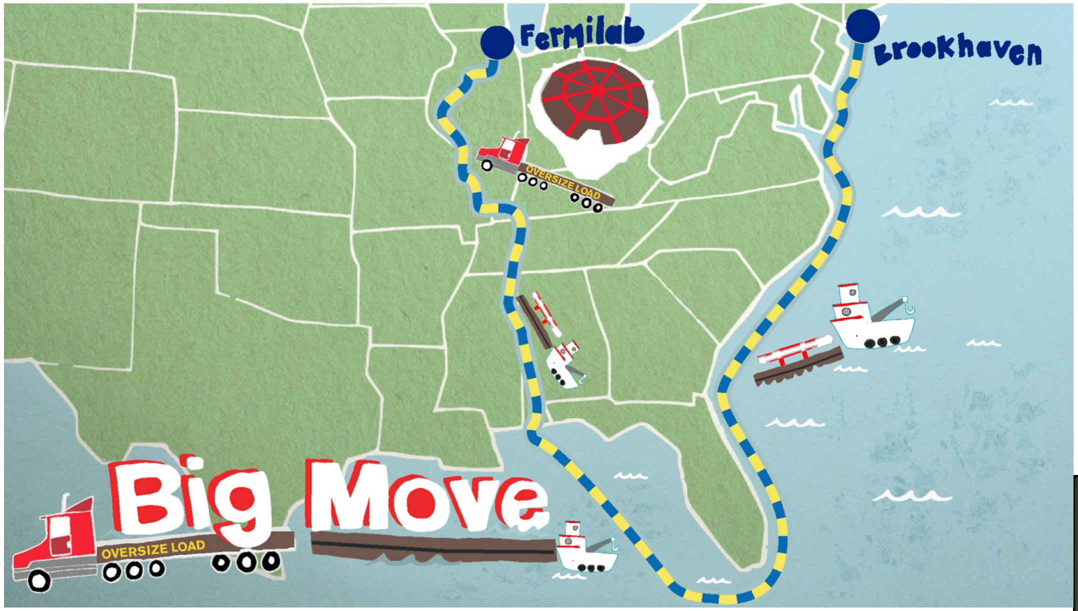
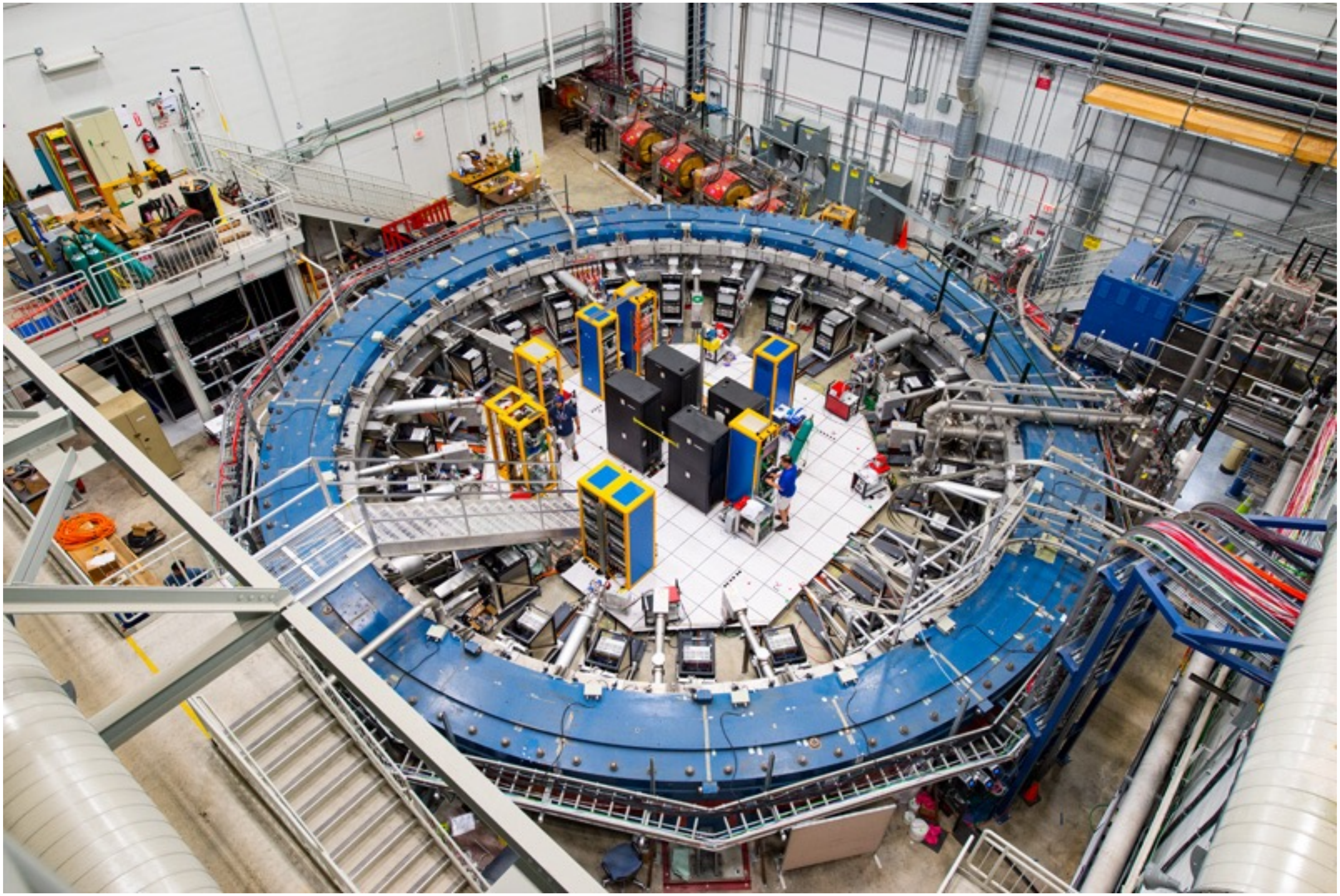


Image Credit: Fermilab

The g-2 ring journey

from Brookhaven to Fermilab (2013)

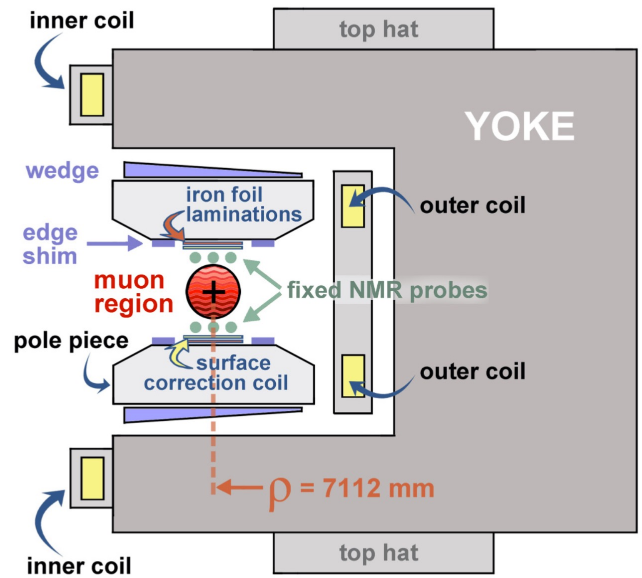
The Muon $g-2$ storage ring



The Muon $g-2$ storage ring

Storage ring magnet

- ~15 m diameter magnet
- Uniform & homogeneous vertical magnetic field $\mathbf{B}$ of 1.45 T.
- 12 C-shaped Yokes



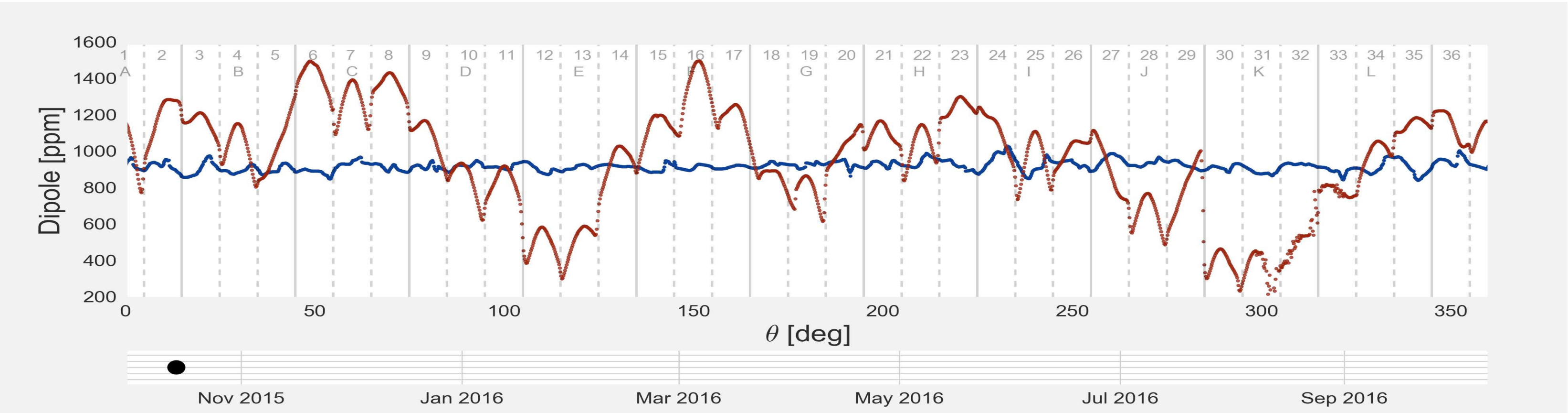
Field Shape

Determined by positioning of pole pieces, wedge-shaped pieces of steel, programmable surface coils

Mechanical shimming

Oct 2015 -> Sept 2016 : Shimming in order to obtain a uniform $\mathbf{B}$

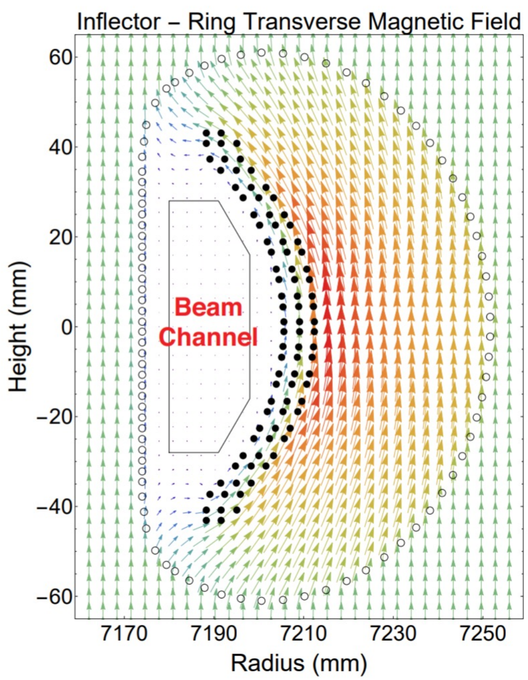
➢ field homogeneity of ~14 ppm RMS (3 times better than BNL)

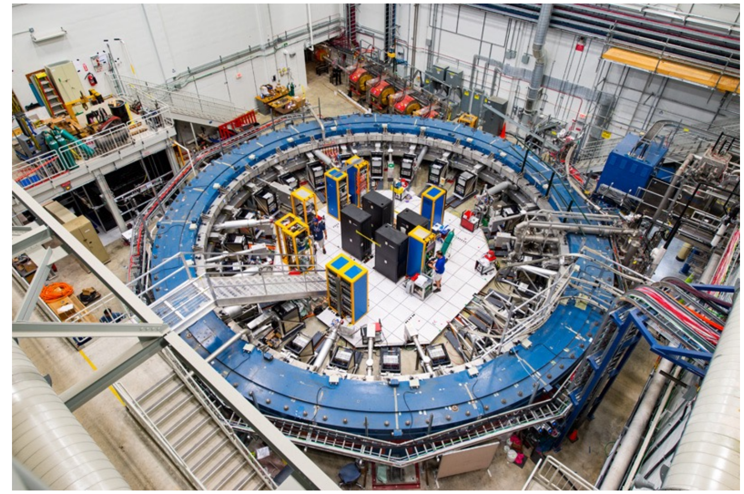


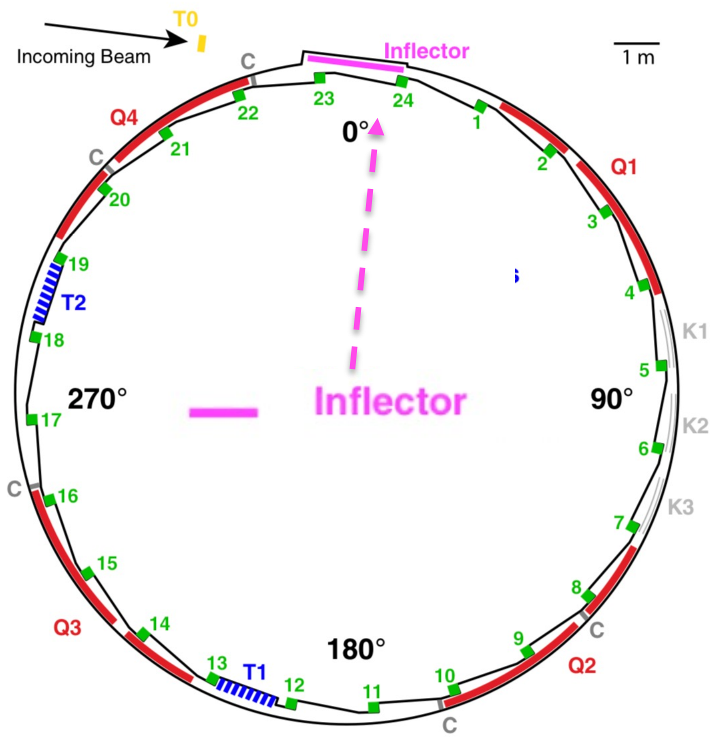
BNL field map
FNAL field map

Overview of the storage ring and its subsystems

- Polarized anti-muon bunch (3.1 GeV) is injected into the storage ring.
- Superconducting **inflector** magnet cancels the main focusing magnetic field (1.5 T) to inject the bunch into the storage ring tangentially.

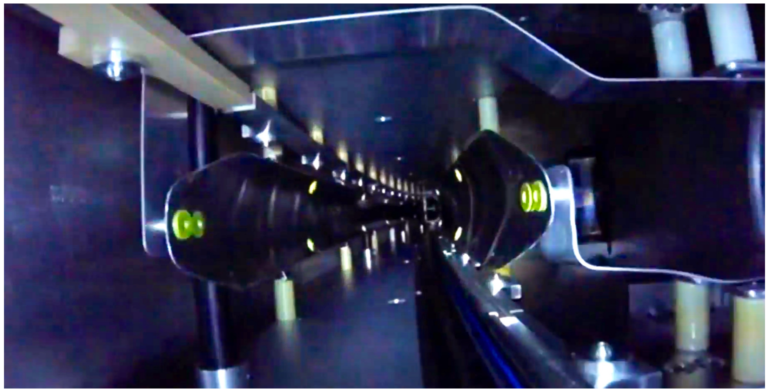


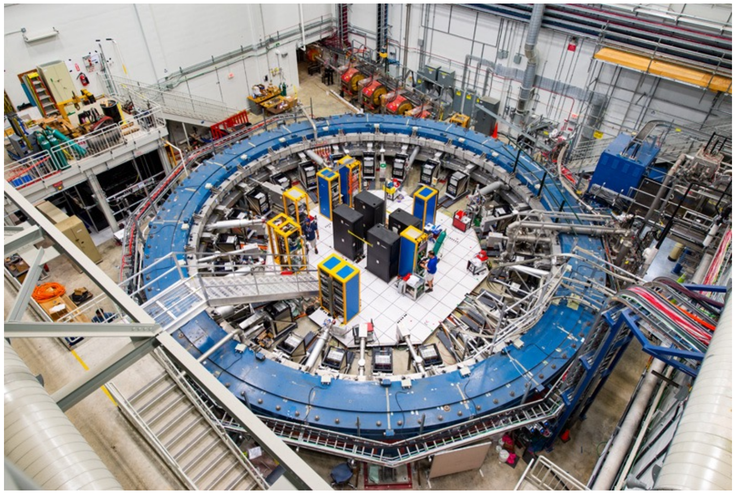


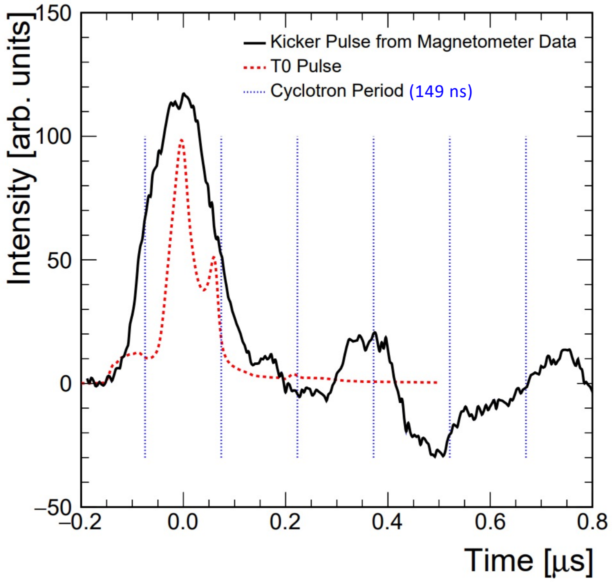


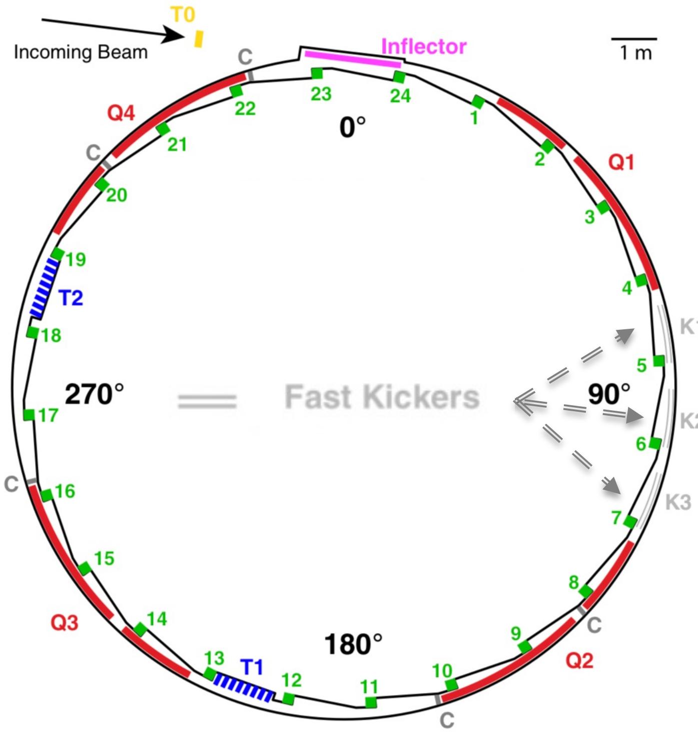
Overview of the storage ring and its subsystems

- The fast non-ferric **kicker magnet system** applies a short magnetic pulse to kick the muons onto the design orbit



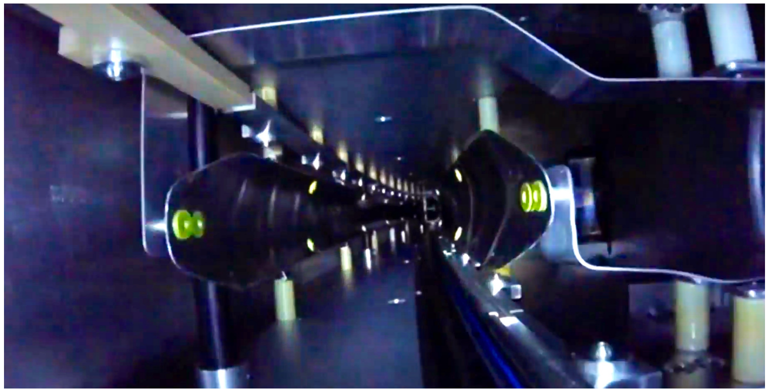


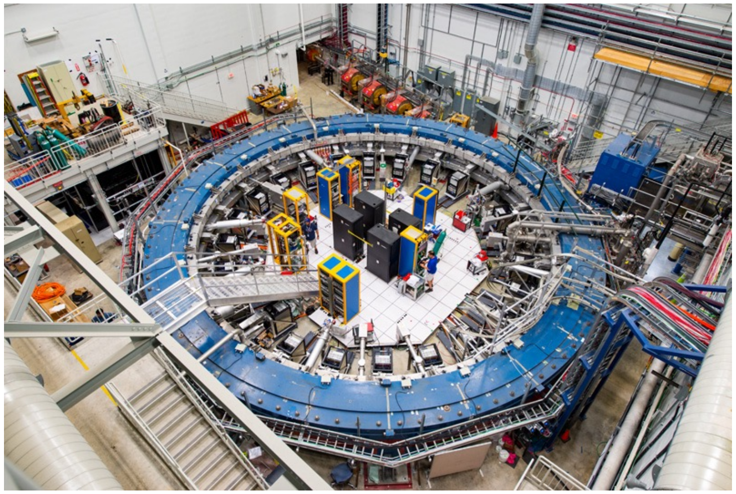


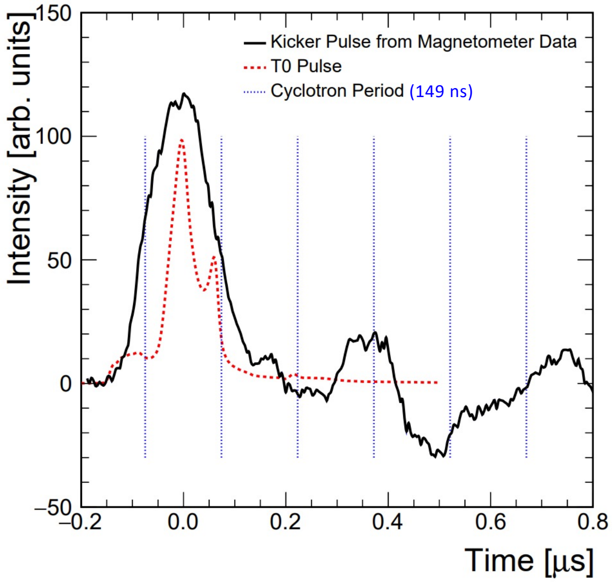


Overview of the storage ring and its subsystems

- The fast non-ferric **kicker magnet system** applies a short magnetic pulse to kick the muons onto the design orbit

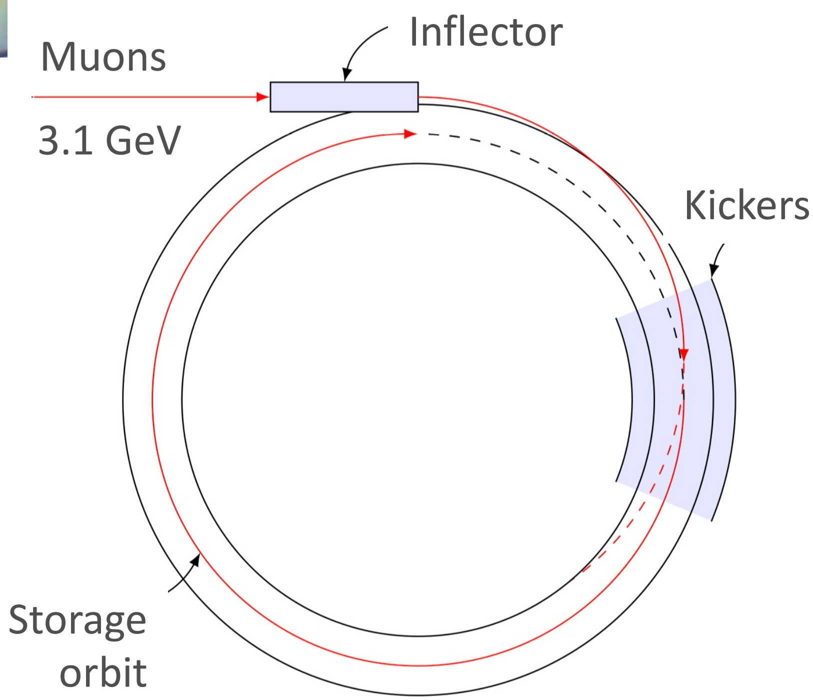






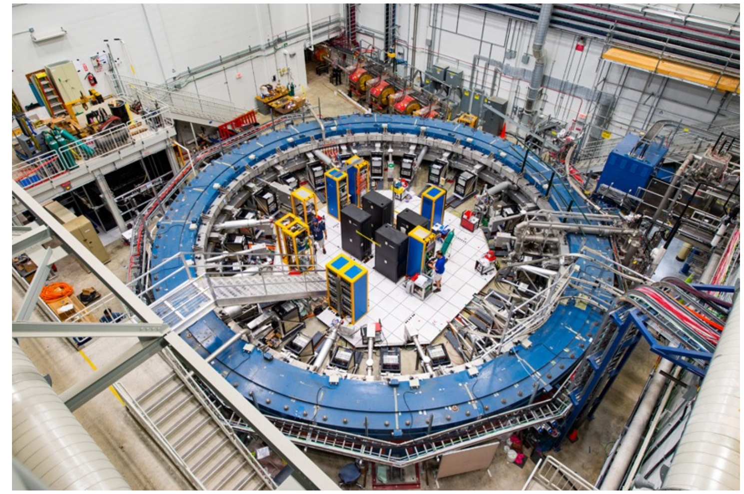
- After the inflectors, muons are 77 mm away from the ideal radius
- Kickers fire on the first turn after injection → ~11 mrad radial deflection

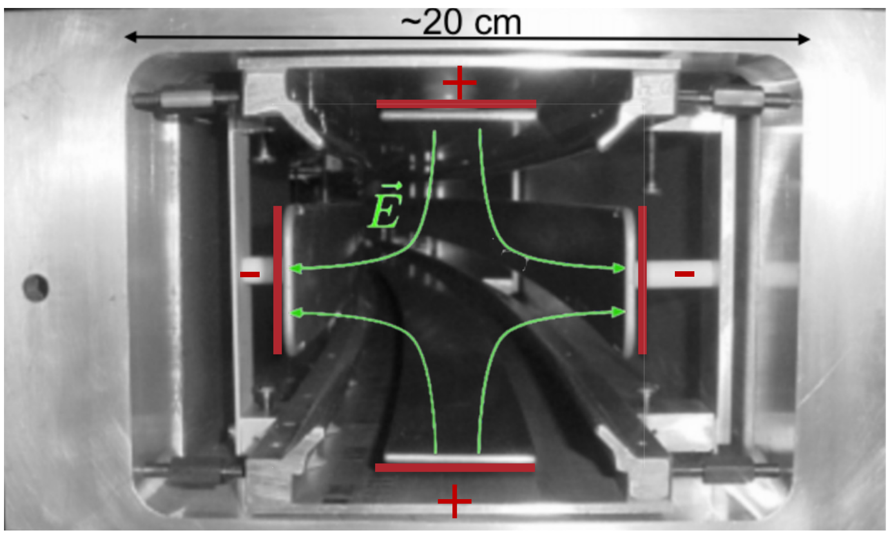
➢ Not all entering muons experience the same magnetic deflection

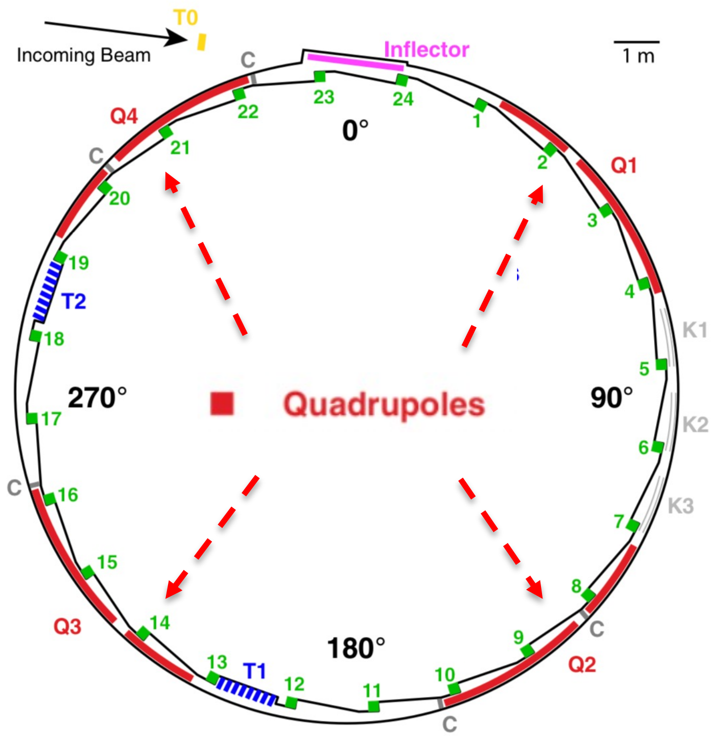


Overview of the storage ring and its subsystems

- **Electrostatic Quadrupoles** (ESQ) focus the beam vertically.
  - 4 Quadrupole sections (long and short for each) occupy 43% of the ring circumference.



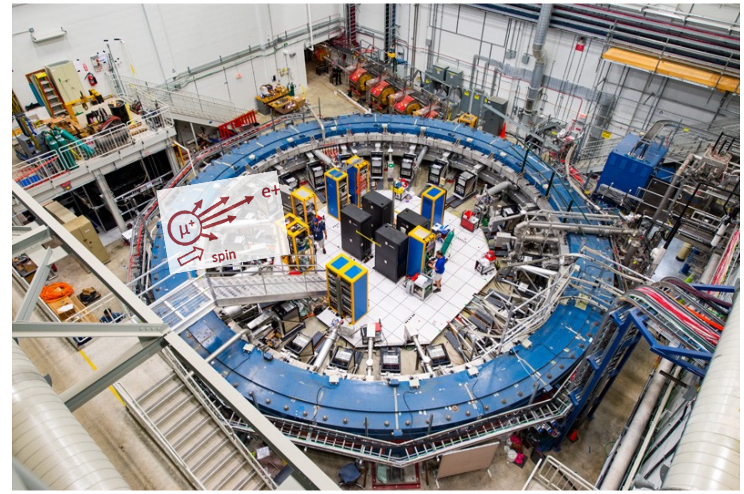


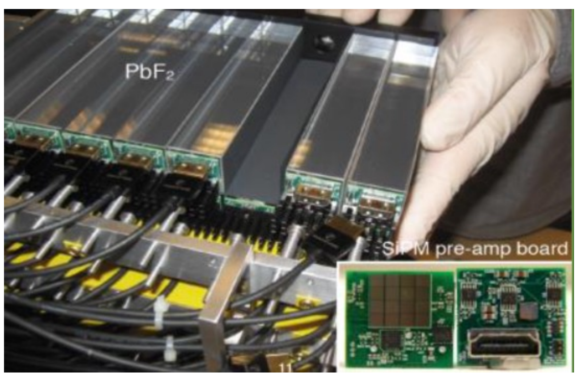


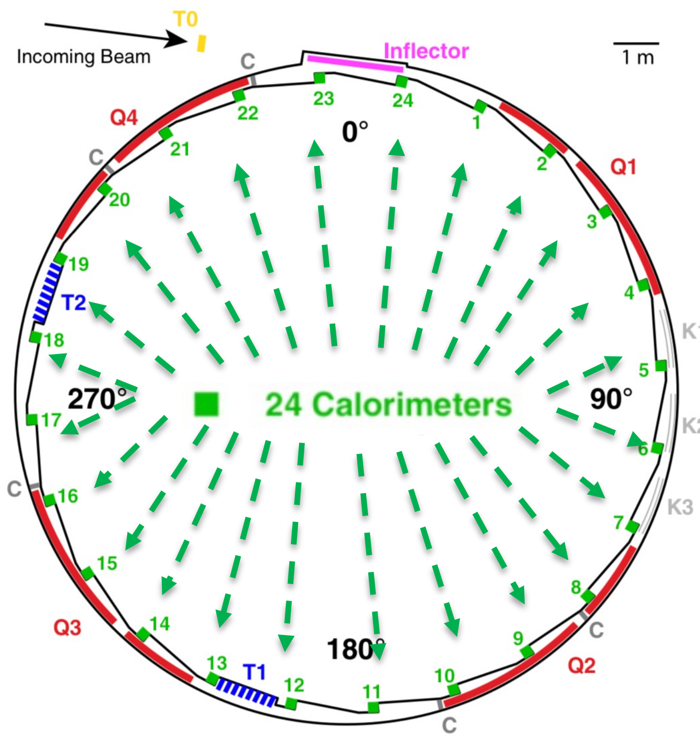
Fermilab

Overview of the storage ring and its subsystems

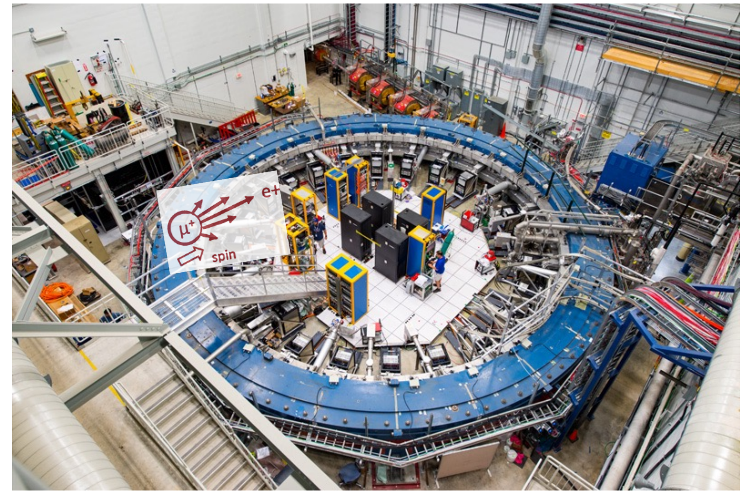
- Detection
  - Muons decay into positrons which curl into **24 electromagnetic calorimeters** surrounding the storage ring:
    - each contains a 9 × 6 array of 2.5 × 2.5 × 14 cm^3 (15 X_0) PbF_2 crystals, readout by SiPMs to 800 MHz WFDs





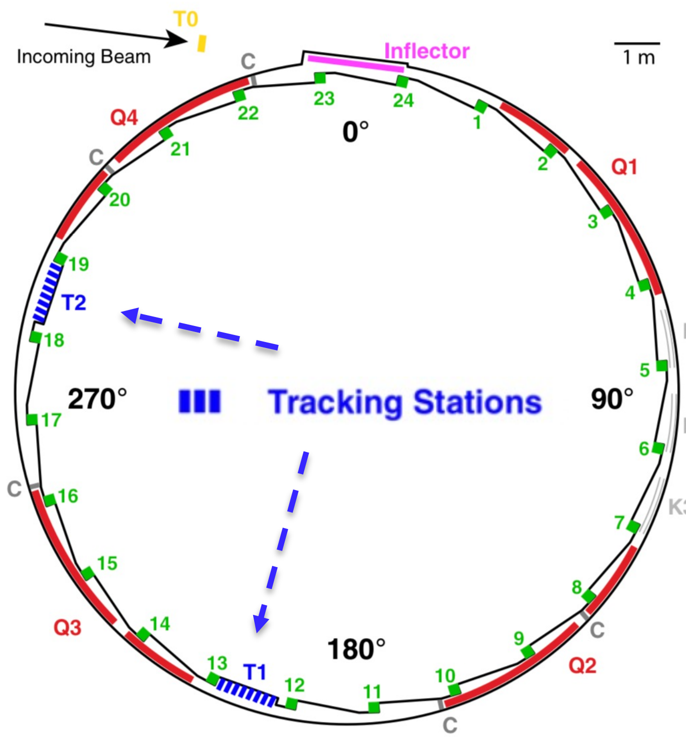


Overview of the storage ring and its subsystems



- Detection
  - **trackers** measures trajectories of the decay positrons and extrapolates to find the muon distribution:
    - each contains 8 modules of 128 gas filled straws

MUON'S VIEW OF A TRACKER



Courtesy of W. Turner and J. Grange

The Muon Anomaly Measurement (1/2)

The magnetic-field experienced by the muons is measured with pulsed proton NMR

$$a_\mu = -\frac{m_\mu}{q}\frac{\vec{\omega}_a}{\vec{B}}$$

→ is related to the **Larmor precession frequency** of protons shielded in a spherical water sample at a reference temperature T_r

THE AVERAGE FIELD OVER THE MUON DISTRIBUTION WEIGHTED BY THE DETECTED DECAYS OVER TIME

$$\tilde{B} = \frac{\hbar\tilde{\omega}'_p(T_r)}{2\mu'_p(T_r)} = \frac{\hbar\tilde{\omega}'_p(T)}{2}\frac{\mu_e(H)}{\mu'_p(T_r)}\frac{\mu_e}{\mu_e(H)}\frac{1}{\mu_e}$$

THE ELECTRON MAGNETIC MOMENT

$$\mu_e = \frac{g_e}{2}\frac{e}{m_e}\frac{\hbar}{2}$$

$$a_\mu = \frac{\omega_a}{\tilde{\omega}'_p(T_r)}\frac{\mu'_p(T_r)}{\mu_e(H)}\frac{\mu_e(H)}{\mu_e}\frac{m_\mu}{m_e}\frac{g_e}{2}$$

$\mu_e(H)/\mu'_p(T_r)$: Measured at $T_r = 34.7\ ^\circ\mathrm{C}$ (**10.5 ppb**)
Metrologia **13**, 179 (1977)

$\mu_e/\mu_e(H)$: Bound-state QED (exact)
Rev. Mod. Phys. **88** 035009 (2016)

m_μ/m_e: From muon hyperfine splitting (**22 ppb**)
Phys. Rev. Lett. **82,** 711 (1999)

$g_e/2$: Measured to **0.28 ppt**
Phys. Rev. A **83**, 052122 (2011)

The Muon Anomaly Measurement (2/2)

In order to determine a_μ the experiment measures two frequencies: $\boldsymbol{\omega_a}$ and $\boldsymbol{\tilde{\omega}'_p(T_r)}$

$$a_\mu = \underbrace{\boxed{\frac{\omega_a}{\tilde{\omega}'_p(T_r)}}}_{\text{Run-1 result: 461 ppb}} \underbrace{\frac{\mu'_p(T_r)}{\mu_e(H)} \frac{\mu_e(H)}{\mu_e} \frac{m_\mu}{m_e} \frac{g_e}{2}}_{< 25 \text{ ppb}}$$

Uncertainty	BNL (ppb)	FNAL goal (ppb)
Statistical	460	100
Systematic (ω_a)	210	70
Systematic ($\tilde{\omega}'_p(T)$)	170	70
Total	**540**	**140**

The Muon Anomaly Measurement

In order to determine a_μ the experiment measures two frequencies: $\boldsymbol{\omega_a}$ *and* $\boldsymbol{\tilde{\omega}'_p(T_r)}$

$$a_\mu = \boxed{\frac{\omega_a}{\tilde{\omega}'_p(T_r)}} \frac{\mu'_p(T_r)}{\mu_e(H)} \frac{\mu_e(H)}{\mu_e} \frac{m_\mu}{m_e} \frac{g_e}{2}$$

Unblinding conversion factor

Measured g-2 frequency

Corrections from the beam dynamics systematic effects

$$a_\mu \propto \frac{\omega_a}{\tilde{\omega}'_p(T_r)} = \frac{f_{clock}\, \omega_a^m \left(1 + C_e + C_p + C_{ml} + C_{pa}\right)}{f_{calib} \langle \omega_p(x,y,\varphi) \times M(x,y,\varphi) \rangle (1 + B_k + B_q)}$$

NMR probe calibration factor

Magnetic field weighted over the muon distribution and azimuthally averaged

Corrections from the transient magnetic field

The Muon Anomaly Measurement

In order to determine a_μ the experiment measures two frequencies: $\boldsymbol{\omega_a}$ and $\boldsymbol{\tilde{\omega}'_p(T_r)}$

$$a_\mu = \boxed{\frac{\omega_a}{\tilde{\omega}'_p(T_r)}} \frac{\mu'_p(T_r)}{\mu_e(H)} \frac{\mu_e(H)}{\mu_e} \frac{m_\mu}{m_e} \frac{g_e}{2}$$

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$$a_\mu \propto \frac{\omega_a}{\tilde{\omega}'_p(T_r)} = \frac{f_{clock}\ \omega_a^m \left(1 + C_e + C_p + C_{ml} + C_{pa}\right)}{f_{calib} \langle \omega_p(x,y,\varphi) \times M(x,y,\varphi) \rangle (1 + B_k + B_q)}$$

NMR probe calibration factor

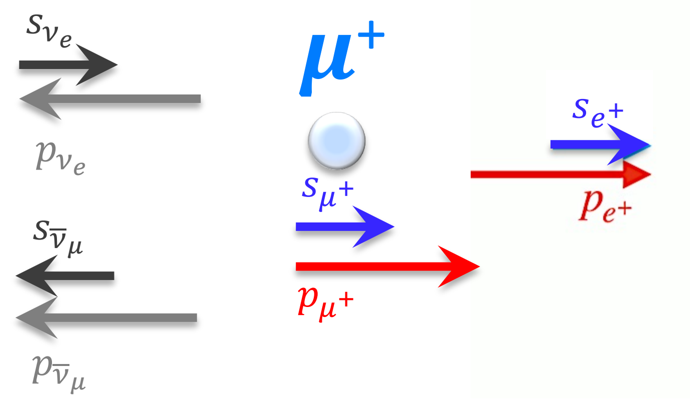
Magnetic field weighted over the muon distribution and azimuthally averaged

Corrections from the transient magnetic field

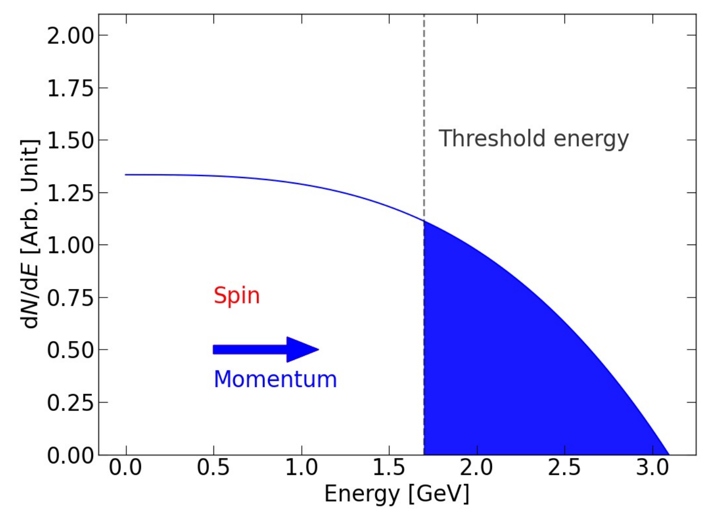
$$\boldsymbol{\omega_a} \approx \omega_a^m \left(1 + C_e + C_p + C_{ml} + C_{pa}\right)$$

Determination of ω_a^m

- Muons decay primarily through the three-body decay channel $\mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu$
- Muon spin information is encoded in parity violating decay
- The decay positrons (e^+) with high energies are preferentially emitted along the direction of muon spin
- The number of high momentum positrons above a fixed energy threshold oscillates at precession frequency
- Simply measure the time and energy of decay positrons and count the number above an energy threshold

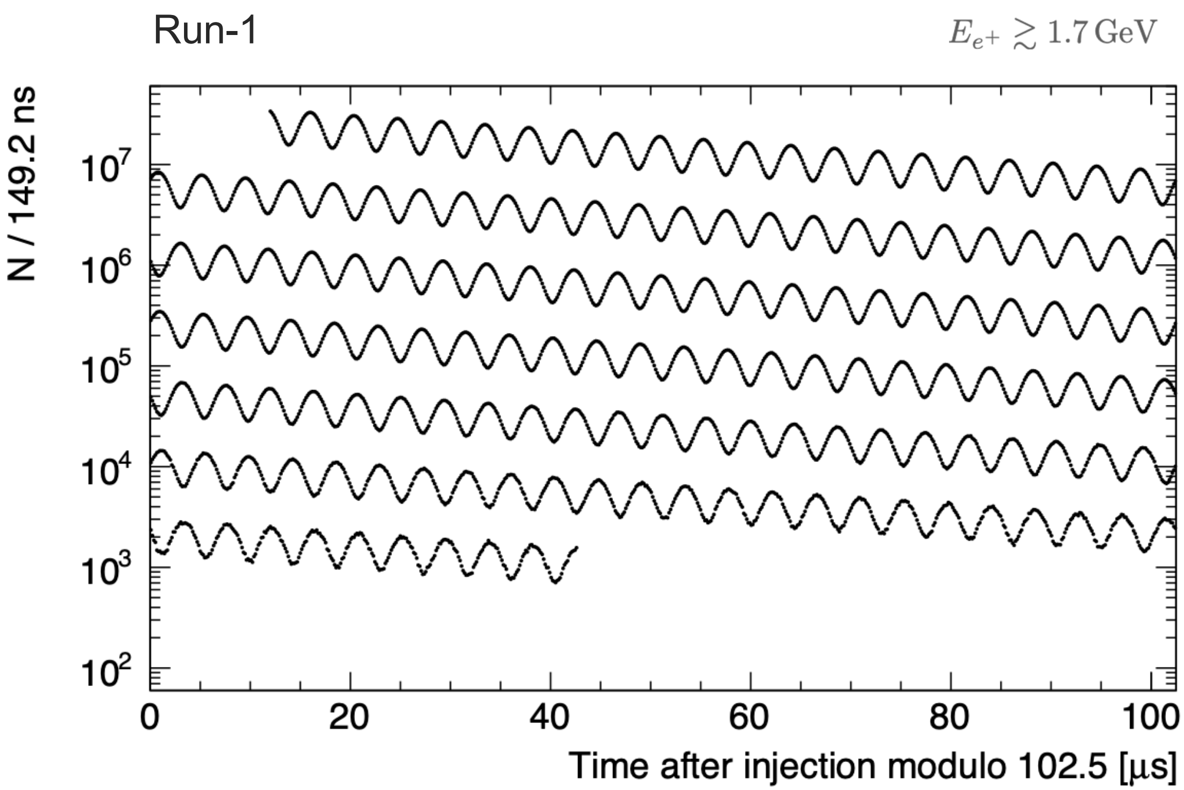


DECAY e^+ ENERGY SPECTRUM IN THE LAB FRAME



Fermilab

Determination of ω_a^m (1/2)



Positrons time spectrum over an optimized energy threshold oscillates with the anomalous spin-precession frequency ω_a^m

Ideally a 5-parameter fit would make sense

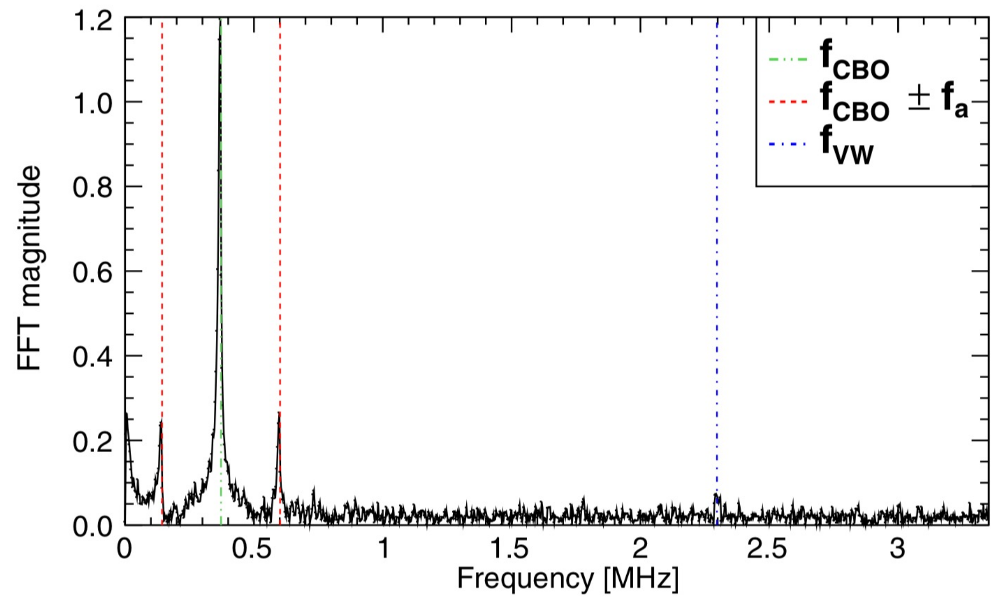
$$N_0\, e^{-\frac{t}{\gamma\tau}}\,(1 + A\cos(\omega_a^m t + \varphi))$$

Determination of ω_a^m (2/2)

5-parameter fit

$$N_0\, e^{-\frac{t}{\gamma\tau}}\,(1 + A\cos(\omega_a^m\, t + \varphi))$$

Run-1



- A Fourier transform of the residuals to the fit shows contributions from the movements of the beam, pileup and muon losses
  - Final fit, with 24 parameters to account for these effects

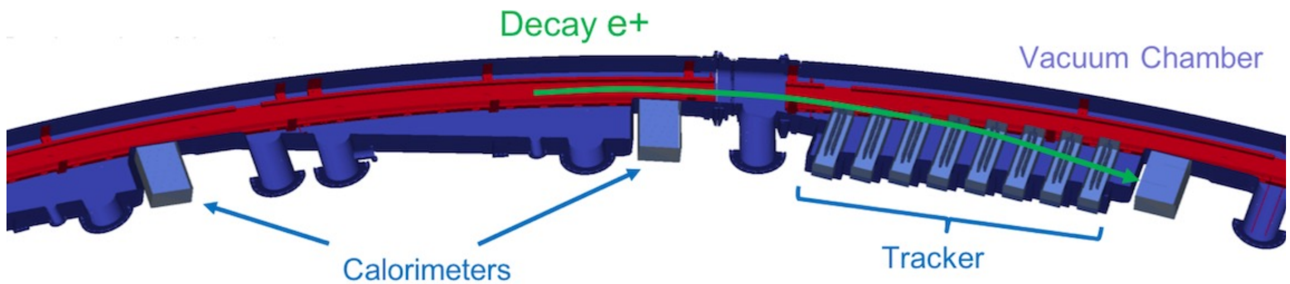
CBO = Radial Mean Oscillations
VW = Vertical Width Oscillations

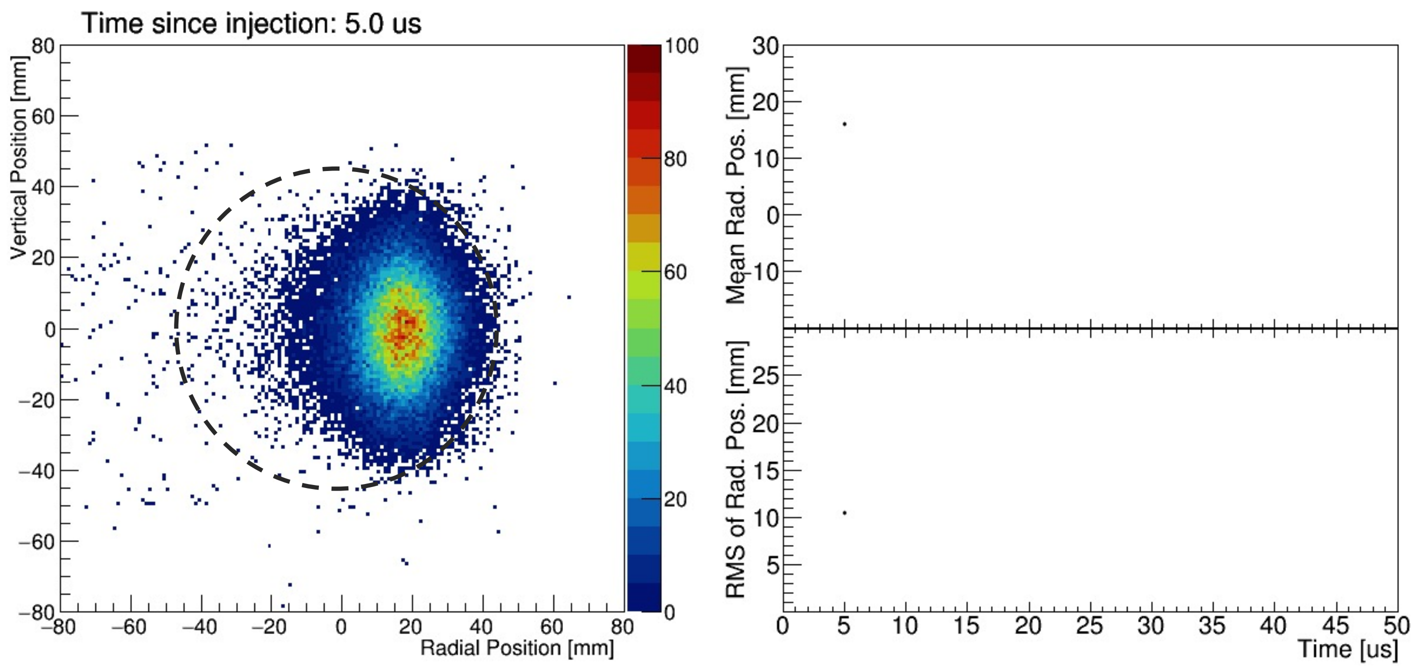
Coherent Betatron Oscillations

- The acceptance of the detector system couples with the beam motion resulting from coherent betatron oscillations (CBO)

  → introduce additional time modulation into the rate of detected positrons and into the shape of the positron energy spectrum.

- Trackers measure trajectories of the decay positrons and extrapolates to find the muon distribution.





Full Fit Function

More terms in the fitting function to take into account detector effects, muon loss, beam oscillations,…

Full Fit function:

$$N_0\, e^{-\frac{t}{\gamma\tau}}\,(1 + A \cdot A_{BO}(t)\cos(\omega_a^m\, t + \varphi + \varphi_{BO}(t)\,))\cdot N_{\mathrm{CBO}}(t)\cdot N_{\mathrm{VW}}(t)\cdot N_y(t)\cdot N_{2\mathrm{CBO}}(t)\cdot \Lambda(t)$$

$$A_{\mathrm{BO}}(t) = 1 + A_A\cos(\omega_{\mathrm{CBO}}(t)\cdot t + \varphi_A)e^{-\frac{t}{\tau_{\mathrm{CBO}}}}$$

$$\varphi_{\mathrm{BO}}(t) = A_\varphi\cos(\omega_{\mathrm{CBO}}(t)\cdot t + \varphi_\varphi)e^{-\frac{t}{\tau_{\mathrm{CBO}}}}$$

$$N_{\mathrm{CBO}}(t) = 1 + A_{\mathrm{CBO}}\cos(\omega_{\mathrm{CBO}}(t)\cdot t + \varphi_{\mathrm{CBO}})e^{-\frac{t}{\tau_{\mathrm{CBO}}}}$$

$$N_{2\mathrm{CBO}}(t) = 1 + A_{2\mathrm{CBO}}\cos(2\omega_{\mathrm{CBO}}(t)\cdot t + \varphi_{2\mathrm{CBO}})e^{-\frac{t}{2\tau_{\mathrm{CBO}}}}$$

$$N_{\mathrm{VW}}(t) = 1 + A_{\mathrm{VW}}\cos(\omega_{\mathrm{VW}}(t)\cdot t + \varphi_{\mathrm{VW}})e^{-\frac{t}{\tau_{\mathrm{VW}}}}$$

$$N_y(t) = 1 + A_y\cos(\omega_y(t)\cdot t + \varphi_y)e^{-\frac{t}{\tau_y}}$$

$$\Lambda(t) = 1 - k_{LM}\int_{t_0}^{t} L(t')e^{t'/\tau}dt'$$

$$\omega_{\mathrm{CBO}}(t) = \omega_0^{CBO} + \frac{A}{t}e^{-\frac{t}{\tau_A}} + \frac{B}{t}e^{-\frac{t}{\tau_B}}$$

$$\omega_y(t) = F\omega_{\mathrm{CBO}}(t)\sqrt{2\omega_c/F\omega_{\mathrm{CBO}}(t) - 1}$$

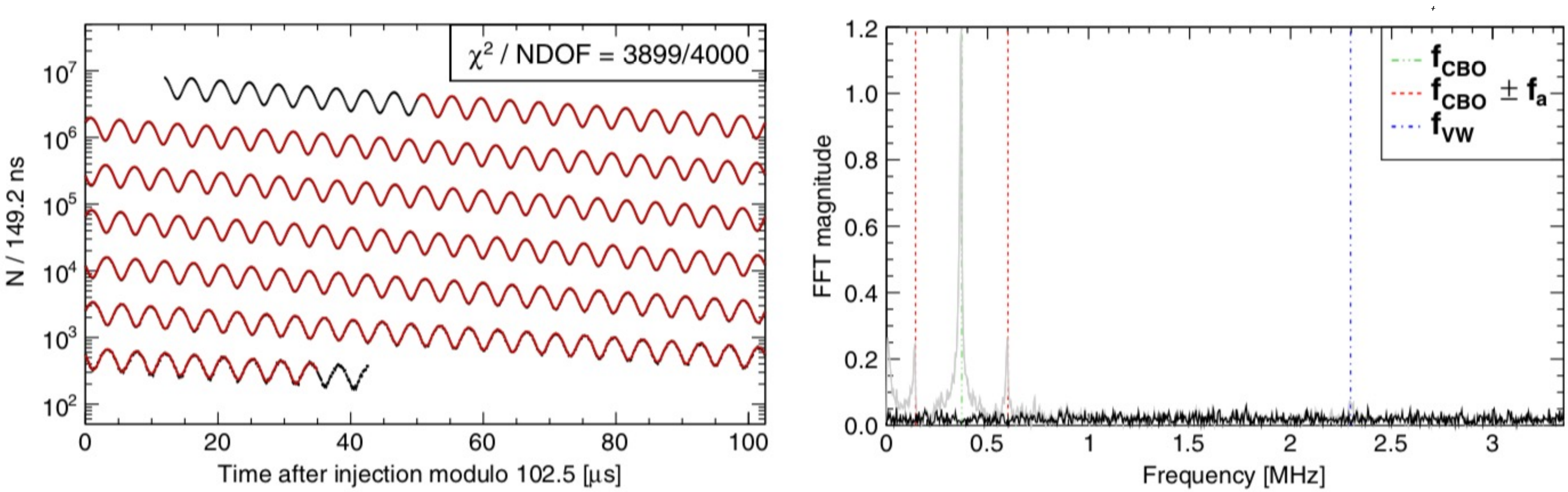
$$\omega_{\mathrm{VW}}(t) = \omega_c - 2\omega_y(t)$$

Full Fit Function

More terms in the fitting function to take into account detector effects, muon loss, beam oscillations,…

Full Fit function:

$$N_0\, e^{-\frac{t}{\gamma\tau}}\,(1 + A \cdot A_{BO}(t)\cos(\omega_a^m\, t + \varphi + \varphi_{BO}(t)))\cdot N_{\text{CBO}}(t)\cdot N_{\text{VW}}(t)\cdot N_y(t)\cdot N_{\text{2CBO}}(t)\cdot \Lambda(t)$$



Fermilab

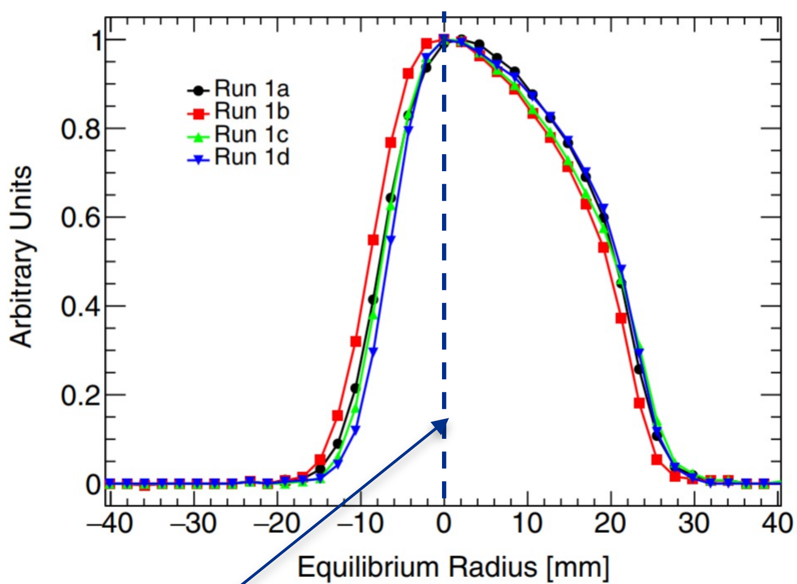
E-field Correction, C_e, and Pitch Correction, C_p

The anomalous precession frequency has additional terms due to

- the electric focusing fields, needed for vertical muon confinement
- The muon motion, not entirely perpendicular to |**B**|

$$\vec{\omega}_a = -\frac{q}{m_\mu}\left[a_\mu \vec{B} - \frac{a_\mu \gamma}{\gamma + 1}(\vec{\beta} \cdot \vec{B})\vec{\beta} - \left(a_\mu - \frac{1}{\gamma^2 - 1}\right)\frac{\vec{\beta} \times \vec{E}}{c}\right]$$

0 *if* γ = 29.3



for p = 3.09 GeV/c, γ = γm

- The electric field term vanishes for muons having the "magic" momentum p_μ = 3.094 GeV/*c* (γ =29.3).
  - For a 1.45T field, this sets the radius of the ring to 7.11m
- Not all muons are at the 'magic' momentum of 3.094 GeV/*c*
  - **E-field correction, C_E,** determined from momentum distribution measured by calorimeters

$$C_e = (489 \pm 53)\ \mathbf{ppb}$$

$\omega_a \approx \omega_a^m(1 + C_e + C_p + C_{ml} + C_{pa})$

E-field Correction, C_e, and Pitch Correction, C_p

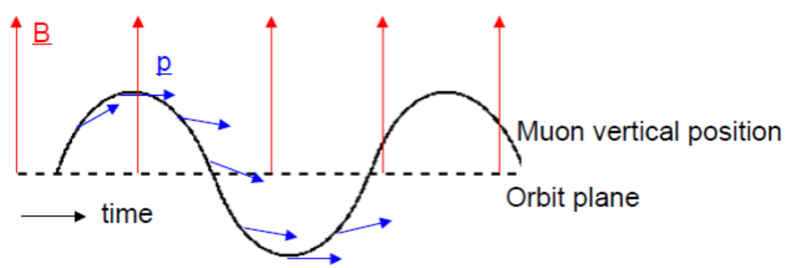
The anomalous precession frequency has additional terms due to

- the electric focusing fields, needed for vertical muon confinement
- The muon motion, not entirely perpendicular to |**B**|

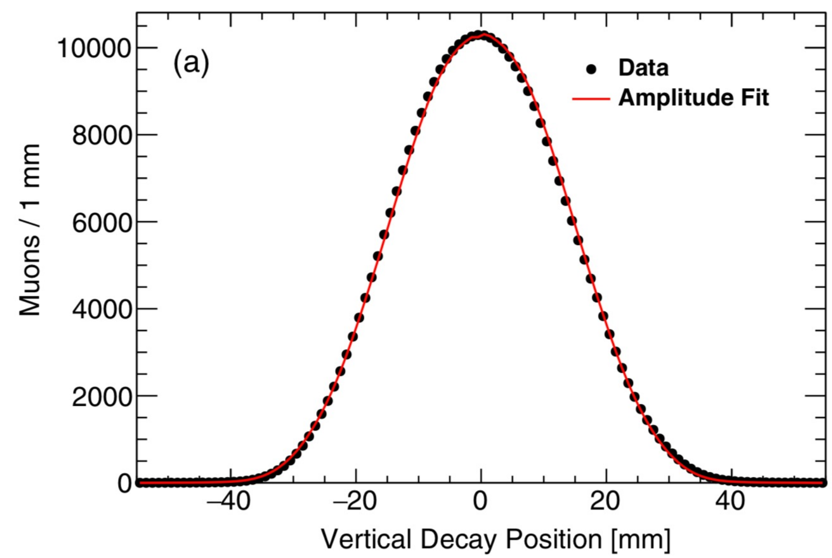
$$\vec{\omega}_a = -\frac{q}{m_\mu}\left[a_\mu \vec{B} - \frac{a_\mu \gamma}{\gamma + 1}(\vec{\beta}\cdot\vec{B})\vec{\beta} - \left(a_\mu - \frac{1}{\gamma^2 - 1}\right)\frac{\vec{\beta}\times\vec{E}}{c}\right]$$

$0\ if\ \gamma = 29.3$

- Vertical momentum component aligned with B field
  - The **pitch correction, C_p,** can be determined from straw tracker data



$C_p = (180 \pm 13)$ **ppb**

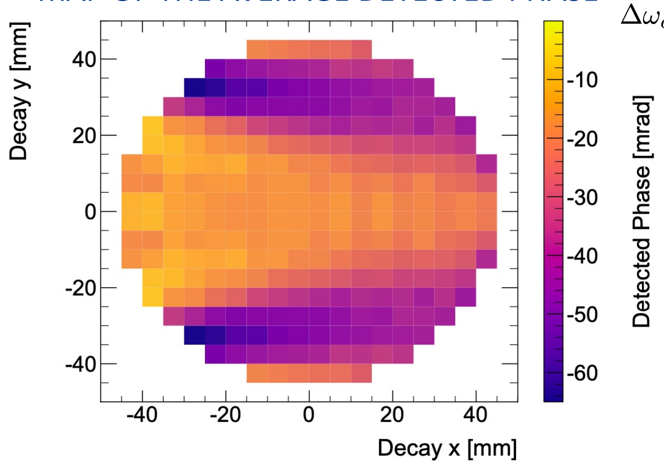


Fermilab

$\omega_a \approx \omega_a^m(1 + C_e + C_p + C_{ml} + C_{pa})$

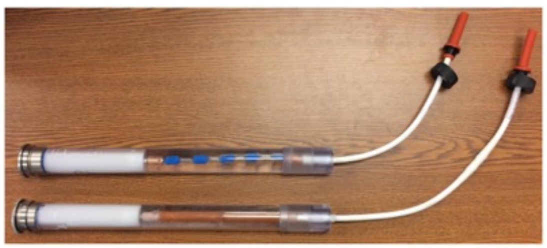
Phase Acceptance Correction, C_{pa}

MAP OF THE AVERAGE DETECTED PHASE



$$\Delta\omega_a = \frac{d\phi}{dt} = \frac{dY_{\mathrm{RMS}}}{dt}\frac{d\phi}{dY_{\mathrm{RMS}}} \neq 0$$

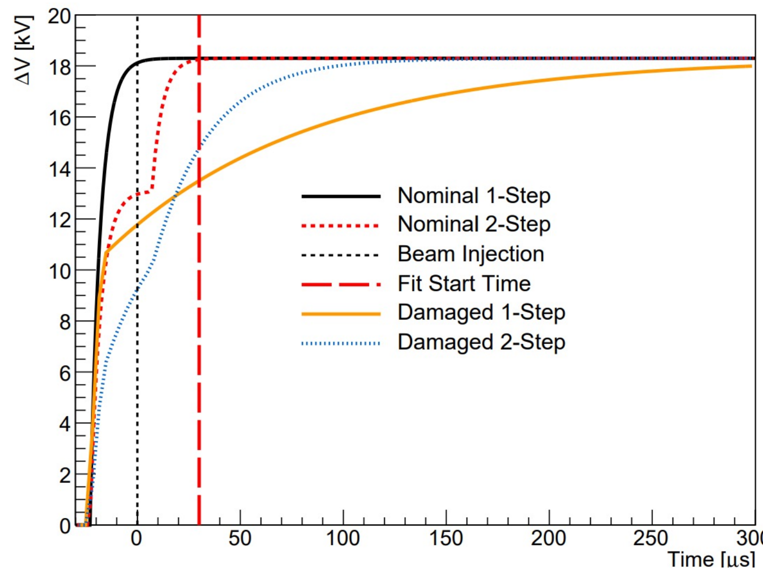
- Due to acceptance, ϕ depends on muon decay position (x,y)
- Not an issue if the muon distribution doesn't change shape over a fill

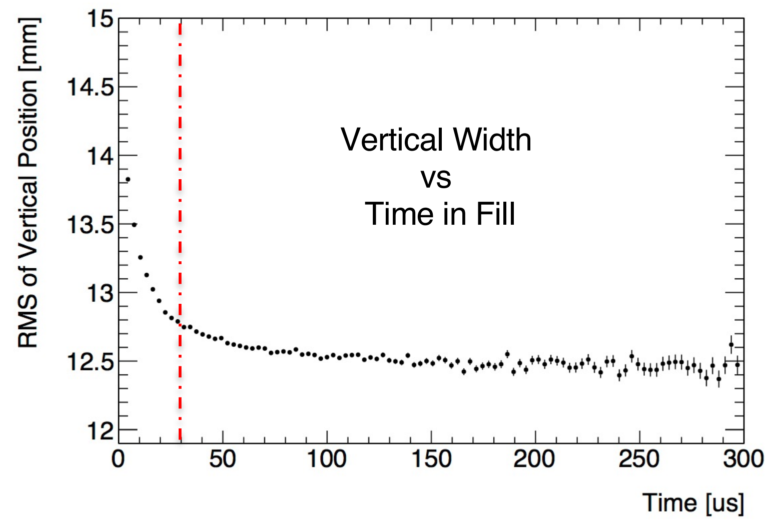


Run-1 equipment malfunction:

2 out of 32 quadrupole HV resistors became damaged → RC time constant altered
→ E-field change → beam vertical width change

$$C_{pa} = (-158 \pm 75\ )\ \mathbf{ppb}$$





Fermilab

ω_a^m Corrections and Uncertainties

The systematic error on ω_a^m for Run-1 is **56 ppb**.
The statistical error is **434 ppb**

	Correction (ppb)	Uncertainty (ppb)
ω_a^m(statistical)	…	**434**
ω_a^m(systematic)	…	**56**
C_e	489	53
C_p	180	13
C_{ml}	-11	5
C_{pa}	-158	75
C_{Tot}	**499**	**93**

C_e : kickers upgrades
→ achieved the ideal mean equilibrium radius during the Run-3
→ expected a smaller effect

C_{pa}: expected systematic effect < 50 ppb in Run-2 after ESQ resistor fix

$$\widetilde{\omega}'_p(T_r) = f_{calib}\langle\omega_p(x,y,\varphi)\times M(x,y,\varphi)\rangle(1+B_k+B_q)$$

Measuring the B-Field: overview

$\boldsymbol{B}$ is expressed in terms of the proton Larmor precession frequency, $\omega'_p = 2\mu_p\left|\vec{B}\right|/\hbar$

THE AVERAGE FIELD OVER THE MUON DISTRIBUTION WEIGHTED BY THE DETECTED DECAYS OVER TIME

$$\tilde{B} = \frac{\hbar\tilde{\omega}'_p(T)}{2\mu'_p(T)} = \frac{\hbar\tilde{\omega}'_p(T)}{2}\frac{\mu_e(H)}{\mu'_p(T)}\frac{\mu_e}{\mu_e(H)}\frac{1}{\mu_e}$$

THE ELECTRON MAGNETIC MOMENT

$$\mu_e = \frac{g_e}{2}\frac{e}{m_e}\frac{\hbar}{2}$$

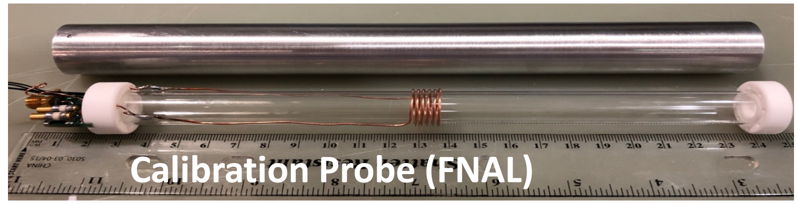
- To determine $\tilde{\omega}'_p(T_r)$ we perform a sequence of measurements.

  - The NMR probes of the in-vacuum trolley are calibrated in terms of the equivalent $\omega'_p(T_r)$ — $\boldsymbol{f_{calib}}$
  - The trolley maps the magnetic field in the muon storage volume — $\boldsymbol{\omega_p(x,y,\varphi)}$
  - The fixed NMR probes are synchronized to the trolley measurements
  - The magnetic-field maps are weighted by the temporal and spatial muon distributions — $\boldsymbol{\langle\omega_p(x,y,\varphi)\times M(x,y,\varphi)\rangle}$
  - Fast transient fields corrections are applied — $\boldsymbol{B_k}$ *and* $\boldsymbol{B_q}$

Calibration of the B-Field measurements

- The trolley and fixed probes use **petroleum jelly** as an NMR sample.
- Detailed calibration procedure to relate our measurements to a well-characterized NMR standard is required (equivalent precession frequency of a proton shielded in a spherical sample of water at a reference temperature T_r = 34.7 °C.)

$$\tilde{B} = \frac{\hbar\tilde{\omega}'_p(T_r)}{2\mu'_p(T_r)} = \frac{\hbar\tilde{\omega}'_p(T_r)}{2}\frac{\mu_e(H)}{\mu'_p(T_r)}\frac{\mu_e}{\mu_e(H)}\frac{1}{\mu_e}$$

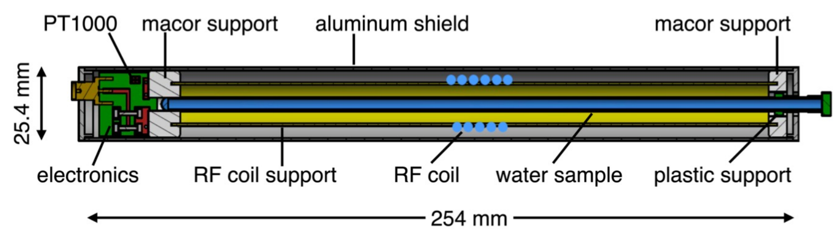


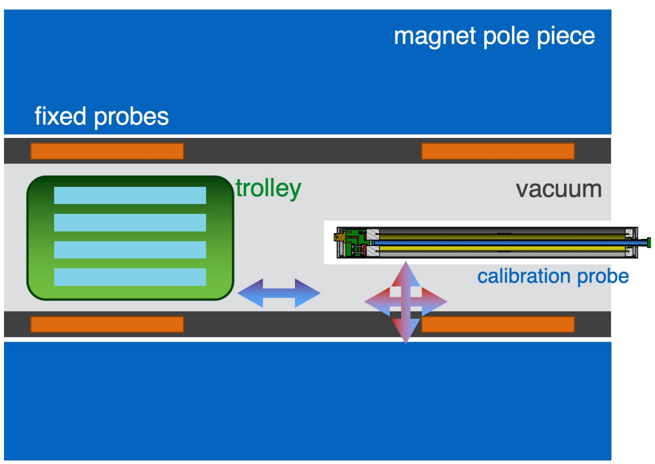
ABSOLUTE CALIBRATION

Cylindrical water sample probe is cross-calibrated to

- a spherical water sample known to high precision
- an absolute ^3He magnetometer

➢ **15 ppb** systematic uncertainty from absolute calibration





TROLLEY CALIBRATION

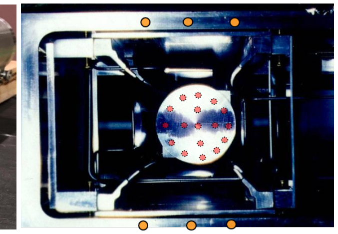
The calibration of this probe is transferred to the trolley probes

➢ **28 ppb** systematic uncertainty from trolley calibration

Measuring the B-Field: the in-vacuum trolley

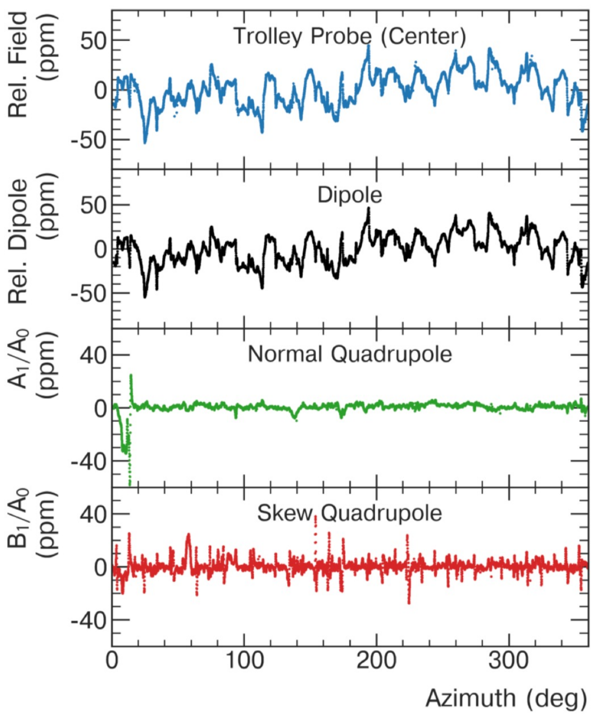
17 NMR probe in-vacuum trolley maps the magnetic field every ~3 days.

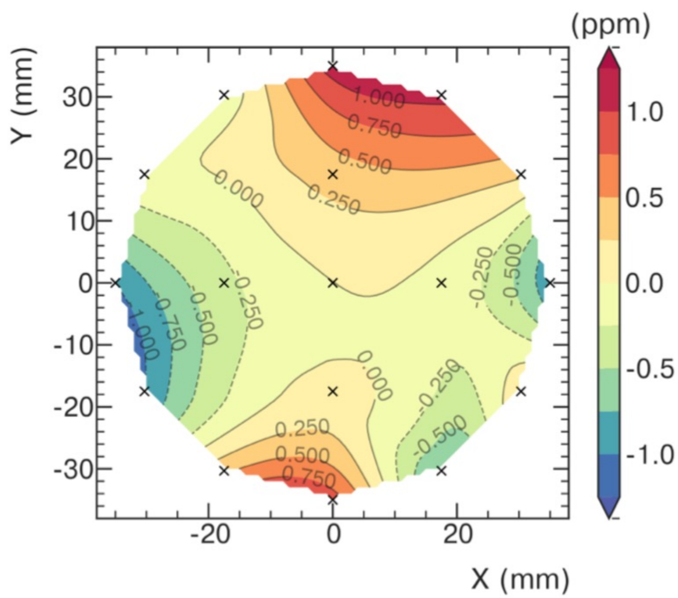
Trolley measures field gradients at different azimuthal slices (~ 9k)



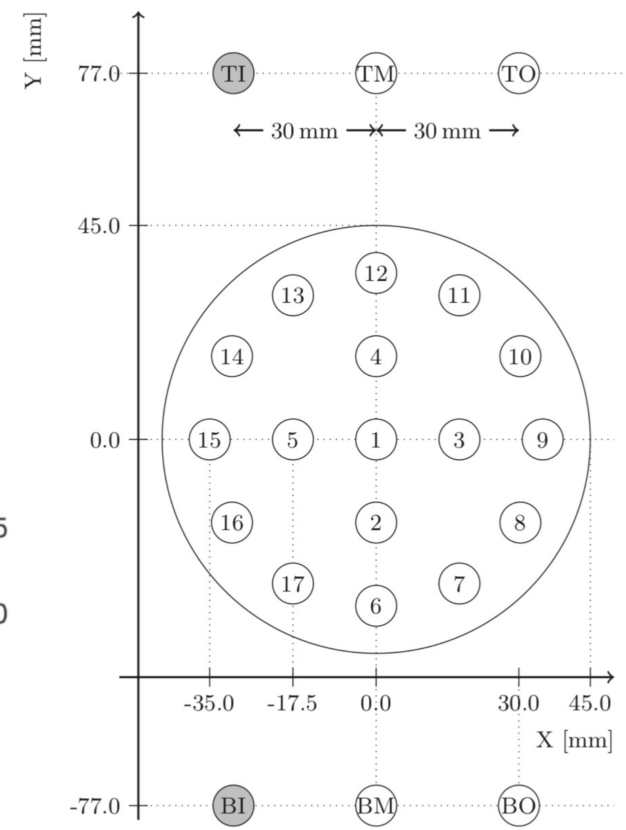
- For the trolley probe geometry, B in a slice is parametrized as

$$B \approx B_y = \underbrace{A_0}_{\text{normal dipole}} + \sum_{n=1} \left(\frac{r}{r_0}\right)^n [\underbrace{A_n \cos(n\theta)}_{\text{normal multipoles}} + \underbrace{B_n \sin(n\theta)}_{\text{skew multipoles}}],$$



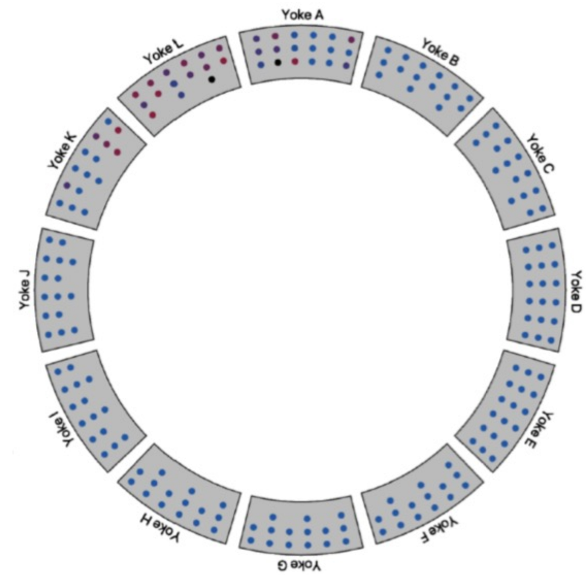


TYPICAL AZIMUTHAL AVERAGE

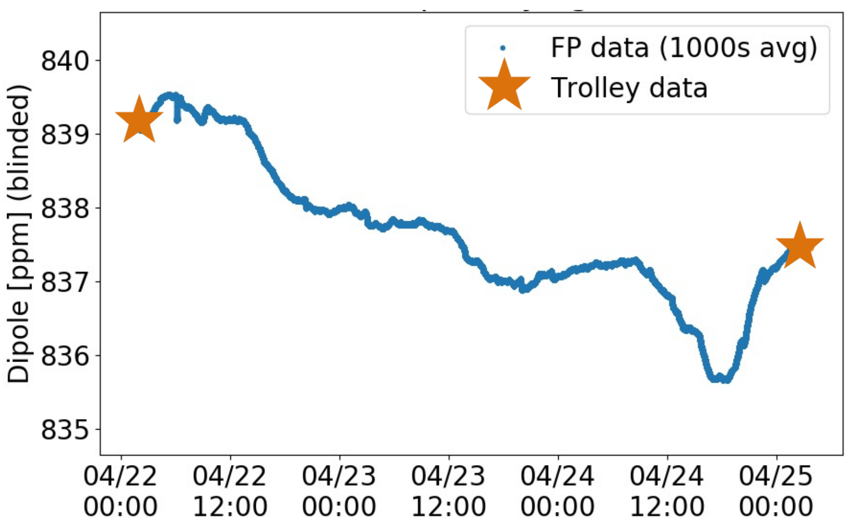


Measuring the B-Field: fixed NMR probes

- 378 fixed NMR probes
  - mounted above and below the storage region
  - located in 72 azimuthal stations (with 6/4 probes each)
- Fixed probes (FB) continuously monitor the field changes during data taking
- FB data used to interpolate between trolley runs



INTERPOLATION (EXAMPLE FROM ONE LOCATION)



FB calibrated using the trolley measurements either side of the data-taking time

Calibration is not constant over time

Drifts in moments not tracked by FP leads to tracking offset

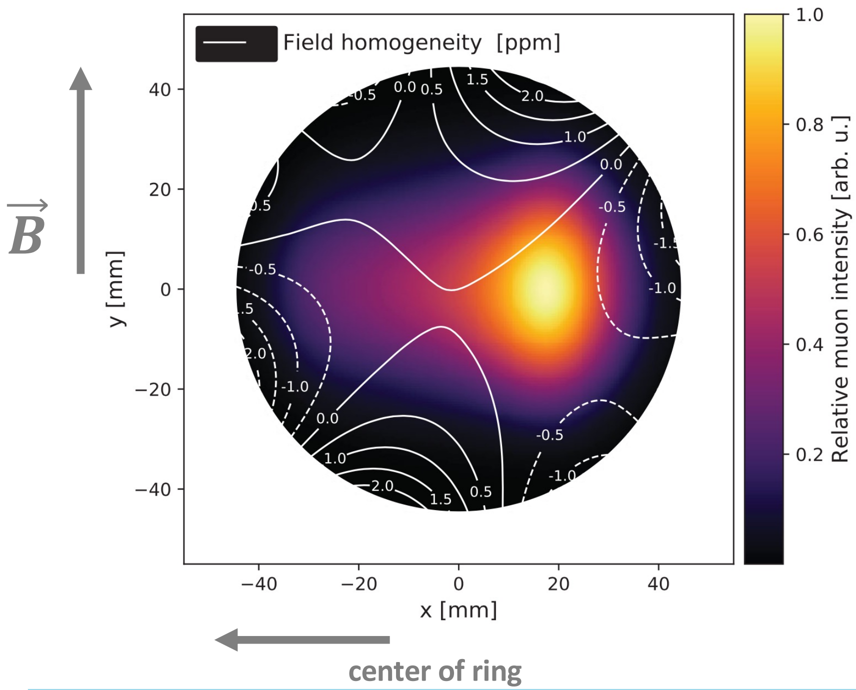
Muon distribution, M

The frequency maps are weighted with the muon distribution and averaged over space and time.

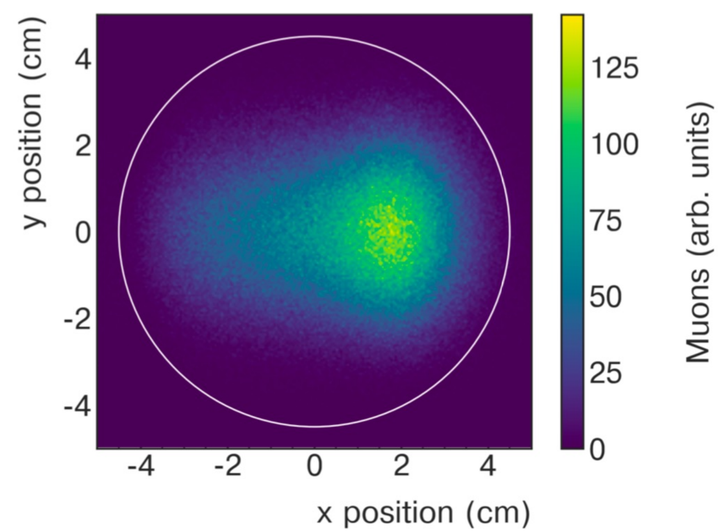
$$\tilde{\omega}'_p = \left\langle \frac{\int dx\, dy\, \omega'_p(x,y,\phi) M(x,y,\phi)}{\int dx\, dy\, M(x,y,\phi)} \right\rangle_\phi$$

The muon distribution is reconstructed from tracker profiles and propagated to other azimuthal locations using beam dynamics simulations.

FIELD CONTOURS OVERLAID



MUON BEAM PROFILE OVER TIME

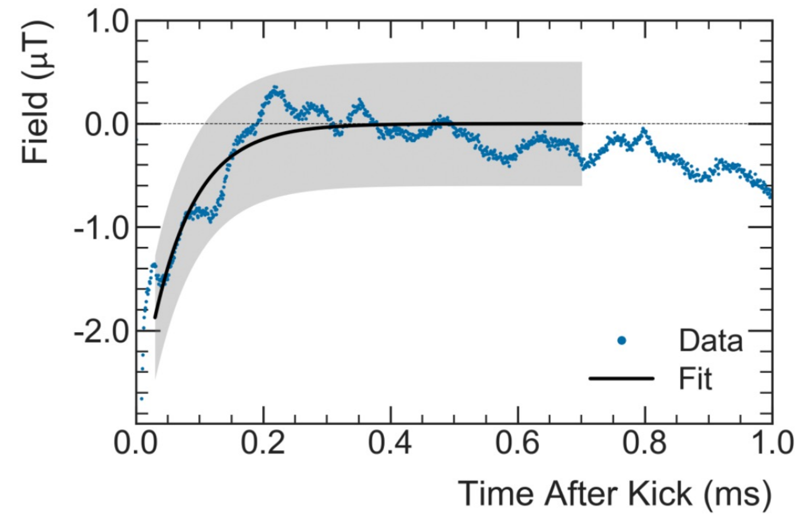


Kicker transient (B_k) and ESQs transient (B_q)

The kicker system induced an eddy current locally in the vacuum
→ transient magnetic field in the storage volume

- Fixed probes see reduced effect due to shielding
- A Faraday magnetometer determines the correction B_k.

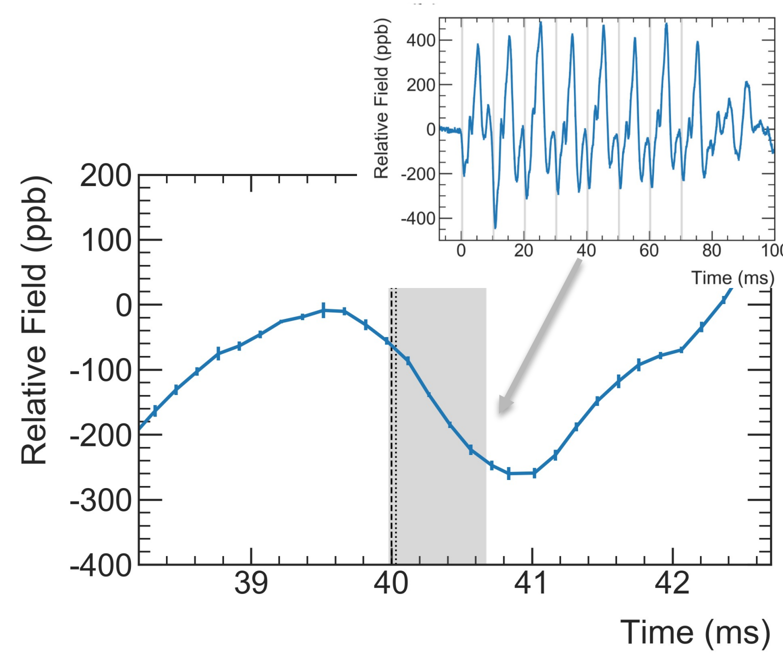
- $B_k = (-\mathbf{27} \pm \mathbf{37})$ **ppb**



As muons are injected, the ESQs are pulsed
→ the induced mechanical vibrations in the plates generates magnetic perturbations

- Affects the fixed probe measurements
- Customized NMR probes measure these transient fields at several positions.

- $B_q = (-\mathbf{17} \pm \mathbf{92})$ **ppb**



Fermilab

$\tilde{\omega}'_p(T_r)$ Corrections and Uncertainties

The total systematic error on $\tilde{\omega}'_p(T_r)$ for Run-1 is **114 ppb**.

- The contributions from calibration, field tracking, and muon weighting total **56 ppb**.
- The contribution from the ESQ transient is **92 ppb**.
  - The ESQ-Transient uncertainty is the dominant uncertainty
  - Additional measurements taken after Run-1 will reduce the systematic effect of the transient by a factor of 2-3
- The contribution from the kicker transient is **37 ppb**.

Run-1 Corrections and Uncertainties

The Run-1 measurement is statistics dominated

Uncertainty	BNL (ppb)	FNAL goal (ppb)	FNAL Run-1 (ppb)
Statistical	460	100	**434**
ω_a systematic	210	70	**109**
$\tilde{\omega}'_p(T)$ systematic	170	70	**114**
Total	**540**	**140**	**462**

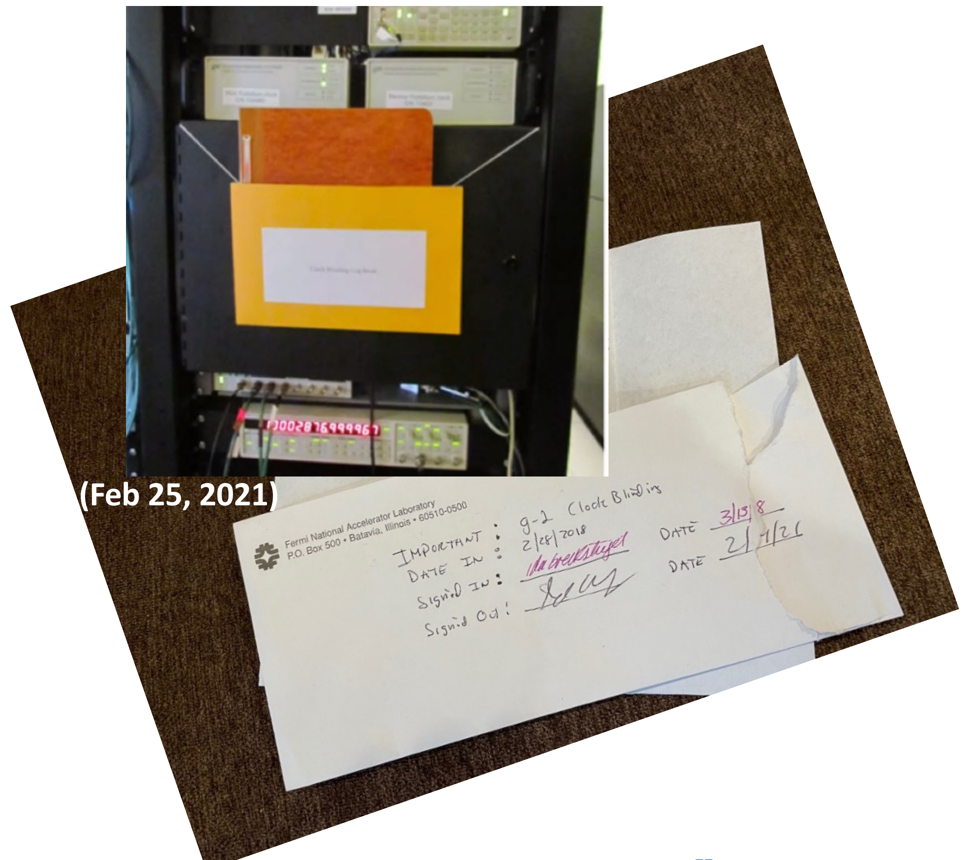
	Correction (ppb)	Uncertainty (ppb)
ω_a^m statistical	–	434
ω_a^m systematic	–	56
clock base	–	2
C_e	489	53
C_p	180	13
C_{ml}	-11	5
C_{pa}	-158	75
ω_a beam dynamics corrections	499	93
ω_a total systematic	499	109
$\omega'_p(T)(x, y, \varphi)$	–	54
$M(x, y, \varphi)$	–	17
$\langle\omega'_p(T)(x, y, \varphi) \times M(x, y, \varphi)\rangle$	–	56
B_q	17	92
B_k	27	37
$\tilde{\omega}'_p(T)$ transient fields corrections	44	99
$\tilde{\omega}'_p(T)$ total	44	114
$\omega_a/\tilde{\omega}'_p(T)$ total systematic	544	157
$g_e/2$ HFG08	–	0
$\mu_e/\mu'_p(T)$ PCK77	–	10
m_μ/m_e CODATA 2018	–	22
external measurements	–	25
Total [correction is for $\omega_a/\tilde{\omega}'_p(T)$]	**544**	**462**

Clock Blinding

The master clock **blinded** until we were ready to publish

- The clock hardware blinded to a frequency of $(40 - \epsilon)$ MHz
- Offset set and known by 2 people (independent of the Collaboration)
- Analysis performed w.r.t. blind clock ticks

Kept secret until completion of all systematic evaluations, cross-checks, …



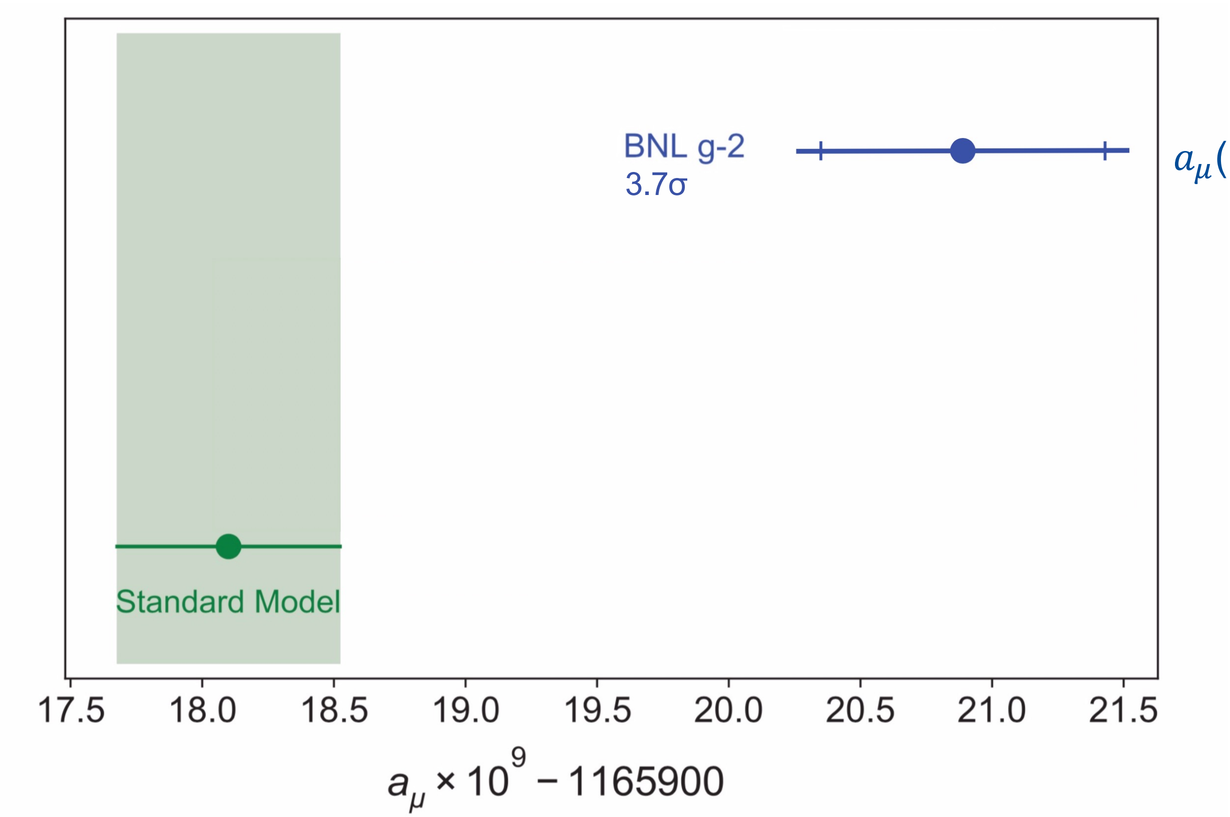
(Feb 25, 2021)

Results

& FUTURE PROSPECTS

Muon $g-2$: Run-1 Result

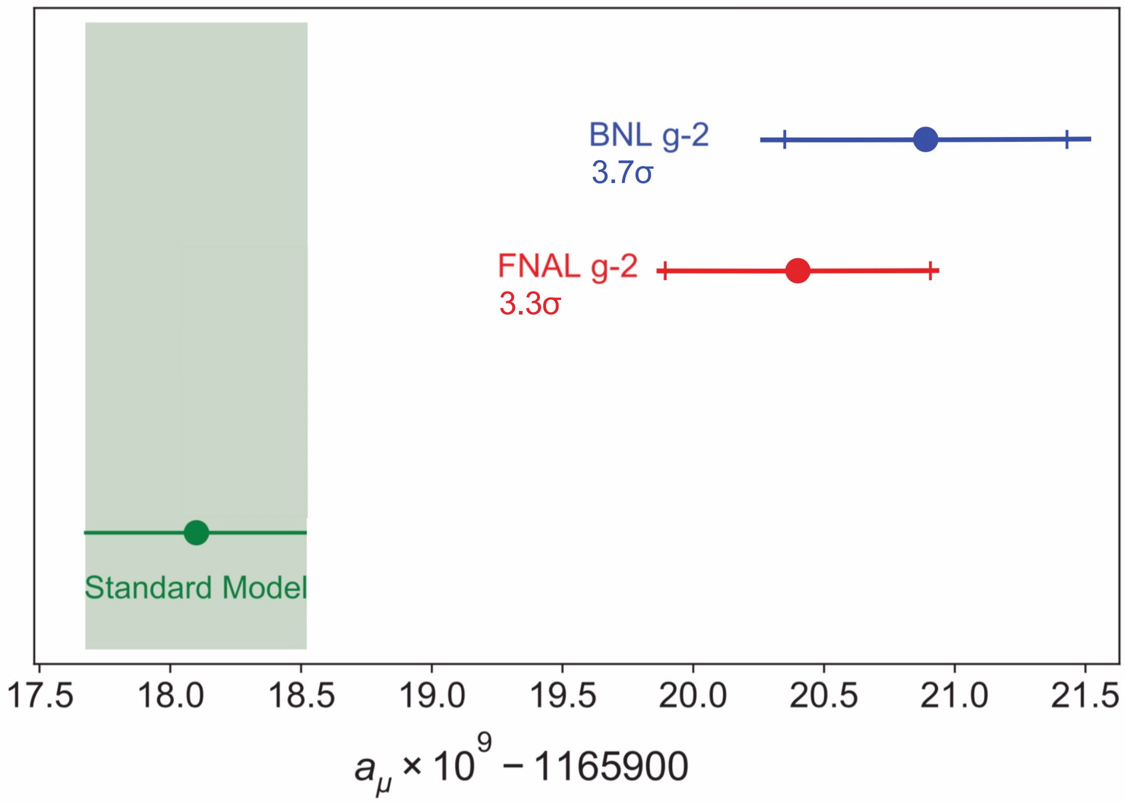
- Before our measurement …



a_μ(BNL) = 116,592,089(63) × 10^{-11} (**0.54 ppm**)

Muon $g-2$: Run-1 Result

- We measured a_μ to a precision of 462 ppb ($\mathbf{434} \oplus 157\ ppb$)
  - ➢ 3.3σ tension with SM prediction
  - ➢ 6% of final data sample
  - ➢ 15% smaller error than BNL
- In good agreement with the Brookhaven (BNL) result
- **Both dominated by statistical error**

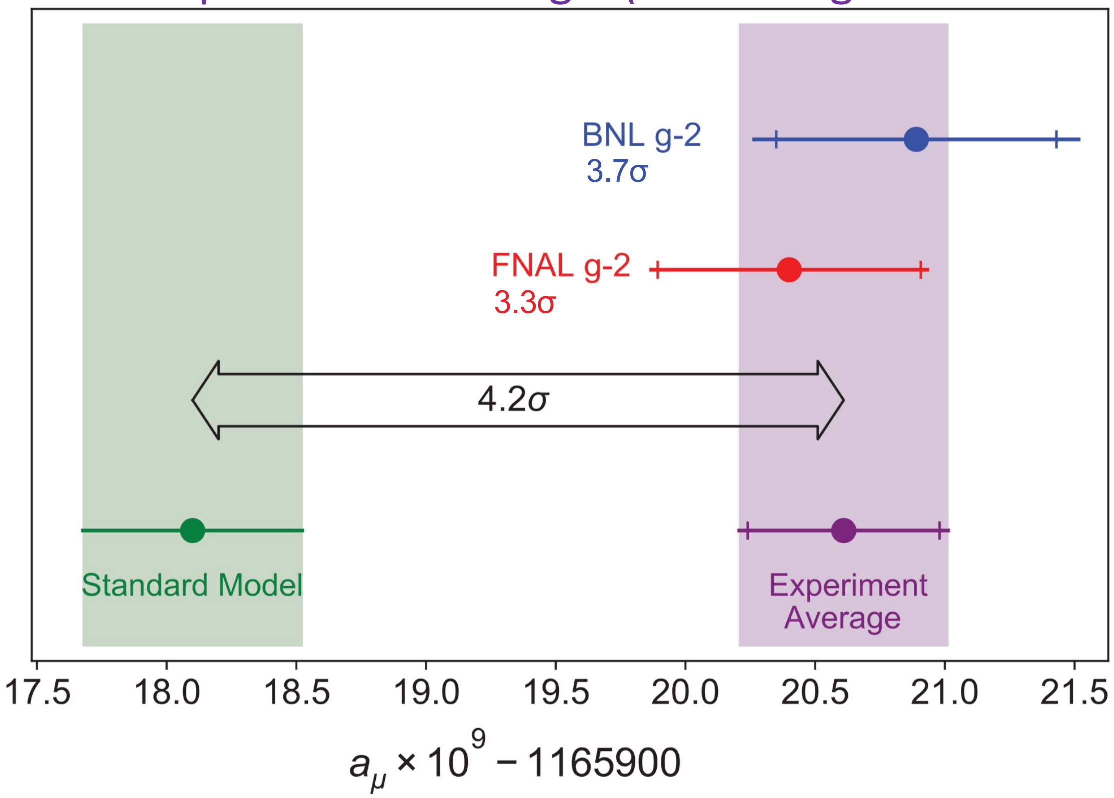


a_μ(BNL) = 116,592,089(63) × 10^{-11} (**0.54 ppm**)

a_μ(FNAL) = 116,592,040(54) × 10^{-11} (**0.46 ppm**)

Muon $g-2$: Run-1 Result

- We measured a_μ to a precision of 462 ppb ($\mathbf{434} \oplus 157\ ppb$)
  - ➢ 3.3σ tension with SM prediction
  - ➢ 6% of final data sample
  - ➢ 15% smaller error than BNL
- In good agreement with the Brookhaven (BNL) result
- **Both dominated by statistical error**
- Experimental average (combining FNAL & BNL) 4.2σ tension with the SM prediction



a_μ(BNL) = 116,592,089(63) × 10^{-11} (**0.54 ppm**)

a_μ(FNAL) = 116,592,040(54) × 10^{-11} (**0.46 ppm**)

a_μ(Exp) = 116,592,061(41) × 10^{-11} (**0.35 ppm**)

What's next

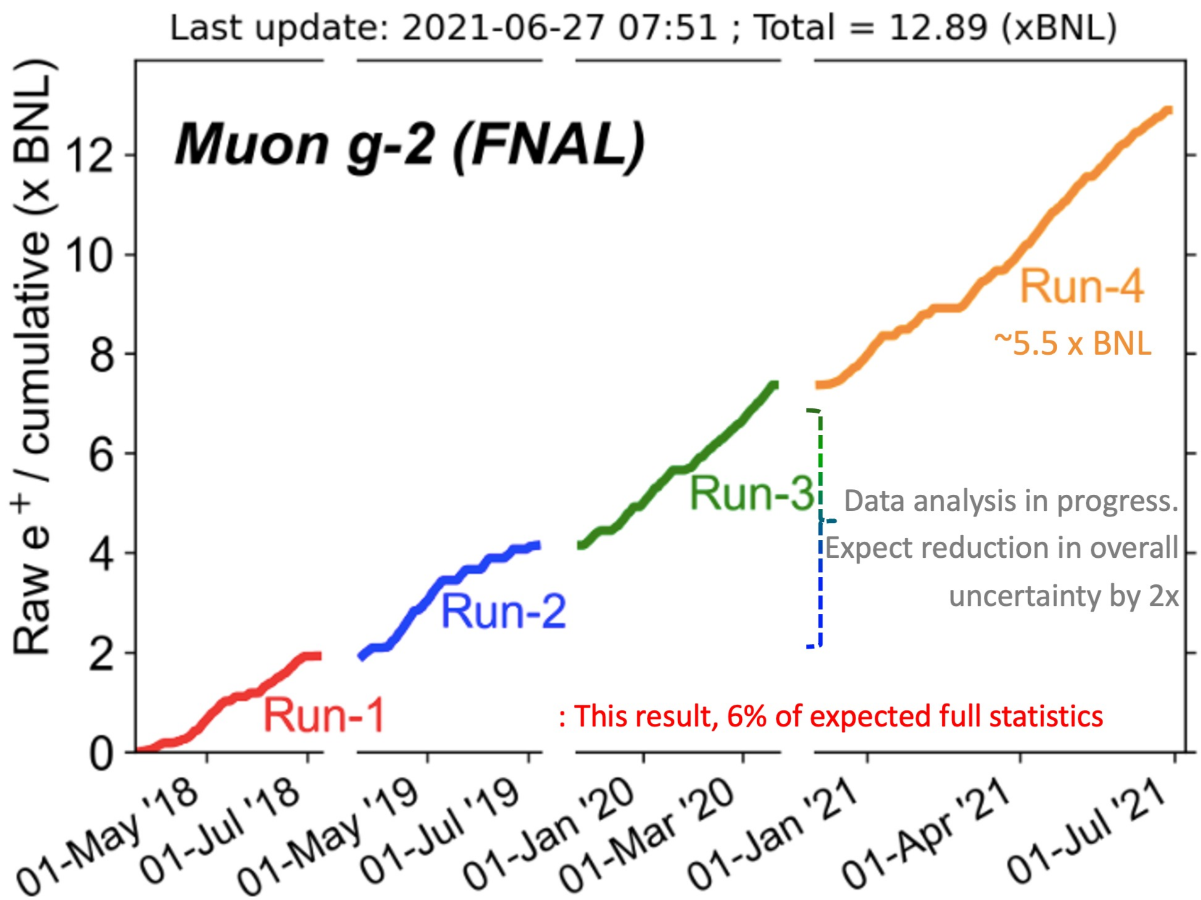
Analyzed a subset of the collected data
(Tot data target ~20x BNL)

With Run 4-5:

We expect an overall reduction in uncertainty by 4x

Many improvements since Run-1

- damaged ESQs resistors replaced
- improved kickers
- improved knowledge of transients
- ....



This result represents the work of

7 Countries,
35 Institutions,
~200 Members

SEE ALSO

Measurement of the Positive Muon Anomalous Magnetic Moment to 0.46 ppm
B. Abi *et al.* (Muon *g–2* Collaboration)
Phys. Rev. Lett. **126**, 141801 (2021)

Measurement of the anomalous precession frequency of the muon in the Fermilab Muon g–2 Experiment
T. Albahri *et al.* (Muon *g–2* Collaboration)
Phys. Rev. D **103**, 072002 (2021)

Magnetic-field measurement and analysis for the Muon g–2 Experiment at Fermilab
T. Albahri *et al.* (The Muon *g–2* Collaboration)
Phys. Rev. A **103**, 042208 (2021)

Beam dynamics corrections to the Run-1 measurement of the muon anomalous magnetic moment at Fermilab
T. Albahri *et al.* (Muon *g–2* Collaboration)
Phys. Rev. Accel. Beams **24**, 044002 (2021)