Time variability of Cosmic Rays

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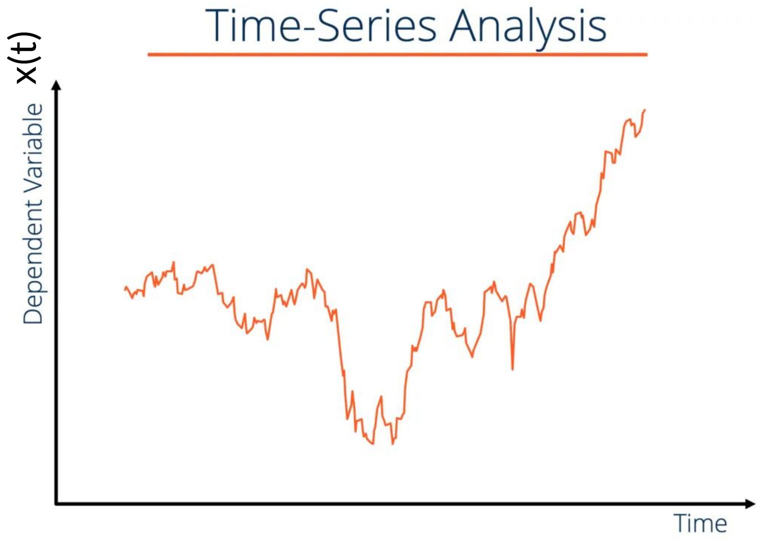
Time variable data

• space data: sun activity, neutron monitors, cosmic rays fluxes;

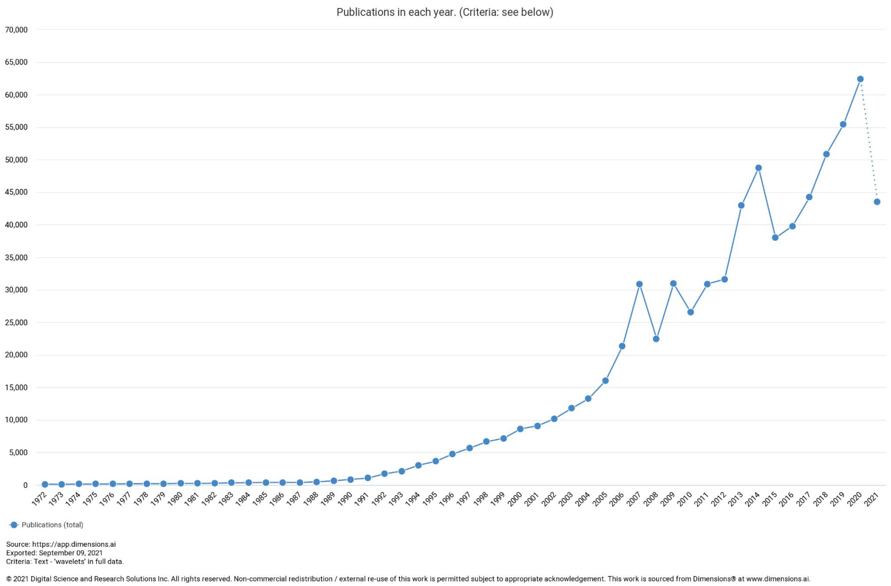
• earth data: meteorological (temperature, sea currents, . . .)

• finance: stock options



Most common tools for data characterization

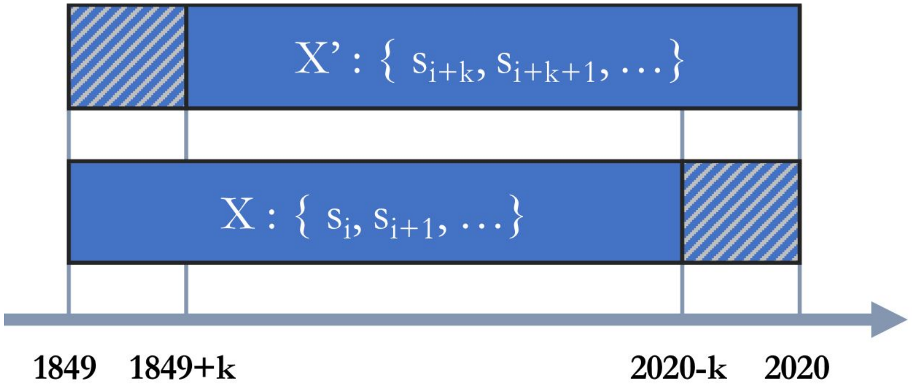
- autocorrelation methods;
- Fourier series;
- wavelets: emerged as powerful tool.



Autocorrelation in short...

Autocorrelation is a mathematical representation of the **degree of similarity** between a given **time series** and a **lagged version of itself** over successive time intervals.

$$R = \frac{\sum_i \left[\left(x_i - \mu_1 \right) \left(x_{i+k} - \mu_2 \right) \right]}{\sigma_1 \sigma_2}$$



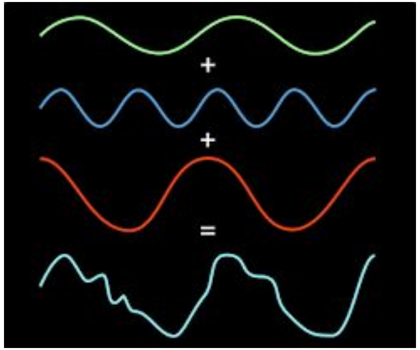
R:Autocorrelation coefficient
μ_1: average of sample X
μ_2: average of sample X'
σ_1, σ_2 : Standard deviation of the two samples
k: time-shift

Fourier Series in short...

- In 1807, a French mathematician (Joseph Fourier) discovered that a periodic function can be represented by an infinite sum of complex exponentials;
- The Fourier Transform(FT) is probably the most widely used signal analysis method;
- The Fourier transform retrieves the global information of the frequency content of a signal.
- Analyses time series as a whole, but **what if the frequency varies with time?**

$$X_{FT}(f) = \int_{-\infty}^{\infty} x(t)\, e^{-2\pi ift} dt$$

$$x(t) = \int_{-\infty}^{\infty} X_{FT}(f)\, e^{2\pi ift} df$$



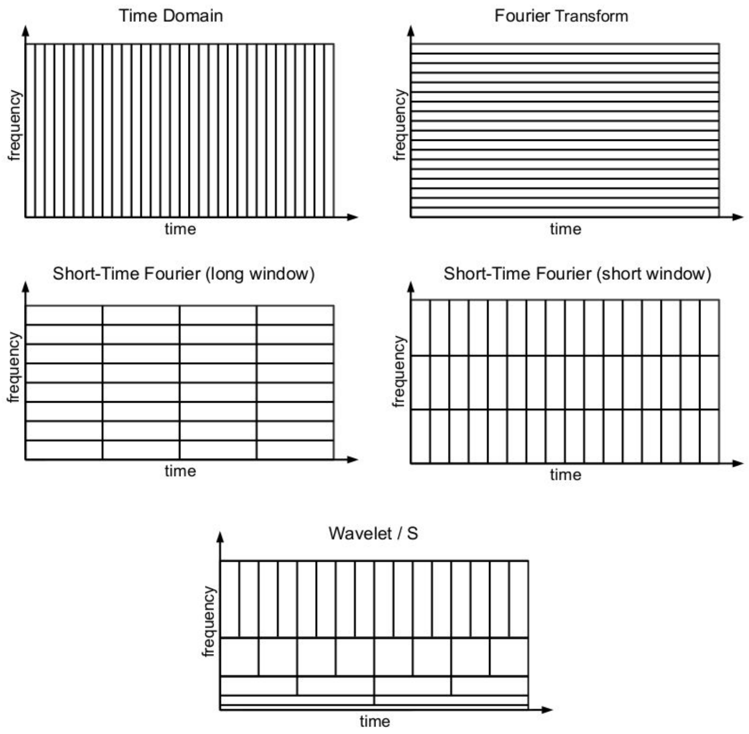
Wavelets

• The analysis of a non-stationary signal using the FT or the STFT does not give satisfactory results;

• **Better results** can be obtained using wavelet analysis(basically an extension of fourier analysis);

• One advantage of wavelet analysis is the ability to perform **local analysis**(trends, breakdown points, discontinuities,...), so there's no problem if the frequency changes throughout the time series;

•While the **Fourier** transform creates a representation of the signal in the **frequency** domain, the **wavelet** transform creates a representation of the signal in **both the time and frequency domain**, thereby allowing efficient access of **localized** information about the signal.



Wavelets

$$X_{WT}(\tau, s) = \frac{1}{\sqrt{s}} \int_{-\infty}^{\infty} x(t)\, \psi^{*}\left(\frac{t-\tau}{s}\right) dt$$

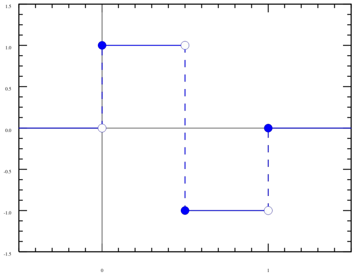
τ: translation parameter

s: scale parameter

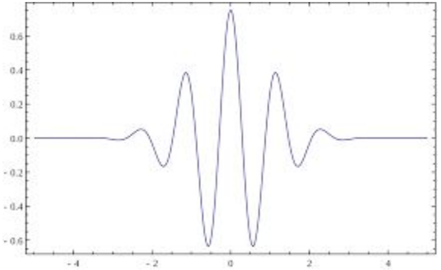
Ψ: mother wavelet

* : complex conjugate

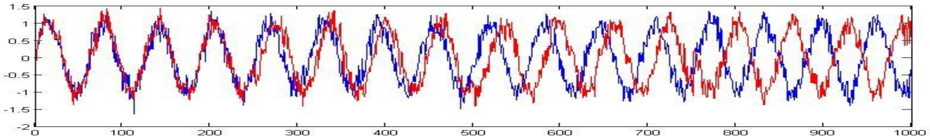
Haar Wavelet:

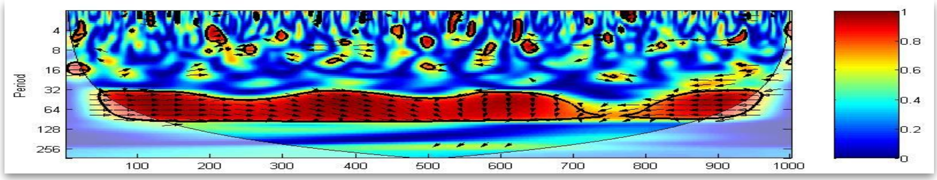


Morlet Wavelet:



Note: Mother wavelets must have null mean, amongst other criteria.

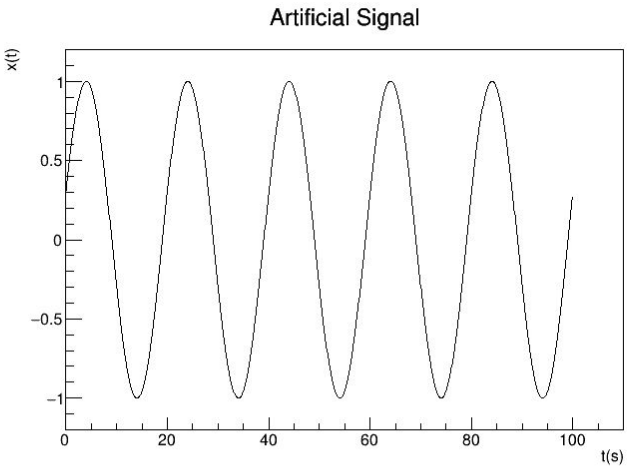


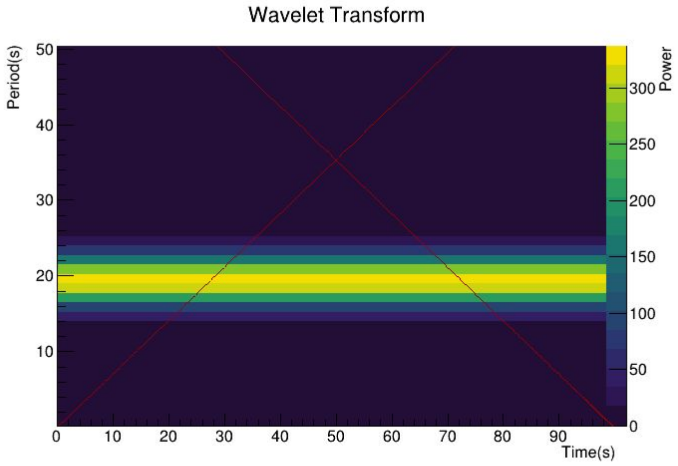


Wavelets for a monochromatic signal

$$x(t) = \sin\left(\frac{2\pi}{20}t + 0.3\right)$$

Parameter	value
N_points	1000
Δt	0.1 s

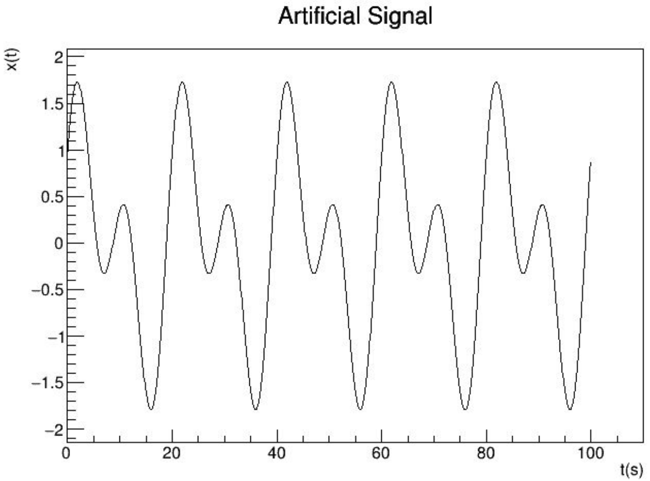


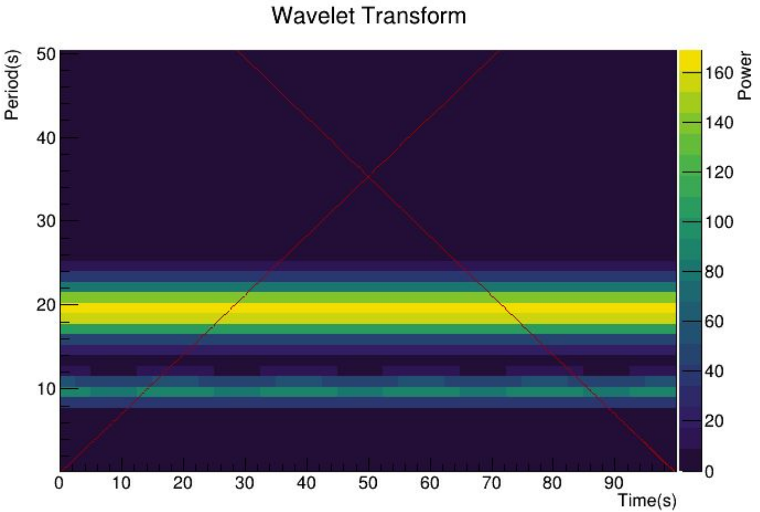


Wavelets for an artificial signal

$$x(t) = \sin\left(\frac{2\pi}{10}t + 0.7\right) + \sin\left(\frac{2\pi}{20}t + 0.3\right)$$

Parameter	value
N_points	1000
Δt	0.1 s

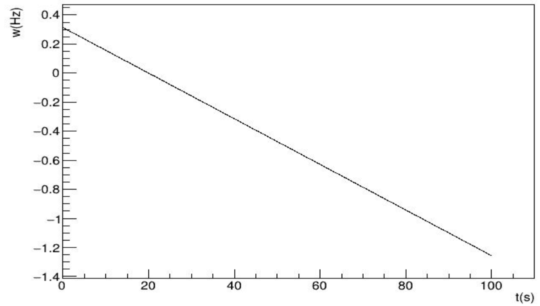


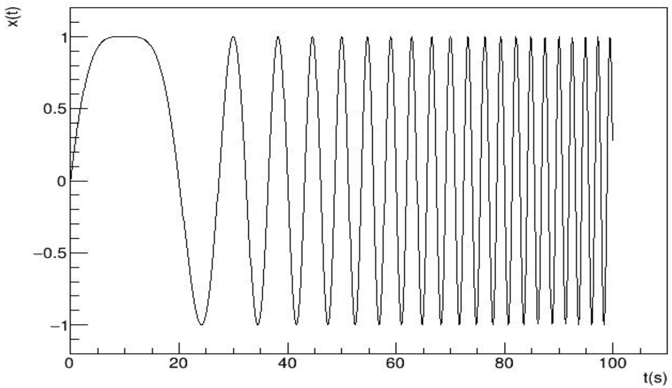


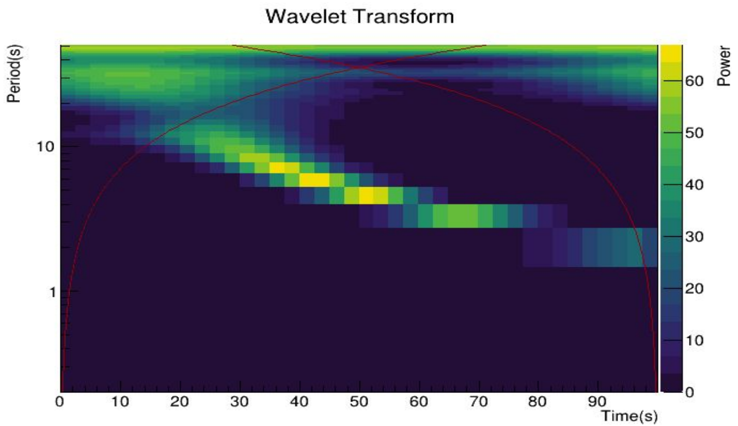
Wavelets for ...

$$x(t) = \sin\left(t \cdot \underbrace{\left(\frac{2\pi}{20} + \left(\frac{2\pi}{40} - \frac{2\pi}{20}\right)\frac{t}{10}\right)}_{\text{Frequency w}}\right)$$

Parameter	value
N_points	1000
Δt	0.1 s



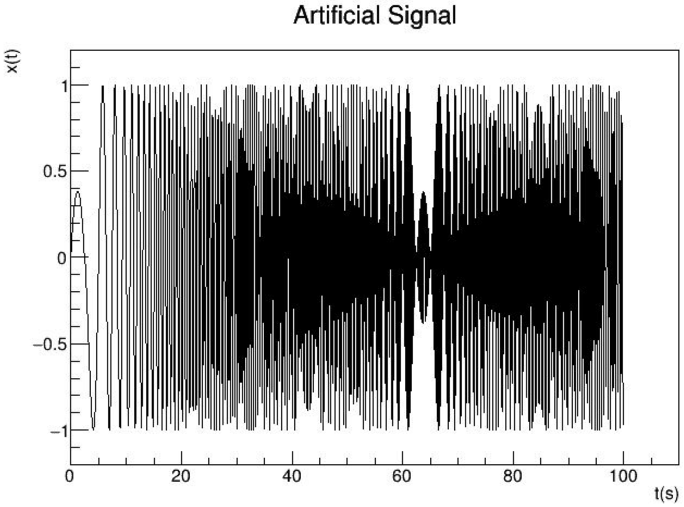


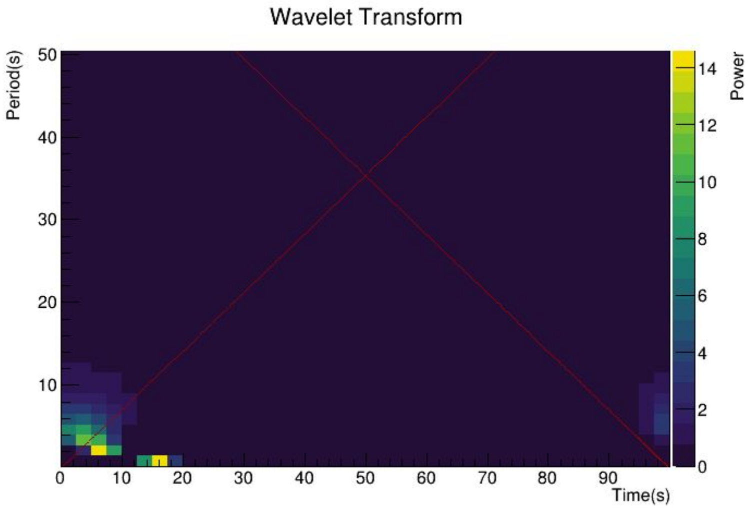


Wavelets for an artificial signal (done wrong)

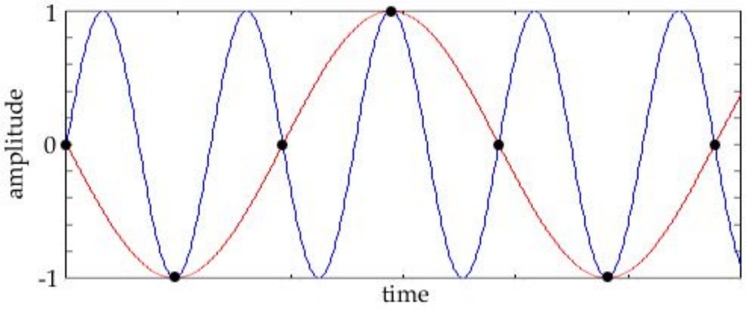
$$x(t) = \sin\left(t \cdot \left(\frac{2\pi}{10} + \left(\frac{2\pi}{10} - \frac{2\pi}{2}\right)\frac{t}{10}\right)\right)$$

Parameter	value
N_points	1000
Δt	0.1 s



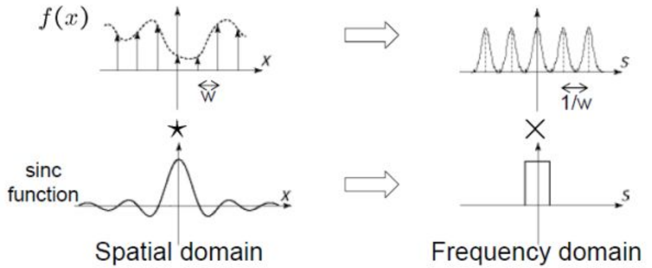


Nyquist rate...



Nyquist Rate

What's the minimum Sampling Rate *1/w* to get rid of overlaps?

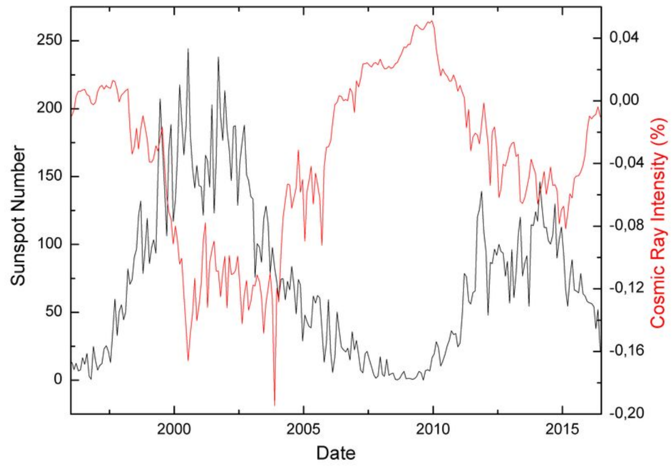


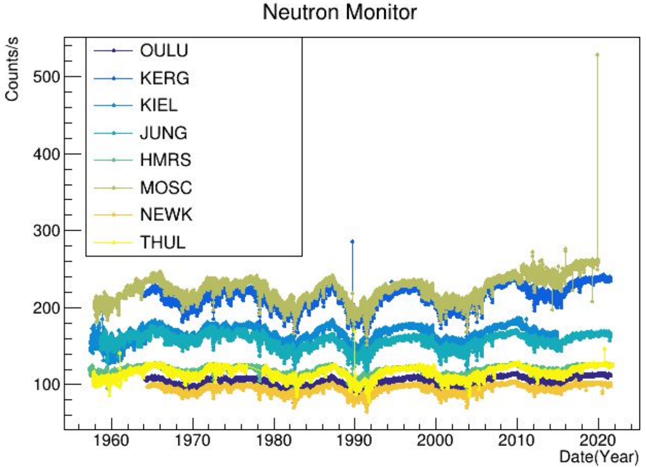
Sampling Rate ≥ 2 * max frequency in the image

- this is known as the Nyquist Rate

NMReader Class

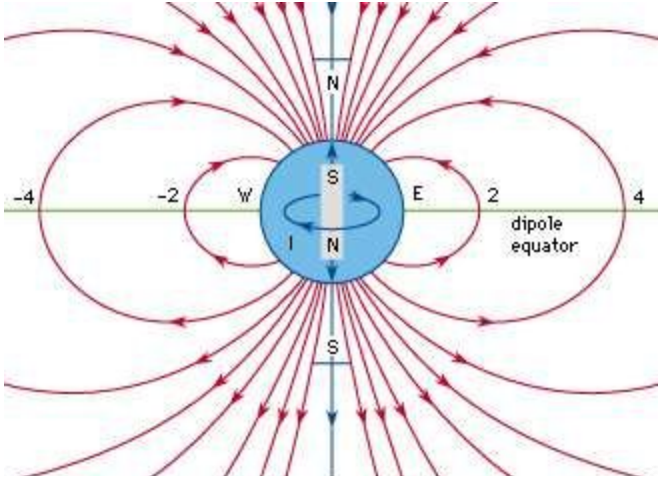
- What are neutron monitors ?
- Neutron monitors and Solar Sunspot number are anti-correlated;
- The 8 stations with over 20,000 entries were chosen;
- Why are there different counts/s in different stations?

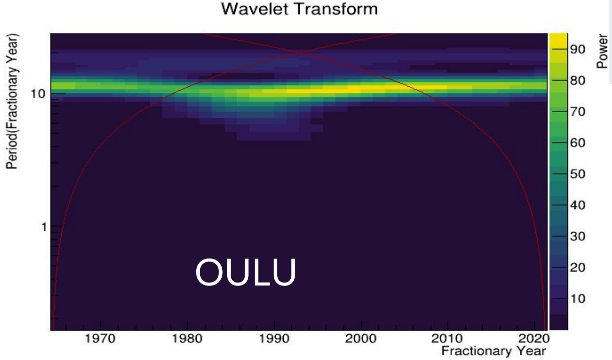


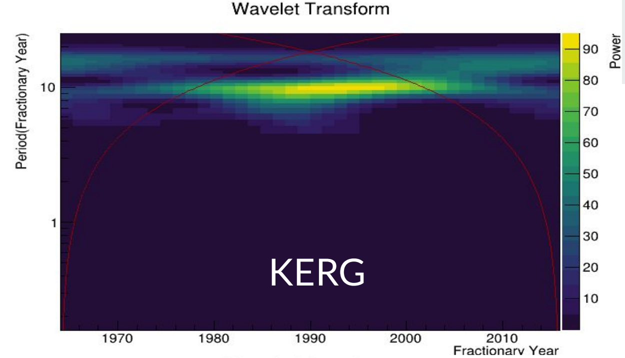


Neutron Monitor and the Earth's Magnetic Field

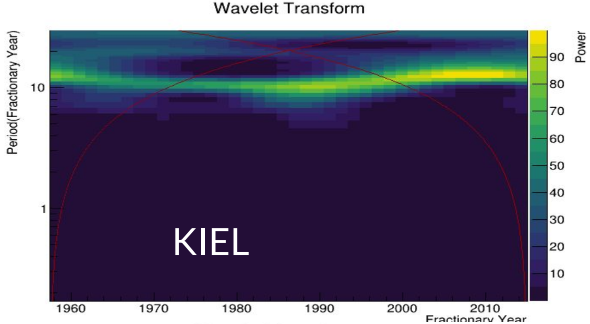
What actually happens depends on the **latitude** and the **inclination** at which the cosmic ray encounters the magnetosphere: around the **poles** is a small region where the magnetic field lines are more or less **radial**. If the particle comes in radially there, it will have **unhindered access** to the atmosphere. If it encounters the magnetosphere in the **equatorial** plane, it hits the magnetic field where its **shielding is most effective**, and the cutoff energy E0 the highest. Particles with energies just above the cutoff may have a very complex orbit before they reach the atmosphere.

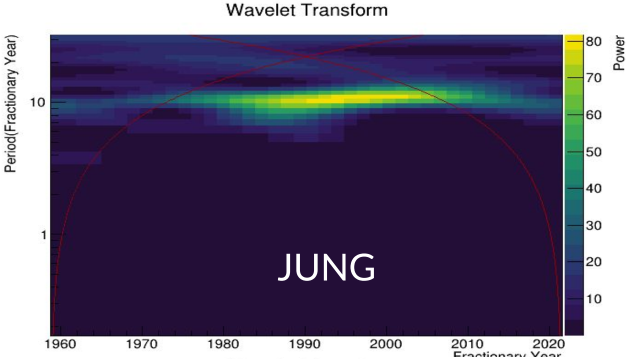


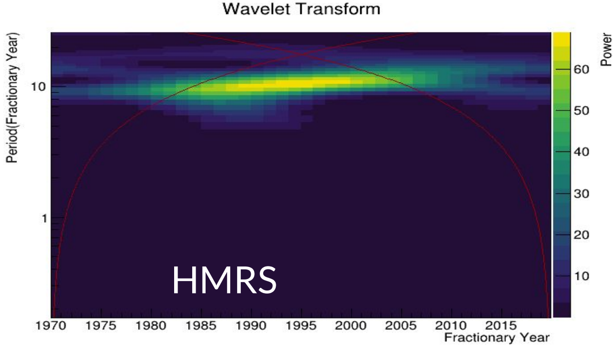


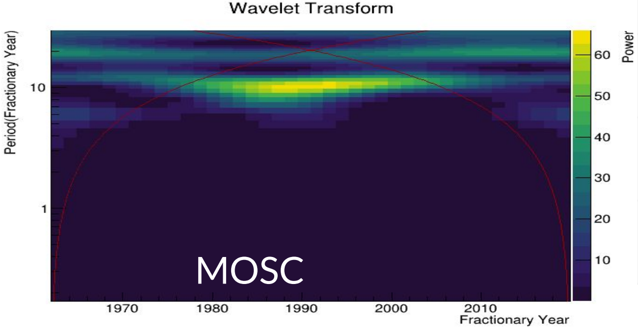


Neutron Monitor Periodograms







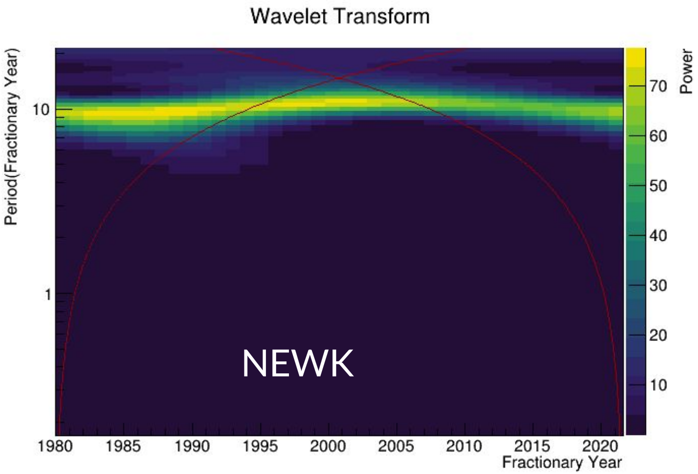


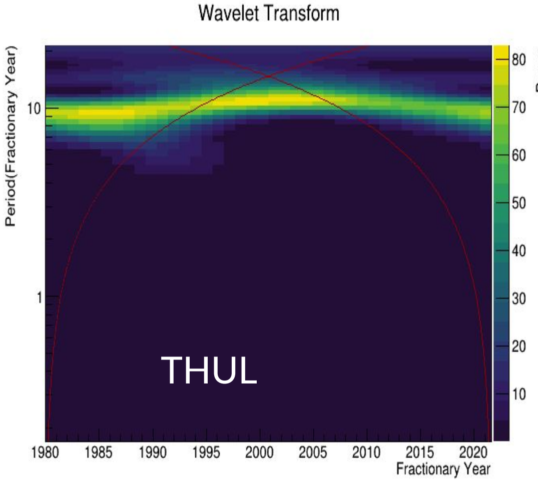
Station	Initial Date	Date Ending
OULU	31/3/1964	17/7/2021
KERG	29/1/1964	1/1/2016
KIEL	1/7/1957	1/9/2021
JUNG	30/9/1958	1/9/2021
HMRS	1/1/1970	1/1/2020
MOSC	1/1/1962	1/9/2021

Neutron Monitor Periodograms

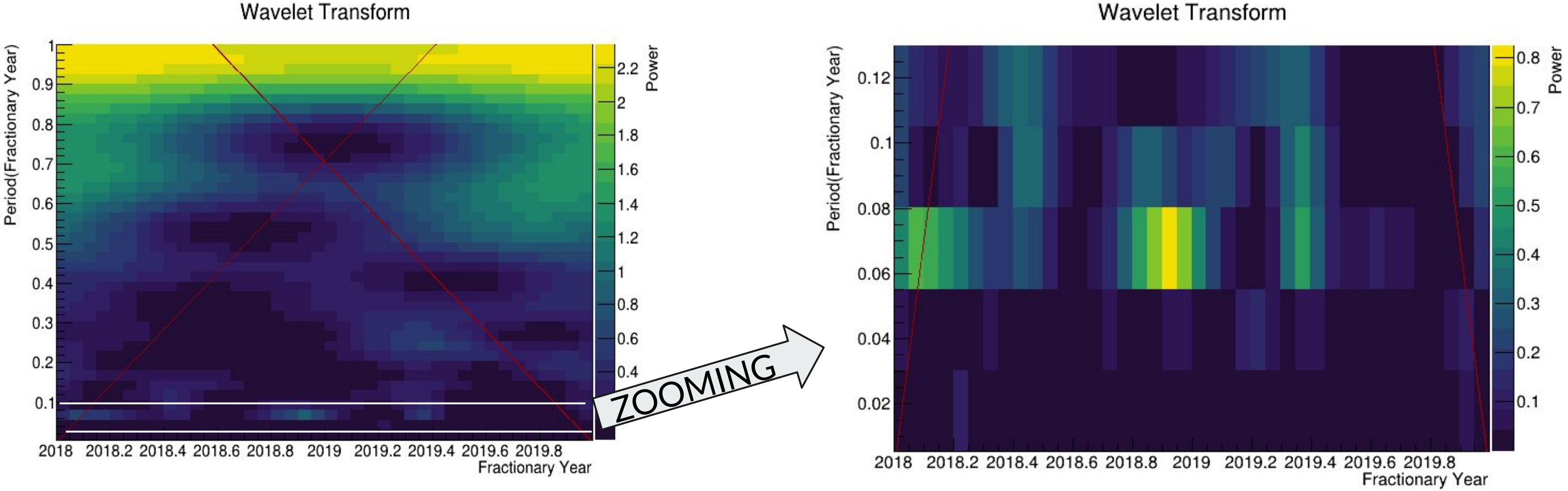
Station	Initial Date	Date Ending
NEWK	1/1/1980	1/1/2021
THUL	1/1/1980	1/1/2021

- All stations provide us with the same periodicity of ~11 years;
- Only 40-60 years of data, which means only 4-6 cycles, more data would improve this study;
- This periodicity is coherent with 11 year sunspot cycle.



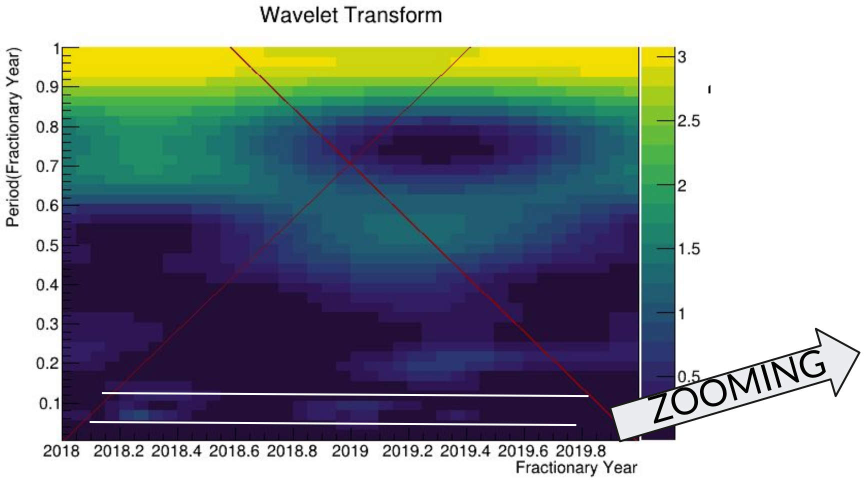


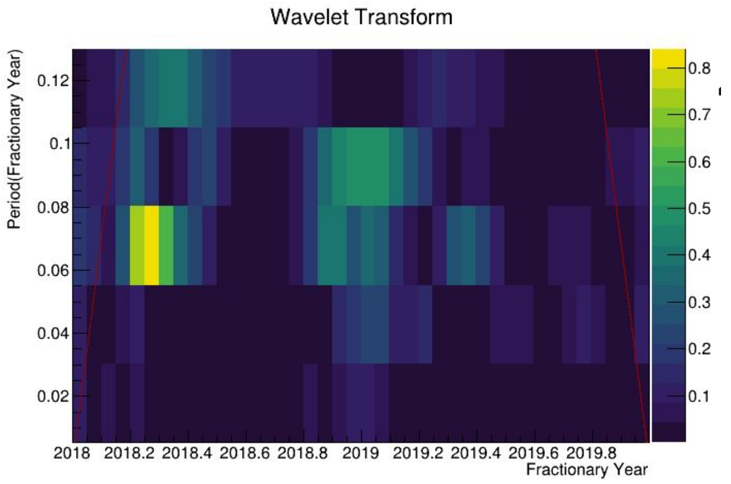
Different timeframes (for OULU)



- Searching for the sun rotation period(27 days).
- Note that 27/365.25~0.07

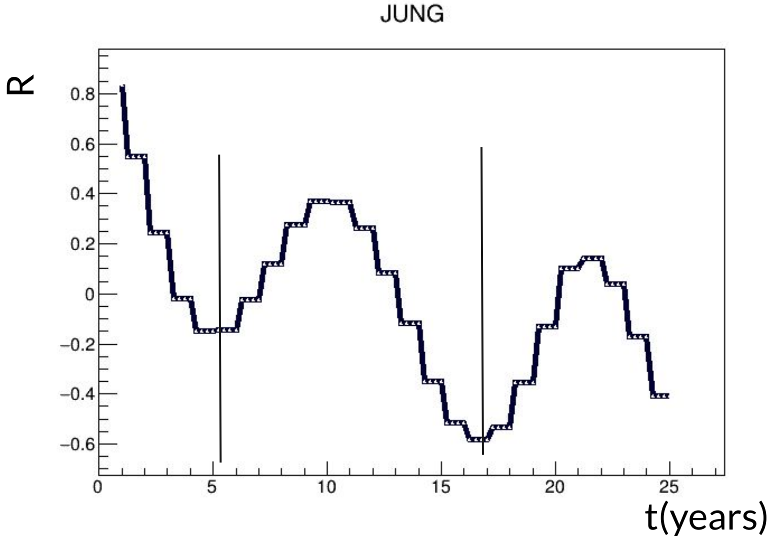
Different timeframes (for JUNG)

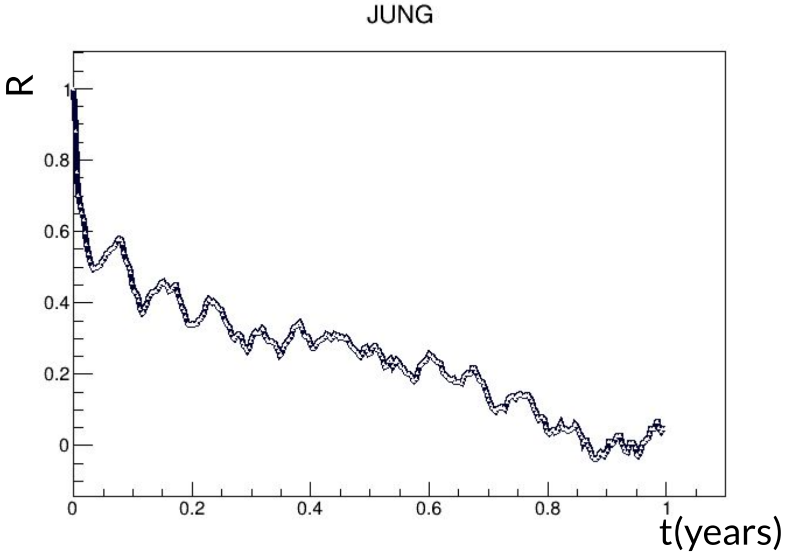




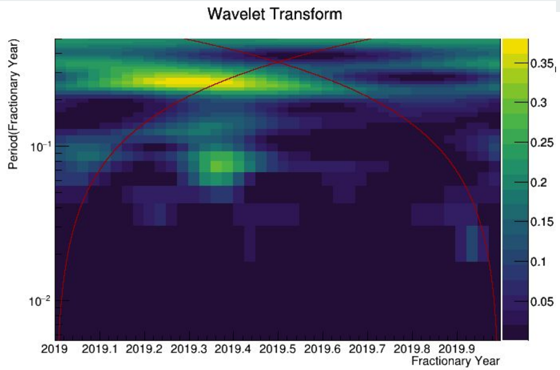
- Searching for the sun rotation period(27 days).
- Note that 27/365.25~0.07

Autocorrelation vs Wavelets
(Autocorrelation graphs for JUNG station)





~11 years



Next Steps...

- Create TimeSeries class to store time series from classes such as NMReader and others(SSNReader, IMFReader,...) and feed them to the Wavelets analysis class;
- Statistical treatment of data could be better, by estimating significance levels of our results;
- With better statistical treatment try to validate the 27 days periodicity;
- Analise AMS Data with Wavelets class;
- Add Background Noise(red noise/white noise);

Conclusions

- Although it has its limitations, wavelets is a very powerful tool ;
- It has multiple applications and I am looking forward to continue to improve the class and to study much more phenomena with it ;

Special thank you to my supervisors: Fernando Barão and Miguel Orcinha !