Simultaneous QCD analysis of collinear distributions

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Introduction

Introduction

• The d.o.f of QCD, quarks and gluons, cannot be directly measured

• All we can see are the asymptotic states of QCD, i.e. hadrons

• Experimental data can be interpreted in terms of quarks and gluons

• How? QCD factorization

$$d\sigma^{e^-p \to hX} = \sum_{ik} f_i(x_1, \mu^2) \otimes d\sigma_{i \to k} \otimes D_{k \to h}(z, \mu^2) + \mathcal{O}\left(\frac{\Lambda_{QCD}^2}{\mu^2}\right)$$



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Universality

PDFs and FFs are universal

$$\sigma_{lp \to lX}^{exp} = \mathbf{f} \otimes \hat{\sigma}_{lp \to lX}$$

$$\sigma_{lp \to hX}^{exp} = \mathbf{f} \otimes \hat{\sigma}_{lp \to hX} \otimes \mathbf{d}$$

$$\sigma_{pp \to l\bar{l}X}^{exp} = \mathbf{f} \otimes \mathbf{f} \otimes \hat{\sigma}_{pp \to l\bar{l}X}$$

$$\sigma_{l\bar{l} \to hX}^{exp} = \hat{\sigma}_{l\bar{l} \to hX} \otimes \mathbf{d}$$

• Their evolution is perturbative: **DGLAP**

$$Q^{2} \frac{\partial f_{i}(x, Q^{2})}{\partial Q^{2}} = \sum_{j} P_{ij} \otimes f_{j}(x, Q^{2})$$
Splitting functions (pQCD)

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Global analysis

• Parametrize the PDFs and FFs at an input scale Q₀

$$f_i(x, Q_0) = N_i x^{a_i} (1 - x)^{b_i} P(x; w_i)$$

 $\overrightarrow{a} = (N_i, a_i, b_i, w_i, \dots)$

- Evolve using DGLAP up to the experiment scale
- Compute the experimental cross sections

• Evaluate the χ^2

• Minimize the χ^2 (or maximize the likelihood)

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Bayesian approach

• The probability distribution *P* is given by

$$P(\overrightarrow{a} | \text{data}) \propto \mathscr{L}(\overrightarrow{a} | \text{data}) \pi(\overrightarrow{a})$$

Likelihood

Likelihood function

$$\mathscr{L}(\overrightarrow{a} | \text{data}) = exp\left\{-\frac{1}{2}\chi^2(\overrightarrow{a})\right\}$$

Observables and their uncertainties are given by

$$E\left[\mathcal{O}\right] = \frac{1}{N} \sum_{k} \mathcal{O}\left(\overrightarrow{a_{k}}\right) \qquad \qquad V\left[\mathcal{O}\right] = \frac{1}{N} \sum_{k} \left[E\left[\mathcal{O}\right] - \mathcal{O}\left(\overrightarrow{a_{k}}\right)\right]^{2}$$

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• Maximize P by finding the best fit parameters \vec{a}_0 that minimize the χ^2

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• Used by CT, MMHT, CJ...

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Maximizing the likelihood II

Data resampling method to build the posterior distribution

• Generate N resampled data $\tilde{\sigma}_i$, i = 1,...N



• Perform *N* fits to the resampled data starting with flat priors

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Why MC?

• Typical PDF parametrization:

$$\chi^{2} = \sum_{e}^{N_{exp}} \sum_{i}^{N_{data}} \frac{(D_{i}^{e} - T_{i})^{2}}{(\sigma_{i}^{e})^{2}}$$

 $x\Delta f(x) = Nx^a(1-x)^b(1+c\sqrt{x}+dx)$

- Perform single *x*²fit: → Multiple local minima!
 Parameters difficult to constrain
 Hessian method for uncertainties → Introduces tolerance criteria
 Unsuitable for simultaneous analysis of collinear distributions
- Monte Carlo methods:
 - Allows efficient exploration of the parameter space
 - Uncertainties directly obtained from MC replicas



What is JAM19?

First simultaneous MC analysis of unpolarized PDFs and FFs

Why JAM19?

To focus on the study of the strange quark distribution

Motivation

• Testing the universality of PDFs ,FFs...

• All the data must be studied using the **same** theoretical framework

First step: (first) simultaneous analysis of unpolarized PDFs and
 FFs ----> Strange quark distribution

• The strange PDF is less known than the non-strange light flavors



CT18, arXiv:1912.10053 [hep-ph]

- Traditionally: neutrino-(heavy) nucleus DIS data used to extract the strange PDF.
 - Drawbacks: nuclear effects on PDFs.

- W and Z inclusive production in p-p collisions also sensitive to flavor separation
 - Drawbacks: tension between CMS and ATLAS results?





ATLAS Collaboration, <u>arXiv:1612.03016</u> [hep-ex]

Why don't we use SIDIS?

Setup: data

DIS : $l + (p, d) \rightarrow l' + X$ DY : $l + (p, d) \rightarrow l\bar{l} + X$ SIDIS : $l + d \rightarrow l' + h + X$ $W^2 > 10 \,\mathrm{GeV^2}$ $Q^2 > \mathrm{m_c^2}$

4366 data points

SIA : $e^+ + e^- \rightarrow h + X$



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JAM19 methodology

Setup: theory

- All observables computed at NLO in pQCD
- DGLAP truncated evolution at order α_s in Mellin space
- DIS/SIDIS/SIA cross sections computed at leading twist
- Nuclear smearing for deuterium DIS
- Heavy quark treatment : ZM-VFN
- Fitting methodology:
 - MC (multi-steps), k-means clustering, extended reduced χ^2



JAM 19: multi-step fitting

$\begin{array}{c} 2.5 \\ 2.0 \\ 1.5 \\ 1.0 \\ 0.0 \\ 1.0^{-1} \\ 1.0^{-1$



X

+ DIS data

JAM 19: multi-step fitting



 ${\mathcal X}$

+ DIS data + DIS + DY data

JAM 19: multi-step fitting PDFs PION FF



 ${\mathcal X}$

+ DIS data
+ DIS + DY data

JAM 19: multi-step fitting PDFs PION FF KAON FF



 ${\mathcal X}$

Z

+ DIS data
+ SIA pion data
+ SIA kaon data
+ DIS + DY data

JAM 19: multi-step fitting **PION FF PDFs KAON FF**



X

+ DIS data + SIA pion data + SIDIS pion data + DIS + DY data

+ SIA kaon data + SIDIS kaon data

+ SIDIS data

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Discrminating multiple solutions



X


X



X



X



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k-means clustering E.g. $f(x) = x^{\alpha} (1 - x)^{\beta}$











+ DIS data

+ DIS + DY data

+ SIDIS data



+ DIS data

+ DIS + DY data

+ SIDIS data

$$R_s = \frac{s + \bar{s}}{\bar{u} + \bar{d}}$$

PDFs



X

+ DIS data

$$R_s = \frac{s + \bar{s}}{\bar{u} + \bar{d}}$$

PDFs



X

+ DIS data

 $\frac{s+\bar{s}}{\bar{u}+\bar{d}}$ $R_s =$

PDFs



$$R_s = \frac{s + \bar{s}}{\bar{u} + \bar{d}}$$

PDFs



X

+ DIS data + DY data

$$R_s = \frac{s + \bar{s}}{\bar{u} + \bar{d}}$$

PDFs



X

+ DIS data + DY data



PDFs



Constraints on R_s

$$R_s = \frac{s + \bar{s}}{\bar{u} + \bar{d}}$$

PDFs



X

+ DIS data + DY data + SIA + SIDIS data

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$$R_s = \frac{s + \bar{s}}{\bar{u} + \bar{d}}$$

PDFs



X

+ DIS data + DY data + SIA + SIDIS data





Z



Z



Z



Z



Z



Z



Z





Z



Z





JAM19: Selection Criteria

Apply k-means clustering

• Classify clusters by increasing order in 'extended' reduced χ^2

$$\frac{\chi^2}{N_{\rm tot}} + \sum_{exp} \frac{\chi^2_{\rm exp}}{N_{\rm exp}}$$

• Perform a new sampling with flat priors around the best cluster



PDF results

JAM19 PDFs



JAM19 PDFs





JAM19 PDFs


JAM19 PDFs



JAM19 PDFs



FF results

JAM19: FF



 $Q = m_c$

JAM19: FF



$Q = m_c$



 $Q = m_c$

JAM19: FF



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JAM19: FF



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 $Q = m_c$



Impact of SIDIS data









Strong strange suppression









 $Q = m_c$







 $Q = m_c$



 $Q = m_c$





$Q = m_c$



 $Q = m_c$



Summary

• First MC fit of PDFs and FFs using DIS, DY, SIDIS and SIA data

 MC statistical methods are important for a robust extraction of nonperturbative collinear distributions

- JAM19 Methodology: MC (multi-steps), k-means clustering, 'extended' reduced χ^2
- Strange PDF strongly suppressed

Thanks

Chi2

 $\widehat{}$

Reaction	$N_{\rm dat}$	χ^2	$\chi^2/N_{ m dat}$	Reaction	$N_{\rm dat}$	χ^2	$\chi^2/N_{\rm dat}$
SIDIS	992	1243.12	1.25	SIDIS (π^{\pm})	498	585.48	1.18
SIA	444	562.80	1.27	$\operatorname{SIDIS}(K^{\pm})$	494	657.64	1.33
DIS	2680	3437.96	1.28	$SIA(\pi^{\pm})$	231	247.27	1.07
DY	250	416.29	1.67	SIA (K^{\pm})	213	315.53	1.48

Experiment	target	hadron	$N_{\rm dat}$	$\chi^2/N_{ m dat}$
COMPASS	d	π^+	249	1.26
COMPASS	d	π^{-}	249	1.09
COMPASS	d	K^+	247	1.24
COMPASS	d	K^{-}	247	1.43

SIA: pions



SIA: kaons



DIS: proton



DIS: proton



DIS: deuteron






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