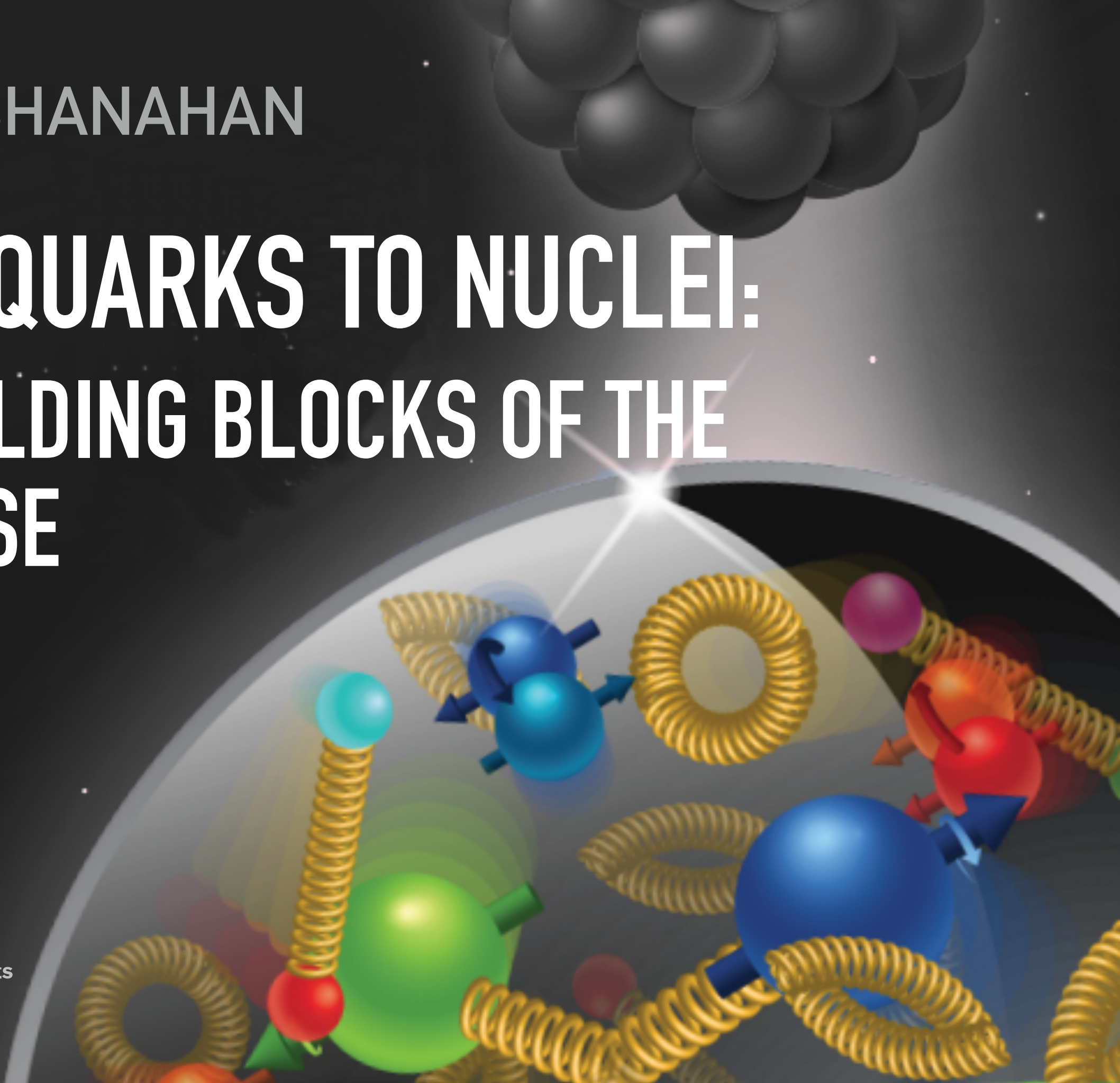


PHIALA SHANAHAN

# FROM QUARKS TO NUCLEI: THE BUILDING BLOCKS OF THE UNIVERSE



Massachusetts  
Institute of  
Technology

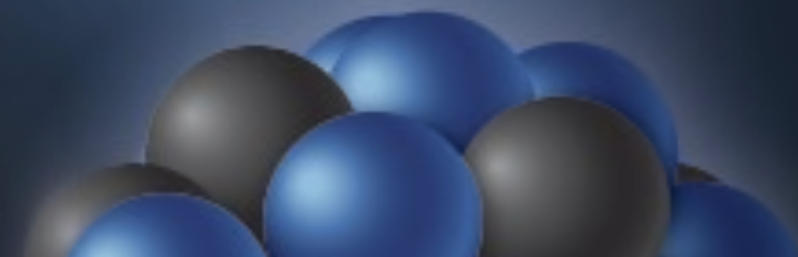




**MATTER**



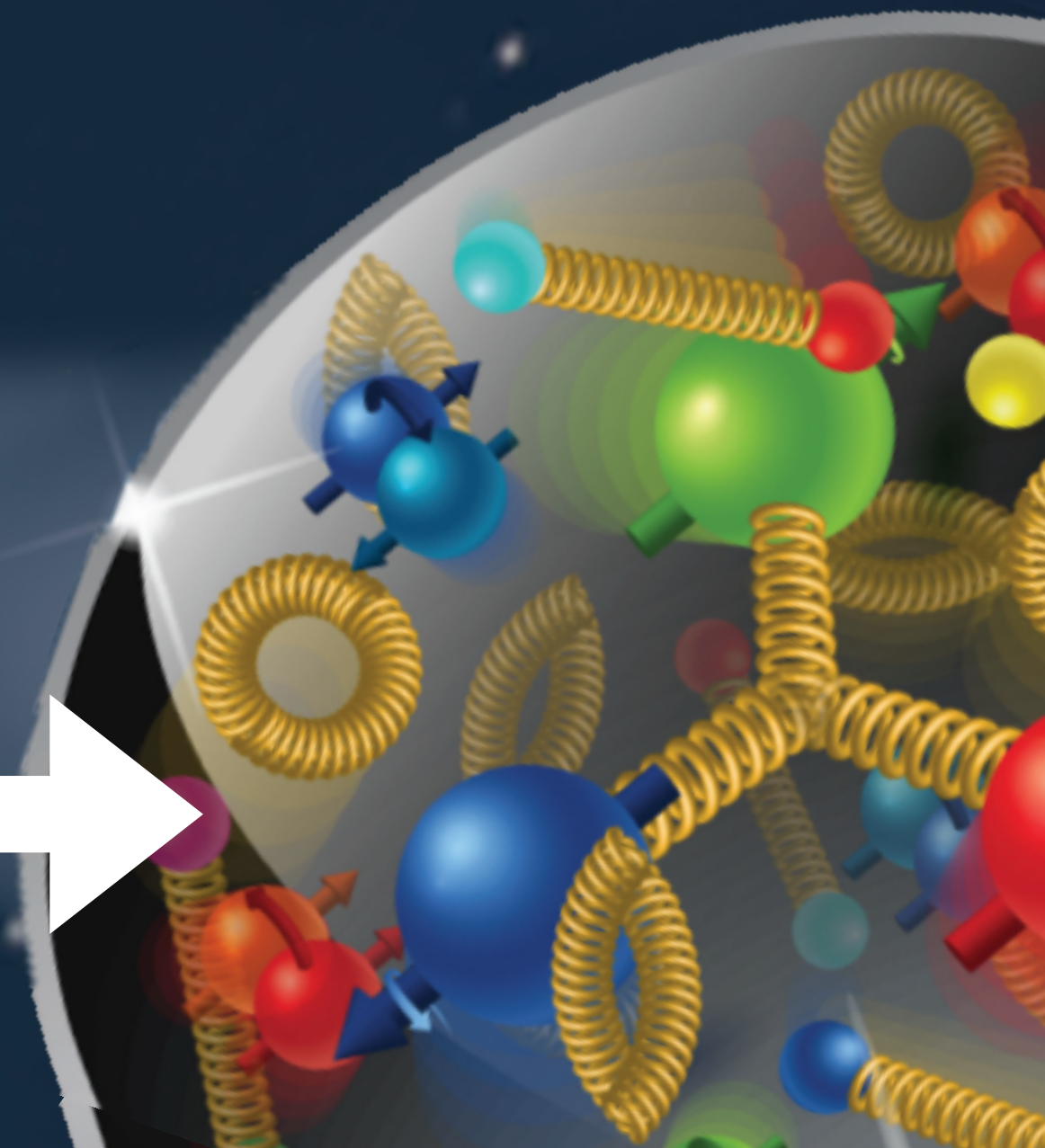
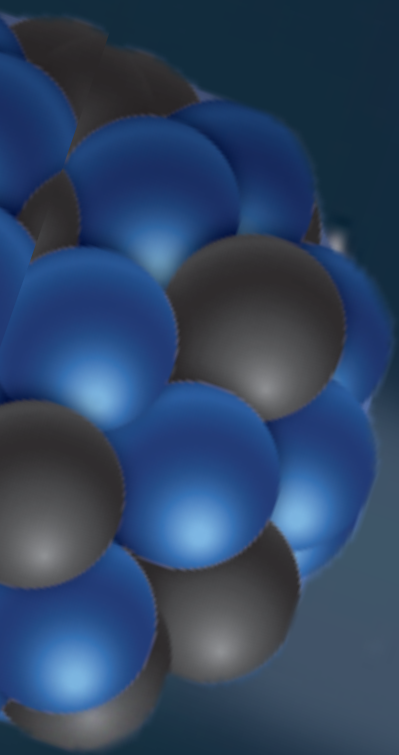
**ATOMS**





**PROTONS &  
NEUTRONS**

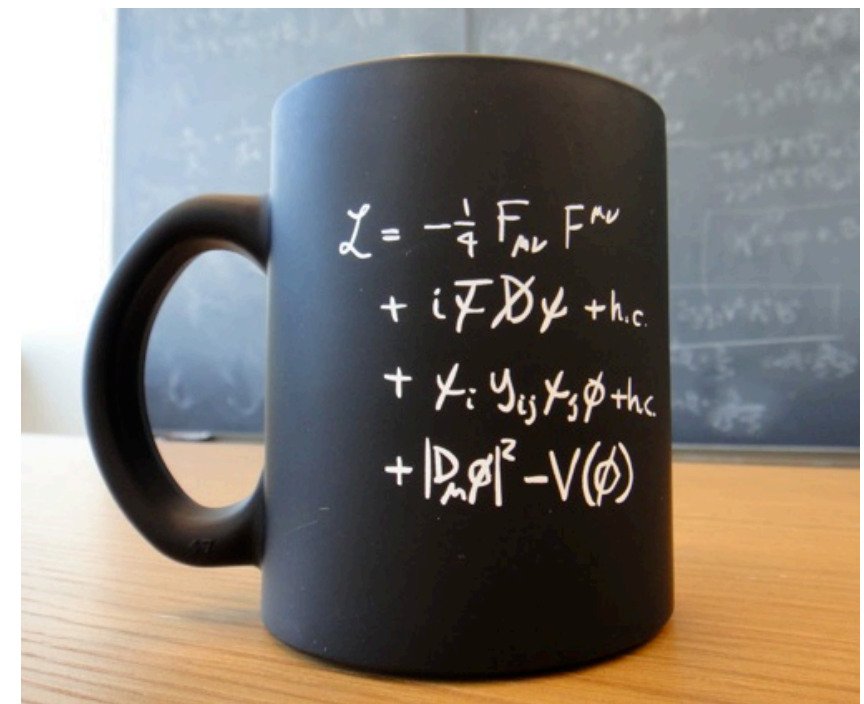
**QUARKS, GLUONS  
AND THE QUANTUM  
VACUUM**



# The structure of matter

The Standard Model of nuclear and particle physics

	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	
<b>Quarks</b>	$u$ up	$c$ charm	$t$ top	<b>Gauge Bosons</b>
	$d$ down	$s$ strange	$b$ beauty	
	$e$ electron	$\mu$ muon	$\tau$ tau	
<b>Leptons</b>	$\nu_e$ neutrino electron	$\nu_\mu$ neutrino muon	$\nu_\tau$ neutrino tau	
				$W^\pm$ W boson
				$Z^0$ Z boson
			$g$ gluon	
			$H$ Higgs Boson	



# The structure of matter

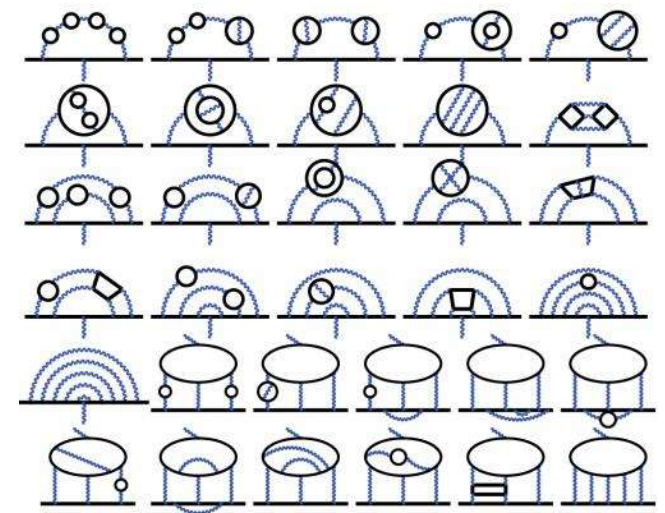
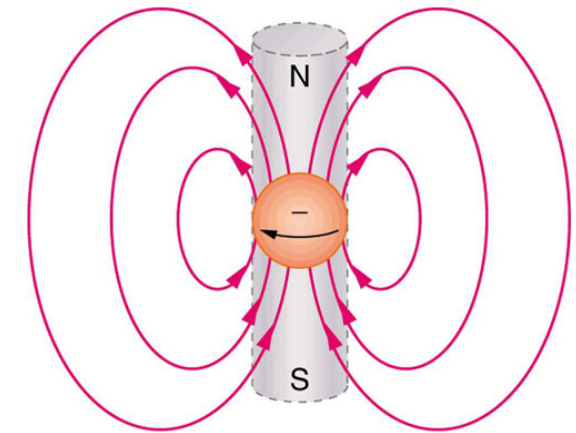
The Standard Model is successful

Magnetic moment of the electron:  
(torque an electron feels in a magnetic field)  $a_e = (g - 2)/2$

**Most accurately verified prediction in  
the history of physics**

**Theory**  $a_e = 0.001159652181643(764)$

**Exp.**  $a_e = 0.00115965218073(28)$



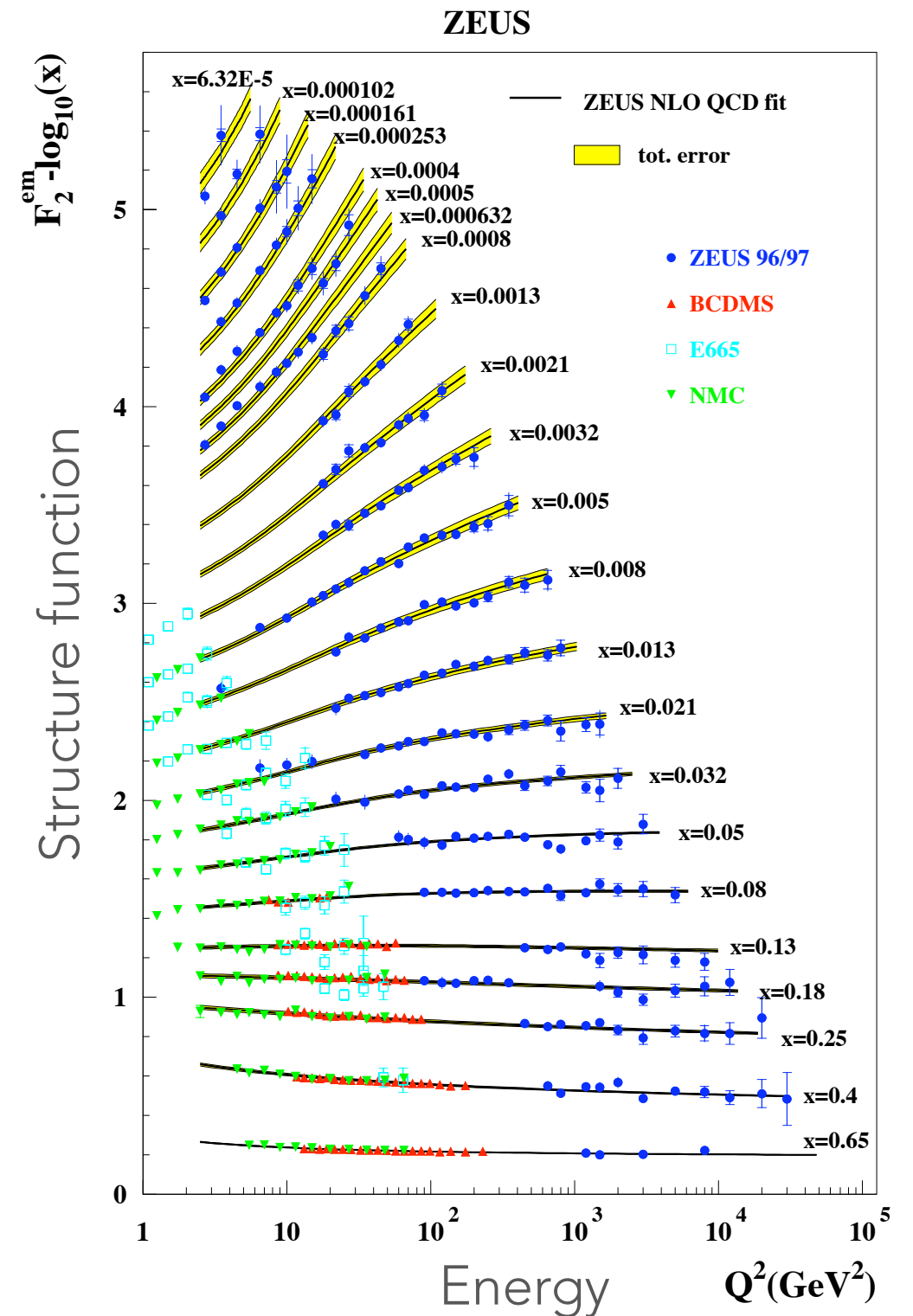
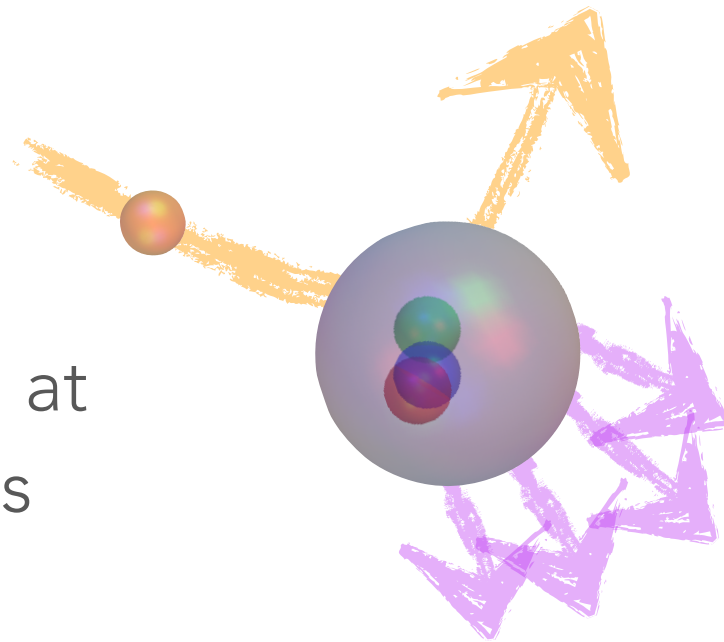


# The structure of matter

The Standard Model  
is successful

Deep inelastic scattering of  
electron on proton (hits and breaks  
proton apart)

Verification at  
many scales

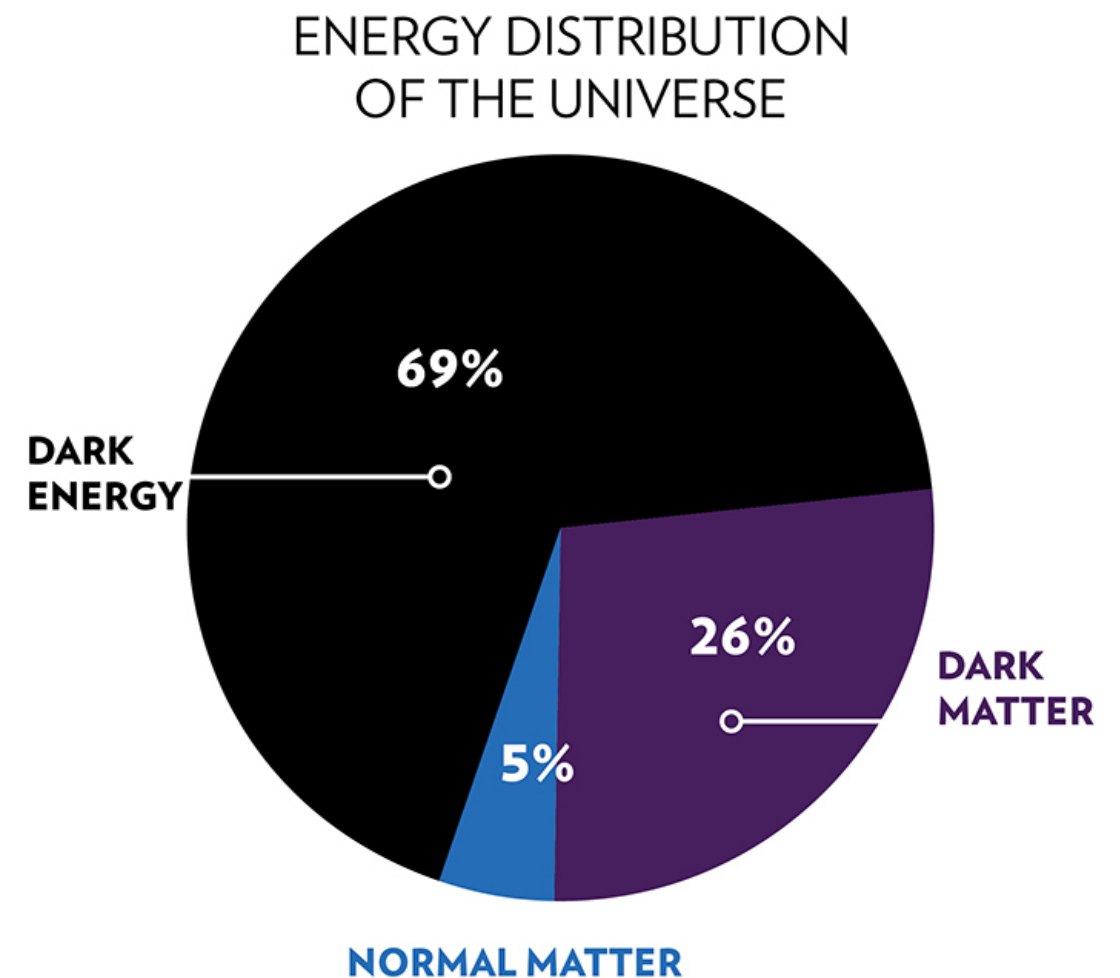


# The structure of matter

**BUT** The Standard Model isn't everything

## Example: Dark matter

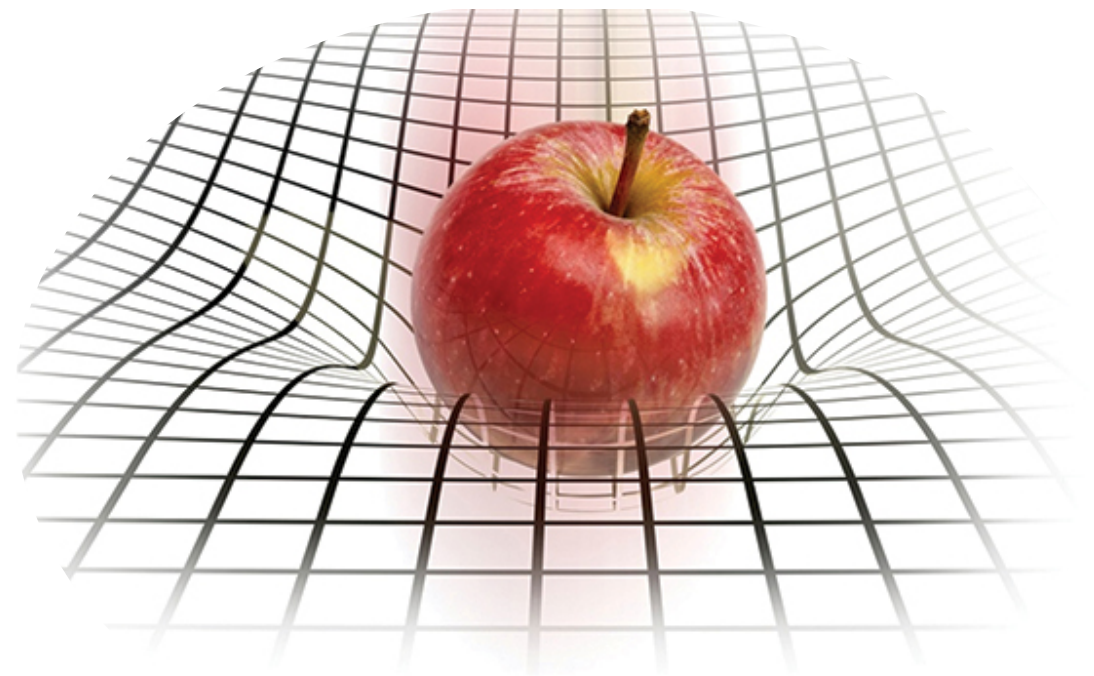
Despite the success of the Standard Model, most of the matter in the universe is something else!



# The structure of matter

**BUT** The Standard Model isn't everything

- Dark matter and dark energy
- Neutrino masses
- Matter–antimatter asymmetry
- Gravity
- Naturalness problems
- ...



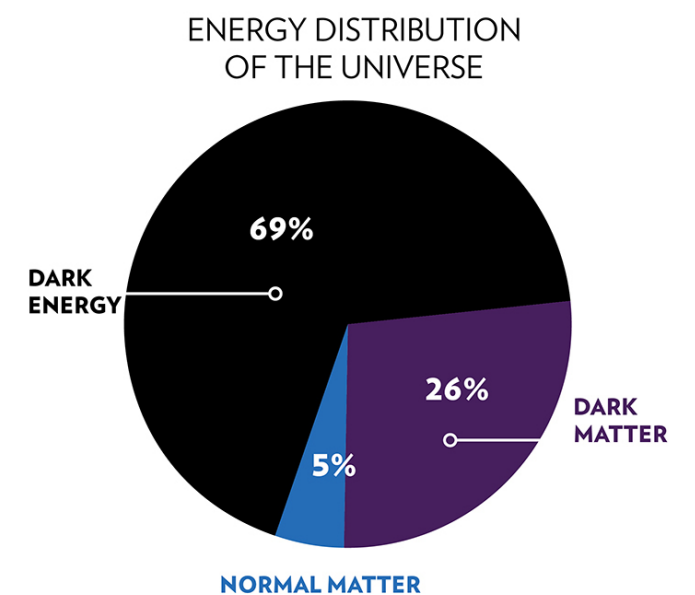
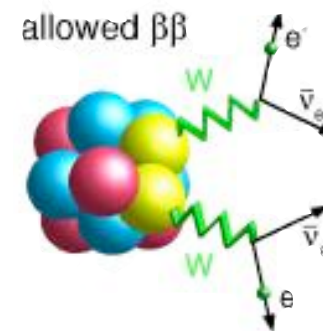


# The search for new physics

## Precise experiments seek new physics at the “Intensity Frontier”

- Sensitivity to probe the rarest Standard Model interactions
- Search for beyond—Standard-Model effects

- Dark matter direct detection
- Neutrino physics
- Charged lepton flavour violation,  $\beta\beta$ -decay, proton decay, neutron-antineutron oscillations...



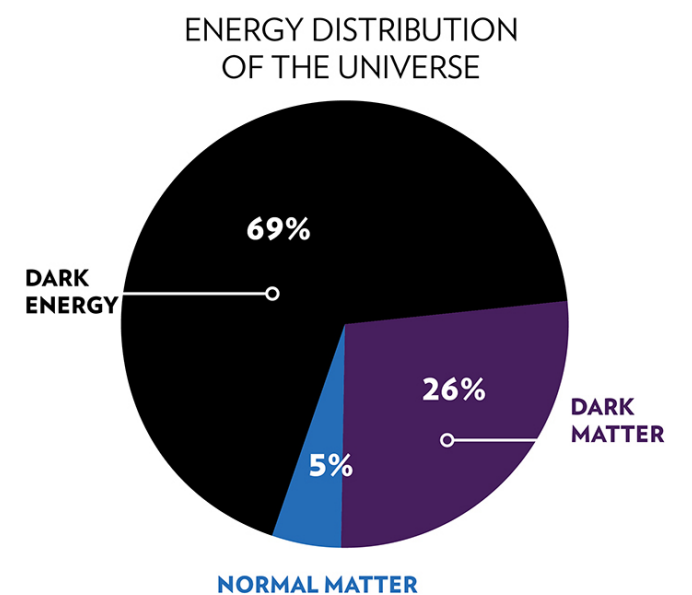
# The search for new physics

Precise experiments seek new physics  
at the “Intensity Frontier”

- Sensitivity to probe the rarest Standard Model interactions
- Search for beyond—Standard-Model effects


EXPERIMENTS USE NUCLEAR  
TARGETS

NEED TO UNDERSTAND  
STANDARD MODEL PHYSICS  
OF PROTONS & NUCLEI




# The structure of matter

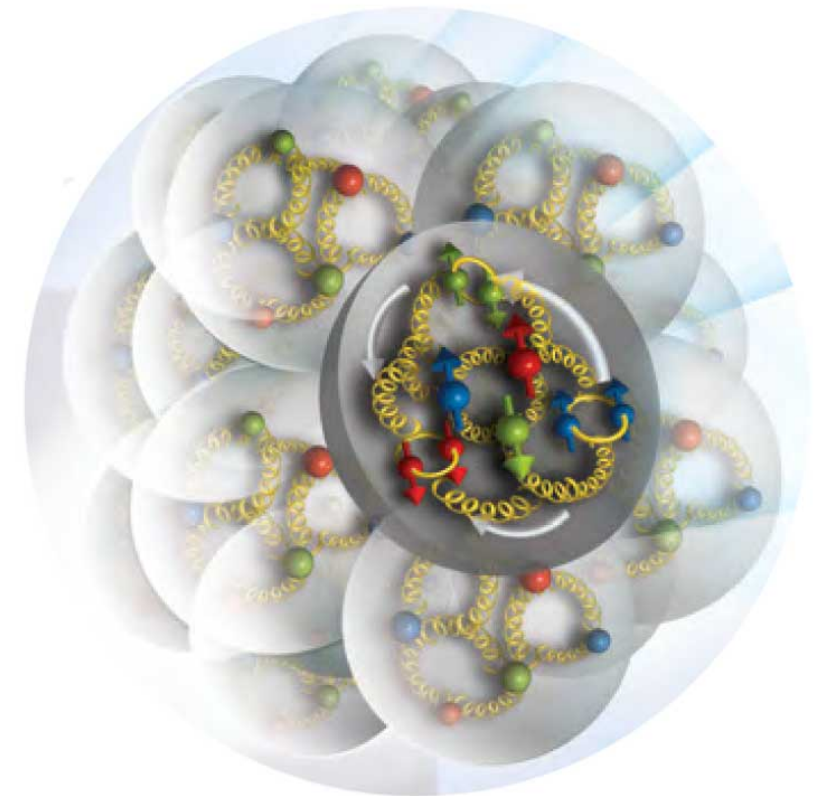
Understanding the quark and gluon structure of matter



Emergence  
of complex  
structure in  
nature



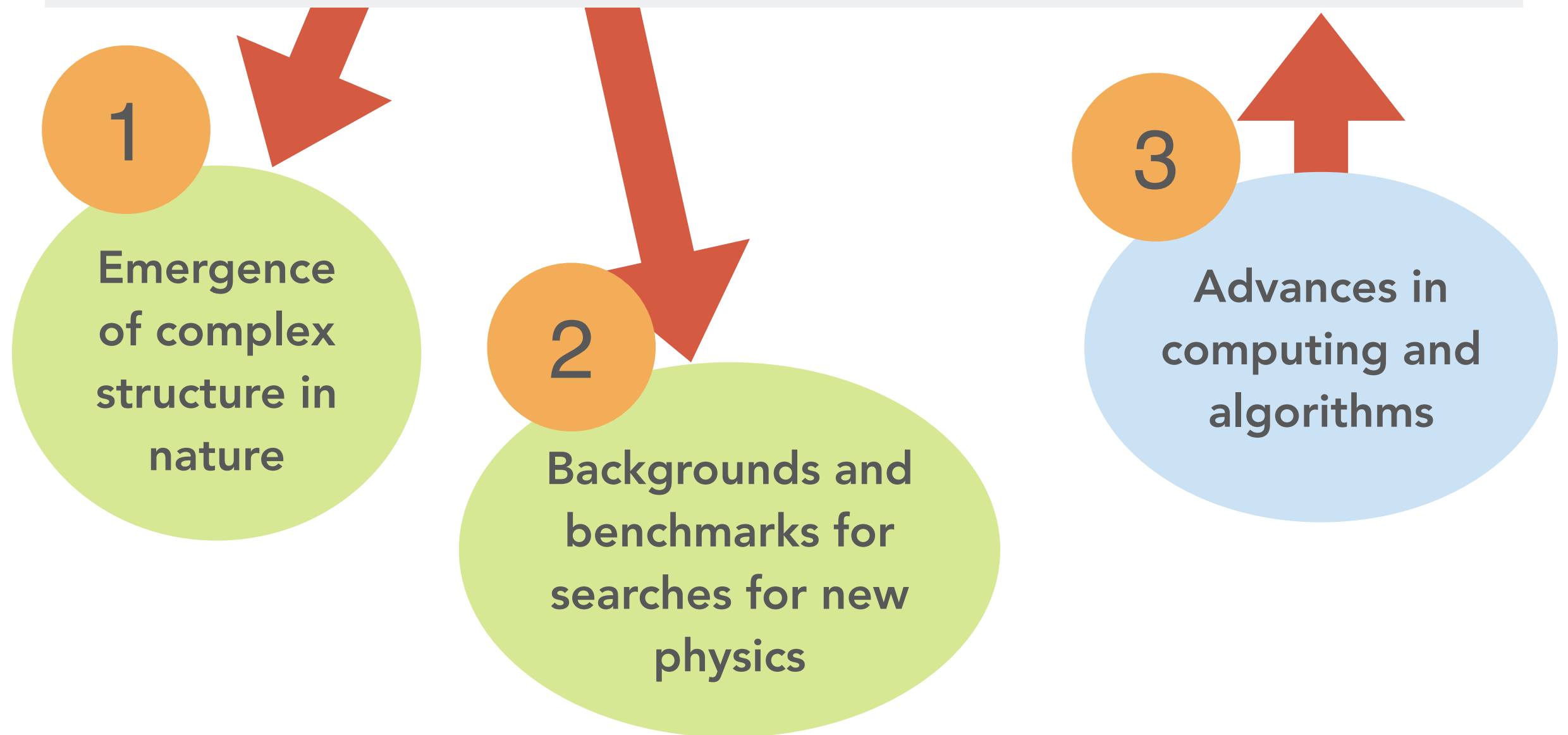
Backgrounds and  
benchmarks for  
searches for new  
physics





# The structure of matter

Understanding the quark and gluon structure of matter

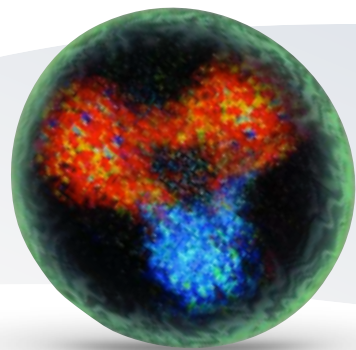


# Strong interactions

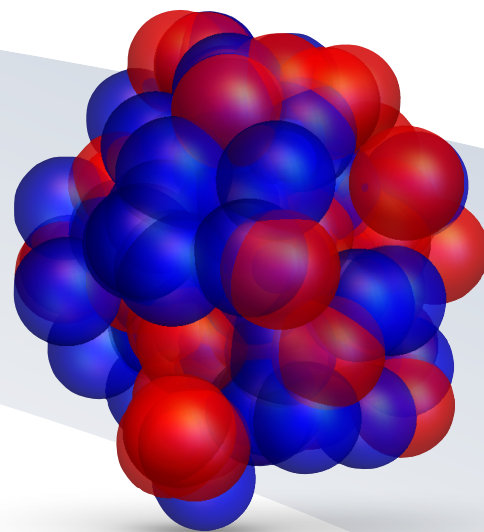
Study nuclear structure from the strong interactions

## Quantum Chromodynamics (QCD)

Strongest of the four forces in nature

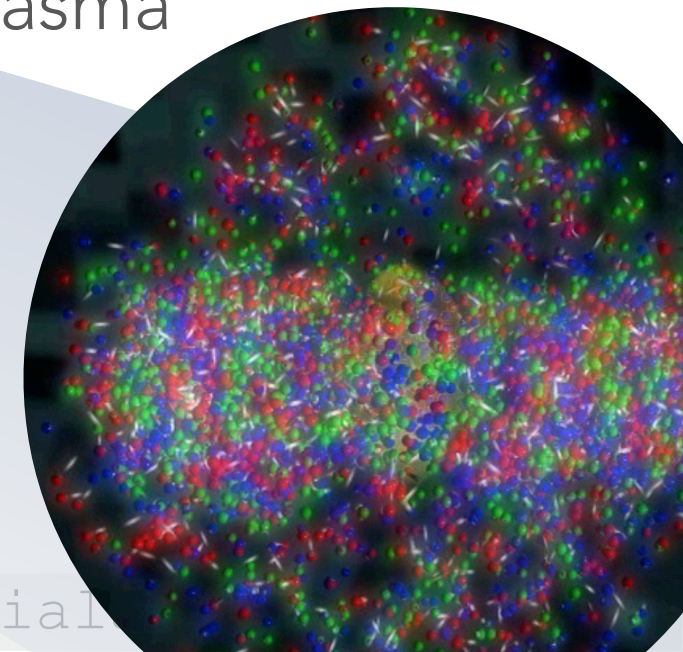


Binds quarks and gluons into protons, neutrons, pions etc.



Binds protons and neutrons into nuclei

Forms other types of exotic matter e.g., quark-gluon plasma



# Strong interactions

Interaction strength depends on energy

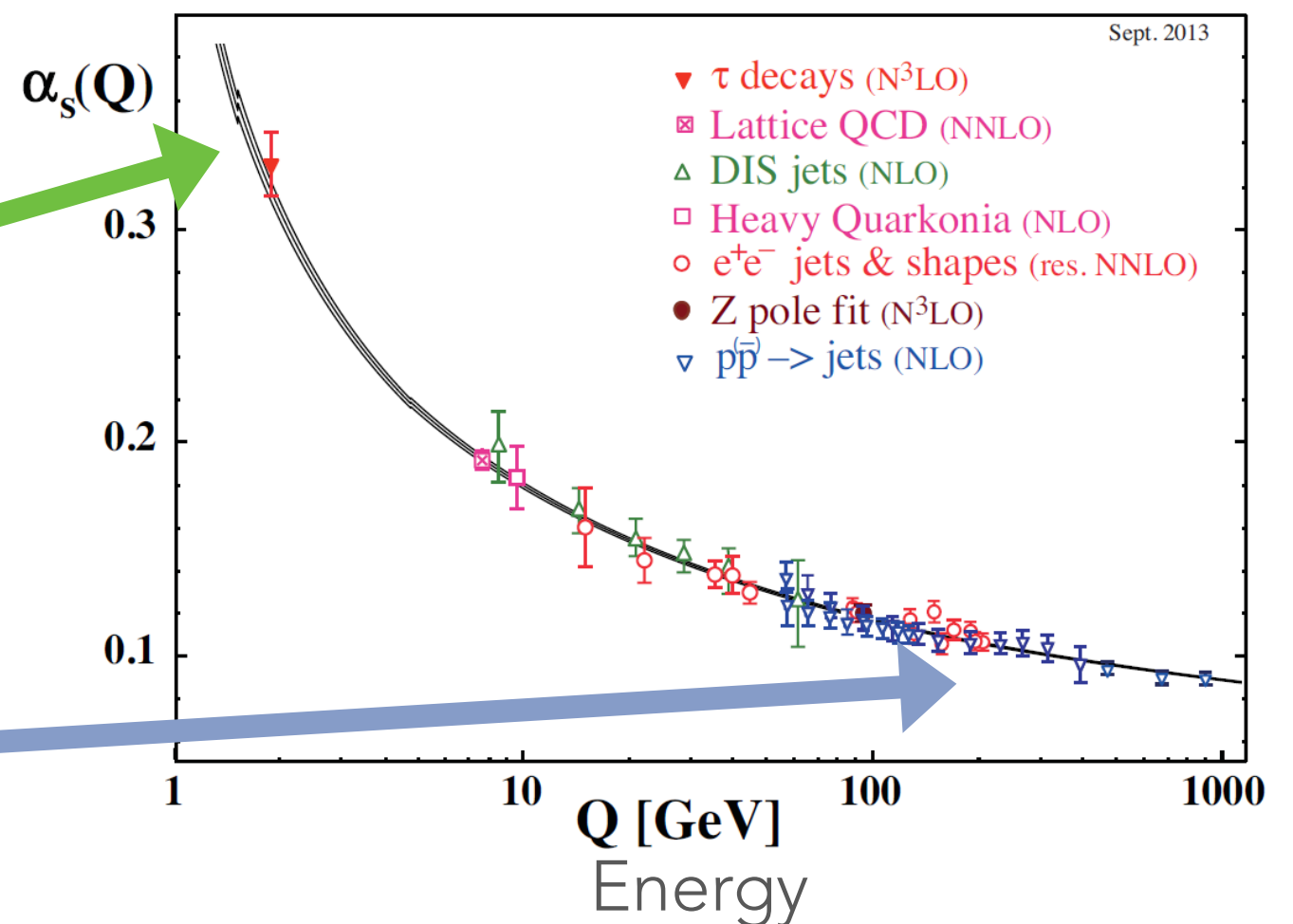
[Gross, Politzer, Wilczek, Nobel 2004]

Low-energy QCD is non-perturbative

Perturbation theory at high energies

$$\mathcal{O}_{\text{exact}} = \mathcal{O}_0 + \mathcal{O}_1\alpha_s + \mathcal{O}_2\alpha_s^2 + \dots$$

Strong coupling

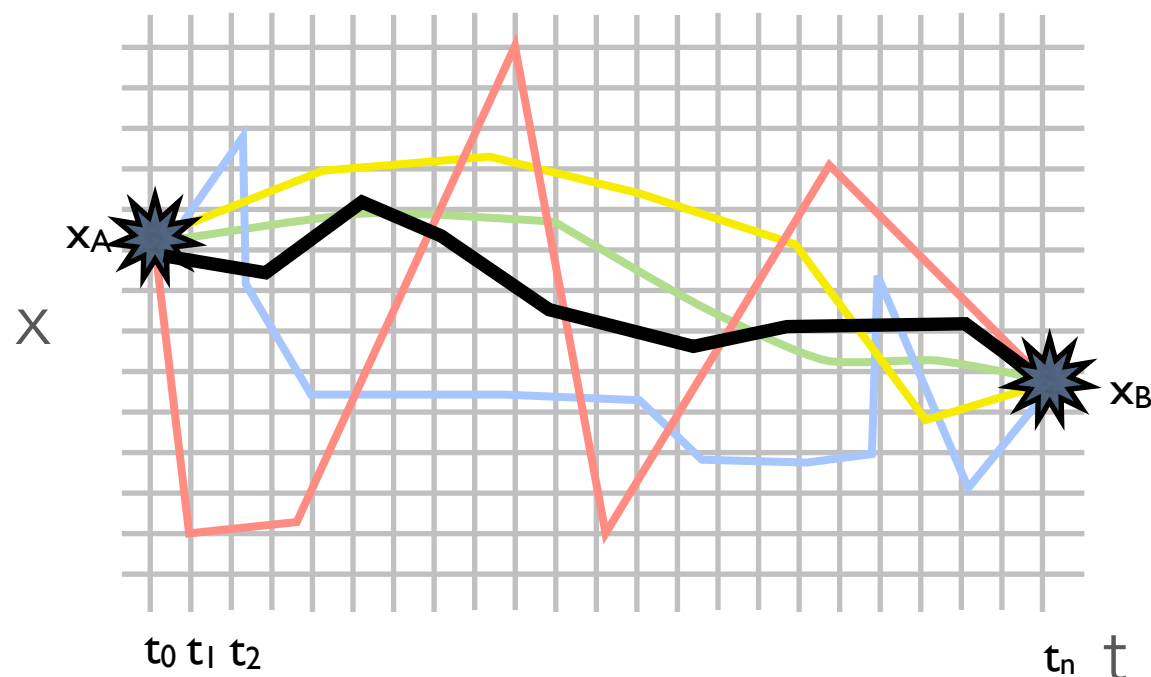




# Lattice QCD

Numerical first-principles approach to non-perturbative QCD

- QCD equations  $\longleftrightarrow$  integrals over the values of quark and gluon fields on each site/link (QCD path integral)
- $\sim 10^{12}$  variables (for state-of-the-art)

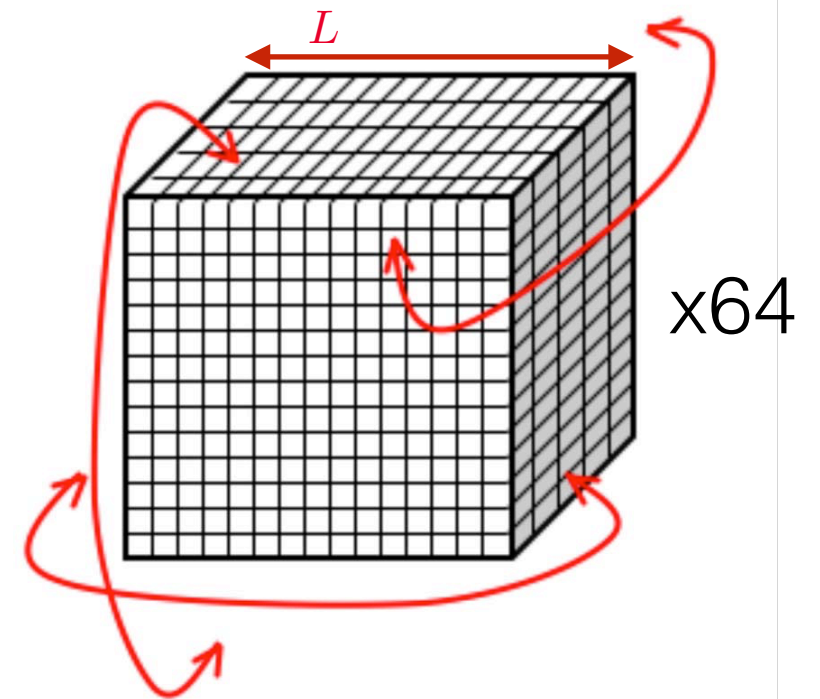


- Evaluate by importance sampling
- Paths near classical action dominate
- Calculate physics on a set (ensemble) of samples of the quark and gluon fields

# Lattice QCD

Numerical first-principles approach to non-perturbative QCD

- Euclidean space-time
  - Non-zero lattice spacing  $a$
  - Volume  $L^3 \times T \approx 32^3 \times 64$
- Some calculations use larger-than-physical quark masses (cheaper)



Approximate the QCD path integral by **Monte Carlo**

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}A \mathcal{D}\bar{\psi} \mathcal{D}\psi \mathcal{O}[A, \bar{\psi}\psi] e^{-S[A, \bar{\psi}\psi]} \rightarrow \langle \mathcal{O} \rangle \simeq \frac{1}{N_{\text{conf}}} \sum_i^{N_{\text{conf}}} \mathcal{O}([U^i])$$

with field configurations  $U^i$  distributed according to  $e^{-S[U]}$

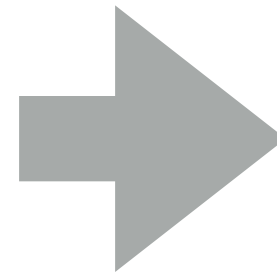
# Lattice QCD

Numerical first-principles approach to non-perturbative QCD

## INPUT

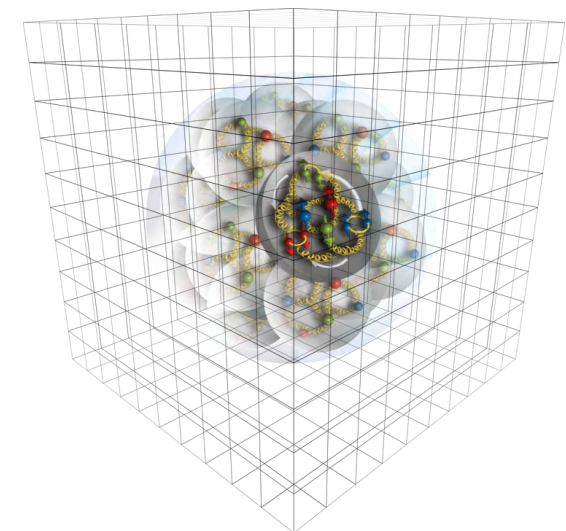
Lattice QCD action has same free parameters as QCD: quark masses,  $\alpha_S$

- Fix quark masses by matching to measured hadron masses, e.g.,  $\pi, K, D_s, B_s$  for  $u, d, s, c, b$
- One experimental input to fix lattice spacing in GeV (and also  $\alpha_S$ ), e.g.,  $2S-1S$  splitting in  $\Upsilon$ , or  $f_\pi$  or  $\Omega$  mass



## OUTPUT

Calculations of all other quantities are QCD predictions



# Lattice QCD

Numerical first-principles approach to non-perturbative QCD

**Calculations use world's largest computers**

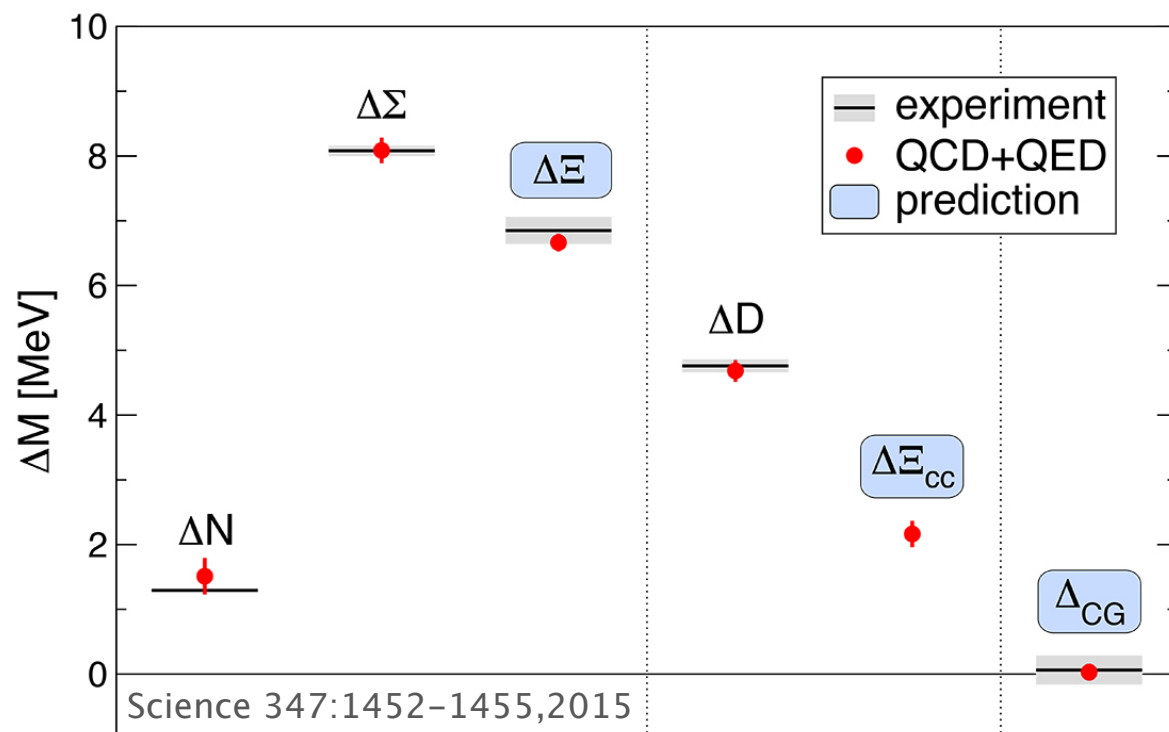
- Many millions of CPU/GPU hours
- Specifically designed processors for QCD



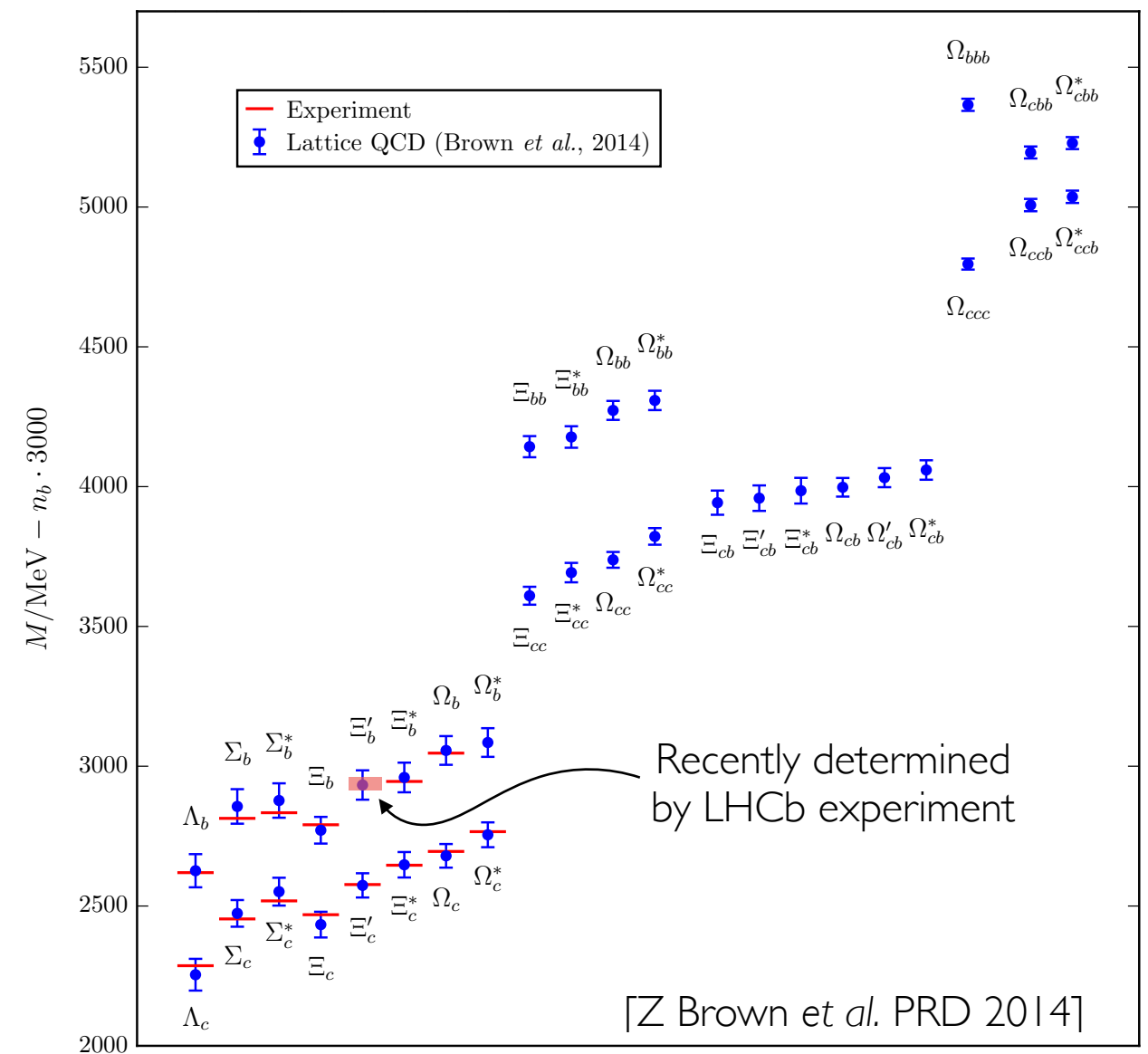


# Lattice QCD works

- Ground state hadron spectrum reproduced
- p-n mass splitting reproduced
- ...



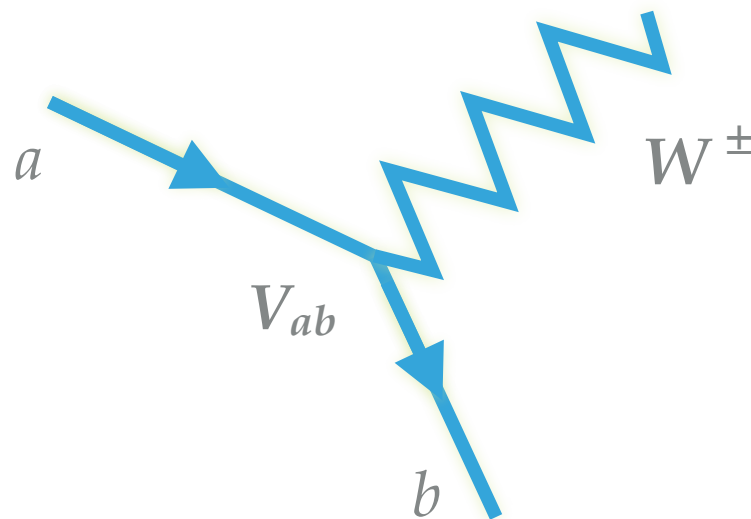
- Predictions for new states with controlled uncertainties



# Lattice QCD works

Essential input for flavour physics by constraints on CKM matrix

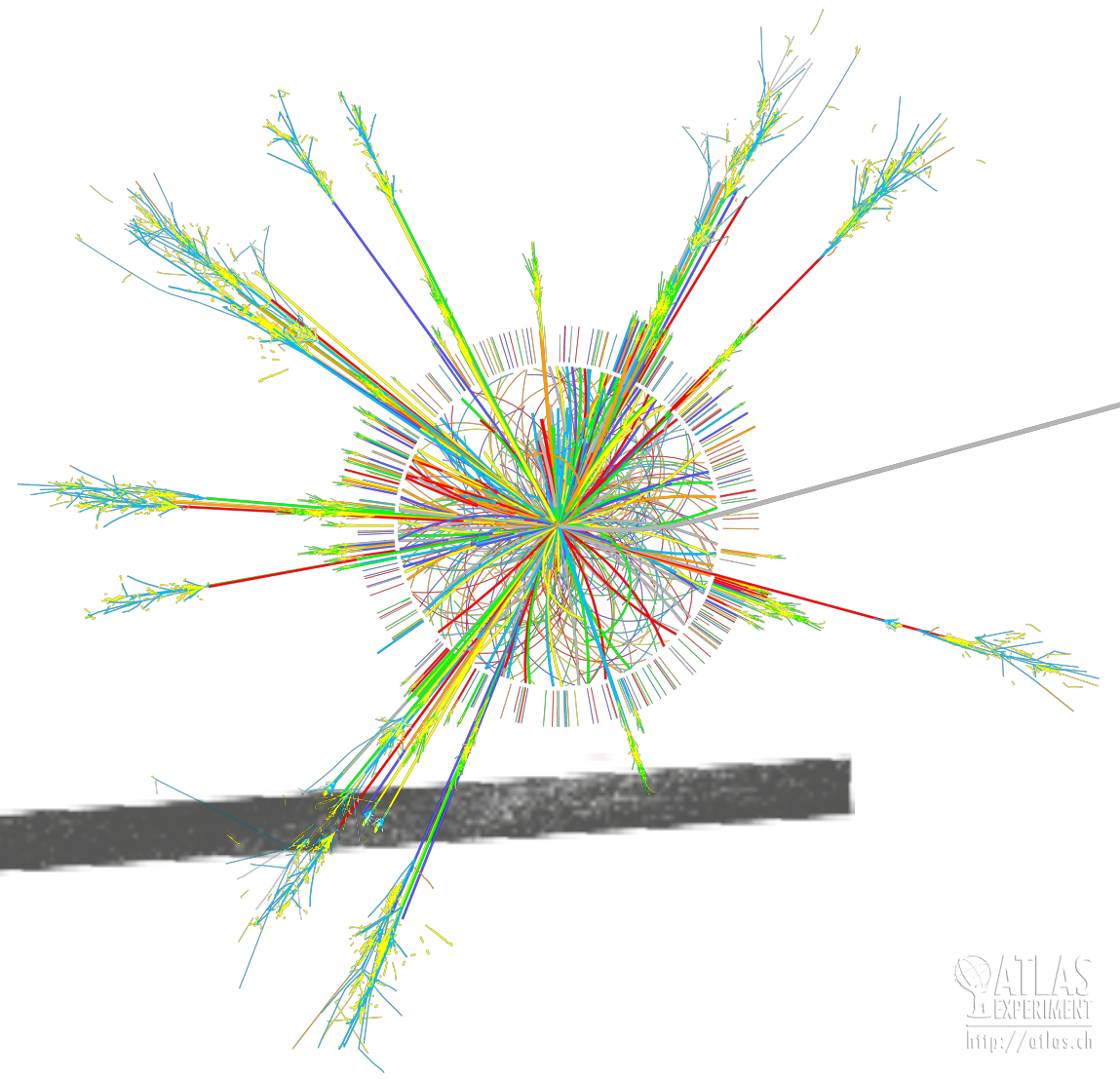
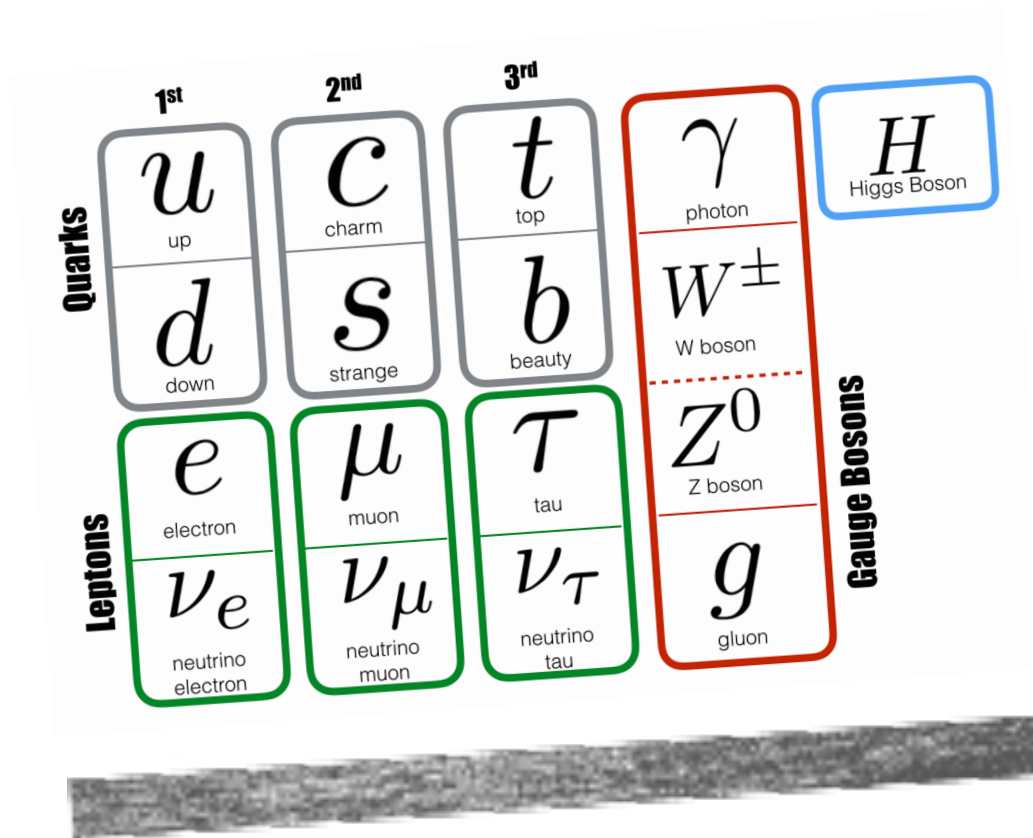
- CKM matrix determines strength of weak interactions between quark flavours



- Parameters are inputs to Standard Model  
→ overdetermining CKM tests Standard Model
- LQCD critical to extraction of all but  $V_{td}$ : a synergy between theory and experiment

$$[\text{experiment}] = [\text{known}] \times [\text{CKM}] \times [\text{hadronic matrix element}]$$

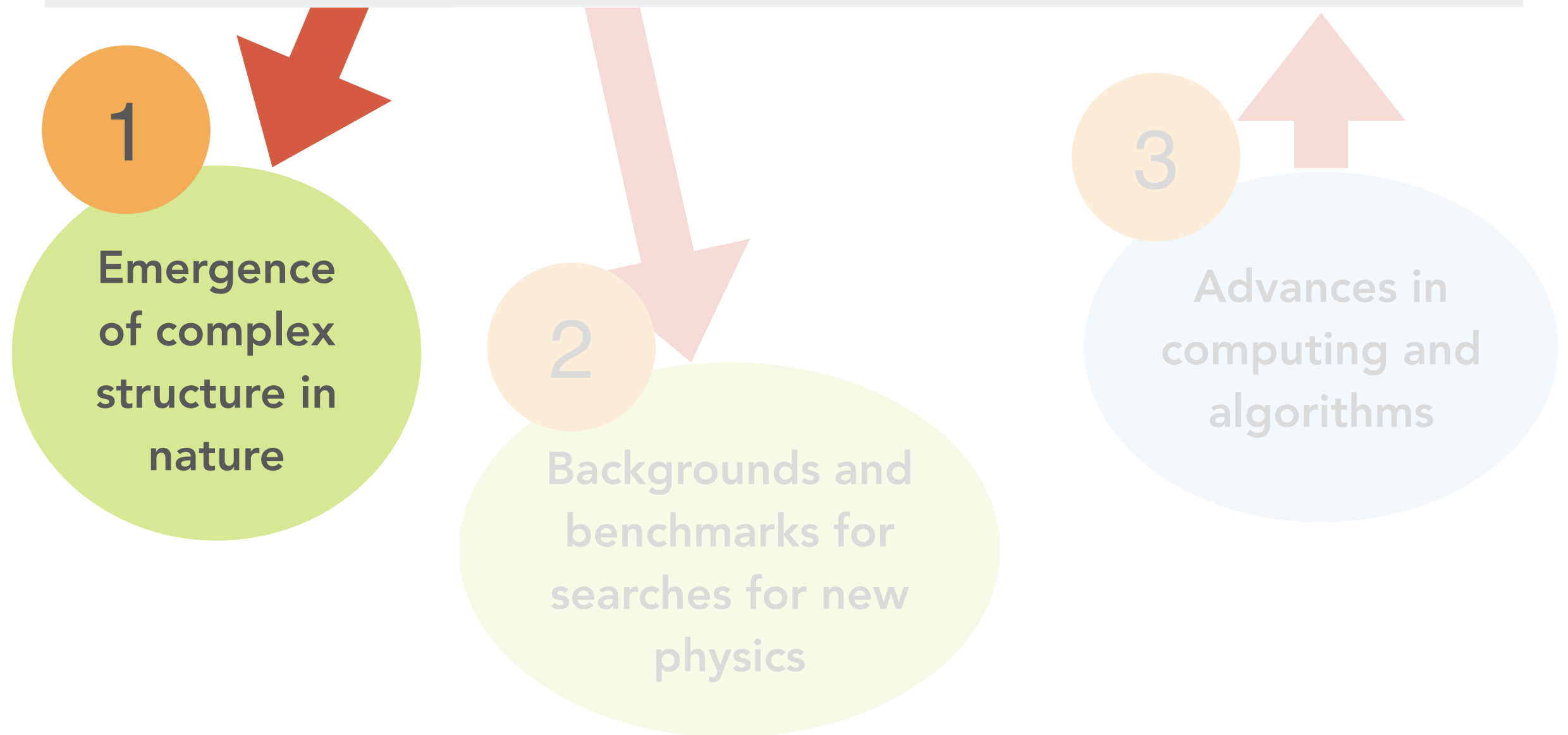
# Theory complements experiment



ATLAS  
EXPERIMENT  
<http://atlas.ch>

# The structure of matter

Understanding the quark and gluon structure of matter





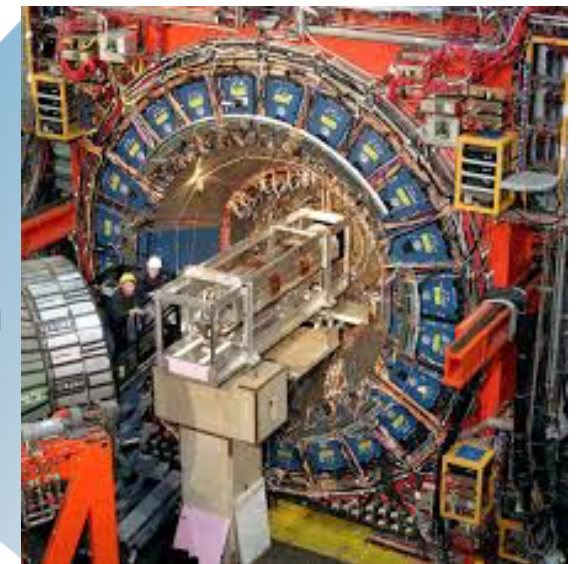
# The structure of matter

Understanding the quark and gluon structure of matter

Defining effort in experiment



JLab



Tevatron



DESY



LHC



EIC

**Electron Ion Collider:  
The Next QCD Frontier**

Understanding the glue  
that binds us all

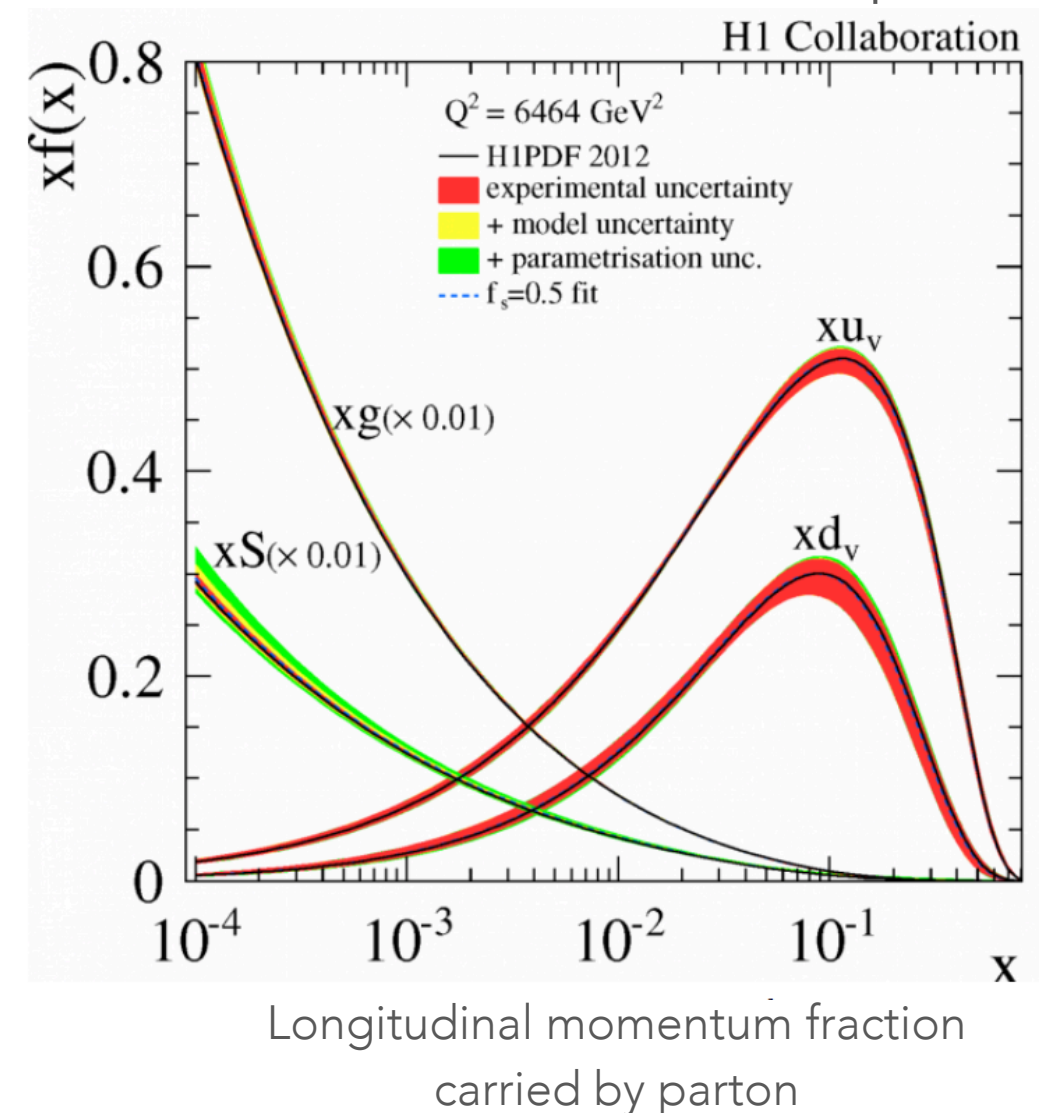


# The gluon structure of the nucleon

Gluons offer a new window on proton/nuclear structure

- Past 60+ years: detailed view of quark structure of nucleons
- Gluon structure also important
  - Unpolarised gluon parton distribution function dominant at small longitudinal momentum fraction
- Other aspects of gluon structure relatively unexplored

Parton distributions in the proton



# The gluon structure of the nucleon

How much do gluons contribute to the proton's

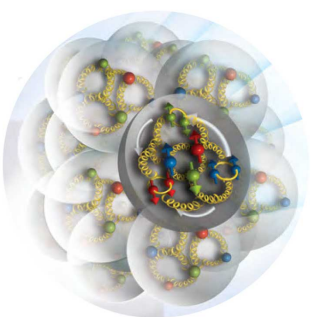
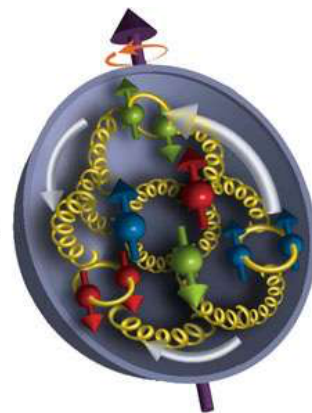
- Momentum
- Spin
- Mass
- D-term

What is the 3D gluon distribution of a proton

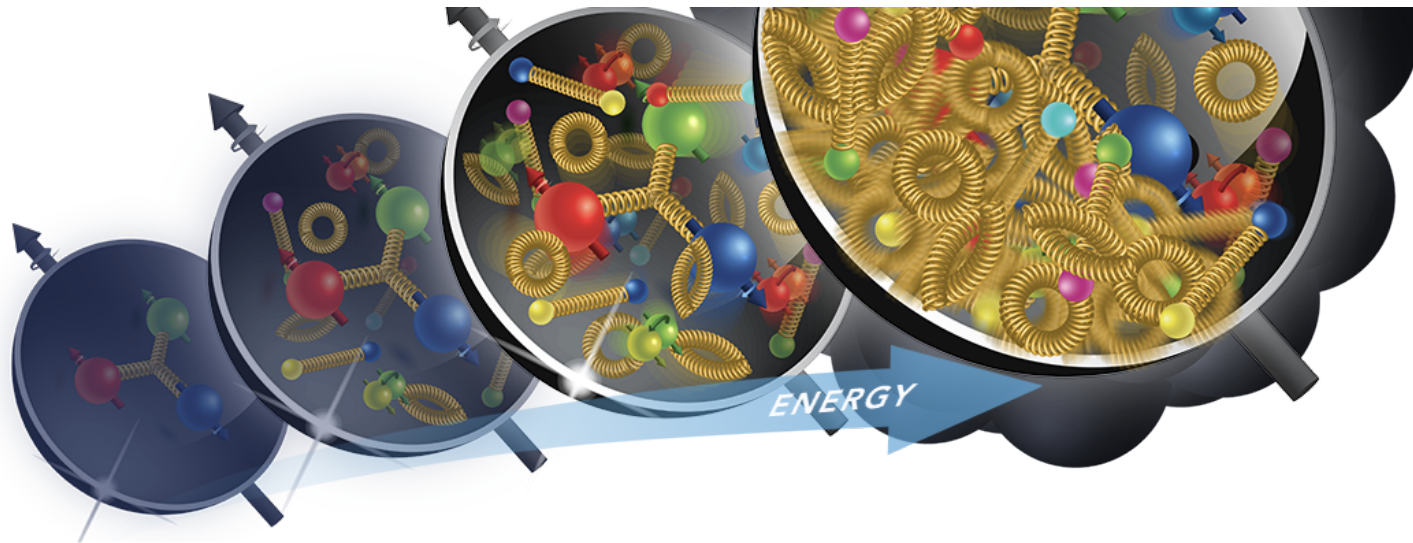
- Encoded in gluon distribution functions and form factors
- Pressure distribution

How is the gluon structure of a proton modified in a nucleus

- Gluon 'EMC' effect
- Exotic glue



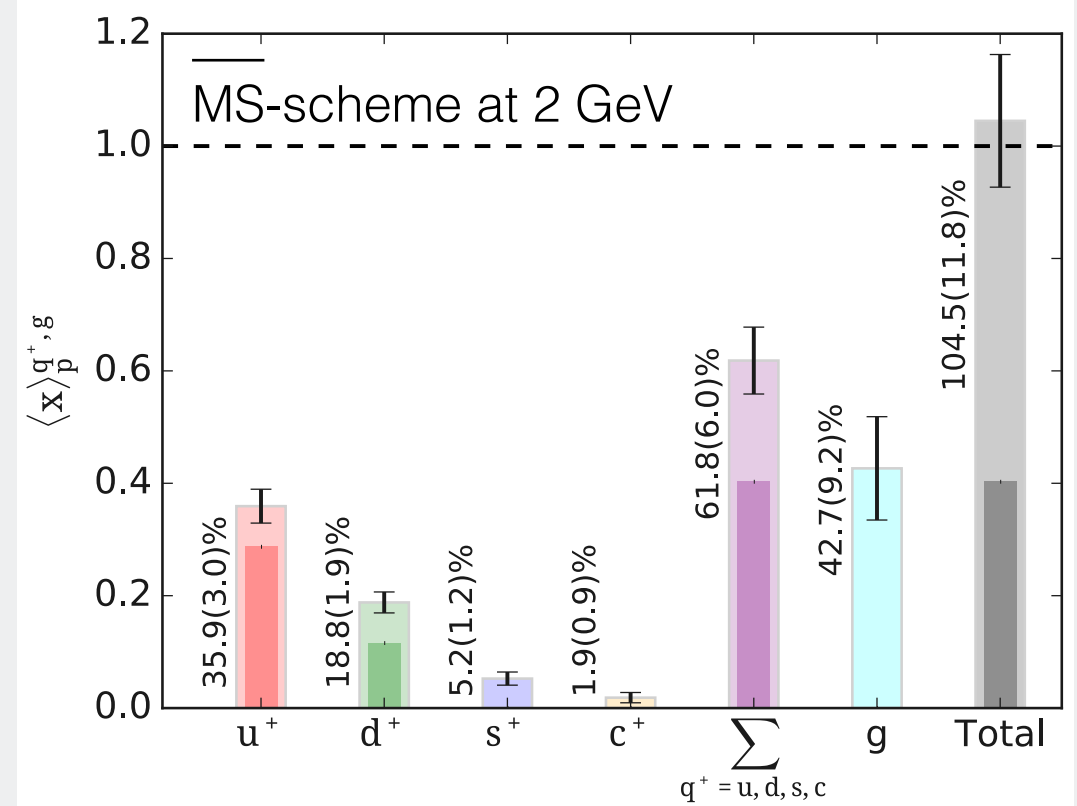
# Decomposition of proton momentum



State-of-the-art calculations for simple quantities have:

- Fully-controlled systematic uncertainties competitive with or better than experiment for some quantities
- Separate contributions from
  - Strangeness and light flavours
  - Gluons

2020 Highlight: All terms of proton momentum decomposition calculated



[C. Alexandrou et al., PRD 101 (2020)]

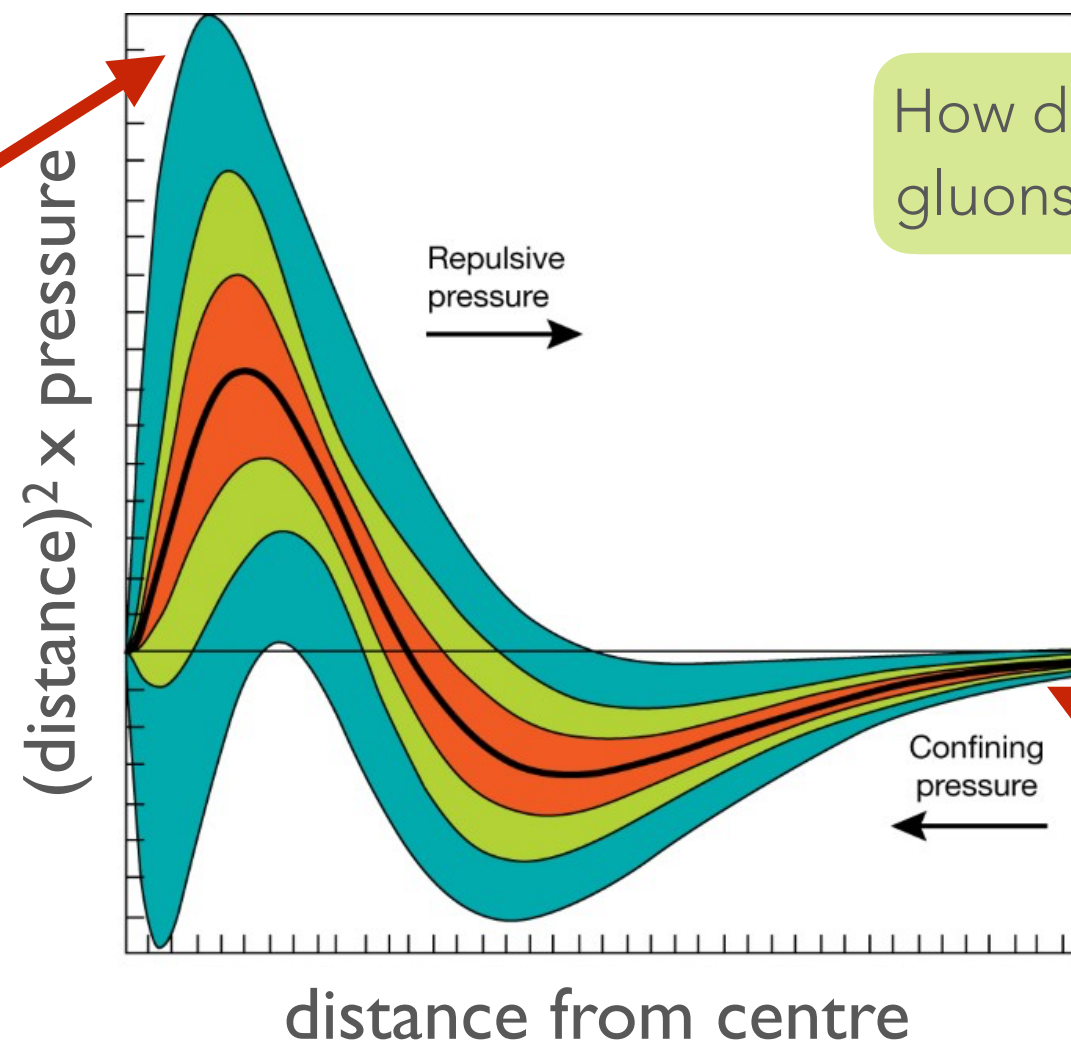
# Pressure inside the proton

Quarks are confined inside protons

➔ compressive forces must cancel expanding force for stability

What is the peak pressure?

Bike tyre  $\sim 10^5$  Pa  
Ocean floor  $\sim 10^8$  Pa  
Neutron star  $\sim 10^{34}$  Pa  
Proton  $\sim ??$



# Pressure inside the proton

## Recent experimental extraction of proton's pressure distribution

[V. D. Burkert et al, Nature 557, 396 (2018)]

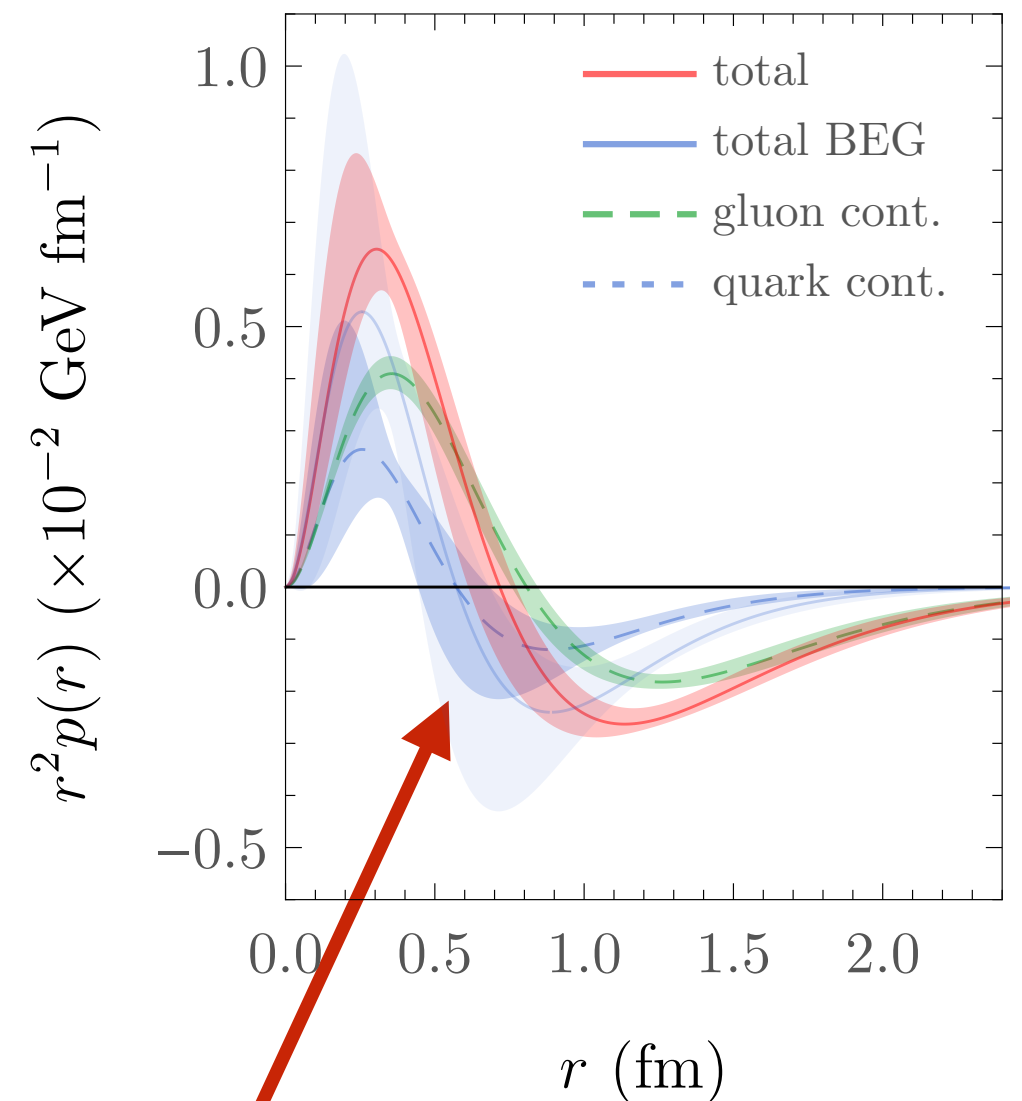
- Peak pressure near the centre  $\sim 10^{35}$  Pascal  $>$  pressure estimated for neutron stars
- Experiment only sensitive to quark contributions, not to gluons

### EXPERIMENT + LQCD

[Shanahan, Detmold PRL 122 072003 (2019)]

- First complete pressure determination
- Gluons change the picture!
- Explore kinematics needed in future experiments

## Radial pressure distribution



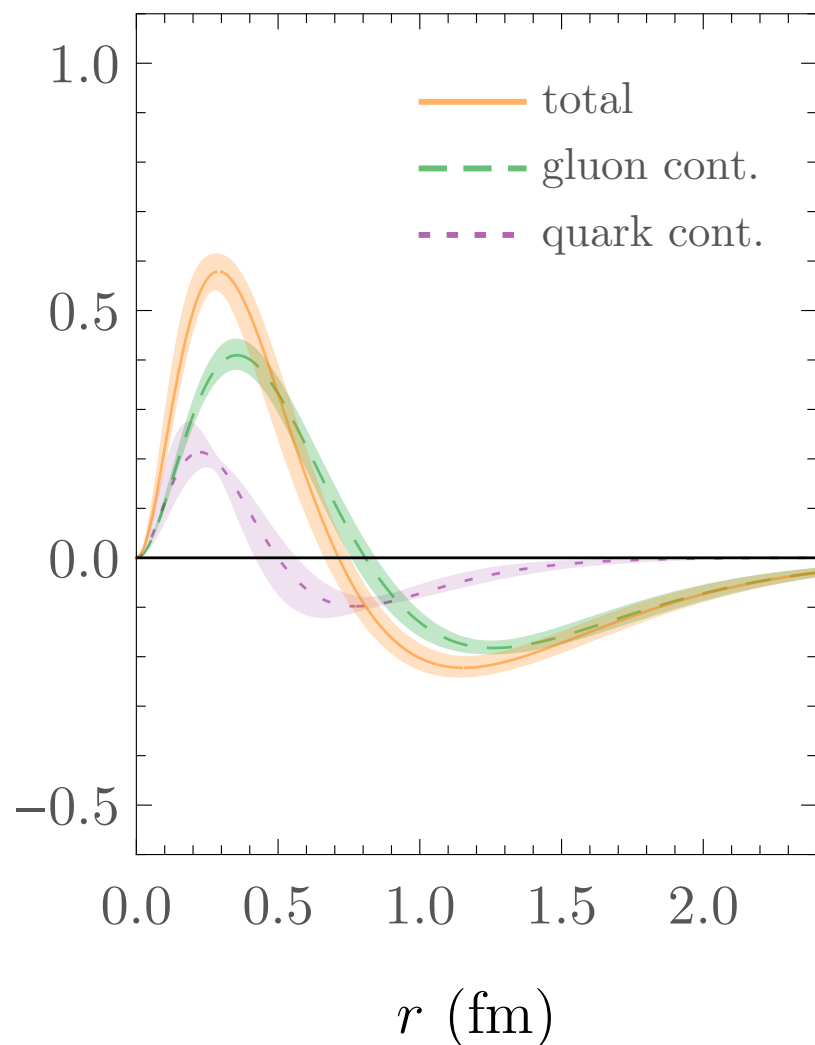
Blue  $\rightarrow$  Red:  
gluon contribution shifts peaks, extends region over which pressure is non-zero



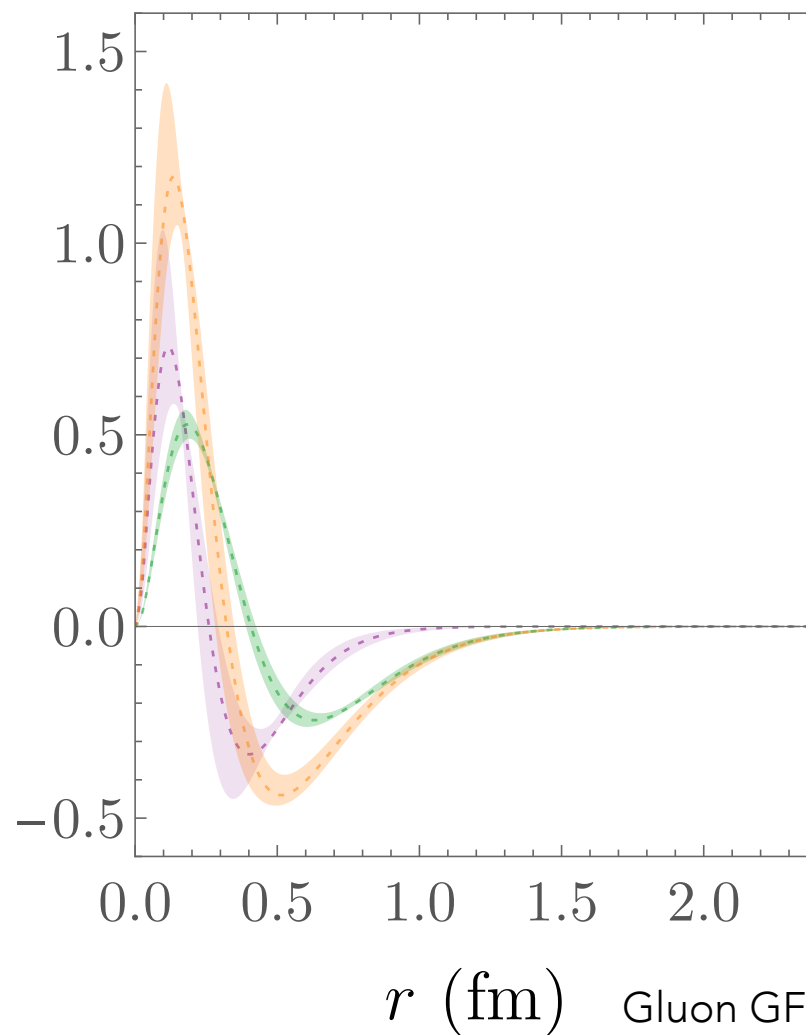
# Next: pressure distribution in nuclei

Pressure in light nuclei  
c.f. pressure in the nucleon?

## Nucleon



## Pion



Pion & Nucleon  
quark and gluon  
momentum fractions  
consistent within  
uncertainties, but  
very different  
pressure  
distributions

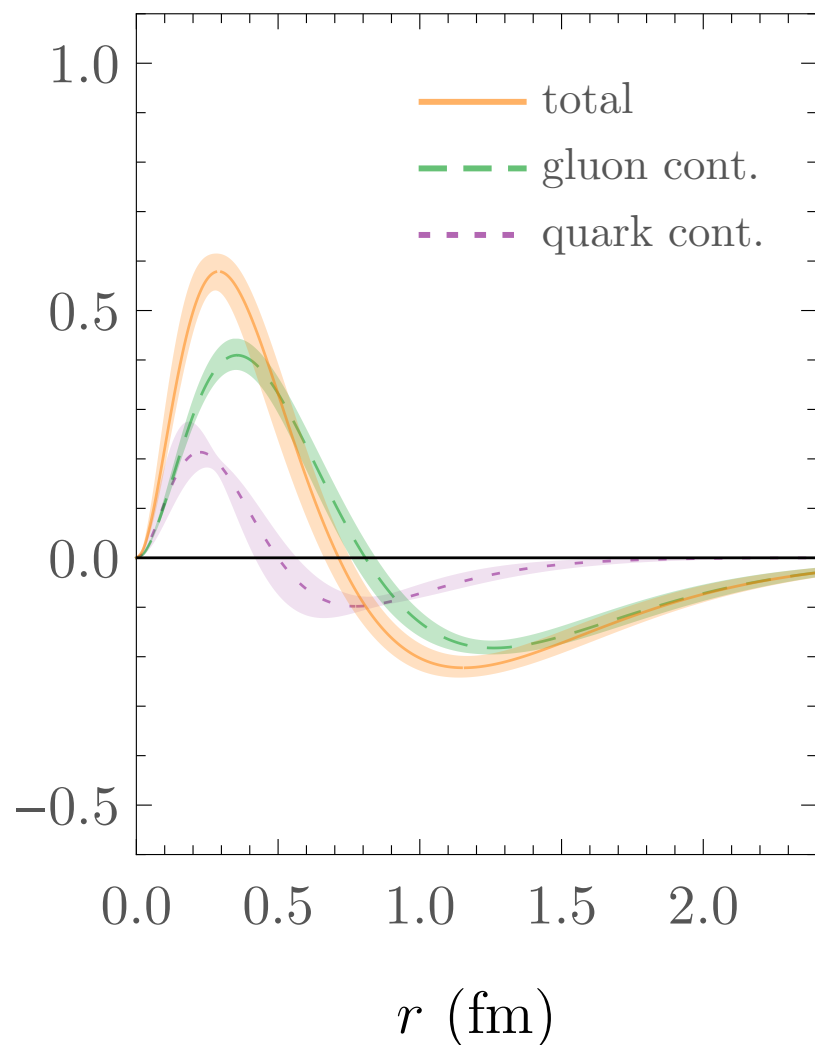
Gluon GFFs: [Shanahan, Detmold, PRD 99, 014511](#)

Quark GFFs: [P. Hägler et al. \(LHPC\), PRD77, 094502 \(2008\)](#)

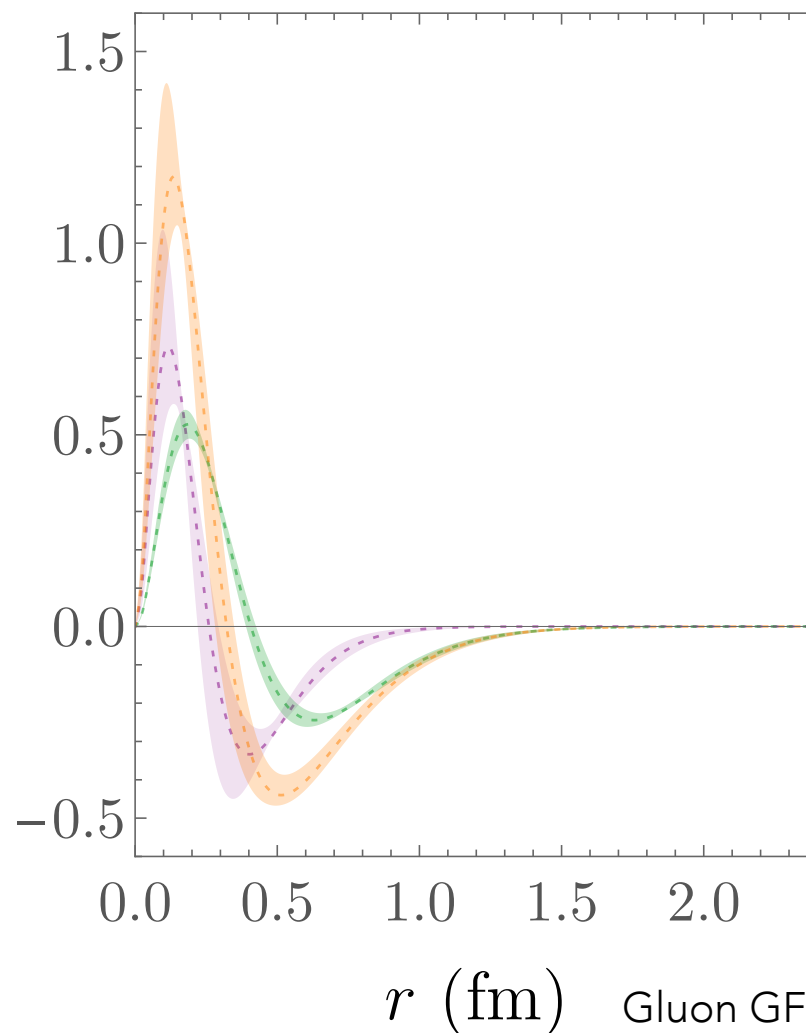
# Next: pressure distribution in nuclei

Pressure in light nuclei  
c.f. pressure in the nucleon?

### Nucleon



### Pion



### Light nuclei

?

**coming soon**

Insights into equation of state of neutron stars?

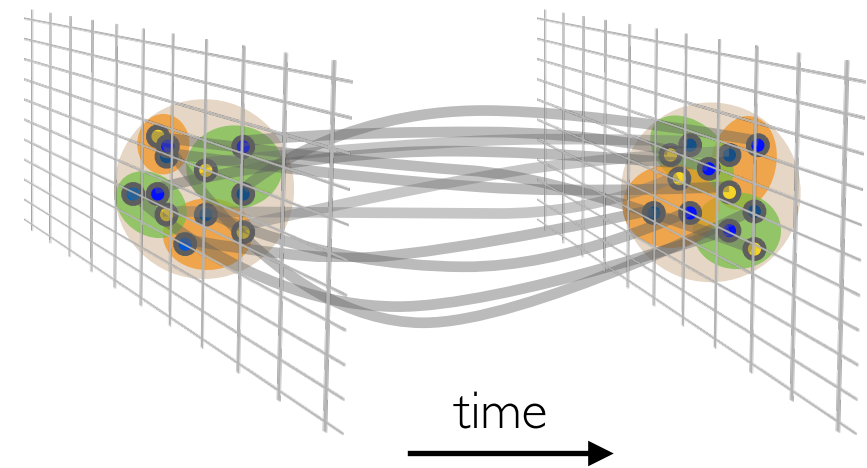
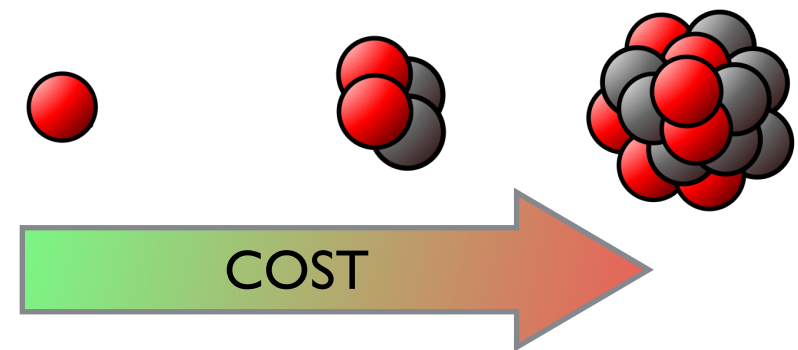
Gluon GFFs: [Shanahan, Detmold, PRD 99, 014511](#)

Quark GFFs: [P. Hägler et al. \(LHPC\), PRD77, 094502 \(2008\)](#)

# Nuclear physics from lattice QCD

Nuclei on the lattice are  
**HARD**

- **Noise:**  
Statistical uncertainty grows exponentially with number of nucleons
- **Complexity:**  
Number of contractions grows factorially



Calculations possible for  $A < 5$

# Nuclear physics from lattice QCD

- Nuclear physics from lattice QCD Collaboration



- Nuclei with  $A < 5$  unphysical quark masses
- First calculation of spectrum of light nuclei in 2013

## Other lattice studies of nuclei:

**PACS-CS** e.g., Yamazaki et al, Phys.Rev.D 92 (2015);

**Callatt** e.g., E Berkowitz et al, Phys.Lett.B 765 (2017) 285

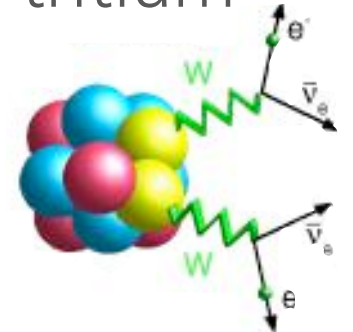
**Mainz** e.g., A. Francis et al, Phys.Rev.D 99 (2019)

**HALQCD** e.g., Ishii et al, Phys.Rev.Lett. 99 (2007) (potential approach)

## Recent highlights

- Proton-proton fusion and tritium  $\beta$ -decay

[PRL 119, 062002 (2017)]



- Double  $\beta$ -decay

[PRL 119, 062003 (2017), PRD 96, 054505 (2017)]

- Gluon structure of light nuclei

[PRD 96 094512 (2017)]

- Scalar, axial, tensor MEs

[PRL 120 152002 (2018)]

- Baryon-baryon interactions, including QED

[2003.12130, 2009.12357]

- EMC-type effects in light nuclei

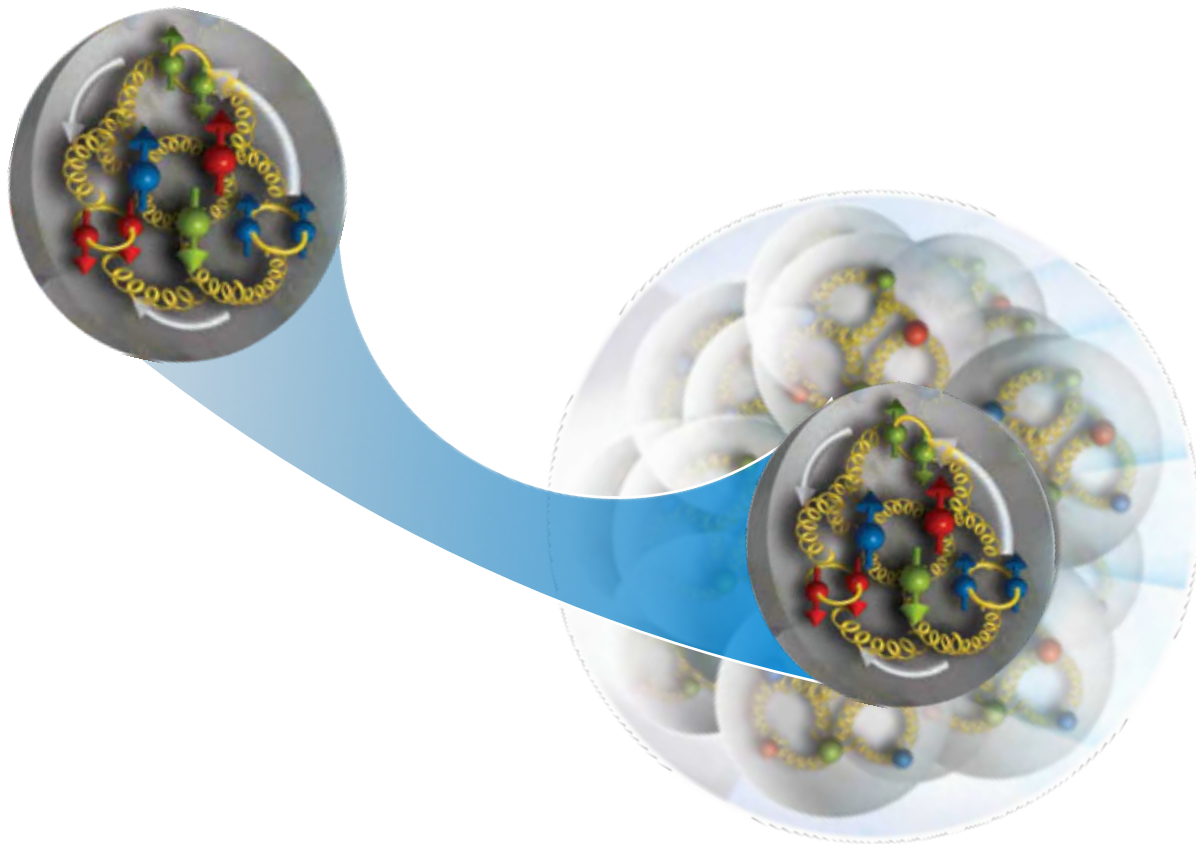
[2009.05522]



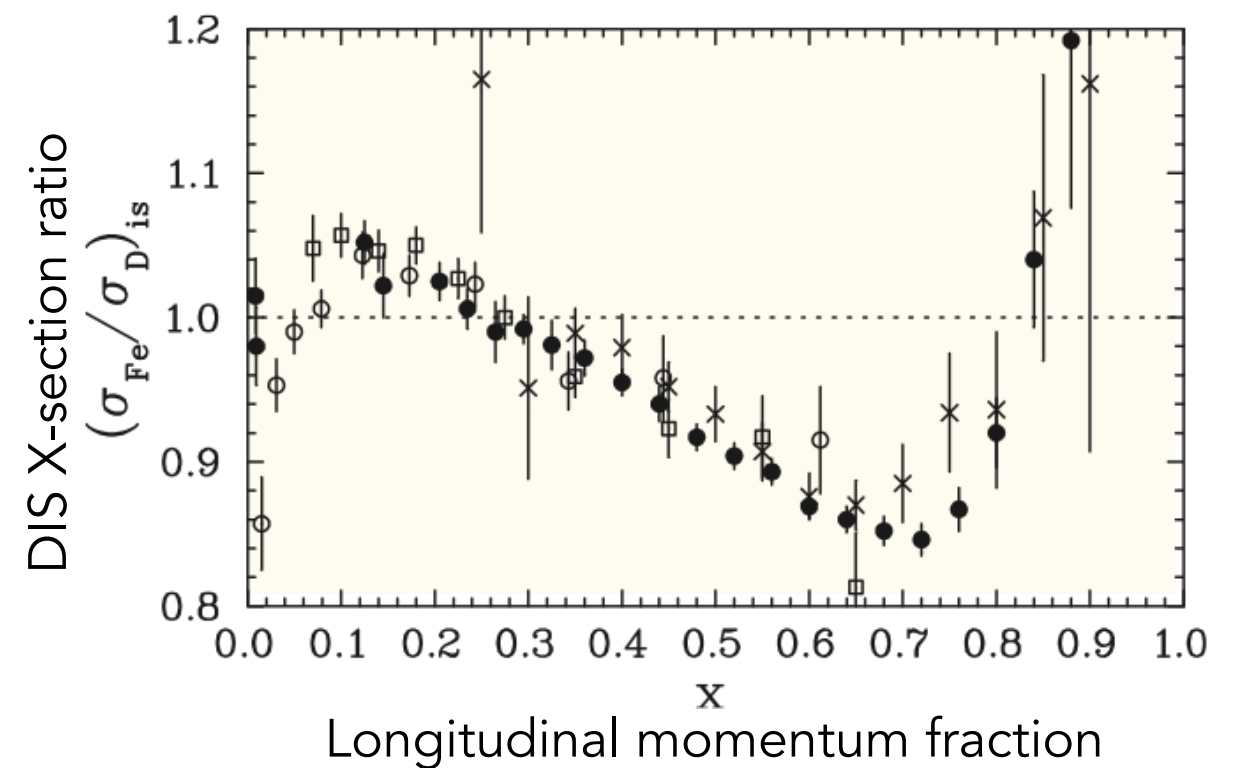
# The structure of matter

Understanding the quark and gluon structure of matter

How is the partonic structure of nucleons modified in nuclei?



Encoded in EMC-type effects

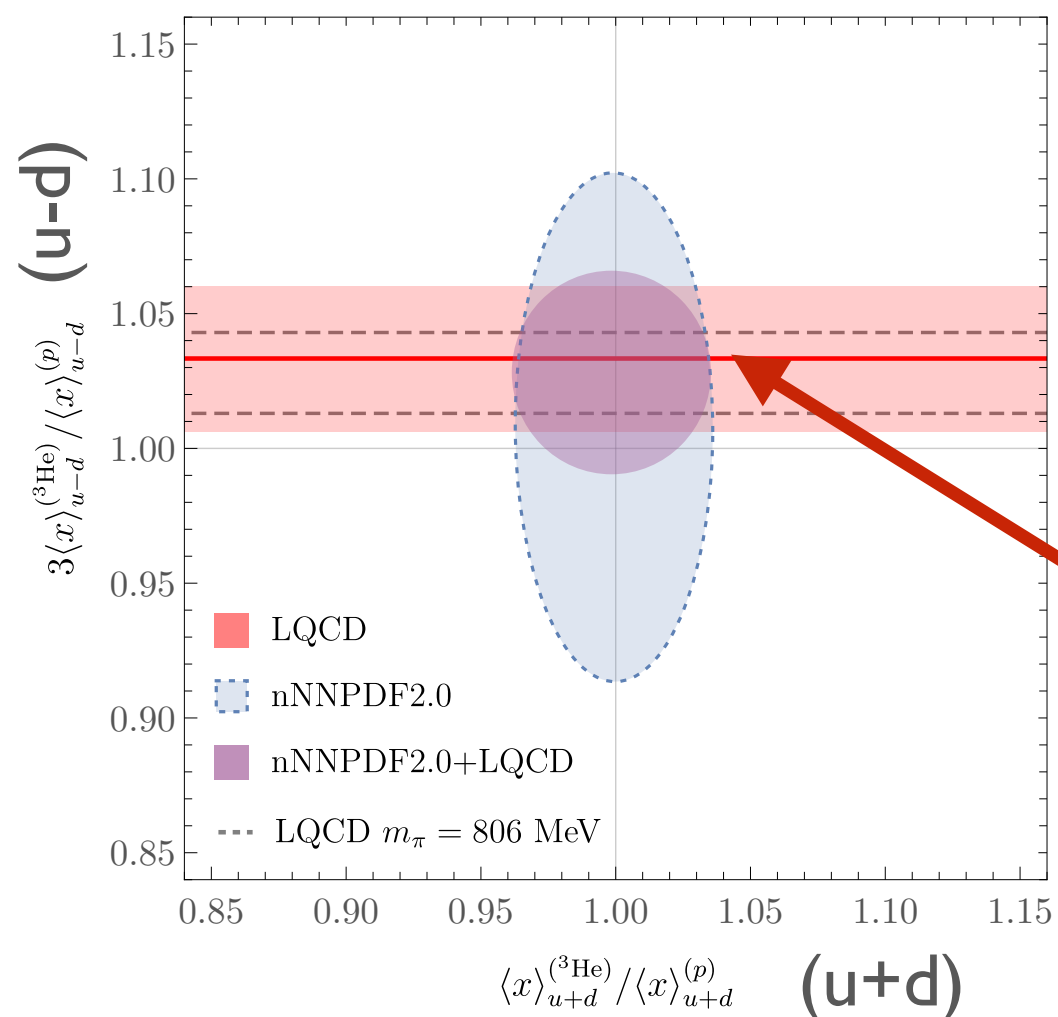


(EMC: Aubert et al., 1983)

# Momentum fraction of ${}^3\text{He}$

Study nuclear effects in the breakdown of momentum carried by quarks in nuclei

## Ratio of ${}^3\text{He}$ to proton momentum fractions



- Match isovector (u-d quark combination) momentum fraction to low-energy constants of effective field theory, extrapolate to physical quark masses
- Include into nNNPDF global fits of experimental lepton-nucleus scattering data

Blue  $\rightarrow$  Purple:  
Improvement using theory constraints

[NPLQCD 2009.05522 (2020)]

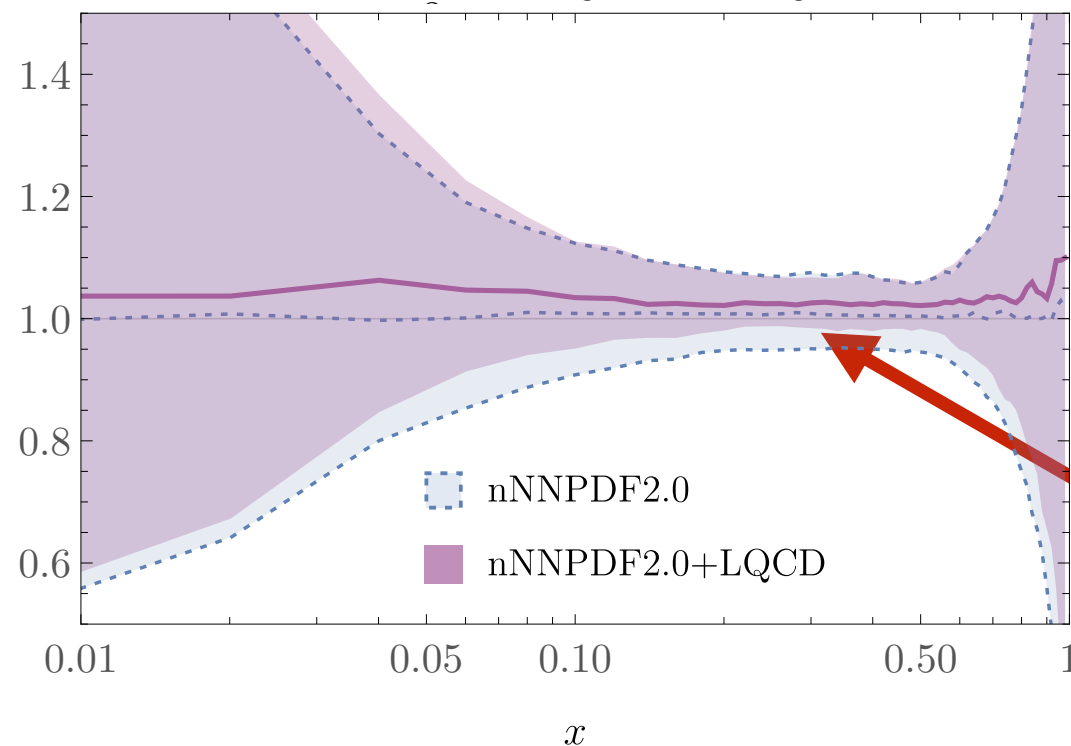
Phiala Shanahan, MIT

# Momentum fraction of $^3\text{He}$

Study nuclear effects in the breakdown of momentum carried by quarks in nuclei

Ratio of  $^3\text{He}$  to proton parton distributions

$$R^{(^3\text{He})}(x) = 3q_3^{(^3\text{He})}(x)/q_3^{(p)}(x)$$



- Match isovector (u-d quark combination) momentum fraction to low-energy constants of effective field theory, extrapolate to physical quark masses
- Include into nNNPDF global fits of experimental lepton-nucleus scattering data

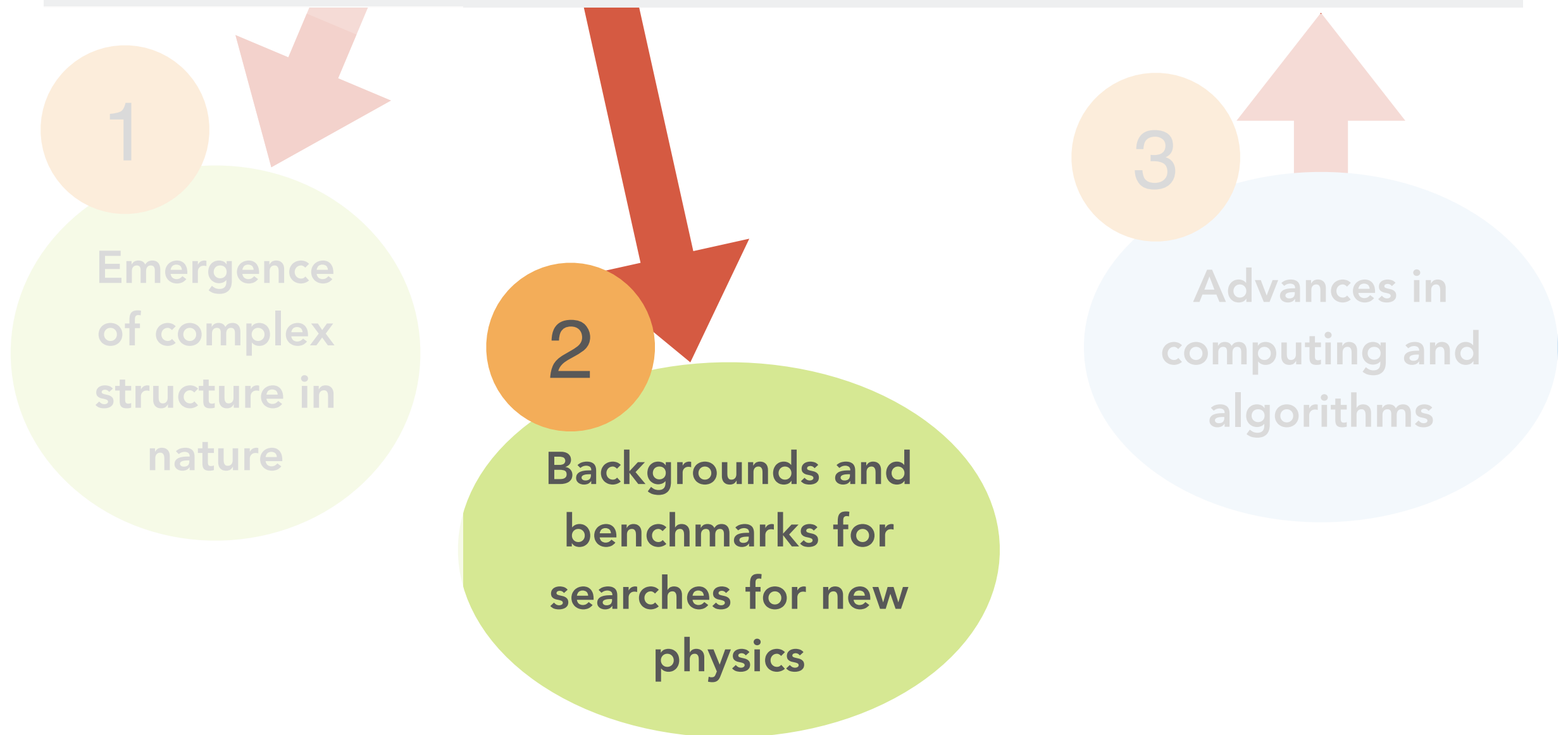
Blue  $\rightarrow$  Purple:  
Improvement using theory constraints

[NPLQCD 2009.05522 (2020)]

Phiala Shanahan, MIT

# The structure of matter

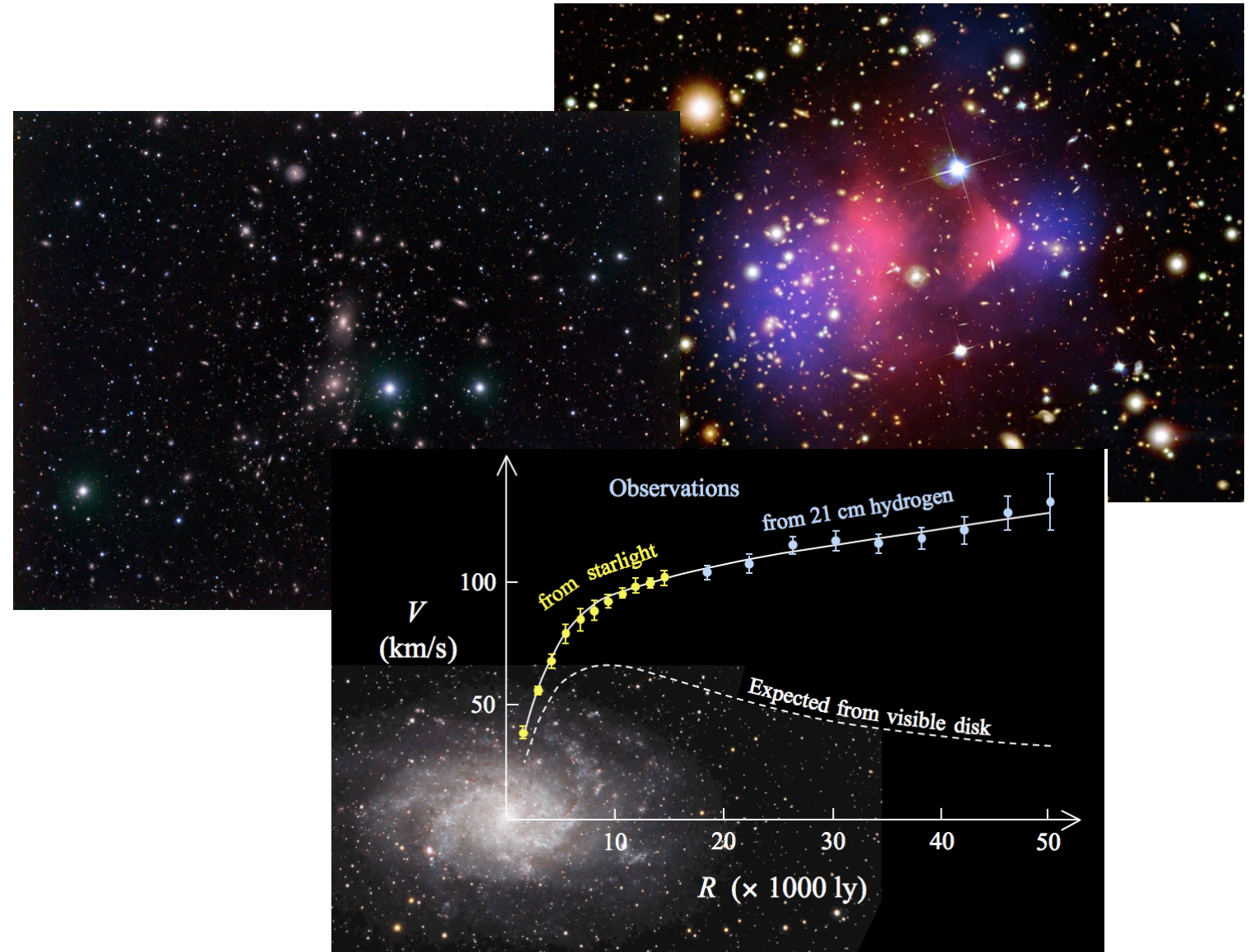
Understanding the quark and gluon structure of matter



# The structure of matter

Compelling evidence for dark matter

- Rotation curves
- Lensing
- Structure formation
- Cosmic microwave background





# Dark matter

5%

## Normal matter

The building blocks of stars and planets (and everything on Earth).

26 %

## Dark matter

Matter that we don't understand.

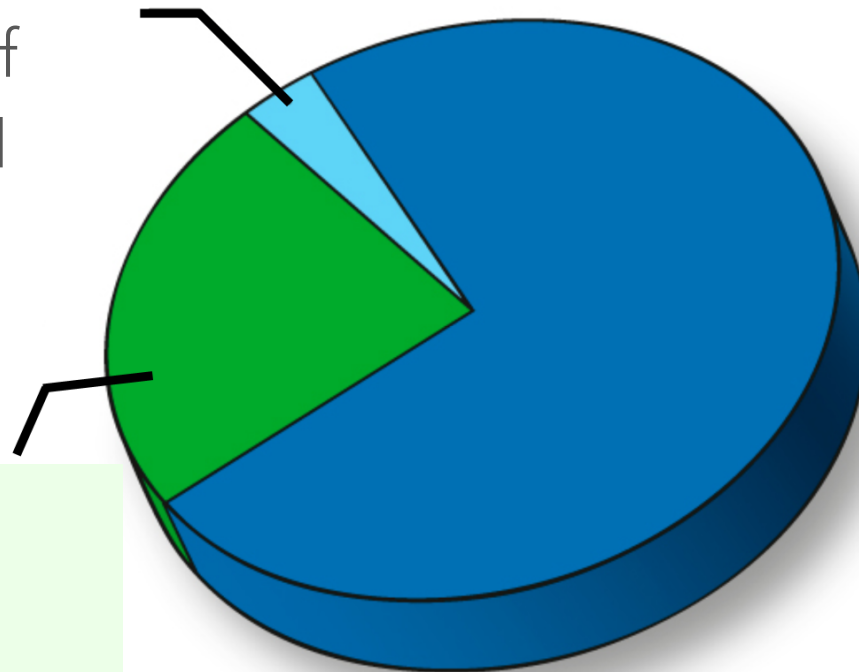
Doesn't emit or absorb light. Interacts through gravity.

69 %

## Dark energy

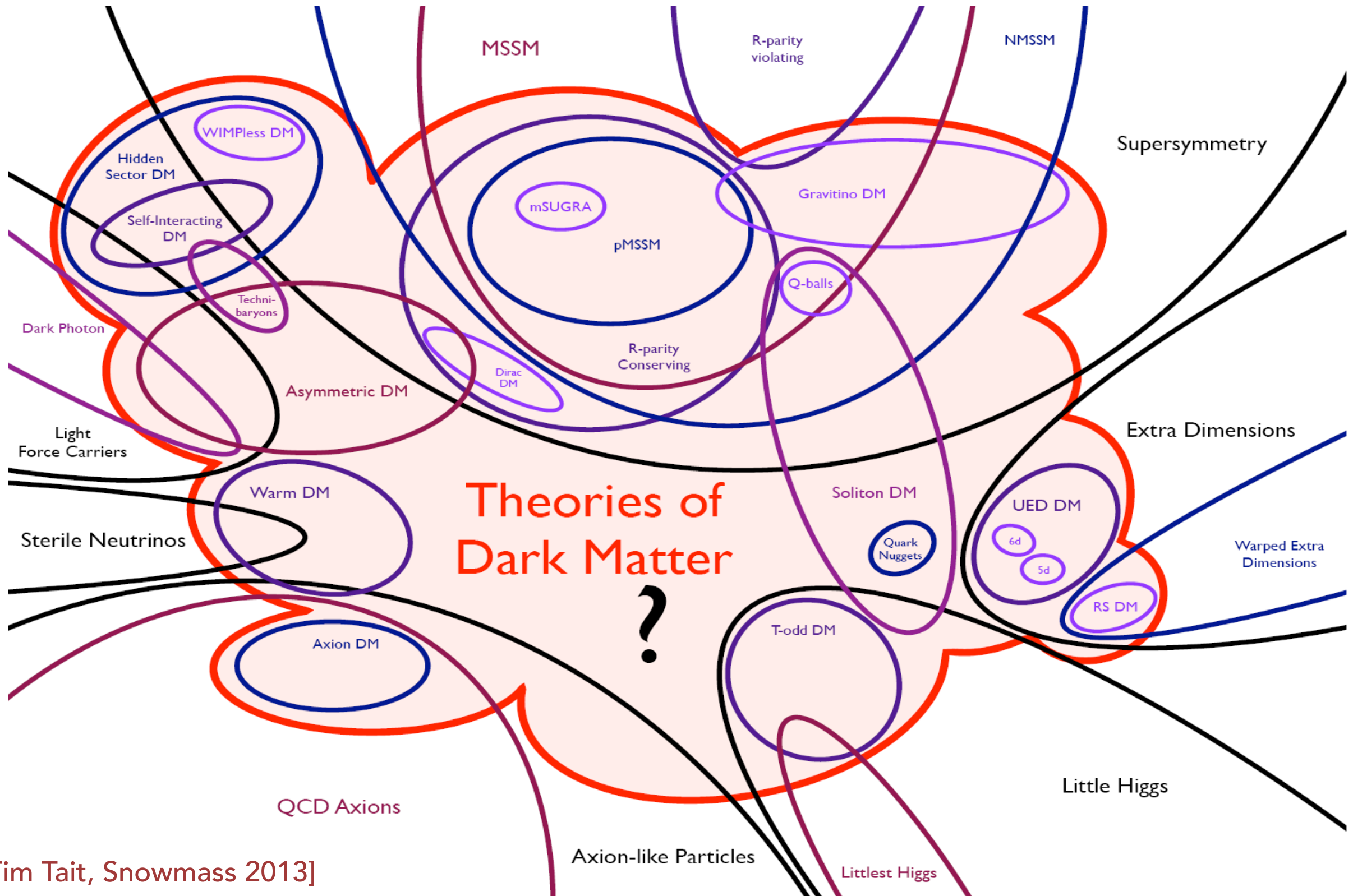
Something that we don't understand... but it's not matter.

Responsible for the accelerating expansion of the universe



Many models for dark matter!  
**BUT few constraints**

# Dark matter



[Tim Tait, Snowmass 2013]

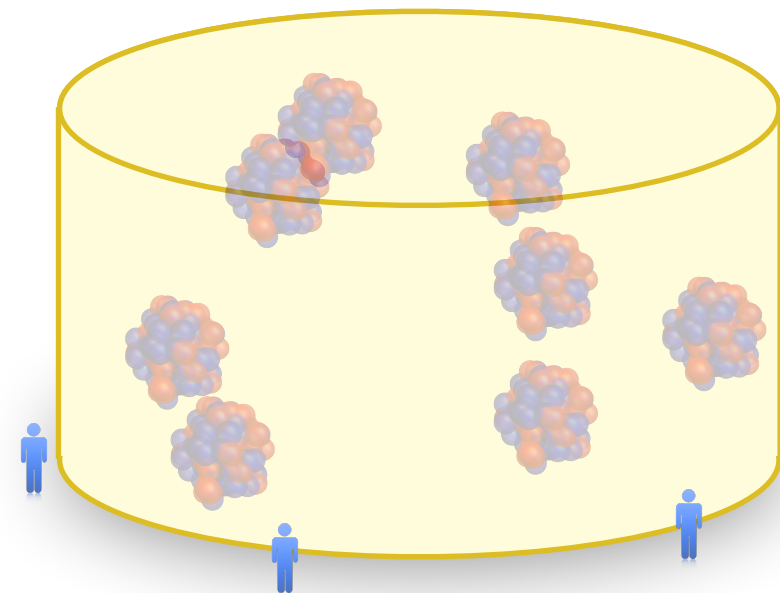
# Dark matter

How do we find dark matter?

- Dark (does not interact with light)
- Interacts through gravity

**WIMP**  
Weakly-interacting  
massive particles

Direct detection  
Wait for DM to hit us

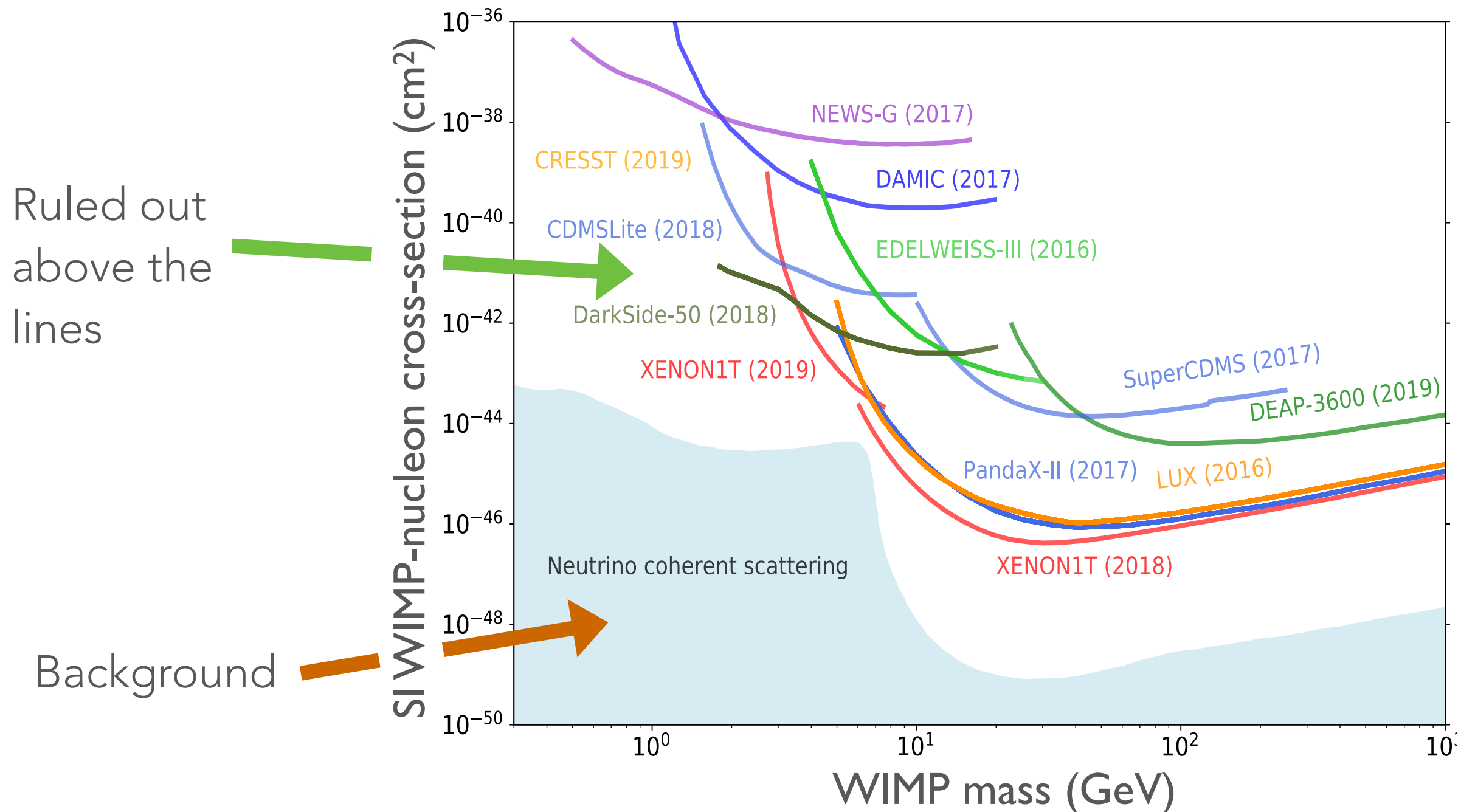


Detection rate depends on

- Dark matter properties
- Probability for interaction with nucleus

# Dark matter direct detection

Limits on WIMP-nucleon interaction from direct detection experiments



# Dark matter direct detection

Direct detection experiments use nuclear targets e.g., Xenon

Determine interaction cross-section (with nucleus) for a given dark matter model

- Born approximation – interacts with a single nucleon

$$\sigma \sim |A \langle N | DM | N \rangle|^2$$

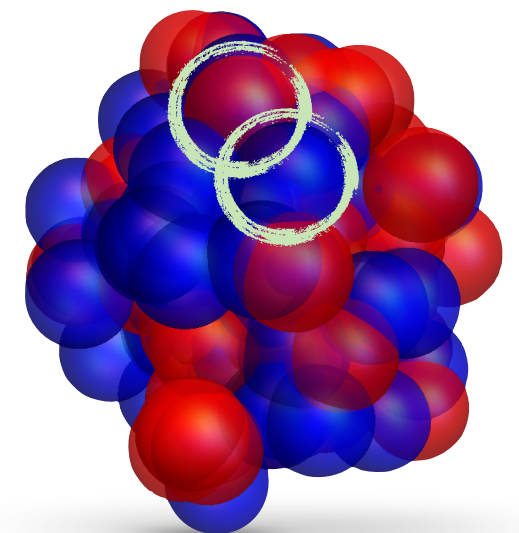
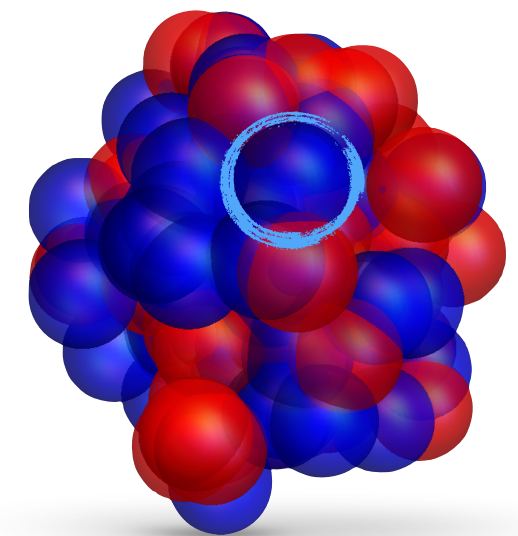
known from lattice QCD

- Interacts non-trivially with multiple nucleons

$$\sigma \sim |A \langle N | DM | N \rangle + \alpha \langle NN | DM | NN \rangle + \dots|^2$$

Not known!

Second term may be significant!

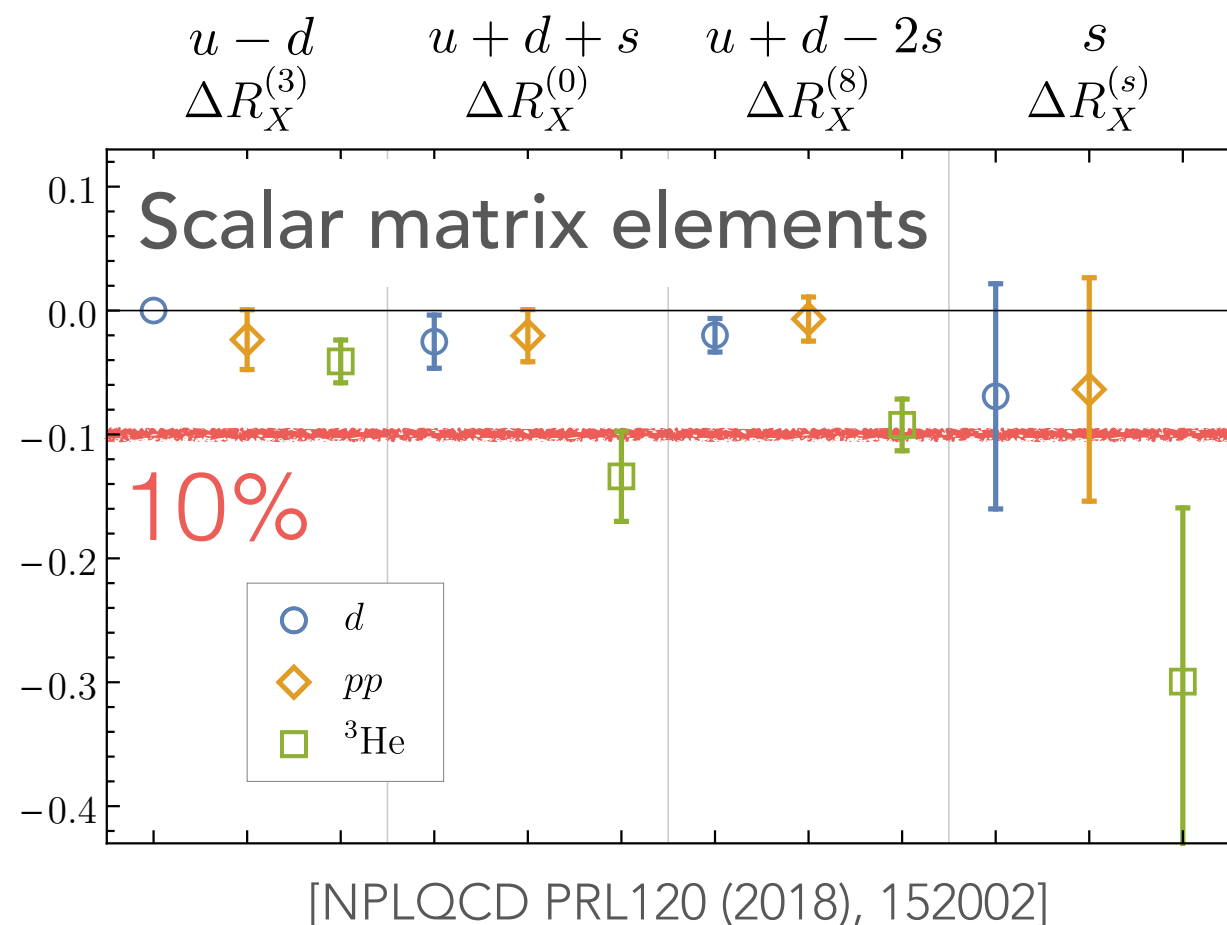




# Dark matter direct detection

Spin-independent scattering of WIMP candidates is governed by scalar matrix elements

- Lattice QCD calculation with  $m_\pi \sim 800$  MeV shows 10% nuclear effects in  ${}^3\text{He}$   $\longrightarrow$  potentially very significant effects in e.g., Xenon
- Same calculation gives axial and tensor nuclear effects around  $\sim 1\%$



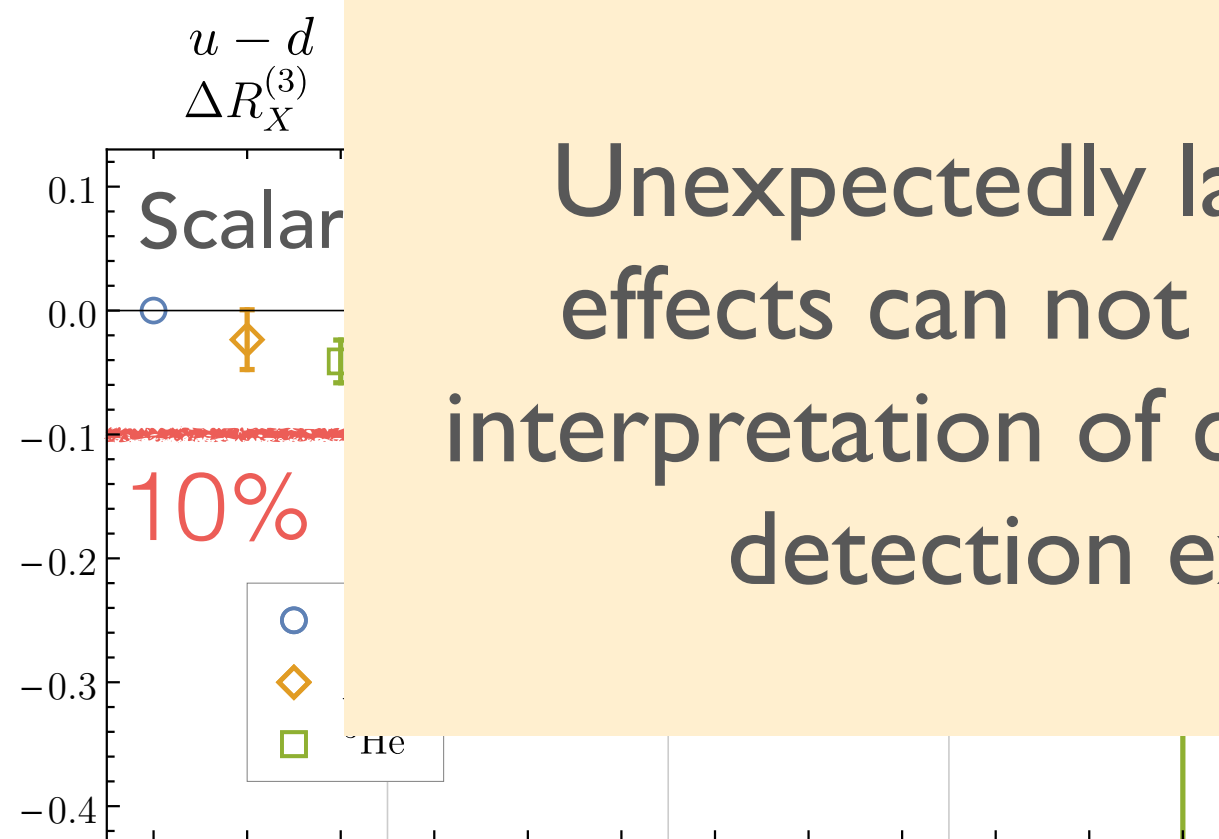
$$\frac{\text{ME}}{A(\text{Nucleon ME})} - 1$$

Isolate "nuclear effect"

# Dark matter direct detection

Spin-independent scattering of WIMP candidates is governed by scalar matrix elements

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[NPLQCD PRL120 (2018), 152002]

Unexpectedly large multi-body effects can not be neglected in interpretation of dark matter direct detection experiments

1

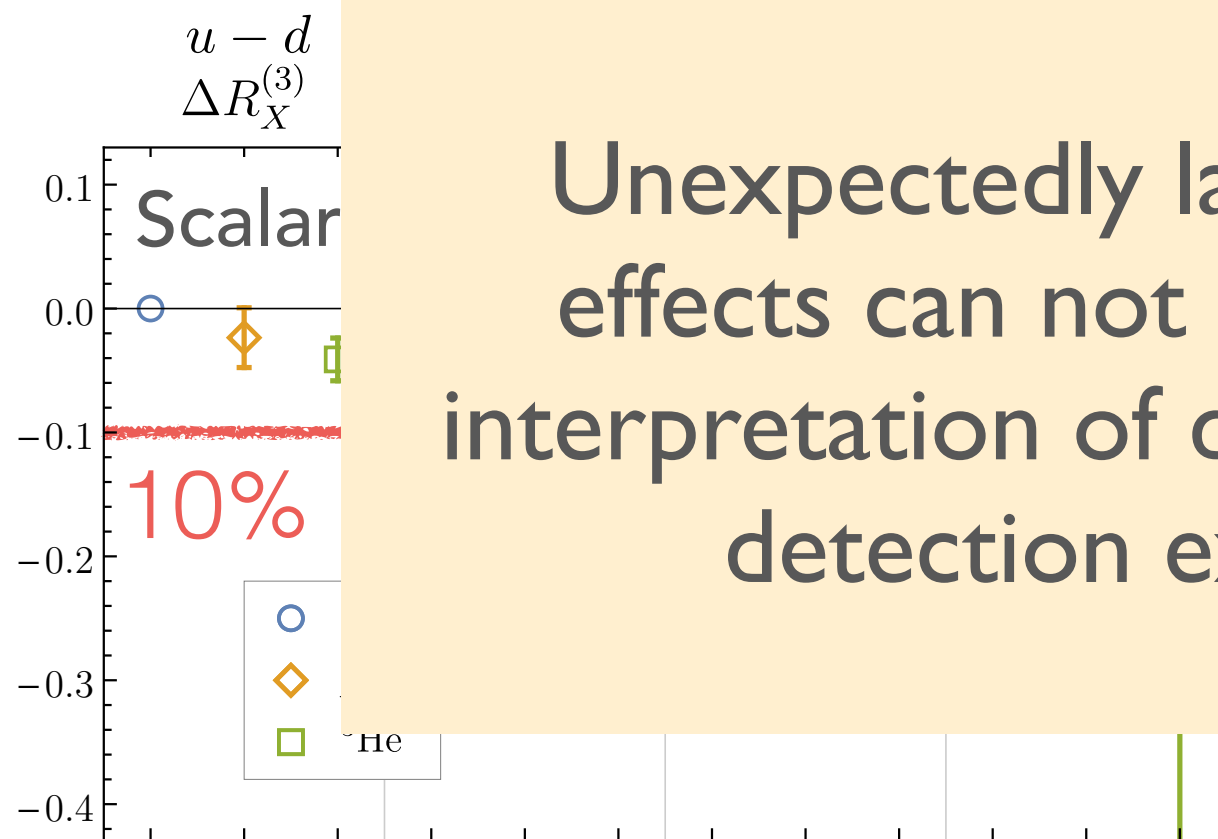
# Dark matter direct detection

Spin-independent scattering of WIMP candidates is governed by scalar matrix elements

- Lattice QCD calculation with  $m_\pi \sim 800$  MeV shows 10% nuclear effects in  ${}^3\text{He}$   $\longrightarrow$  potentially very significant effects in Xenon
- Same calculation gives axial and tensor

Calculation at  $\sim$ physical quark masses in progress Stay tuned!

Unexpectedly large nuclear effects can not be neglected in interpretation of dark matter direct detection experiments

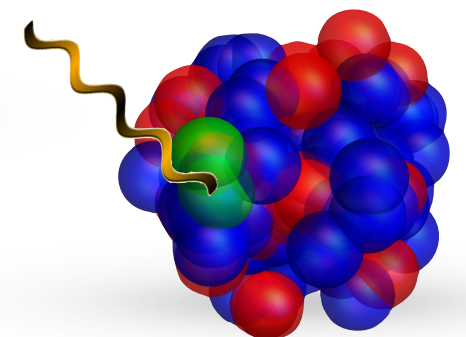
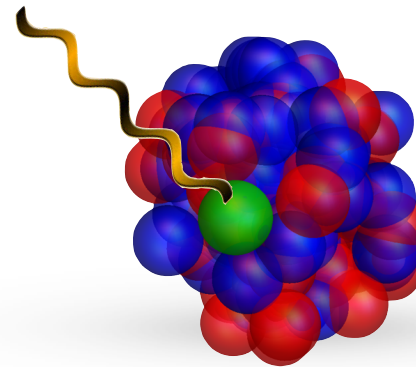


[NPLQCD PRL120 (2018), 152002]

# Larger nuclei

What about larger  
(phenomenologically-relevant) nuclei?

- Nuclear effective field theory:
  - 1-body currents are dominant
  - 2-body currents are sub-leading  
*but non-negligible*
- Determine one body contributions from single nucleon
- Determine few-body contributions from  $A=2,3,4\dots$
- Match effective theory and many body methods to lattice results to make predictions for larger nuclei

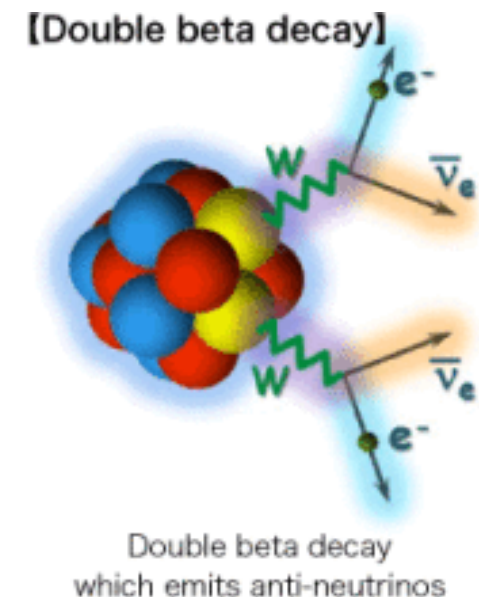


# Pushing the boundaries

Will we ever achieve first-principles nuclear physics beyond  $A=4$ ?

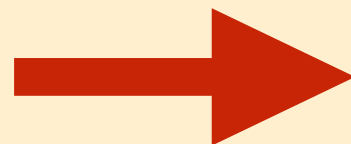
## Interpretation of intensity-frontier experiments

- Axial form factors of Argon  $A=40$   
DUNE long-baseline neutrino expt.
- Double-beta decay rates of Calcium  $A=48$
- Scalar matrix elements in  $A=131$   
XENON1T dark matter direct detection search



How finely tuned is the emergence of nuclear structure in nature?

Exponentially harder  
problem

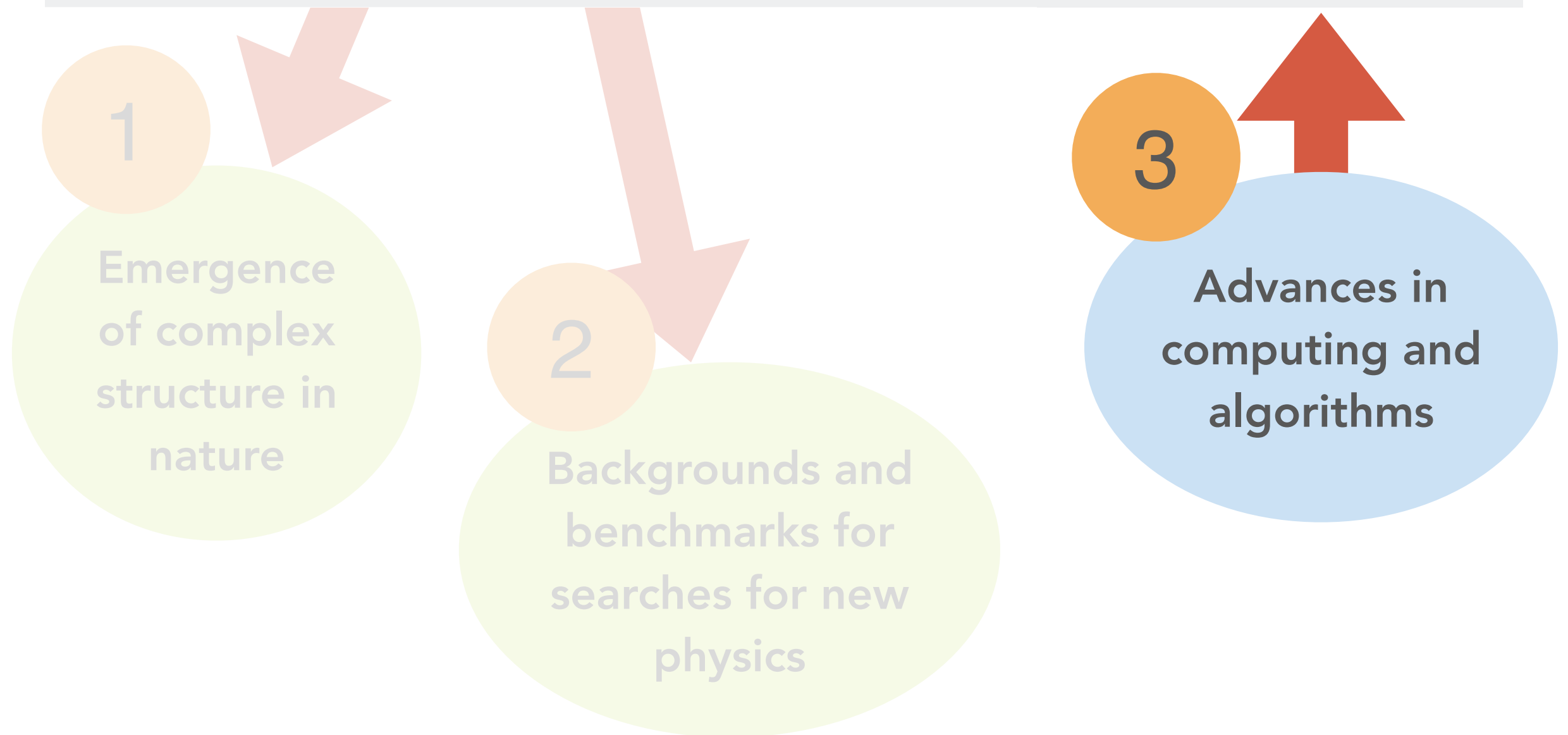


Need exponentially  
improved algorithms



# The structure of matter

Understanding the quark and gluon structure of matter

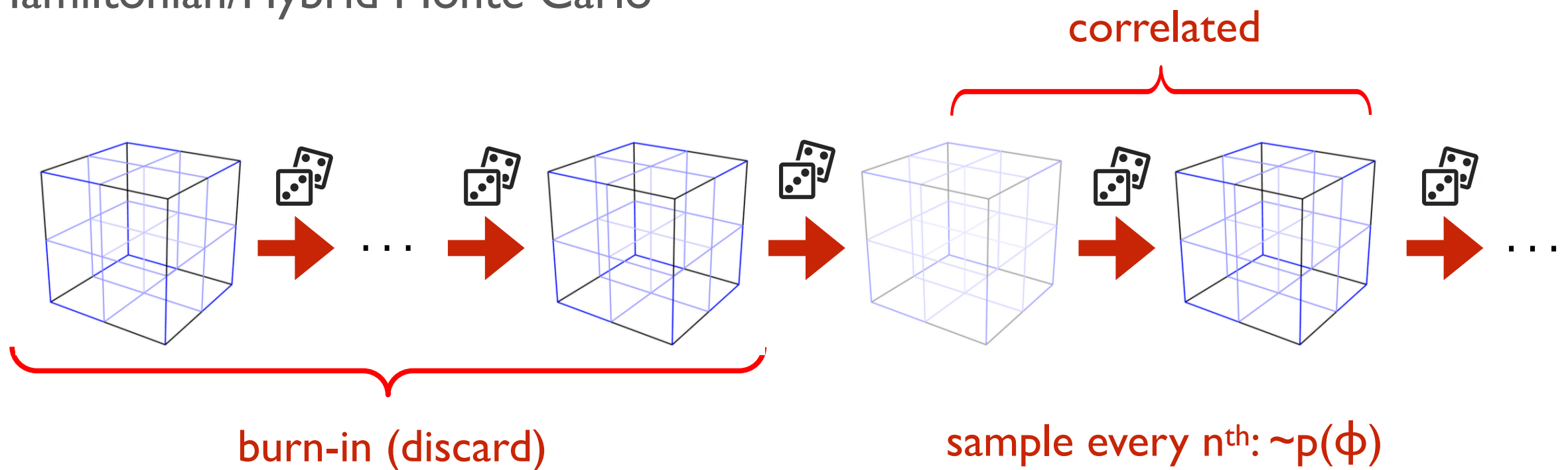


# Generate QCD gauge fields

Generate field configurations  $\phi(x)$  with probability

$$P[\phi(x)] \sim e^{-S[\phi(x)]}$$

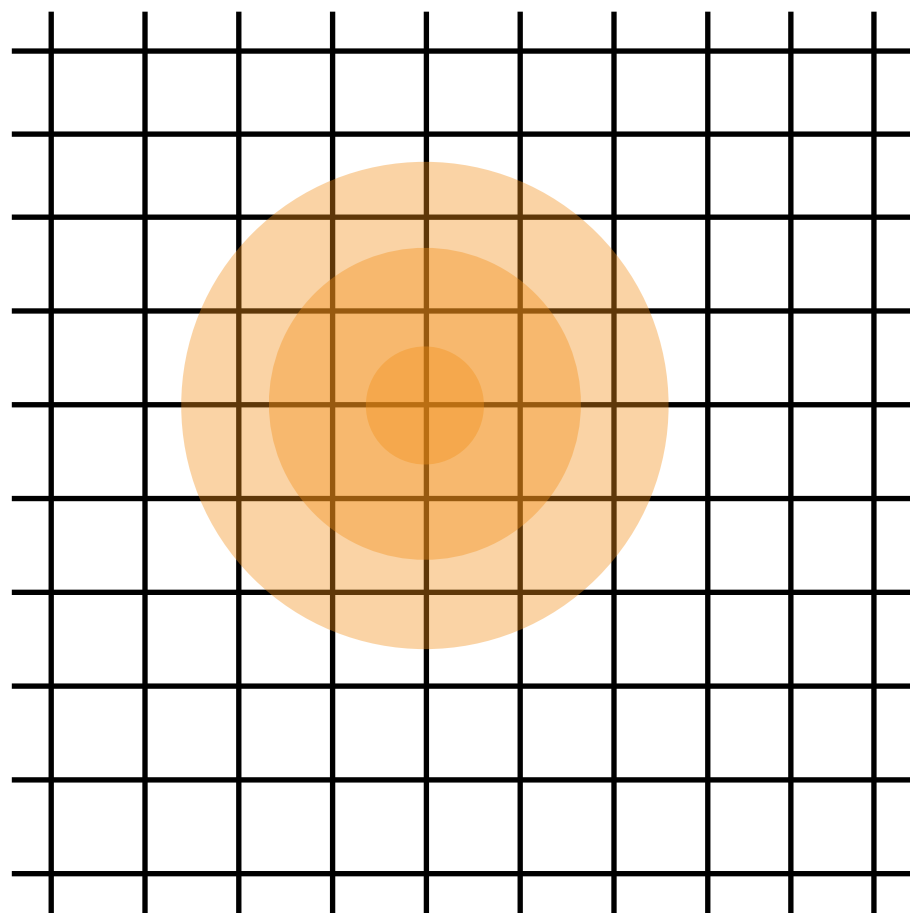
Hamiltonian/Hybrid Monte Carlo



Burn-in time and correlation length dictated by Markov chain 'auto-correlation time': shorter autocorrelation time implies less computational cost

# Generate QCD gauge fields

QCD gauge field configurations sampled via  
Hamiltonian dynamics + Markov Chain Monte Carlo



Updates diffusive

Lattice spacing  0

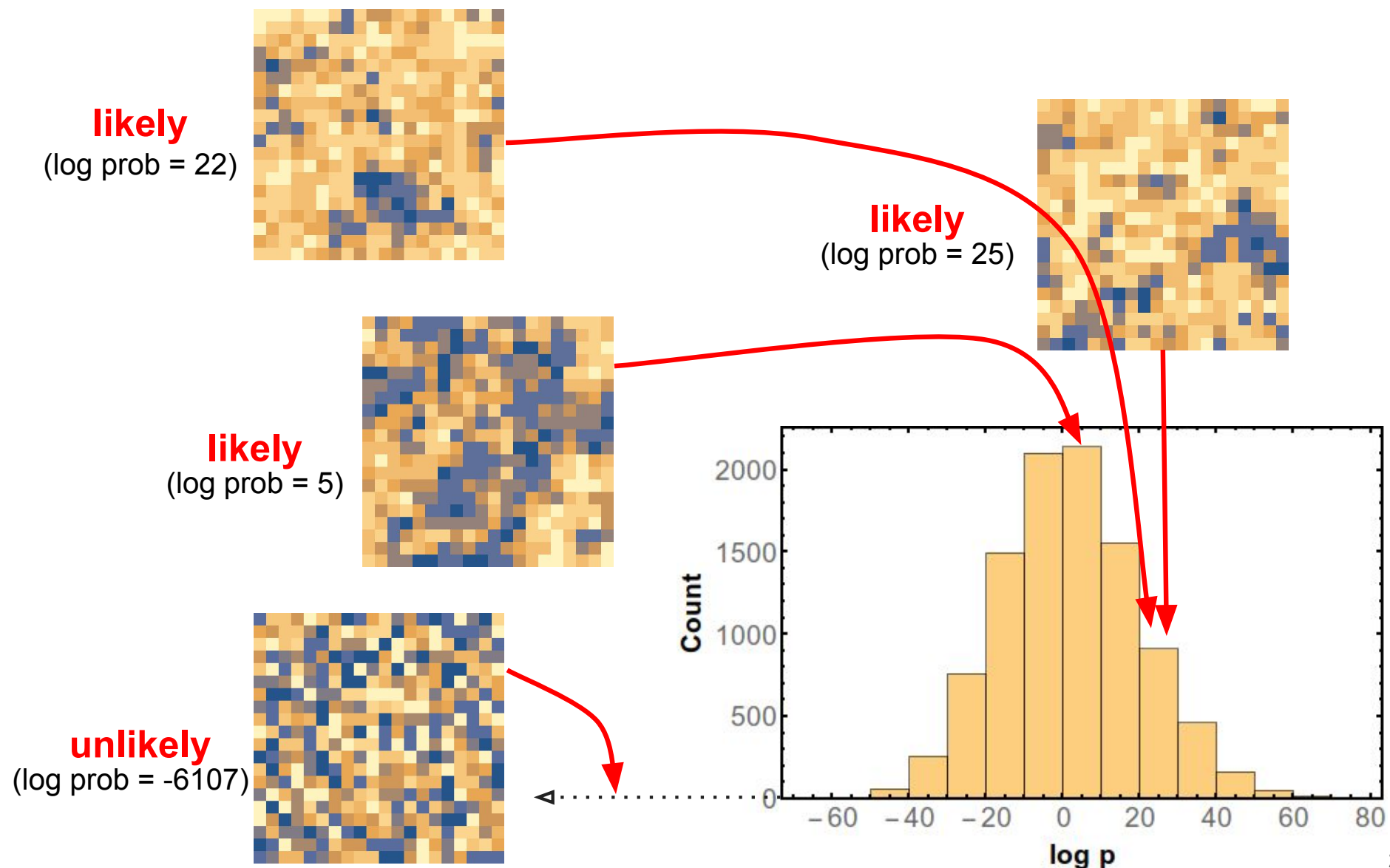
Number of updates  
to change fixed  
physical length scale   $\infty$

“Critical slowing-down”  
of generation of uncorrelated samples

# Generate QCD gauge fields

Generate field configurations  $\phi(x)$  with probability

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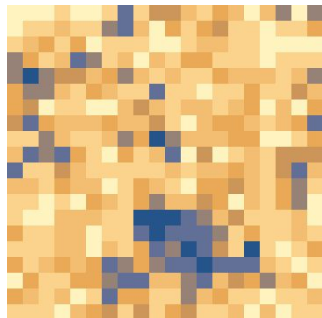
# Generate QCD gauge fields

Generate field configurations  $\phi(x)$  with probability

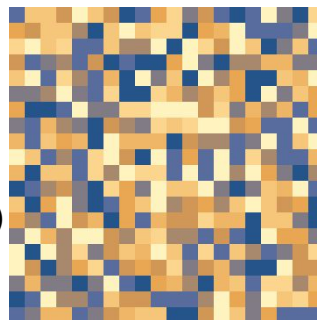
$$P[\phi(x)] \sim e^{-S[\phi(x)]}$$

Parallels with image generation problem

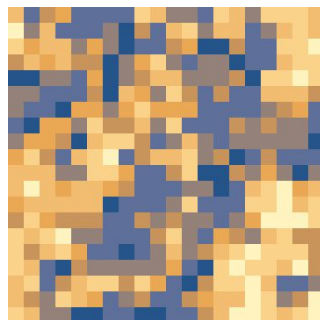
**likely**  
(log prob = 22)



**unlikely**  
(log prob = -6107)



**likely**  
(log prob = 5)



**likely**



[Karras, Lane, Aila / NVIDIA 1812.04948]

**likely**



**unlikely**

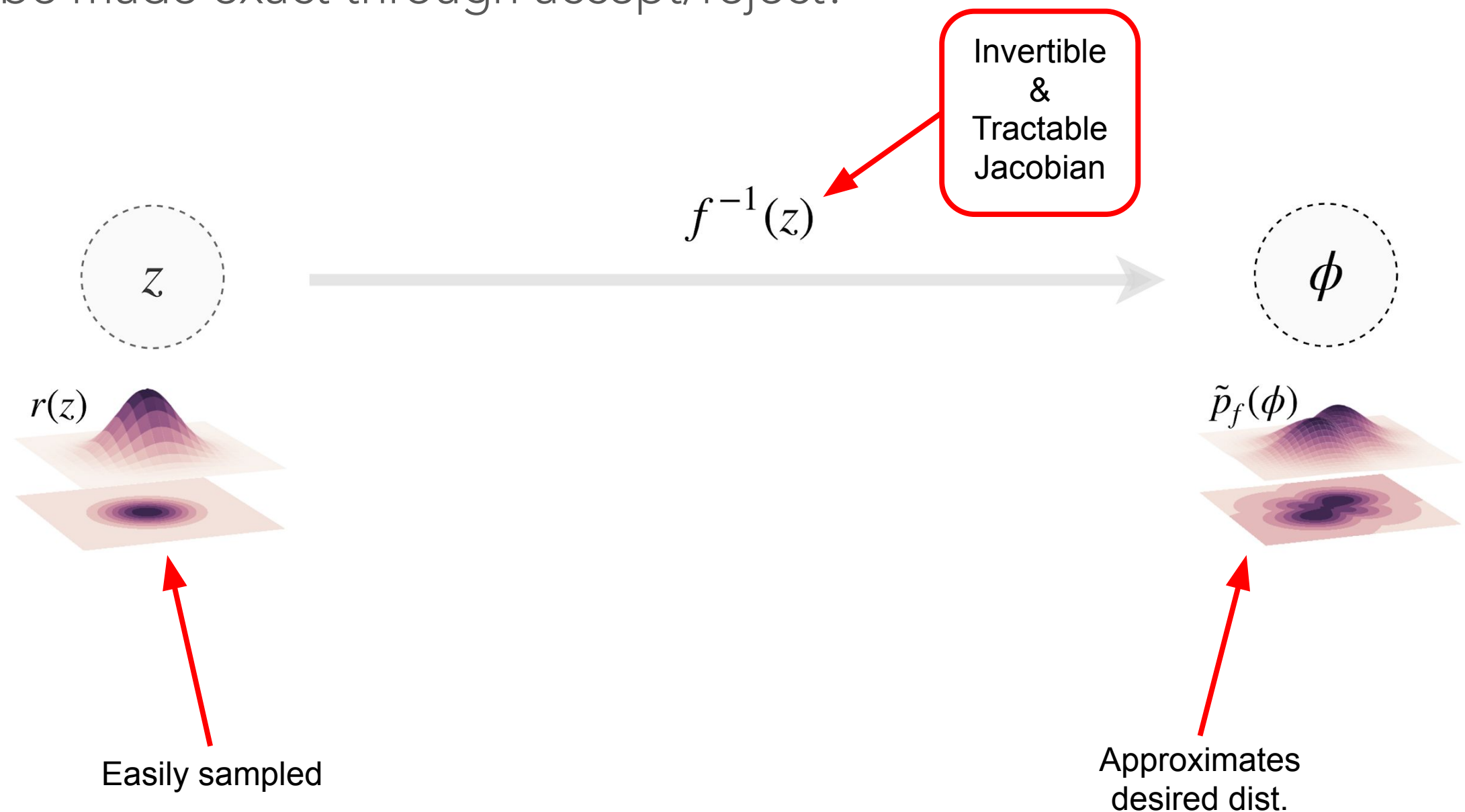




# Generative flow models

Flow-based models learn a change-of-variables that transforms a known distribution to the desired distribution [Rezende & Mohamed 1505.05770]

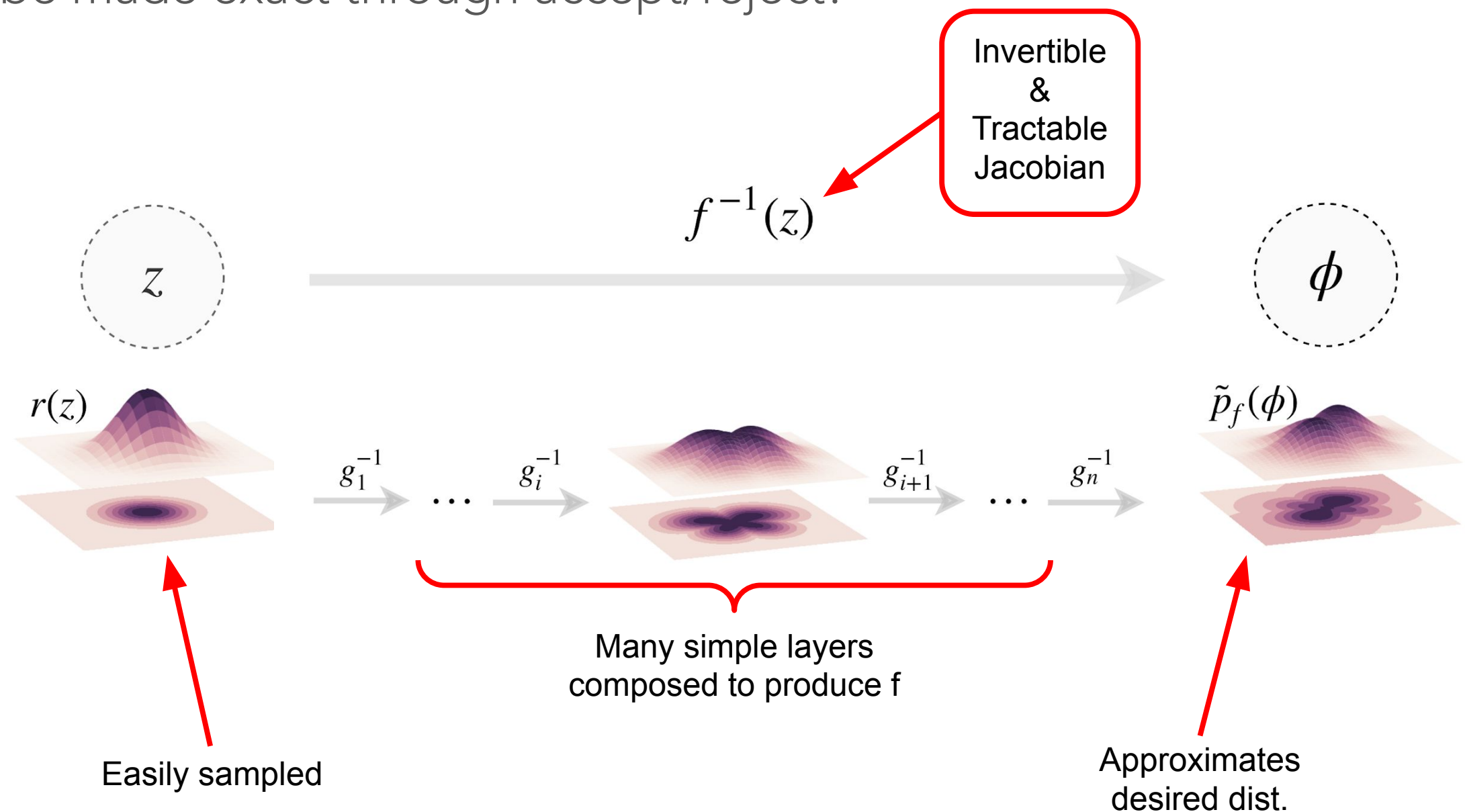
Can be made exact through accept/reject!



# Generative flow models

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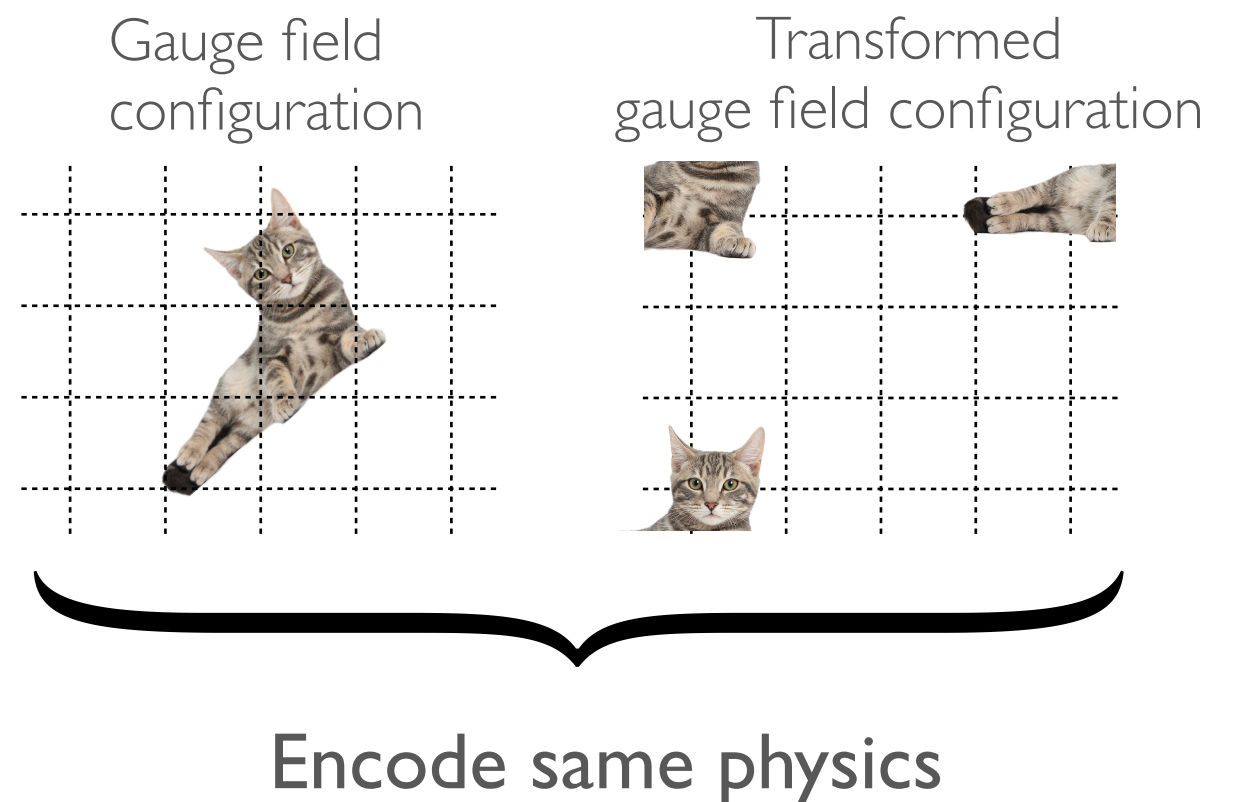
# Machine learning QCD

Ensemble of lattice QCD gauge fields

- $64^3 \times 128 \times 4 \times N_c^2 \times 2 \approx 10^9$  numbers
- $\sim 1000$  samples
- Ensemble of gauge fields has meaning
- Long-distance correlations are important
- Gauge and translation-invariant with periodic boundaries

Physics is invariant under specific field transformations

- Rotation, translation (4D), with boundary conditions



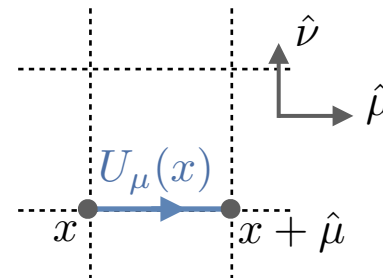
# Machine learning QCD

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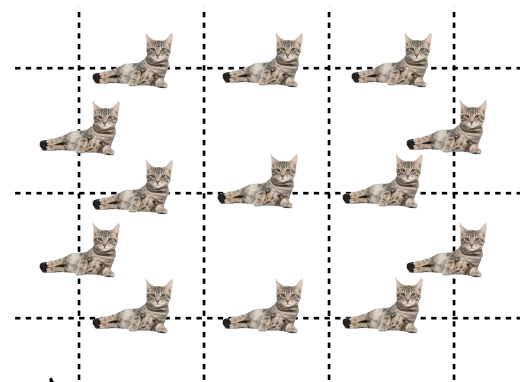
Physics is invariant under specific field transformations

■ Gauge transformation

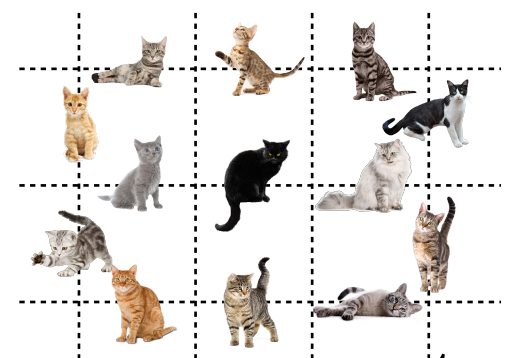

$$U_\mu(x) \rightarrow \Omega(x)U_\mu(x)\Omega^\dagger(x + \hat{\mu})$$

for all  $\Omega(x) \in \text{SU}(3)$

Gauge field configuration



Transformed gauge field configuration



Encode same physics

# Machine learning QCD

## Ensemble of lattice QCD gauge fields

- $64^3 \times 128 \times 4 \times N_c^2 \times 2$   
 $\approx 10^9$  numbers
- $\sim 1000$  samples
- Ensemble of gauge fields has meaning
- Long-distance correlations are important
- Gauge and translation-invariant with periodic boundaries

## CIFAR benchmark image set for machine learning

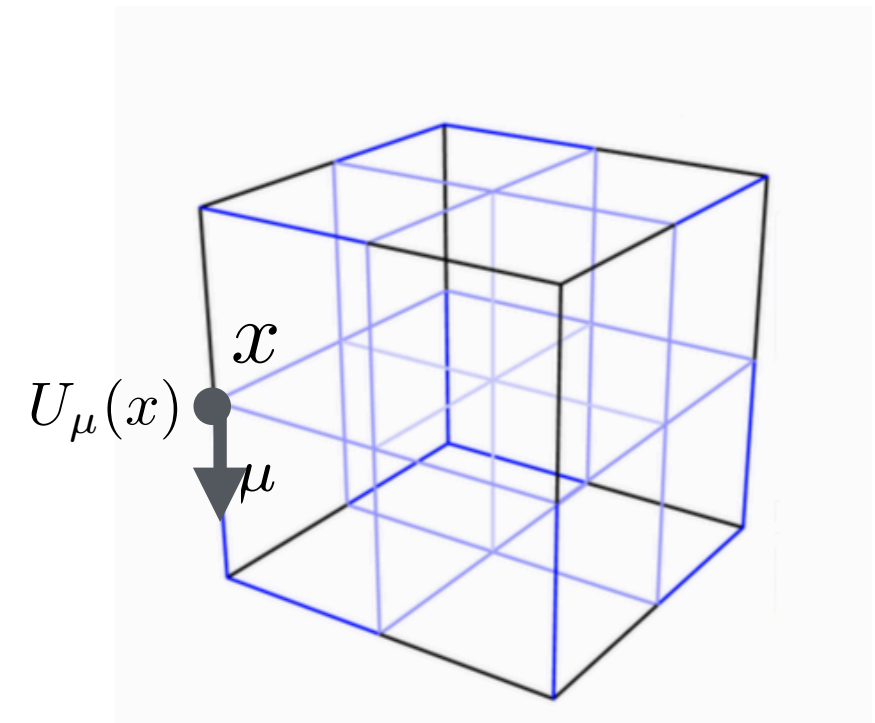
- $32 \times 32$  pixels  $\times$  3 cols  
 $\approx 3000$  numbers
- 60000 samples
- Each image has meaning
- Local structures are important
- Translation-invariance within frame



# Incorporating symmetries

## Gauge field theories

- Field configurations represented by links  $U_\mu(x)$  encoded as matrices
- e.g., for Quantum Chromodynamics, SU(3) matrices (3x3 complex matrices  $M$  with  $\det[M] = 1$ ,  $M^{-1} = M^\dagger$ )
- Group-valued fields live not on real line but on compact manifolds
- Action is invariant under group transformations on gauge fields



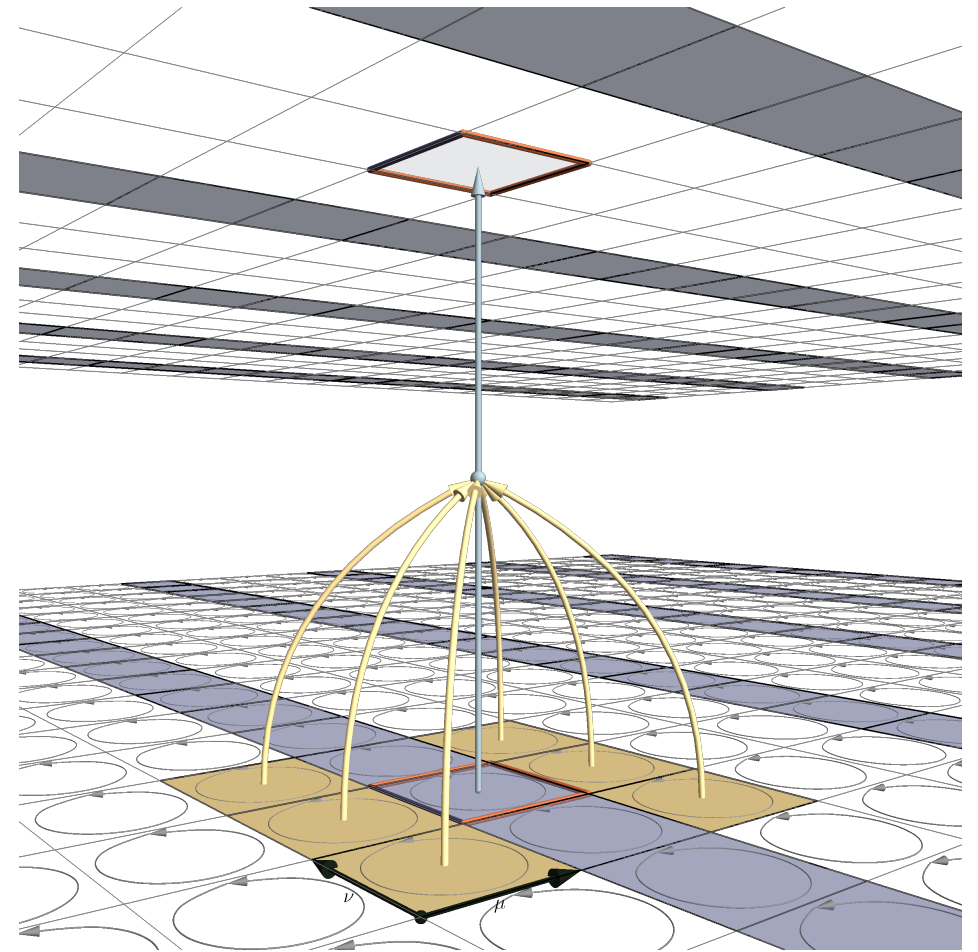
1. Flows on compact, connected manifolds
2. Incorporate symmetries: gauge-equivariant flows

[2008.05456 (2020), PRL 125, 121601 (2020), 2002.02428 (2020)]

# Incorporating symmetries

## Gauge field theories

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[2008.05456 (2020), PRL 125, 121601 (2020), 2002.02428 (2020)]

# Application: U(1) field theory

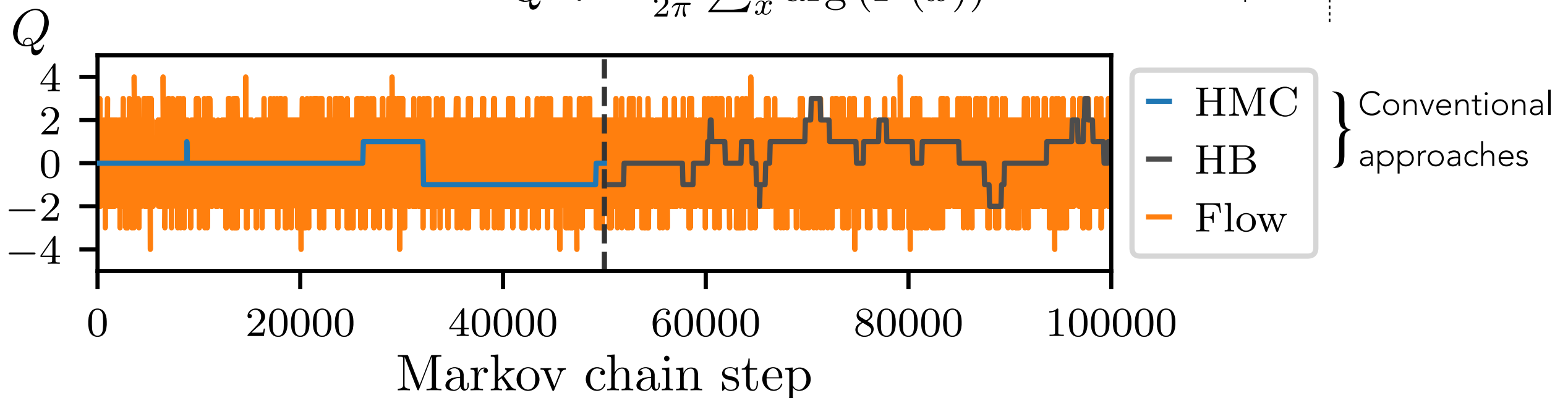
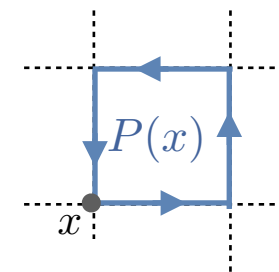
First gauge theory application: U(1) field theory

**Success:** Critical slowing down is significantly reduced

**Cost:** Up-front training of the model

Sampling of the topological charge

$$Q := \frac{1}{2\pi} \sum_x \arg(P(x))$$



2D,  $L=16$ ,  $\beta=6$

[2008.05456 (2020), PRL 125, 121601 (2020), 2002.02428 (2020)]

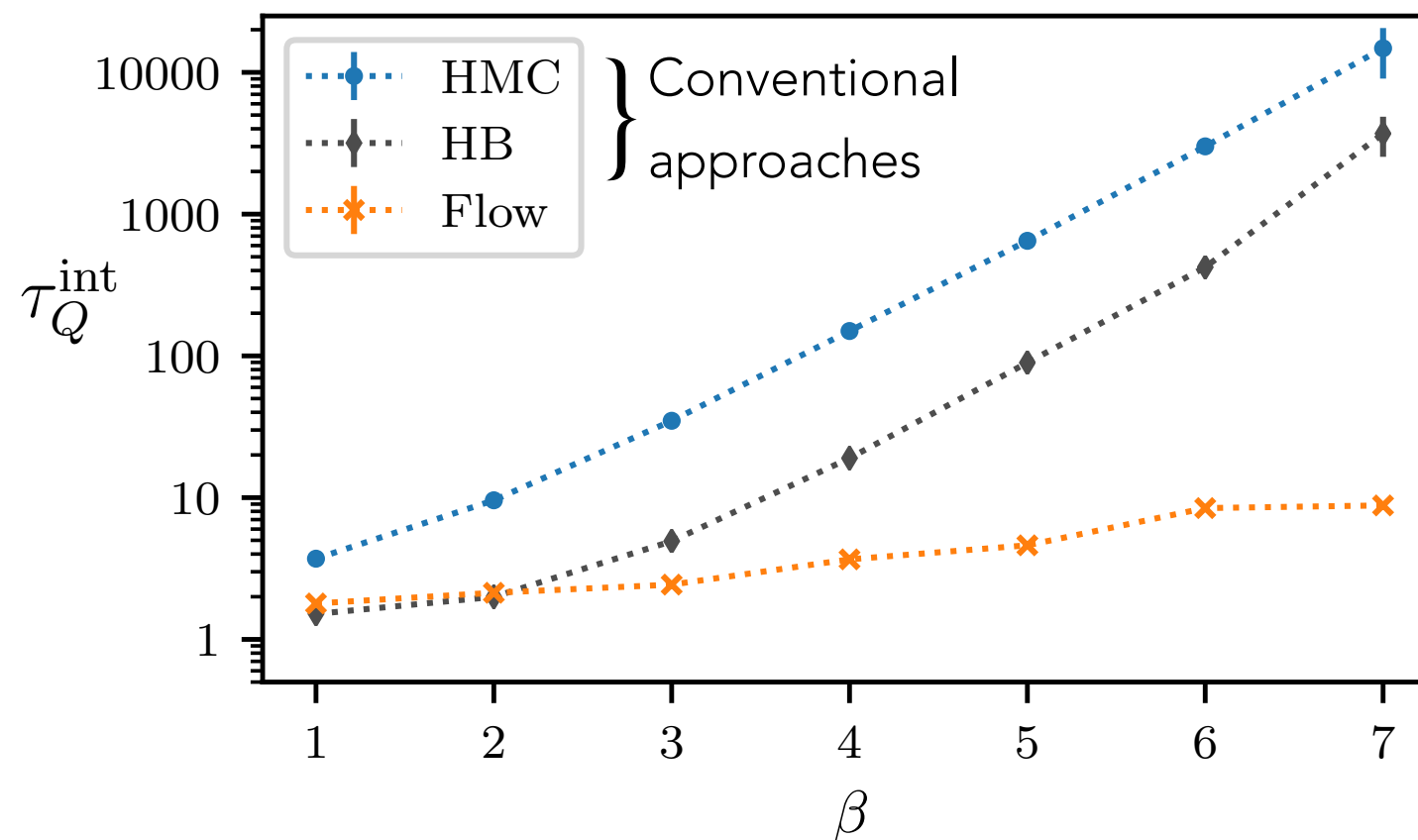
# Application: U(1) field theory

First gauge theory application: U(1) field theory

**Success:** Critical slowing down is significantly reduced

**Cost:** Up-front training of the model

Cost per independent sample



[2008.05456 (2020),  
PRL 125, 121601 (2020),  
2002.02428 (2020)]

2D, L=16

# Application: U(1) field theory

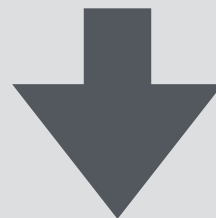
First gauge theory application: U(1) field theory

Success:  $\mathcal{O}(1)$  complexity for ground state energy estimation

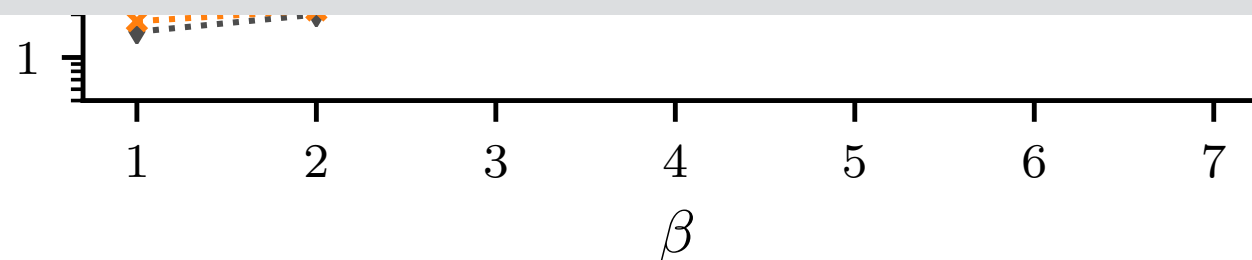
Cost:

**SUCCESS!**

Proof-of-principle of efficient, exact, ML algorithm for LQFT



Already extended to SU(N).  
Significant work required to scale to state-of-the-art



[2008.05456 (2020),  
PRL 125, 121601 (2020),  
2002.02428 (2020)]

2D, L=16



# Interdisciplinary applications

Molecular genetics and drug design

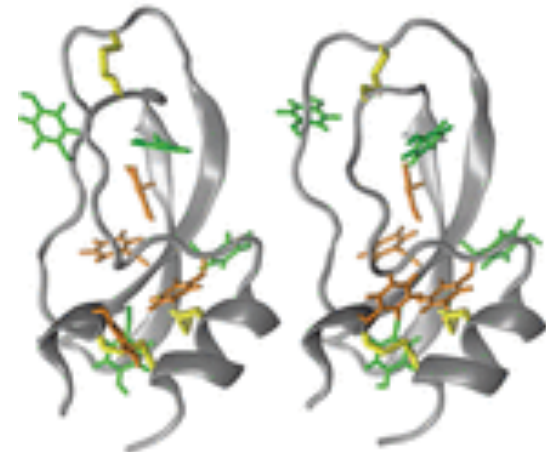
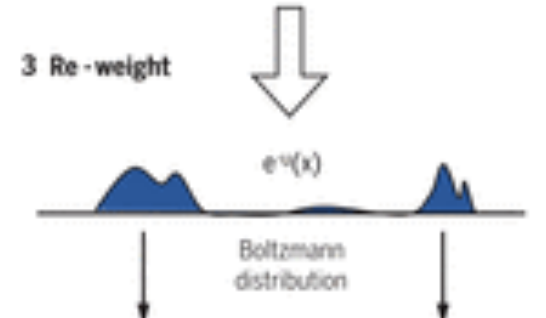
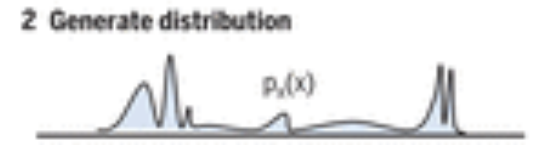
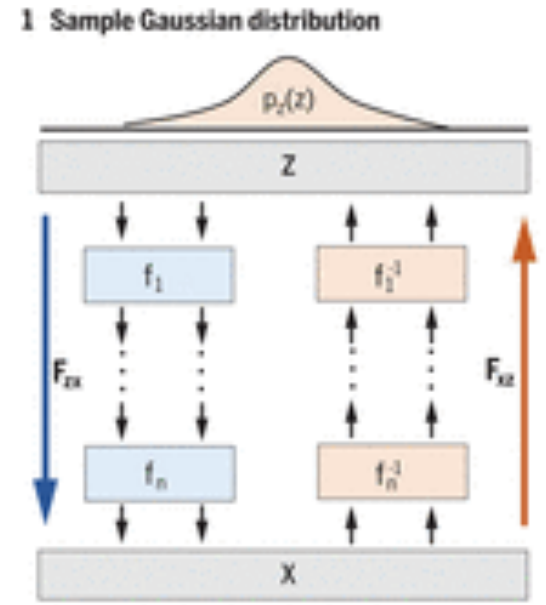


**RESEARCH ARTICLE SUMMARY**

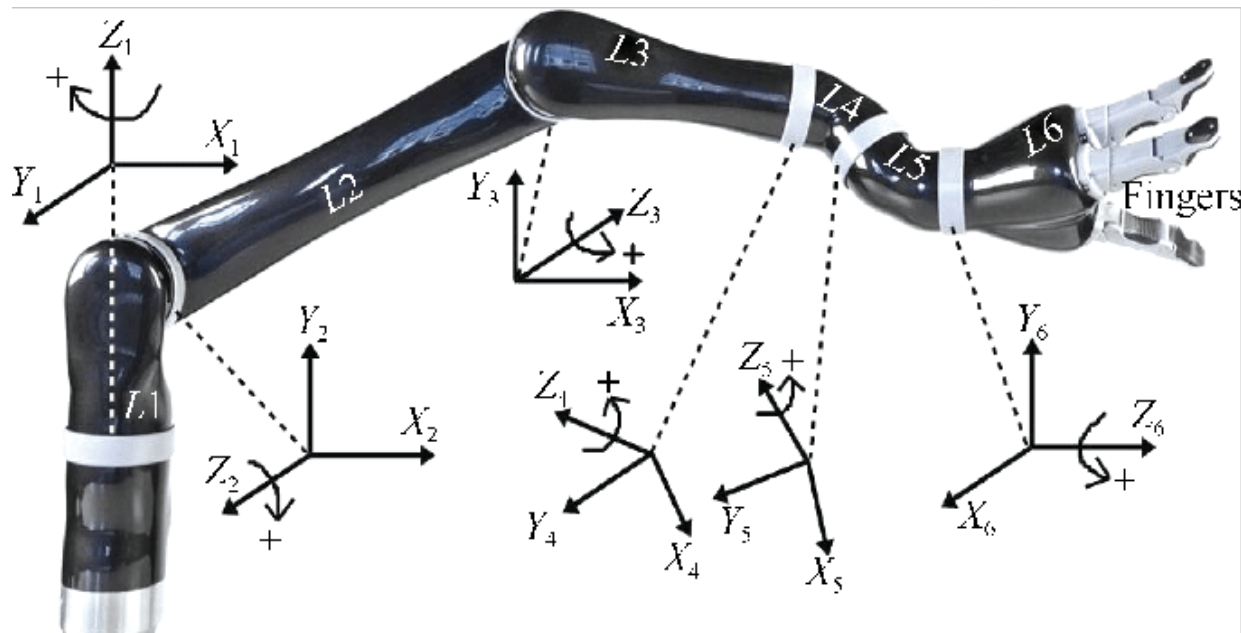
**MACHINE LEARNING**

**Boltzmann generators: Sampling equilibrium states of many-body systems with deep learning**

Frank Noé<sup>\*†</sup>, Simon Olsson<sup>\*</sup>, Jonas Köhler<sup>\*</sup>, Hao Wu



Robotics



**H. Application: Multi-Link Robot Arm**

As a concrete application of flows on tori, we consider the problem of approximating the posterior density over joint angles  $\theta_1, \dots, \theta_6$  of a 6-link 2D robot arm, given (soft) constraints on the position of the tip of the arm. The possible configurations of this arm are points in  $\mathbb{T}^6$ . The position  $r_k$  of a joint  $k = 1, \dots, 6$  of the robot arm is given by

$$r_k = r_{k-1} + \left( l_k \cos \left( \sum_{j \leq k} \theta_j \right), l_k \sin \left( \sum_{j \leq k} \theta_j \right) \right),$$

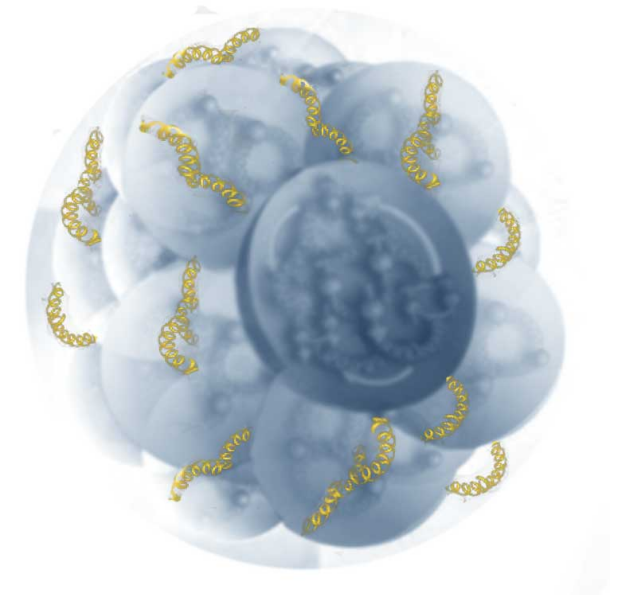
where  $r_0 = (0, 0)$  is the position where the arm is affixed

# Nuclear physics from the Standard Model

1

## New insights into proton and nuclear structure

First-principles QCD calculations to reveal nuclear structure and provide benchmarks for BSM searches

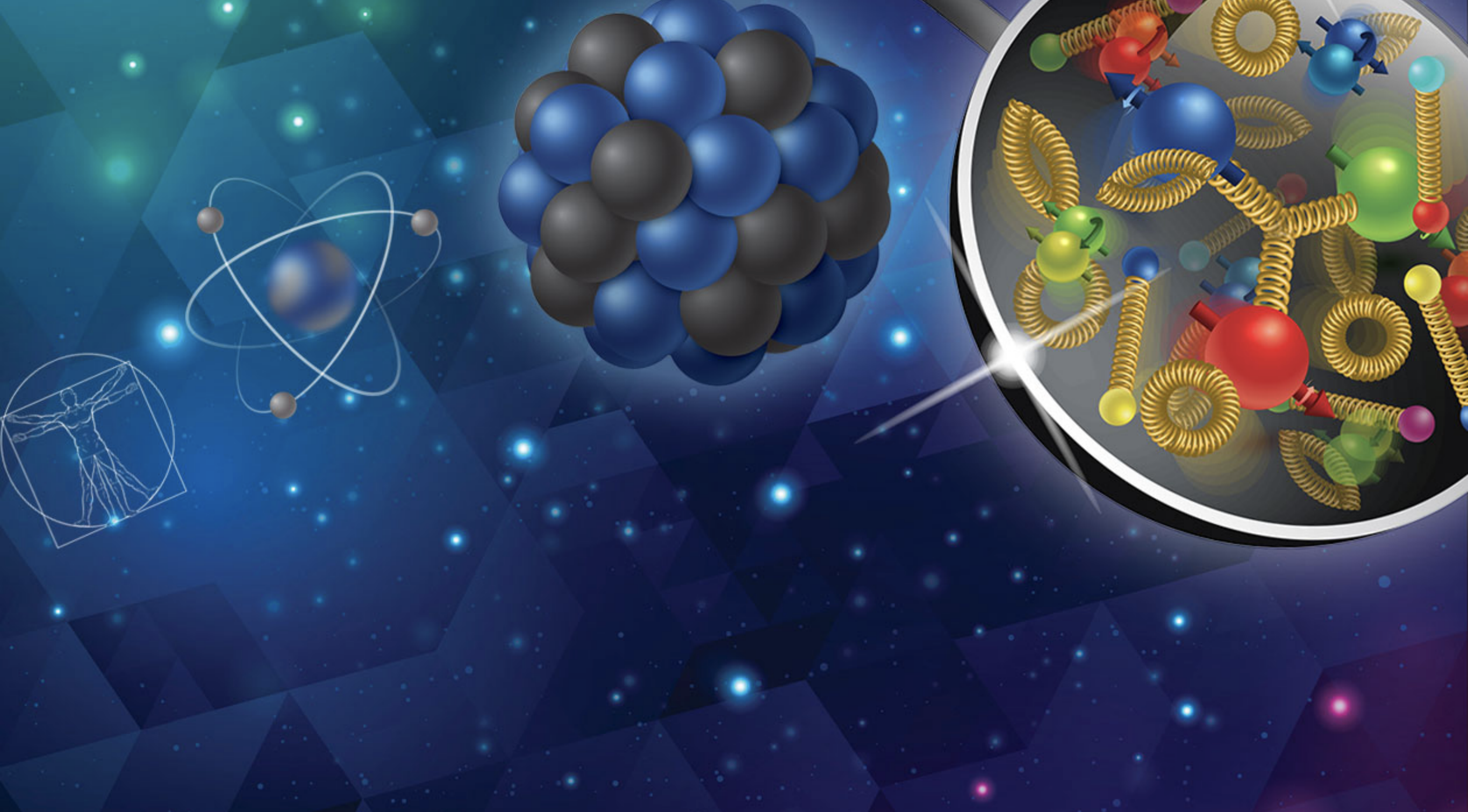


2

## Beyond the frontiers with ML

- Reaching nuclei with  $A > 4$
- Provably-exact physics-informed machine learning algorithms





Massachusetts  
Institute of  
Technology

