From quarks and gluons to hadrons and multiquarks

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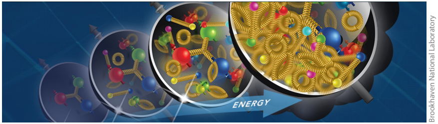
LIP Lisboa

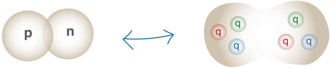
LIP Seminar
June 4, 2020

QCD and nuclear physics

- **Quark-gluon structure of hadrons**

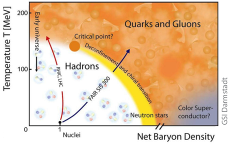


- **From quarks and gluons to nuclei**

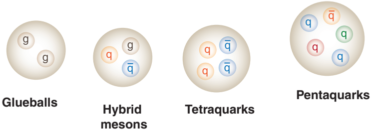


- **Mass generation and confinement**

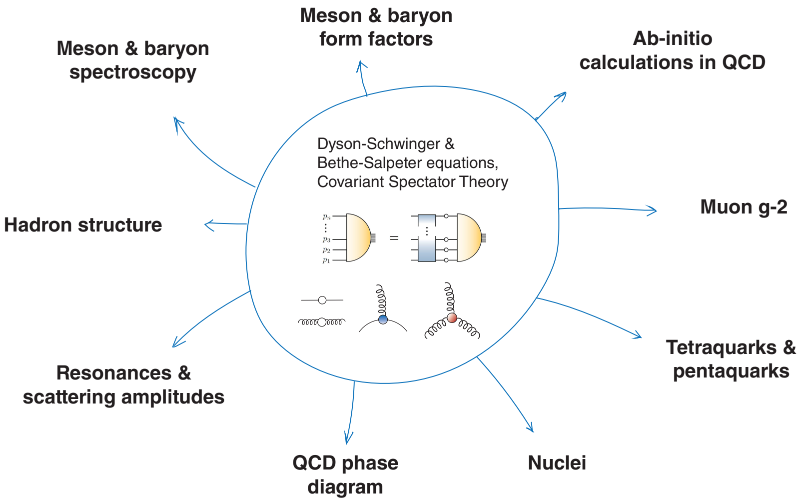
- **The phases of QCD**



- **Understanding exotic hadrons**

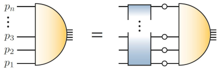


Our research



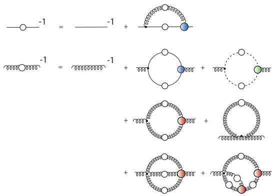
Methods

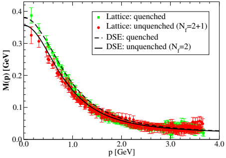
- Hadronic **bound-state equations** (BSEs, Faddeev eqs, ...)



"QFT analogue of Schrödinger eq."
→ hadron masses & "wave functions"
→ **spectroscopy calculations**

- Ingredients: **QCD's n-point functions,** Satisfy Dyson-Schwinger equations (**DSEs**): QCD's quantum eqs. of motion



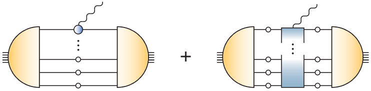


→ running **quark mass**

...

- Similar: **CST** (Covariant spectator theory)
Stadler & Gross, PRL 78 (1997), PRC 78 (2008),
Leitão, Stadler, Peña, Biernat, PLB 764 (2017)

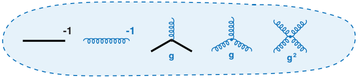
- Structure calculations: form factors, PDFs, GPDs, TMDs, two-photon processes, ...



Everything runs

QCD's classical action:

$$S = \int d^4x \left[\bar{\psi}\,(\partial\!\!\!/ + ig A\!\!\!/ + m)\,\psi + \frac{1}{4}\,F^a_{\mu\nu}\,F^{\mu\nu}_a\right]$$

$=$ 

Tree-level propagators & vertices

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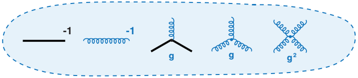
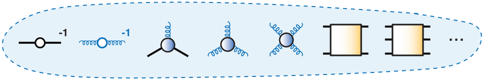
$=$ 

Tree-level propagators & vertices

Quantum "effective" action:

$$\int \mathcal{D}[\psi, \bar{\psi}, A] \, e^{-S} = e^{-\Gamma}$$



Dressed propagators & vertices

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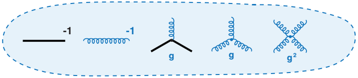
$=$ 

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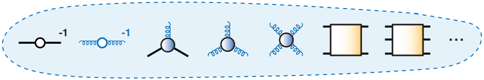
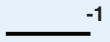


Dressed propagators & vertices

Quantum also means **structure:**

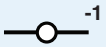


$i p\!\!\!/ + m$

$p^2\,\delta^{\mu\nu} - p^\mu p^\nu$

$ig\gamma^\mu$

→



$A(p^2)\,(i p\!\!\!/ + M(p^2))$

Quark mass function

$Z(p^2)^{-1}(p^2\,\delta^{\mu\nu} - p^\mu p^\nu)$

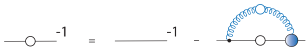
$ig\,\left(F_1(p^2, p\cdot Q, Q^2)\,\gamma^\mu + F_2(p^2, p\cdot Q, Q^2)\,[\gamma^\mu, Q\!\!\!/] + \dots\right)$

→ **everything runs** with momentum

→ **dressing functions** encode everything we can know about quarks & gluons

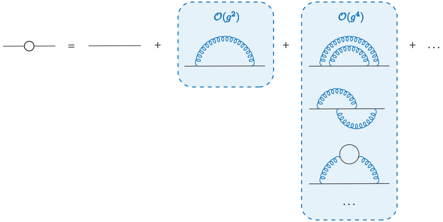
→ encode **full QFT**

Unwrapping the quark DSE

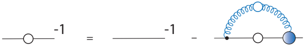


- DSE generates every diagram in **perturbation theory:**

$$S^{-1} = S_0^{-1} - \Sigma \quad \Rightarrow \quad S = S_0 + S_0\,\Sigma\,S$$
$$= S_0 + S_0\,\Sigma\,S_0 + S_0\,\Sigma\,S_0\,\Sigma\,S_0 + \dots$$



Unwrapping the quark DSE



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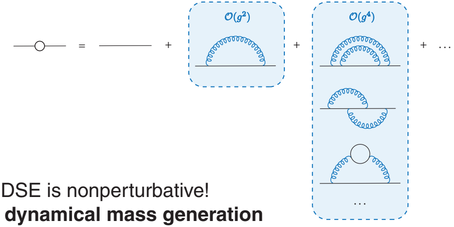
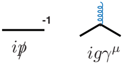


- But DSE is nonperturbative!
  E.g. **dynamical mass generation**

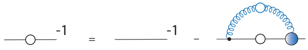
$$S(p)^{-1} = A(p^2)\,(i\not{p} + M(p^2)) \quad \Rightarrow \quad M(p^2) \sim \mathbf{Tr}\,S(p)^{-1}$$

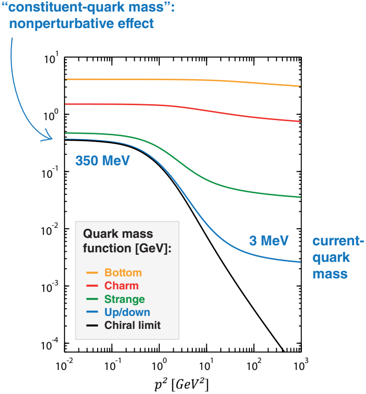
Massless quarks:



Every perturbative diagram contains odd # gamma matrices
⇒ mass function always zero!

Unwrapping the quark DSE





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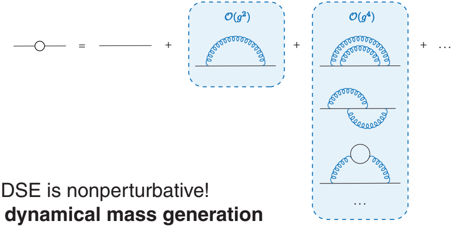

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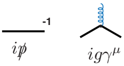

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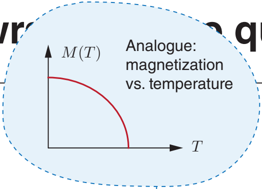
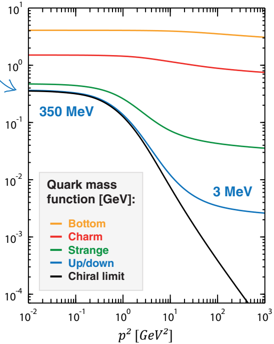
Unwr… quark DSE



"constituent-quark mass": nonperturbative effect



current-quark mass

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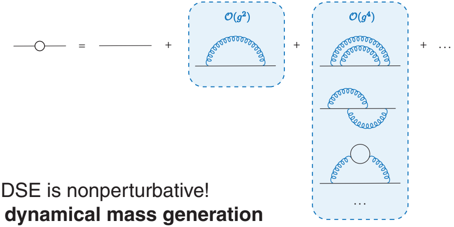

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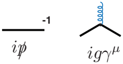

Every perturbative diagram contains odd # gamma matrices
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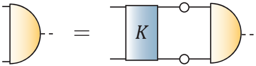
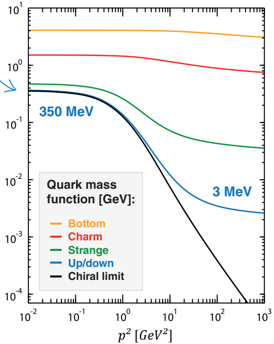
Massless pion

- Bethe-Salpeter equation for **mesons:**



Mass generation for hadrons, but pion is **Goldstone boson**

"constituent-quark mass": nonperturbative effect

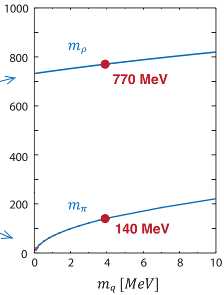


current-quark mass

Effects are connected: Dynamical breaking of chiral symmetry

$$m_\pi^2 \sim m_q$$

Meson masses [MeV]



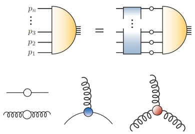
Our research

Meson & baryon spectroscopy

Meson & baryon form factors

Ab-initio calculations in QCD

Dyson-Schwinger &
Bethe-Salpeter equations,
Covariant Spectator Theory



Hadron structure

Muon g-2

Resonances & scattering amplitudes

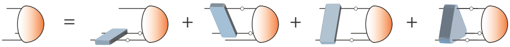
Tetraquarks & pentaquarks

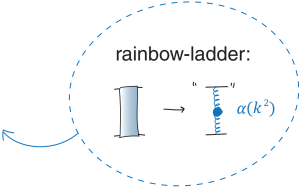
QCD phase diagram

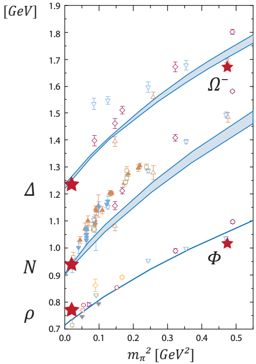
Nuclei

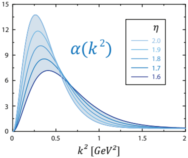
Baryon spectroscopy

- Solution of nucleon's covariant **3-body Faddeev equation**
  GE, Alkofer, Nicmorus, Krassnigg, PRL 104 (2010)









Maris, Tandy, PRC 60 (1999),
Qin et al., PRC 84 (2011)

Baryon spectroscopy

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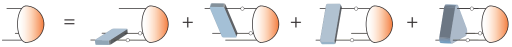
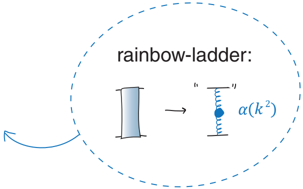
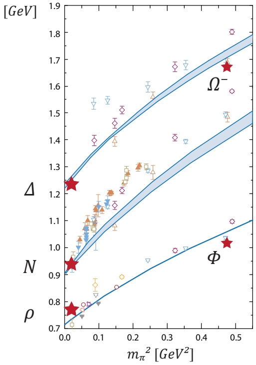
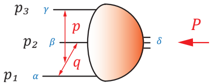






Keep full structure of baryon's Faddeev amplitude:



$$\Psi_{\alpha\beta\gamma\delta}(p,q,P) = \sum_i f_i(p^2, q^2, p\cdot q, p\cdot P, q\cdot P)\; \tau_i(p,q,P)_{\alpha\beta\gamma\delta}$$

Lorentz-invariant dressing functions

Dirac-Lorentz tensors:
64 for J = 1/2,
128 for J = 3/2

Review: GE, Sanchis-Alepuz, Williams, Alkofer, Fischer,
Prog. Part. Nucl. Phys. 91 (2016), 1606.09602

Baryon spectroscopy

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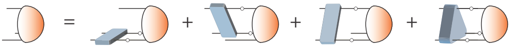
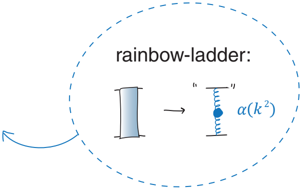
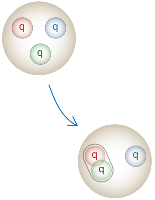
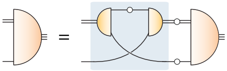
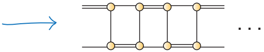




- Simpler: **quark-diquark Faddeev equation**
  Oettel et al., PRC 58 (1998), Cloet et al., FBS 46 (2009), Chen et al., PRD 97 (2018)





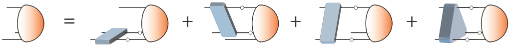


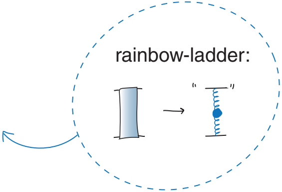
Ping-pong exchange
between quark and diquark

Calculate all ingredients in rainbow-ladder:
can compare **quark-diquark** and **3-body** directly
GE, Krassnigg, Schwinzerl, Alkofer, Ann. Phys. 323 (2008)

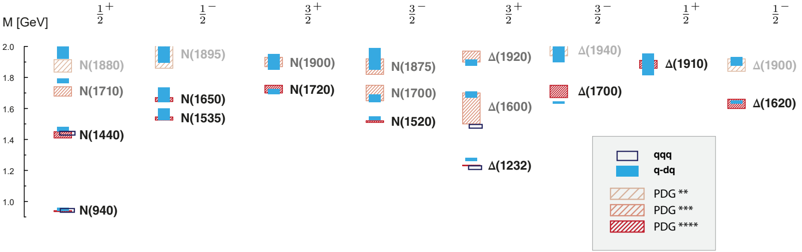
Baryon spectroscopy

- Solution of nucleon's covariant **3-body Faddeev equation**
  GE, Alkofer, Nicmorus, Krassnigg, PRL 104 (2010)



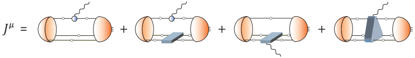


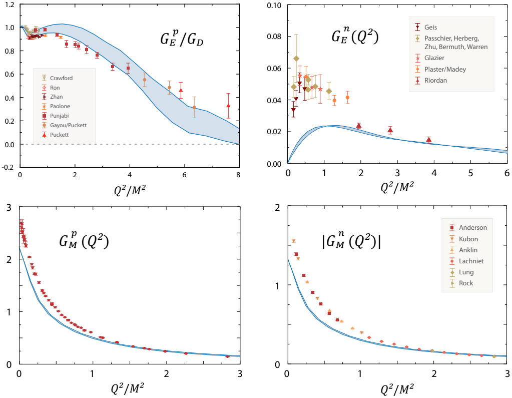
- Similar results in **quark-diquark** description
  GE, Fischer, Sanchis-Alepuz, PRD 94 (2016)



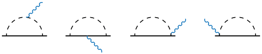
Baryon form factors

Nucleon em. form factors from three-quark equation
GE, PRD 84 (2011)





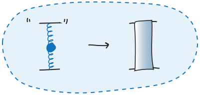
- **“Quark core without pion cloud”**



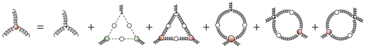
- **similar:** $N \to \Delta\gamma$ transition, axial & pseudoscalar FFs, octet & decuplet em. FFs

Review: GE, Sanchis-Alepuz, Williams, Fischer, Alkofer, PPNP 91 (2016), 1606.09602

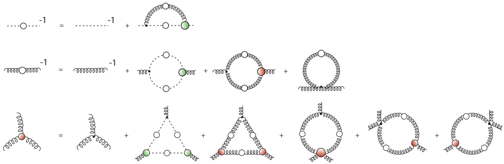
Ab-initio calculations



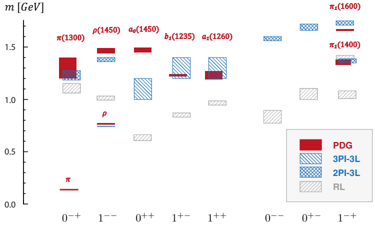
- **Three-gluon vertex** with all tensors: zero crossing at low momenta
  GE, Williams, Alkofer, Vujinovic, PRD 89 (2014)



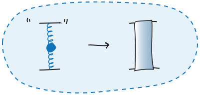
- **Mass generation** in Yang-Mills sector
  GE, Pawlowski, Silva, in preparation



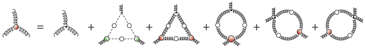
- Beyond rainbow-ladder calculations improve **light-meson spectrum**
  Williams, Fischer, Heupel, PRD 93 (2016)



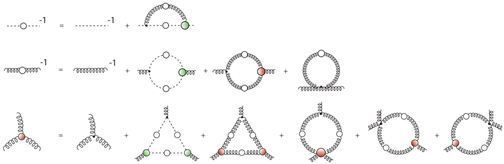
Ab-initio calculations



- **Three-gluon vertex** with all tensors: zero crossing at low momenta
  GE, Williams, Alkofer, Vujinovic, PRD 89 (2014)



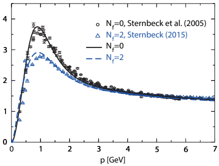
- **Mass generation** in Yang-Mills sector
  GE, Pawlowski, Silva, in preparation



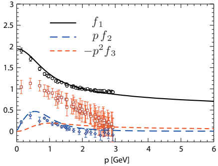
- Beyond rainbow-ladder calculations so far (mostly) only for **mesons**

- Need more interplay between **DSEs, FRG, lattice**
  Huber, 2003.13703 [hep-ph]
  Cyrol, Mitter, Pawlowski, Strodthoff, PRD 97 (2018)
  Oliveira, Silva, Skullerud, Sternbeck, PRD 99 (2019)

Gluon propagator



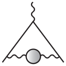
Quark-gluon vertex



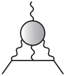
GE, Sanchis-Alepuz, Williams, Alkofer, Fischer,
Prog. Part. Nucl. Phys. 91 (2016)

Muon g-2

- **Muon anomalous magnetic moment:** total SM prediction deviates from exp. by ~4σ, theory uncertainty dominated by **QCD**

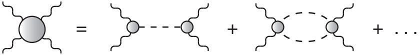


Hadronic vacuum polarization



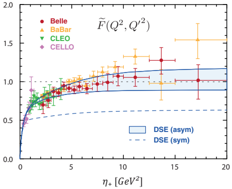
Hadronic light-by-light scattering

- **Dispersion theory:** LbL amplitude through intermediate onshell mesons
  Colangelo, Hoferichter, Kubis, Procura, Stoffer, PLB 738 (2014)



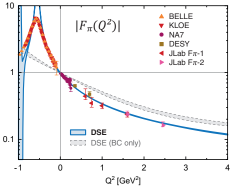
Jegerlehner, Nyffeler, Phys. Rept. 477 (2009), Jegerlehner 1705.00263

	$a_\mu\ [10^{-10}]$		
QED:	11 658 471.9	(0.0)	
EW:	15.4	(0.1)	
Hadronic:			
• VP (LO+HO)	680.1	(3.4)	
• LbL	**10.3**	**(2.9)**	**?**
SM:	11 659 177.6	(4.4)	
Exp:	11 659 208.9	(6.3)	
Diff:	31.3	(7.7)	



Pion transition form factor

Pion em. form factor

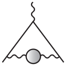


Knowledge of FFs in spacelike region helps constrain theory uncertainty

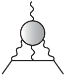
GE, Fischer, Weil, Williams, PLB 774 (2017), PRD 96 (2017), PLB 797 (2019), PRD 101 (2020)

Muon g-2

- **Muon anomalous magnetic moment:**
  total SM prediction deviates from exp. by ~4σ,
  theory uncertainty dominated by **QCD**



Hadronic vacuum polarization



Hadronic light-by-light scattering

Jegerlehner, Nyffeler, Phys. Rept. 477 (2009), Jegerlehner 1705.00263

	a_μ $[10^{-10}]$		
QED:	11 658 471.9	(0.0)	
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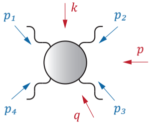
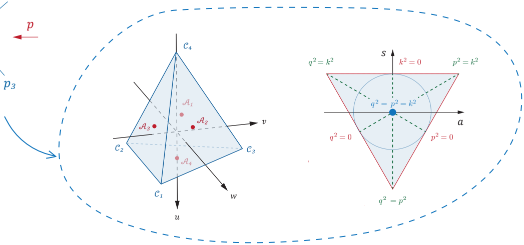
Diff:	31.3	(7.7)	

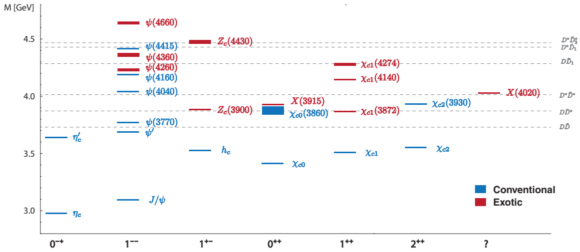
- Structure of LbL amplitude from **permutation group S4**
  GE, Fischer, Heupel, PRD 92 (2015)





136 independent tensors ...

Exotic mesons



Reviews:

Chen, Chen, Liu, Zhu,
Phys. Rept. 639 (2016), 1601.02092

Lebed, Mitchell, Swanson
PPNP 93 (2017), 1610.04528

Esposito, Pilloni, Polosa,
Phys. Rept 668 (2017), 1611.07920

Guo, Hanhart, Meißner et al.,
Rev. Mod. Phys. 90 (2018), 1705.00141

Ali, Lange, Stone,
PPNP 97 (2017), 1706.00610

Olsen, Skwarnicki, Zieminska,
Rev. Mod. Phys. 90 (2019), 1708.04012

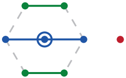
Liu, Chen, Chen, Liu, Zhu,
PPNP 107 (2019), 1903.11976

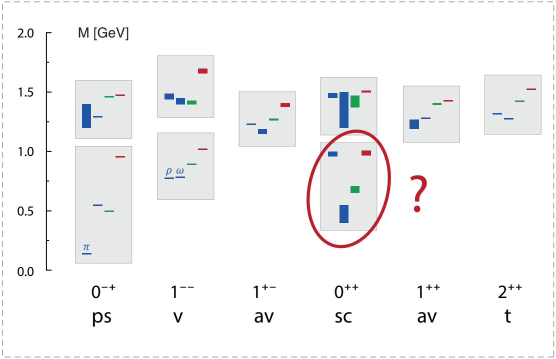
Brambilla, Eidelman, Hanhart et al.,
1907.07583

- Several tetraquark candidates in **charmonium spectrum**: X(3872), X(3915), Zc(3900), ...
- Z states cannot be $c\bar{c}$ since they carry charge
- Oldest tetraquark candidates: **light scalar mesons**

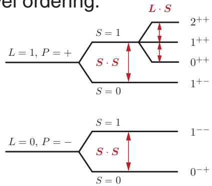
Light scalar mesons

Light meson spectrum (PDG):
grouped with J^{PC} and flavor content

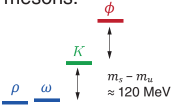




- Nonrelativistic level ordering:



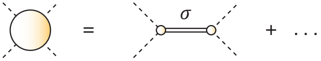
- Vector mesons:

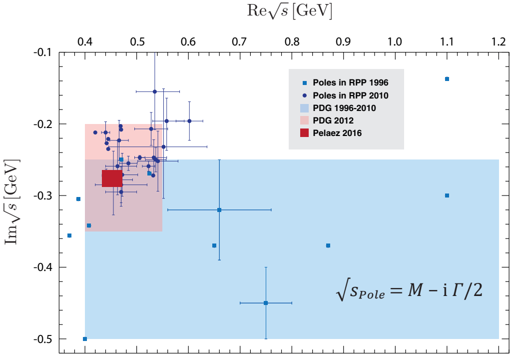


- **Pseudoscalar mesons?** spontaneous chiral symmetry breaking & axial anomaly
- **Scalar mesons..?**

Light scalar mesons

$\sigma/f_0(500)$ is a **resonance** in $\pi\pi$ **scattering:**



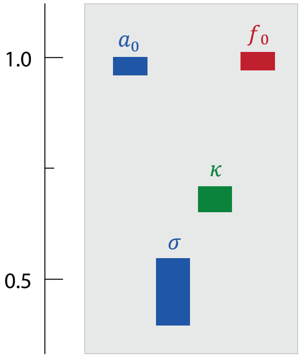


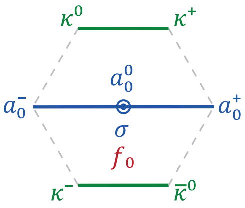
Pelaez, Phys. Rept. 658 (2016) 1

- **PDG 2010:** "$f_0(600)$"
  $\sqrt{s} \sim$ (400...1200) – i (250...500) MeV
- **Dispersive analyses:**
  $\sqrt{s} \sim$ 450(20) – i 275(10) MeV
  Caprini, Colangelo, Leutwyler 2006
  Garcia-Martin, Kaminski, Pelaez, Ruiz de Elvira 2011
  Moussallam 2011
  Masjuan, Ruiz de Elvira, Sanz-Cillero 2014
  Pelaez 2016
- **PDG 2012:** "$f_0(500)$"
  $\sqrt{s} \sim$ (400...550) – i (200...350) MeV
- Pole locations from **lattice QCD**
  Briceno, Dudek, Edwards, Wilson, PRL 118 (2017), PRD 97 (2018)

Light scalar mesons

Light scalar (0^{++}) **mesons** don't fit into the conventional meson spectrum:





f_0 (980 MeV) $s\bar{s}$
κ (680 MeV) $u\bar{s}, d\bar{s}$
a_0 (980 MeV), σ (500 MeV) $\}$ $u\bar{u}, d\bar{d}, u\bar{d}$

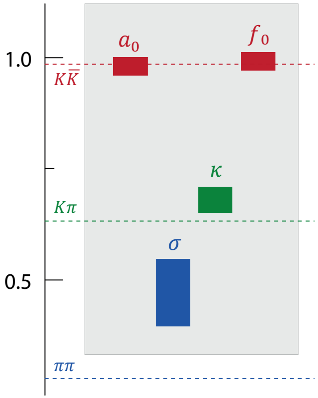
- Why are a_0, f_0 mass-degenerate?
- Why are their **decay widths** so different?

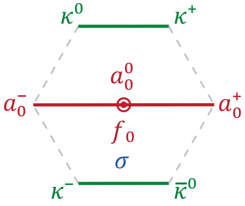
  $\Gamma(\sigma, \kappa) \approx$ 550 MeV
  $\Gamma(a_0, f_0) \approx$ 50–100 MeV

- Why are they so **light**?
  Scalar mesons ~ **p-waves,** should have masses similar to axialvector & tensor mesons ~ 1.3 GeV

Light scalar mesons

What if they were **tetraquarks** (diquark-antidiquark)? Jaffe 1977, Close, Tornqvist 2002, Maiani, Polosa, Riquer 2004



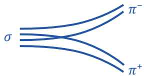


f_0 (980 MeV) } $us\overline{us}$, ...
a_0 (980 MeV) }

κ (800 MeV) $us\overline{ud}$, ...

σ (500 MeV) $ud\overline{ud}$

- Explains **mass ordering** & **decay widths**:
  f_0 and a_0 couple to K$\overline{\text{K}}$, large widths for σ, κ

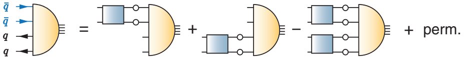


- Alternative: **meson molecules?**
  Weinstein, Isgur 1982, 1990; Close, Isgur, Kumano 1993

- **Non-q$\overline{\text{q}}$ nature** of σ supported by dispersive analyses, unitarized ChPT, large Nc, extended linear σ model, quark models
  Pelaez, Phys. Rept. 658 (2016)

Tetraquarks

- Light scalar mesons (σ, κ, a_0, f_0) as **four-quark states:**
  GE, Fischer, Heupel, PLB 753 (2016)



$$\Gamma(p,q,k,P) = \sum_i f_i\left(p^2, q^2, k^2, \{\omega_j\}, \{\eta_j\}\right) \tau_i(p,q,k,P) \quad \otimes \quad \text{Color} \quad \otimes \quad \text{Flavor}$$

9 Lorentz invariants:

$p^2, \quad q^2, \quad k^2, \quad P^2 = -M^2$

$\omega_1 = q \cdot k \qquad \eta_1 = p \cdot P$
$\omega_2 = p \cdot k \qquad \eta_2 = q \cdot P$
$\omega_3 = p \cdot q \qquad \eta_3 = k \cdot P$

256 Dirac-Lorentz tensors

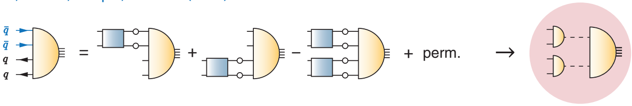
2 Color tensors:

$3 \otimes \overline{3}, \quad 6 \otimes \overline{6}$ or
$1 \otimes 1, \quad 8 \otimes 8$
(Fierz-equivalent)

- Group momentum variables into multiplets of **permutation group S4:** can switch off groups of variables without destroying symmetries
  GE, Fischer, Heupel, PRD 92 (2015)

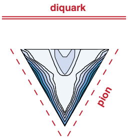
Tetraquarks

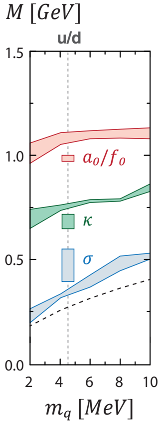
- Light scalar mesons (σ, κ, a_0, f_0) as **four-quark states:**
  GE, Fischer, Heupel, PLB 753 (2016)



- BSE dynamically generates **meson poles** in BS amplitude:

$f_i(\mathcal{S}_0, \ldots)$ → 1500 MeV
$f_i(\mathcal{S}_0, \ldots)$ → 1500 MeV
$f_i(\mathcal{S}_0, \ldots)$ → 1200 MeV
$f_i(\mathcal{S}_0, \ldots)$ → **350 MeV !**





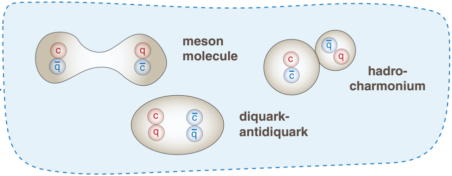
- "Light scalar mesons" look like **meson molecules,** diquark-antidiquark components almost negligible

- Lightness is inherited from pseudoscalar Goldstone bosons!

Tetraquarks

- Heavy-light **four-quark states:** what is their internal decomposition?

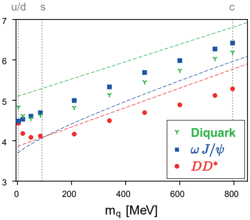
$cq\bar{q}\bar{c}$ →

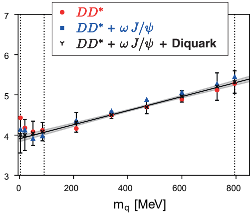


- **Four-quark BSE:** all mix together

$M_{cq\bar{q}\bar{c}}$ [GeV] in X(3872) channel

$m_S + m_A$
$m_\omega + m_{J/\psi}$
$m_D + m_{D^*}$



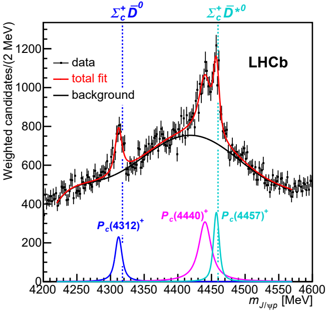


Wallbott, GE, Fischer, PRD 100 (2019), 2003.12407 [hep-ph]

$cq\bar{q}\bar{c}$ → strong meson-meson component: DD* for X(3872), Zc(3900):

$cc\bar{q}\bar{q}$ → diquarks also play role

Pentaquarks?

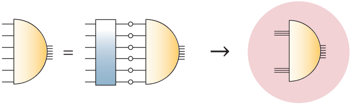


Aaij et al., PRL 112 (2019)

Madalena Lourenço
https://www.youtube.com/user/projetoMEFT

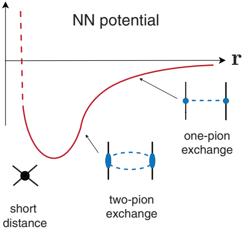
Nucleons in nuclei

Transition from quarks & gluons to **light nuclei:**

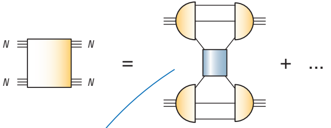


- Relativistic structure of **deuteron**
- Exotic dibaryons and hypernuclei
- **Short-range correlations**
- **EMC effect:** overlapping nucleons in nuclei?

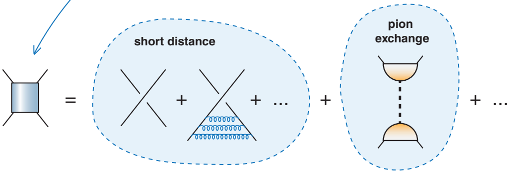
Microscopic origins of **short-range nuclear force?**



Weise, Nucl. Phys. A805 (2008)

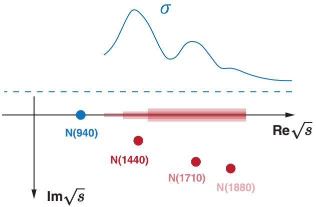


Constrain phenomenological potentials?

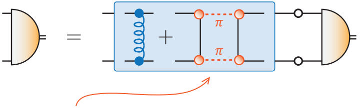


Resonances

- Most hadrons are **resonances** and decay
  ⇔ poles in complex momentum plane

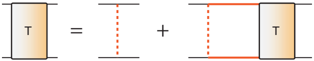


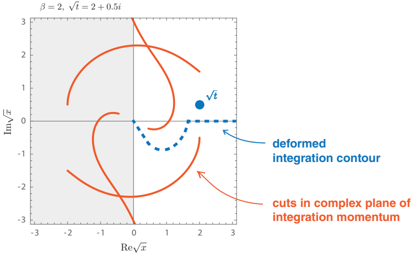
- BSE kernel must be aware of decay channels



real poles produce cuts ⇒ threshold,
ρ meson becomes resonance
Williams, PLB 798 (2019)

- Solution of **4d scattering equation,**
  contour deformations to go beyond thresholds
  GE, Duarte, Peña, Stadler, PRD 100 (2019)

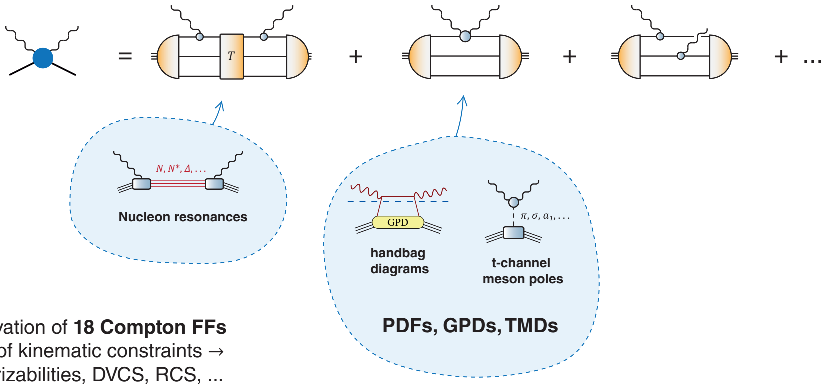




Could be extended to coupled channels...

Hadron structure

- Systematic derivation how external currents couple to hadrons in DSE/BSE approach, e.g. **nucleon Compton scattering** GE, Fischer, PRD 85 (2012), PRD 87 (2013)



- Derivation of **18 Compton FFs** free of kinematic constraints → polarizabilities, DVCS, RCS, ...
  GE, Ramalho, PRD 98 (2018)

Hadron structure

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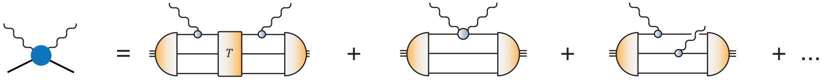
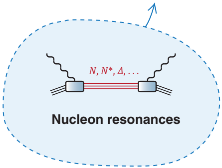
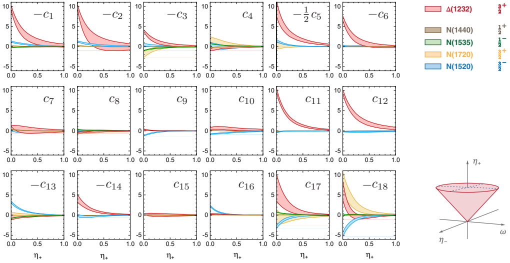




Nucleon resonance contributions:



- Derivation of **18 Compton FFs** free of kinematic constraints → polarizabilities, DVCS, RCS, ...
  GE, Ramalho, PRD 98 (2018)

Thank you!

INT Program INT-20-2c
Accessing and Understanding the QCD Spectra
August 17 - September 4, 2020

Organizers:
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Gernot Eichmann (LIP & IST Lisboa)
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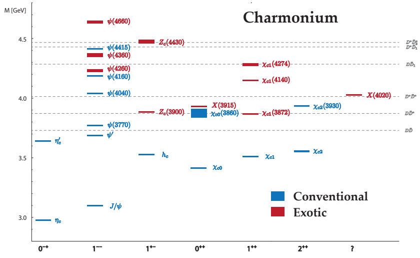
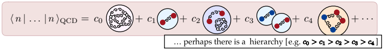
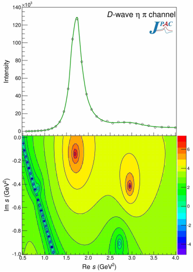
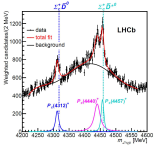


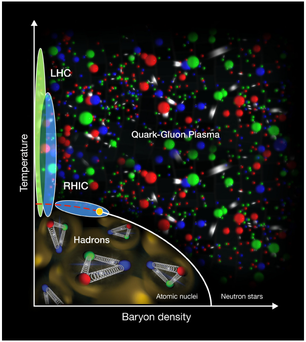






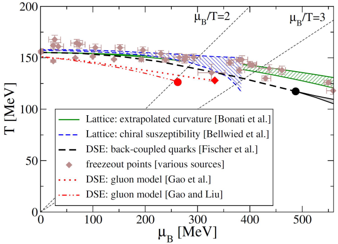
Backup slides

QCD phase diagram



Brookhaven National Laboratory

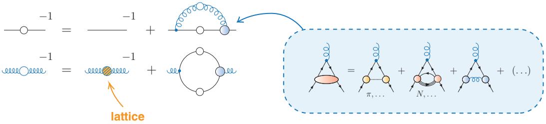
Search for **critical endpoint (CEP)** from DSEs & lattice:



Fischer, Prog. Part. Nucl. Phys. 105 (2019)

Location of CEP sensitive to **baryons**?

GE, Fischer, Welzbacher, PRD 93 (2016)



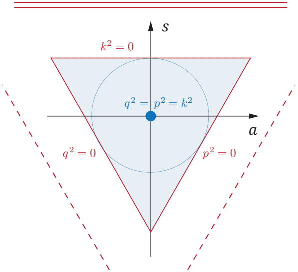
Structure of the amplitude

- **Singlet:** symmetric variable, carries overall scale:

$$\mathcal{S}_0 = \tfrac{1}{4}\,(p^2+q^2+k^2)$$

- **Doublet:** $\mathcal{D}_0 = \dfrac{1}{4\mathcal{S}_0}\begin{bmatrix}\sqrt{3}\,(q^2-p^2)\\ p^2+q^2-2k^2\end{bmatrix}$

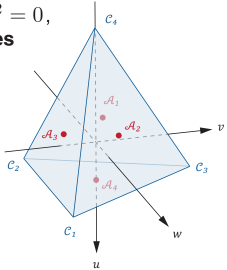
  Mandelstam triangle,
  outside: **meson and diquark poles!**



Lorentz invariants can be grouped into **multiplets of the permutation group S4:**
GE, Fischer, Heupel, PRD 92 (2015)

- **Triplet:** $\mathcal{T}_0 = \dfrac{1}{4\mathcal{S}_0}\begin{bmatrix}2\,(\omega_1+\omega_2+\omega_3)\\ \sqrt{2}\,(\omega_1+\omega_2-2\omega_3)\\ \sqrt{6}\,(\omega_2-\omega_1)\end{bmatrix}$

  tetrahedron bounded by $p_i^2=0$,
  outside: **quark singularities**

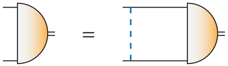


- Second **triplet:**
  3dim. sphere

$$\mathcal{T}_1 = \frac{1}{4\mathcal{S}_0}\begin{bmatrix}2\,(\eta_1+\eta_2+\eta_3)\\ \sqrt{2}\,(\eta_1+\eta_2-2\eta_3)\\ \sqrt{6}\,(\eta_2-\eta_1)\end{bmatrix}$$

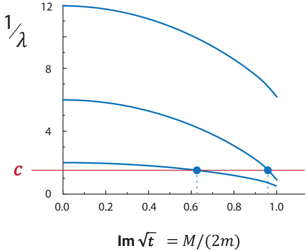
Bound states & resonances

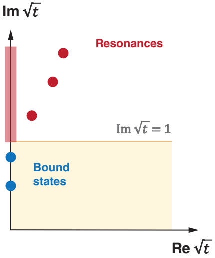
- **Homogeneous BSE:**



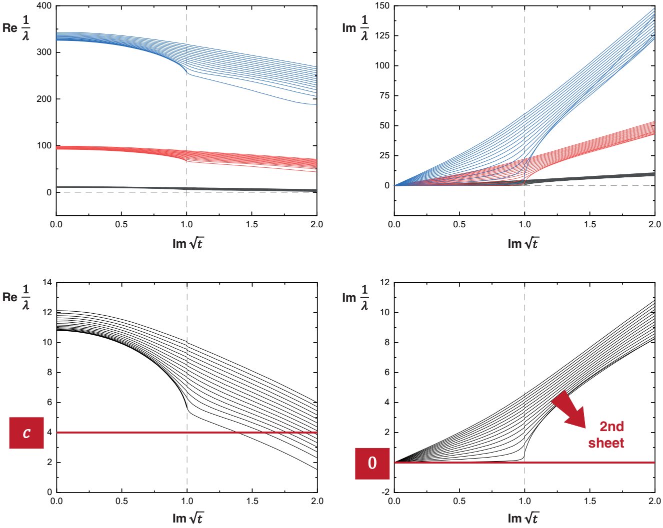
$$\Rightarrow \quad \psi(t) = c\, K G_0(t)\, \psi(t)$$

$$\psi(X,Z,t) = c \int dx \int dz \; K(X,x,Z,z,t)\; G_0(x,z,t)\; \psi(x,z,t) \qquad \Rightarrow \quad \frac{1}{\lambda(t)} \overset{!}{=} c$$





BSE Eigenvalues



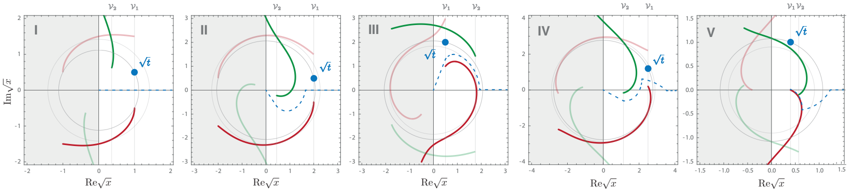
$$\frac{1}{\lambda(t)} \stackrel{!}{=} c + 0 \cdot i$$

still valid for **complex poles:**

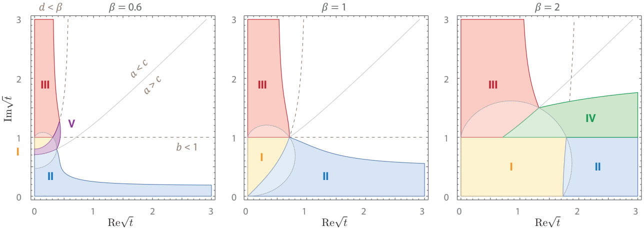
can detect resonances from **homogeneous BSE**

Contour deformation

For **onshell scattering amplitude** more complicated:



Can still cover **parts** of complex t plane:

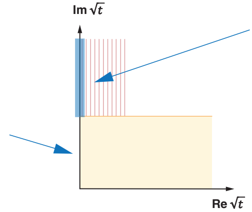


Benchmarks

- **Binding energies**

$c = 1, \beta = 0.5$

$\mathrm{Im}\sqrt{t}$	π/λ_0 this work	π/λ_0 [1, 2]	π/λ_0 [3]
0.999	1.18(3)	1.211	1.216
0.995	1.43(1)	1.440	1.440
0.99	1.623	1.624	1.623
0.95	2.498	2.498	2.498
0.90	3.251	3.251	3.251
0.80	4.416	4.416	4.416
0.75	4.901	4.901	4.901
0.6	6.094	6.096	6.094
0.4	7.205	7.206	7.204
0.2	7.849	7.850	7.849
0	8.061	8.062	8.061

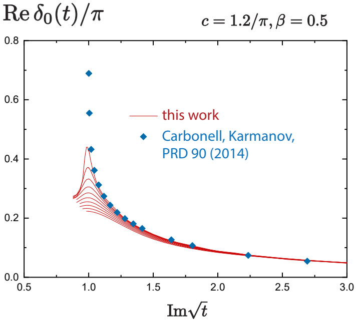


[1, 2] Kusaka, Simpson, Williams, PRD 56 (1997)
Karmanov, Carbonell, EPJ A 28 (2006)

[3] Frederico, Salmè, Viviani, PRD 89 (2014)

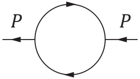
- **Phase shifts**

$$f_l(t) = \frac{1}{2i\,\tau(t)}\left[e^{2i\,\delta_l(t)} - 1\right]$$



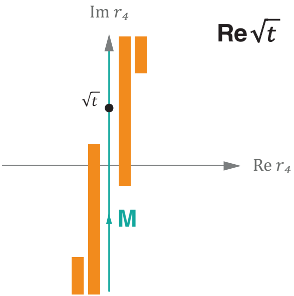
Two poles

Consider two-point function (current correlator, self energy, vacuum polarization, ...)



$$\int d^4k \, \frac{1}{k_+^2 + m^2} \, \frac{1}{k_-^2 + m^2}$$

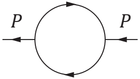
$$\int_{-\infty(i-\epsilon)}^{\infty(i-\epsilon)} dr_4 \ldots$$



Re $\sqrt{t}$ = 0

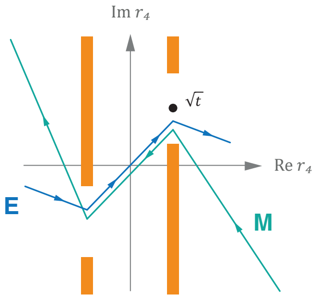
Two poles

Consider two-point function (current correlator, self energy, vacuum polarization, ...)



$$\int d^4k \, \frac{1}{k_+^2 + m^2} \, \frac{1}{k_-^2 + m^2}$$

$$\int_{-\infty(i-\epsilon)}^{\infty(i-\epsilon)} dr_4 \ldots$$



So:

E = M

$\int d^3k \int_{-\infty}^{\infty} dk_4$ ... close contours analytically, pick up **residues**

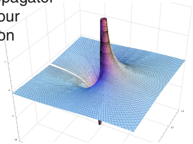
$\int_{-\infty}^{\infty} dk_4 \int d^3k$ ... avoid cuts by numerical **contour deformation**

Suggestions for better wording:

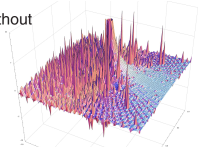
"We need XY ~~in Minkowski space"~~
"We calculate XY ~~directly in Minkowski space"~~

... in the full kinematical domain
... above threshold
... using residue calculus

Quark propagator **with** contour deformation



... and without



Complex eigenvalues?

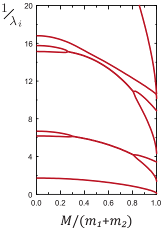
Excited states: some EVs are complex conjugate?

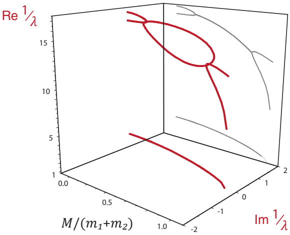
Typical for **unequal-mass** systems, already in Wick-Cutkosky model

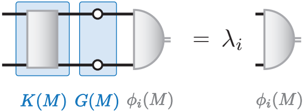
Wick 1954, Cutkosky 1954

Connection with “**anomalous**” **states?**

Ahlig, Alkofer, Ann. Phys. 275 (1999)







K and G are Hermitian (even for unequal masses!) but KG is not

If $G = G^\dagger$ and $G > 0$:
Cholesky decomposition $G = L^\dagger L$

$$K\, L^\dagger L\, \phi_i = \lambda_i\, \phi_i$$
$$(L K L^\dagger)\,(L\phi_i) = \lambda_i\,(L\phi_i)$$

$\Rightarrow$ Hermitian problem with same EVs!

Complex eigenvalues?

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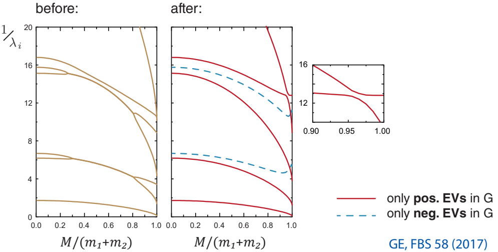
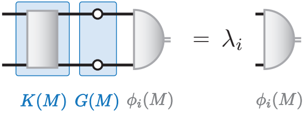


GE, FBS 58 (2017)



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$$(LKL^\dagger)\,(L\phi_i) = \lambda_i\,(L\phi_i)$$

⇒ Hermitian problem with same EVs!

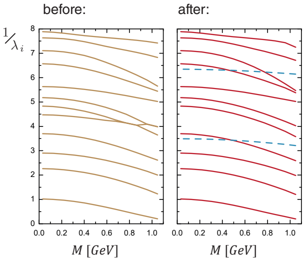
⇒ all EVs strictly **real**
⇒ level repulsion
⇒ "anomalous states" removed?

Complex eigenvalues?

Excited states: some EVs are complex conjugate?

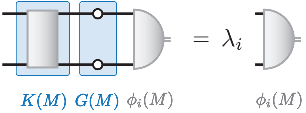
Typical for **unequal-mass** systems, already in Wick-Cutkosky model
Wick 1954, Cutkosky 1954

Connection with "**anomalous" states?**
Ahlig, Alkofer, Ann. Phys. 275 (1999)



Eigenvalue spectrum for pion channel
GE, FBS 58 (2017)

only **pos. EVs** in G
only **neg. EVs** in G



K and G are Hermitian (even for unequal masses!) but KG is not

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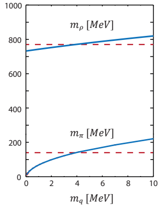
$$(LKL^\dagger)\,(L\phi_i) = \lambda_i\,(L\phi_i)$$

⇒ Hermitian problem with same EVs!

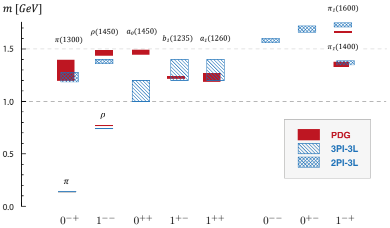
⇒ all EVs strictly **real**
⇒ level repulsion
⇒ "anomalous states" removed?

Mesons

- Pion is **Goldstone boson:** $m_\pi^2 \sim m_q$



- **Light meson spectrum** beyond rainbow-ladder

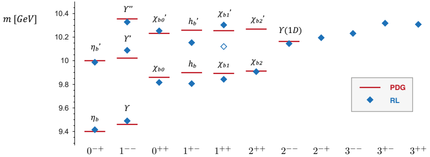


Williams, Fischer, Heupel, PRD 93 (2016)

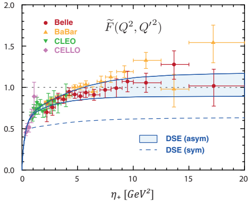
GE, Sanchis-Alepuz, Williams, Alkofer, Fischer, PPNP 91 (2016)

- **Bottomonium spectrum**

Fischer, Kubrak, Williams, EPJ A 51 (2015)



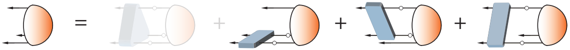
- **Pion transition form factor**



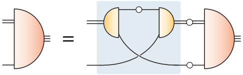
GE, Fischer, Weil, Williams, PLB 774 (2017)

Bound-state equations

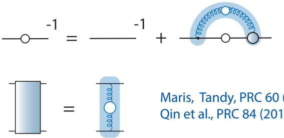
Bethe-Salpeter equation for baryons: GE, Sanchis-Alepuz, Williams, Alkofer, Fischer, PPNP 91 (2016), 1606.09602



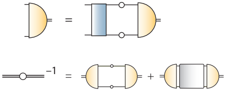
Quark-diquark approximation:



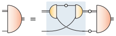
Rainbow-ladder:

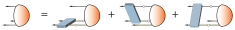


Maris, Tandy, PRC 60 (1999),
Qin et al., PRC 84 (2011)



DSE / BSE / Faddeev landscape



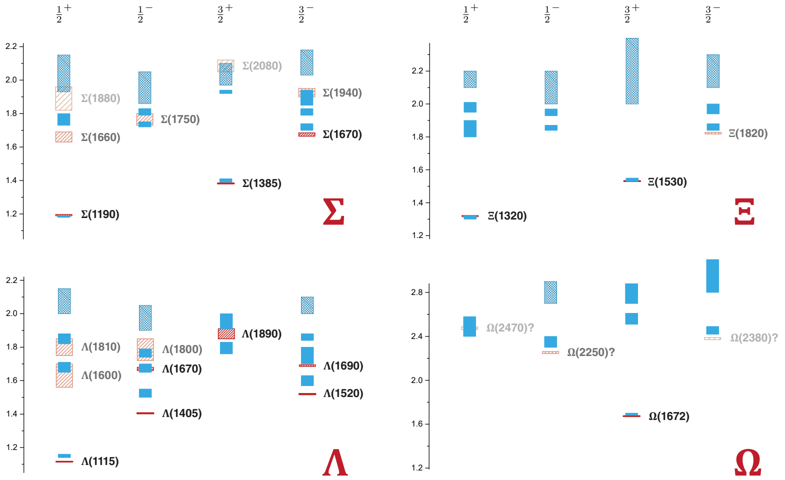


		NJL / contact	q-dq model	DSE (RL)		DSE (bRL)
u/d	N, Δ masses	√ (before 2015)	√ (before 2015)	√ (before 2015)	√ (before 2015)	√ (after 2015)
	N, Δ em. FFs	√ (before 2015)	√ (before 2015)	√ (before 2015)	√ (before 2015)	
	$N \to \Delta\gamma$	√ (before 2015)	√ (before 2015)	√ (before 2015)	√ (after 2015)	
	N^*, Δ^* masses (+)	√ (before 2015)	√ (before 2015)	√ (after 2015)	√ (after 2015)	
	$N \to N^*\gamma$	√ (before 2015)	√ (before 2015)			
	N^*, Δ^* masses (-)	√ (after 2015)	√ (after 2015)	√ (after 2015)	√ (after 2015)	
	$N \to N^*\gamma$					
s	ground states	√ (after 2015)	√ (after 2015)	√ (after 2015)	√ (after 2015)	
	excited states	√ (after 2015)	√ (after 2015)	√ (after 2015)	√ (after 2015)	
	em. FFs & TFFs				√ (after 2015)	
c, b	ground states	√ (after 2015)	√ (after 2015)		√ (after 2015)	
	excited states		√ (after 2015)		√ (after 2015)	
		Cloet, Thomas, Roberts, Bashir, Segovia, Chen, Wilson, Lu, ...	Oettel, Alkofer, Roberts, Cloet, Segovia, Chen, El-Bennich, ...	GE, Alkofer, Nicmorus, Sanchis-Alepuz, Fischer	GE, Sanchis-Alepuz, Fischer, Alkofer, Qin, Roberts	Sanchis-Alepuz, Williams, Fischer

√ . . . before 2015

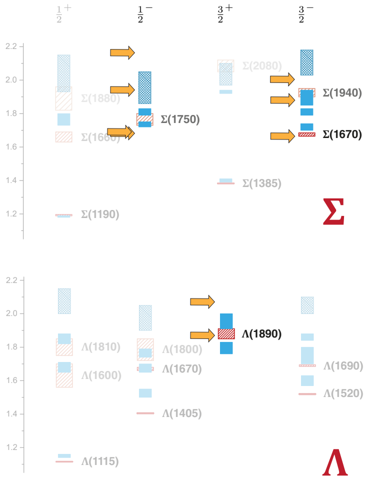
√ . . . after 2015

Strange baryons



GE, Fischer, FBS 60 (2019), Fischer, GE, PoS Hadron 2017

Strange baryons



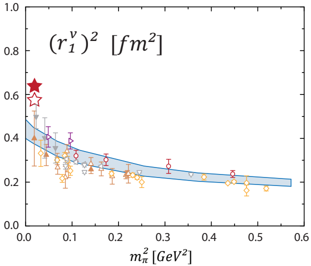
New states from Bonn-Gatchina

Sarantsev et al., 1907.13387 [nucl-ex]

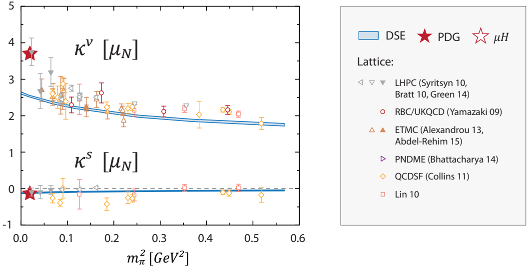
GE, Fischer, FBS 60 (2019), Fischer, GE, PoS Hadron 2017

Nucleon em. form factors

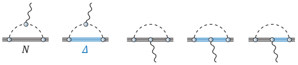
Nucleon charge radii:
isovector (p-n) Dirac (F1) radius



Nucleon magnetic moments:
isovector (p-n), isoscalar (p+n)



- **Pion-cloud effects** missing (⇒ divergence!), agreement with lattice at larger quark masses.



- **But:** pion-cloud **cancels** in κ^s ⟺ **quark core**

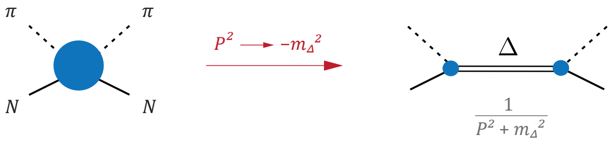
Exp: $\kappa^s = -0.12$

Calc: $\kappa^s = -0.12(1)$

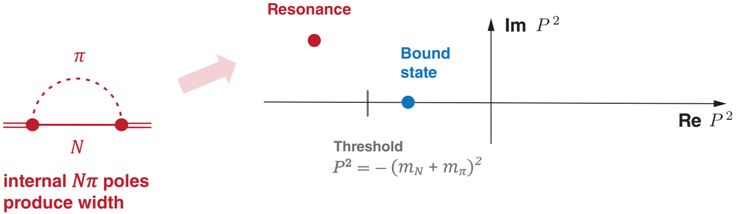
!!

GE, PRD 84 (2011)

Resonances?



π
π
N
N
$P^2 \longrightarrow -m_\Delta{}^2$
Δ
$\frac{1}{P^2 + m_\Delta{}^2}$

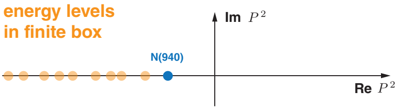


π
N
internal $N\pi$ poles produce width
Resonance
Bound state
Im P^2
Re P^2
Threshold
$P^2 = -(m_N + m_\pi)^2$

Resonances?

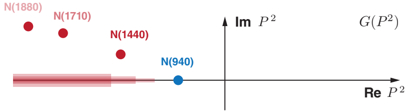
Lattice QCD:

$$\langle \dots \rangle = \int \mathcal{D}[\psi, \bar{\psi}, A]\, e^{-S[\psi, \bar{\psi}, A]} \; (\dots)$$



- **Finite volume:**
  bound states & scattering states

vary volume,
Luescher method

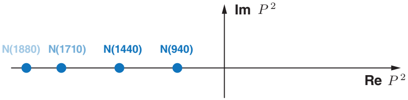


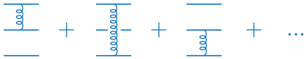
- **Infinite volume:**
  Bound states, resonances,
  branch cuts

Resonances?

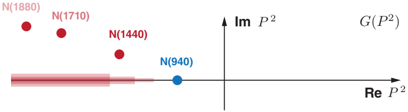
In terms of quarks and gluons?

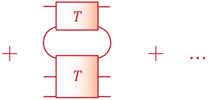
Bound states:



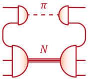


Resonances by **meson-baryon interactions:**





Both **bound states** and **resonances** must be generated from quark-gluon structure!



Analogue for $\rho \to \pi\pi$:

Williams, 1804.11161 [hep-ph], Miramontes, Sanchis-Alepuz, 1906.06227 [hep-ph]