Radiative corrections to the $e + \nu \to e + \nu$ cross section

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LABORATÓRIO DE INSTRUMENTAÇÃO
E FÍSICA EXPERIMENTAL DE PARTÍCULAS

Overview

Overview

Goals of the internship

- Implement a neutrino-electron elastic scattering cross section calculation;
- Introduce different calculations of radiative corrections to permit studies of new physics effects;
- Study the sensibility of these studies in the context of the SNO+ neutrino experiment

Neutrino history

Short chronology of neutrino physics:

- **1930** W. Pauli postulates the existence of the neutrino to explain the energy spectrum of electrons in β decays;
- **1956** first direct detection of the neutrino by F. Reines & C. Cowan (Nobel Prize for Reines in 1995);
- **1968** First detection of solar neutrinos at Homestake Experiment
- **1987** First detection of SuperNova neutrinos at KamiokaNDE-2 (and others)
- **1998** First detection of neutrino oscillations with atmospheric neutrinos at Super-KamiokaNDE
- **2002** Nobel Prize for Koshiba and Davis for the discovery of cosmic neutrinos;
- **2015** Nobel prize for McDonald and Kajita for the detection os neutrino oscillations (Super-K and **SNO** experiments)

The origin of the neutrino: the β decay

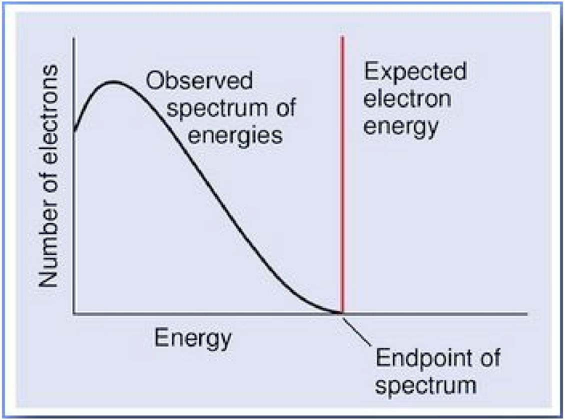
The beta decays are a fundamental part of neutrino's history. So let's introduce alpha and beta decays to understand what made the latter so relevant.

$$ {}^{A}_{Z}\mathrm{X} \rightarrow {}^{A-4}_{Z-2}\mathrm{Y} + {}^{4}_{2}\mathrm{He} $$

$$ {}^{A}_{Z}\mathrm{X} \rightarrow {}^{A}_{Z-1}\mathrm{Y} + e^{(+)} + \overset{(-)}{\nu}_{e} $$

In the alpha decay there is only one ejected particle so energy conservation imposes a very sharp energy spectrum for the alpha particle. In contrast the beta decay produces two particles, so the spectrum for each isolated one can be continuous.

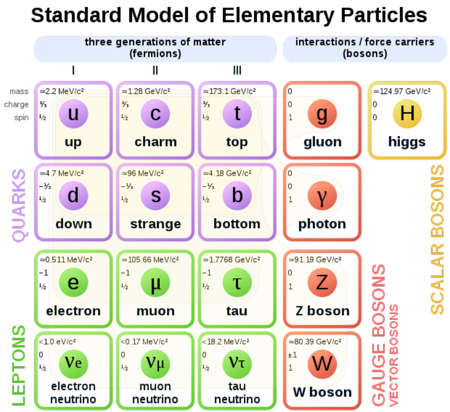
The origin of the neutrino: the β decay



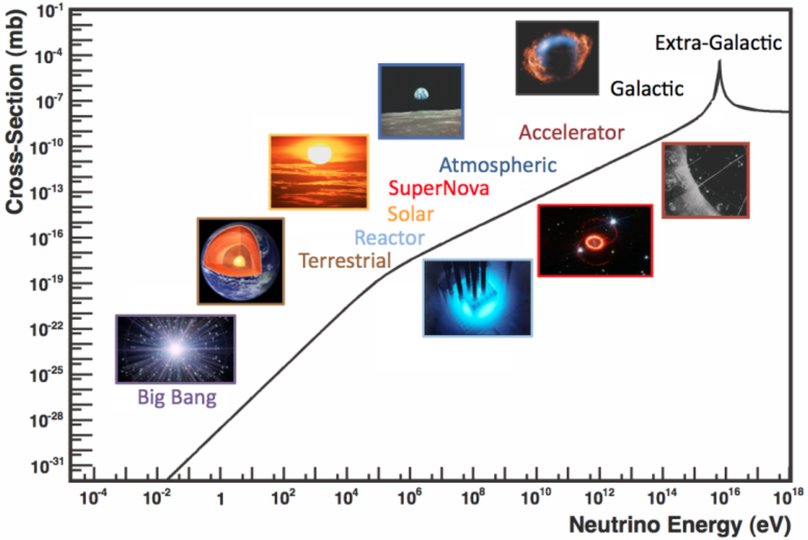
Neutrinos in the Standard Model

What we know about neutrinos:

- are charge and colour neutral, thus only interact via weak force and gravity;
- interact very weakly and therefore are very hard to detect;
- extremely light (thought to be massless for a long time until the discovery of neutrino oscillations);
- come in three leptonic flavors (e, μ, τ).



Where do neutrinos come from

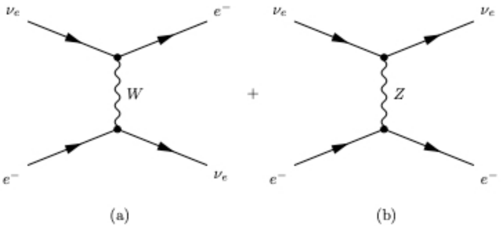


Overview

How do neutrinos scatter

The simplest neutrino interaction are neutrino-lepton scattering. At tree level there are only three ways neutrinos and leptons interact, and these can be splitted into two kinds.

- Neutral current (NC) - corresponding to the exchange of a Z^0 boson, image (b);
- Charged current (CC) corresponding to image (a).



Feynman diagrams for tree level ν_e e elastic scattering

Kinematics

Lets consider an inverse decay of the form $\nu_\ell + e \to \ell + \nu_e$ where $\ell = e, \mu, \tau$. In the lab reference frame (target electron rest frame) the 4-momentum of the particles are:

$$k = \left(E_\nu, \vec{k}\right) \;, \quad p_e = \left(m_e, \vec{0}\right) \;;$$
$$k_\ell = (E_\ell, \vec{p}) \;, \quad k' = \left(E'_\nu, \vec{k}'\right) \;.$$

Energy momentum conservation allows us to construct scalars (Lorentz invariants):

$s = (k + p_e)^2$	Mandelstam variable s, or centre of mass energy
$y = \frac{p_e \cdot q}{p_e \cdot k} = \frac{E_\nu - E_\ell}{E_\nu}$	Inelasticity parameter
$Q^2 = -q^2 = (k - k_l)^2$	Transfered 4-momentum

Kinematical constraints

From the conservation of the 0-th component of the 4-momentum comes:

$$E_\ell = E_\nu - E'_\nu + m_\ell \,.$$

The invariant transferred momentum is:

$$\begin{aligned} Q^2 &= \left(k' - k\right)^2 = -2E_\nu E'_\nu + 2\left|\mathrm{k}\right|\left|\mathrm{k}'\right|\cos\theta_\nu \\ &= \left(p_e - k_\ell\right)^2 = m_e^2 + m_\ell^2 - 2m_e E_\ell \,. \end{aligned}$$

Now neglecting the neutrino mass $p_\nu^2 \simeq 0 \implies E_\nu^{(\prime)} \simeq |\vec{k}^{(\prime)}|$ we obtain the following equation for the neutrino scattering angle θ_ν

$$2E_\nu E'_\nu\left(\cos\theta_\nu - 1\right) = m_e^2 + m_\ell^2 - 2m_e E_\ell$$

Rearranging the above equation the limits for the outgoing neutrino energy follow as

$$\frac{m_e E_\nu}{m_e + 2E_\nu} + \frac{m_e^2 - m_\ell^2}{2\left(m_e + 2E_\nu\right)} \leq E'_\nu \leq E_\nu + \frac{m_e^2 - m_\ell^2}{2m_e}$$

Kinematical constraints

Neutrinos are very hard to detect so E'_ν is not a very useful variable from an experimental point of view. The useful quantity is the scattered lepton energy. Using $E_\ell = E_\nu - E'_\nu + m_\ell$ in the previous equation yields

$$m_e + \frac{m_l^2 - m_e^2}{2m_e} \leq E_\ell \leq m_e + \frac{2E_\nu^2}{m_e + 2E_\nu} + \frac{m_\ell^2 - m_e^2}{2\left(m + 2E_\nu\right)}$$

When $\ell = e$ and using only the kinetic energy $T = E_e - m_e$ which will be the interest from here on out this simplifies to

$$\frac{m_l^2 - m_e^2}{2m_e} \leq T \leq \frac{2E_\nu^2}{m_e + 2E_\nu} + \frac{m_\ell^2 - m_e^2}{2\left(m + 2E_\nu\right)}$$

Scattering angle

A similar reasoning to the one used to express the neutrino scattering angle can be used to determine the charged lepton scattering angle, θ_ℓ, corresponding to the angle between the outgoing charged lepton direction and the incoming neutrino beam. From the conservation of 4-momentum:

$$(k - k_e)^2 = \left(p_e - k'\right)^2$$
$$E_\nu \left|\mathrm{k}_e\right| \cos\theta_e = E_\nu E_e - m_e E'_\nu \, .$$

Rearranging yields

$$\cos\theta_e = \frac{m_e + E_\nu}{E_\nu}\sqrt{\frac{E_e - m_e}{E_e + m_e}} \, .$$

$0 < \cos\theta_e < 1 \implies$ the electron is always scattered in the forward direction.

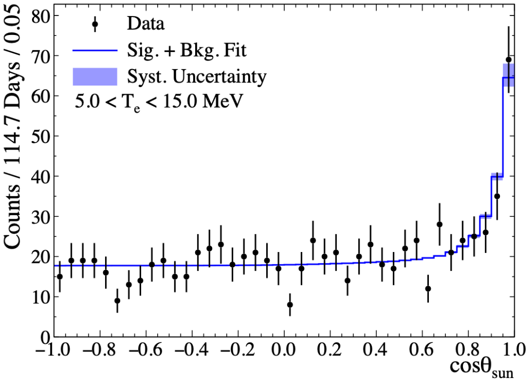
Scattering angle

Considering the small angle approximation and assuming that the outgoing electron's energy is much larger than its mass we get

$$E_e\theta_e^2 = 2m_e y \, .$$

Since $0 \leq y \leq 1$ we obtain an important experimental constraint $E_e\theta_e^2 < 2m_e$. This means the outgoing electron is projected in extremely forward directions, which is used experimentally to subtract background events, with a much broader distribution for $T_e\theta_e^2$.

Scattering angle



Experimental data from solar neutrinos in SNO+. The vertical axis corresponds to the number of detected electrons while the horizontal axis corresponds to the angle between the reconstructed electron trajectory and the direction of the incident neutrinos.

Overview

S matrix

The differential cross section for two body collisions between a neutrino and a stationary electron is given by:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}q^2} = \frac{1}{16\pi} \frac{\left|\mathcal{M}^2\right|}{\left(s-(m_e+m_\nu)^2\right)\left(s-(m_e-m_\nu)^2\right)} .$$

Here $\mathcal{M}$ are the matrix elements for the interaction, derived from the S matrix. Neutrino masses can be neglected to a very good approximation.

Differential cross section

The leading order contribution for the differential cross section for neutrino-lepton elastic scattering is given by:

$$\frac{\mathrm{d}\sigma_{\mathrm{LO}}^{\nu_\ell \ell' \to \nu_\ell \ell'}}{\mathrm{d}E'_\nu} = \frac{m_\ell}{4\pi} \left[\left(c_{\mathrm{L}}^{\nu \ell \ell'} \right)^2 \mathrm{I_L} + c_{\mathrm{R}}^2 \mathrm{I_R} + c_{\mathrm{L}}^{\nu_\ell \ell'} c_{\mathrm{R}} \mathrm{I_R^L} \right] ,$$

Here the c's come from the interaction Lagrangian and the I's are kinematical factors.

$$c_{\mathrm{L}}^{\nu_\ell \ell'} = 2\sqrt{2} G_{\mathrm{F}} \left(\sin^2\theta_W - \frac{1}{2} + \delta_{\ell\ell'} \right) , \quad c_{\mathrm{R}} = 2\sqrt{2} \mathrm{G_F} \sin^2\theta_W$$

$$\mathrm{I_L} = 1 \, , \quad \mathrm{I_R} = \frac{E_\nu'^2}{E_\nu^2} \, , \quad \mathrm{I_R^L} = -\frac{m_\ell}{E_\nu} \left(1 - \frac{E'_\nu}{E_\nu} \right)$$

Total cross section

When considering neutrino electron scattering the cross section is much larger when the scattering occurs with electron neutrinos, due to the possibility of interaction via the charged channel, so other contributions will be neglected. The differential cross section for this scattering is obtained noticing that $\mathrm{d}E'_\nu = \mathrm{d}T$ so the only difference is on the kinematical factors:

$$\mathrm{I}_{\mathrm{R}} = \frac{(E_\nu - T)^2}{E_\nu^2}\,, \quad \mathrm{I}_{\mathrm{R}}^{\mathrm{L}} = m_\ell \frac{T}{E_\nu^2}$$

Total cross section

The outgoing electron's energy is restricted by the kinematics of the collision to by:

$$m_e \leq E_e \leq m_e + \frac{2E_\nu^2}{m_e + 2E_\nu} \, .$$

So the limits of the integral are:

$$0 \leq T \leq T_{\text{max}} = \frac{2E_\nu^2}{m_e + 2E_\nu} \, .$$

Performing the integral:

$$\begin{aligned}\sigma =& \frac{4G_F^2 m_e}{\pi} \left\{ \left(\sin^2\theta_W + \frac{1}{2} \right)^2 \frac{2E_\nu^2}{m_e + 2E_\nu} + \frac{1}{3E_\nu^2} \left[E_\nu^3 - \left(\frac{m_e E_\nu}{m_e + 2E_\nu} \right)^3 \right] \right. \\ & \left. + \left(\sin^2\theta_W + \frac{1}{2} \right) \frac{2m_e E_\nu^2}{(m_e + 2E_\nu)^2} \right\}\end{aligned}$$

Total cross section - units

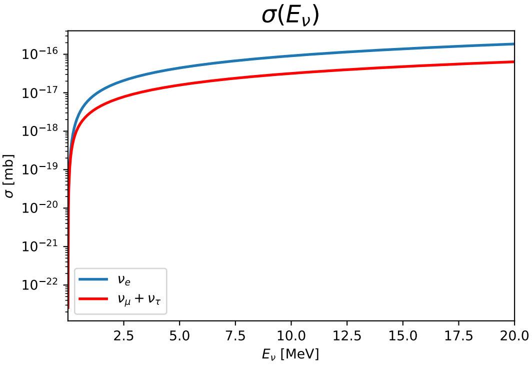
Recalling the total cross section:

$$\sigma = \frac{4G_F^2 m_e}{\pi}\left\{\left(\sin^2\theta_W + \frac{1}{2}\right)^2 \frac{2E_\nu^2}{m_e + 2E_\nu} + \frac{1}{3E_\nu^2}\left[E_\nu^3 - \left(\frac{m_e E_\nu}{m_e + 2E_\nu}\right)^3\right]\right.$$
$$\left. + \left(\sin^2\theta_W + \frac{1}{2}\right)\frac{2m_e E_\nu^2}{(m_e + 2E_\nu)^2}\right\}$$

It is clear to see that in the limit of high energy neutrinos when compared to the electron's mass, $E_\nu \gg m_e$ the cross section is linear with the incident neutrino energy. Here the cross section is in units of $\mathrm{MeV}^{-2}(\hbar c)^2 = 0.3894 \times 10^6\,\mathrm{mb}$.

Plot

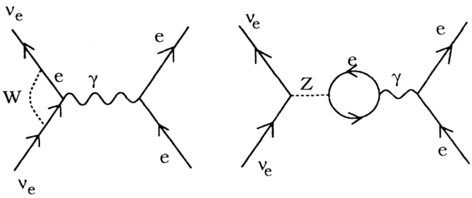
Plotting this cross section as a function of the neutrino energy yields:

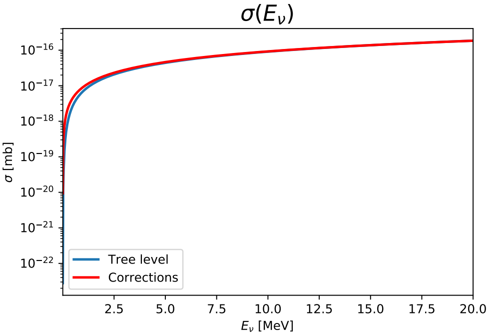


The total cross section for $e + \nu_e \to e + \nu_e$ scattering

This is a very small cross section $1\,\mathrm{mb} = 10^{-27}\,\mathrm{cm}^2$, for comparison *pp* cross sections at LHC are of in the range $40 - 60\,\mathrm{mb}$

This is just the beginning





Overview

Neutrino magnetic dipole moment

If neutrinos are Dirac particles they will have, albeit tiny, magnetic dipole moments (and even smaller electric dipole moments). In terms of electron Bohr magnetons, $e/2m_e$, one finds the SM prediction

$$
\begin{aligned}
\vec{\mu}_{\nu_i} &= \frac{e}{m_e} \kappa_{\nu_i} \vec{S} \\
\kappa_{\nu_i} &= \frac{3}{4} \frac{G_F m_e m_{\nu_i}}{\sqrt{2}\pi^2} \simeq 3 \times 10^{-19} \left(m_{\nu_i}/eV \right) .
\end{aligned}
$$

Since m_{ν_i} are expected to be $< 0.005\,\mathrm{eV}$, neutrino dipole moments appear to be unobservable in the SM. However, in some left-right symmetric models or extended Higgs models, it is possible to have much larger dipole moments. It is, therefore, of interest to ask what direct experimental bounds can be placed on neutrino dipole moments (magnetic, electric, or transition), independent of theory.

Neutrino magnetic dipole moment

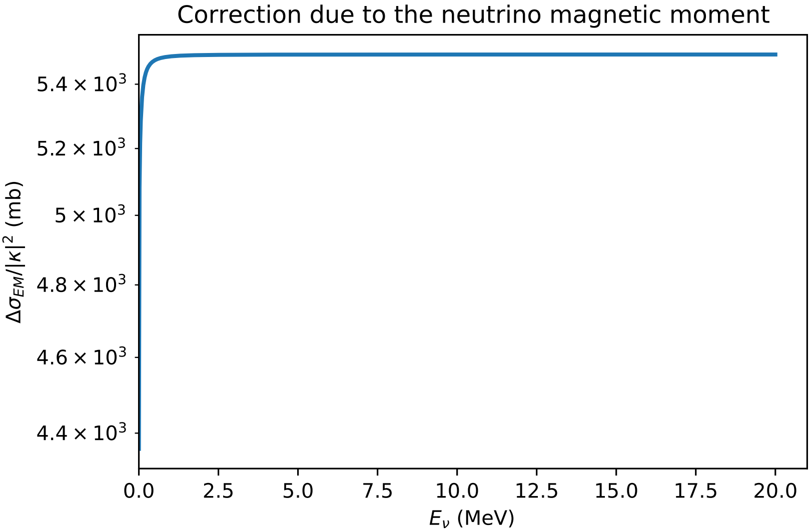
The existence of any neutrino dipole moment (magnetic, electric or transition) of magnitude $\kappa e/2m_e$ will increase νe cross-sections by

$$\Delta \frac{d\sigma}{dy} = |\kappa|^2 \frac{\pi\alpha^2}{m_e^2} \left(\frac{1}{y} - 1 \right) .$$

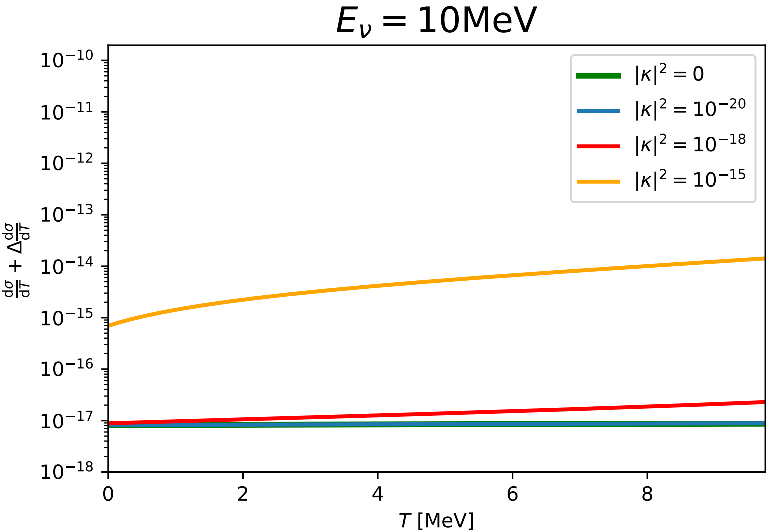
Therefore a larger than expected cross-section, particularly one exhibiting the 1/y behavior could be taken as evidence for a non-vanishing κ. Integrating the equation with the appropriate limits yields a total correction:

$$\Delta\sigma = |\kappa|^2 \frac{\pi\alpha^2}{m_e^2} \left(\ln \left(\frac{y_{\mathrm{max}}}{y_{\mathrm{min}}} \right) - y_{\mathrm{max}} \right) .$$

Cross section with magnetic moment



Differential cross section with magnetic moment



Overview

What's next?

- ~~Implement a neutrino-electron elastic scattering cross section~~;
- ~~Introduce different calculations of radiative corrections to permit studies of additional physics topics~~;
- Study the sensibility of these studies in the context of the SNO+ neutrino experiment.

The end