Gluon interactions from **Lattice Quantum Chromodynamics**

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Abstract

Lattice Quantum Chromodynamics (LQCD) allows for non-perturbative first principles calculations by introducing a finite space-time lattice. The main focus of my research is the computation of the gluon self interaction vertices, namely the propagator and the three and four gluon vertices, using 80^4 and 64^4 lattices with 550 and 2000 gauge field configurations, respectively. The correct tensor description in the lattice is studied using the gluon propagator. In this work it is also shown how it is possible to correct for some lattice artifacts coming from the discretization procedure, which breaks some symmetries of the theory. All the work is done with disregard for the quark fields - we are working in the pure Yang-Mills formulation.

Introduction and Motivation

The theory describing Quarks and Gluons is called Quantum Chromodynamics (QCD). The dynamics is determined by its Lagrangian that, in the boson sector, is given by

$$\mathcal{L}_{\text{QCD}} = -\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} \tag{1}$$

where A^a_μ represents the gluon fields and $F^a_{\mu\nu}$ the field-strength tensor,

$$F^a_{\mu\nu} = (\partial_\mu A^a_\nu - \partial_\nu A^a_\mu) + g f^{abc} A^b_\mu A^c_\nu. \tag{2}$$

Physical results, like correlation functions, are obtained in the path integral quantization from the expectation values of the type,

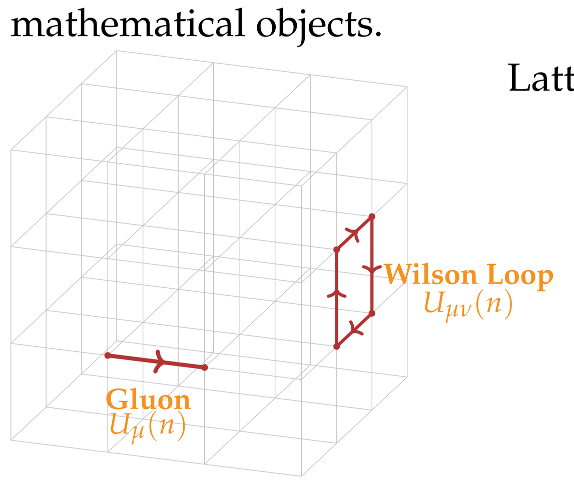
$$\langle \mathcal{O} \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}A_\mu \mathcal{O} e^{-\int d^4x \mathcal{L}} \tag{3}$$

using the generating functional/partition function $\mathcal{Z}$ in the Euclidean space-time. This formulation, in combination with the Euclidean metric creates the correspondence between a quantum field theory and a statistical mechanical problem - allowing for the use of computational methods associated with the latter .

In LQCD expectation values are simply obtained with,

$$\langle \mathcal{O} \rangle \sim \frac{1}{N} \sum_{\substack{U_n \text{with} \\ \text{probability} \\ e^{-S[U_n]}}} \mathcal{O}[U_N]. \tag{4}$$

In addition, to use the lattice approach it is necessary to find discretized version of the various mathematical objects.



Lattice **Wilson Action** [4], and lattice field

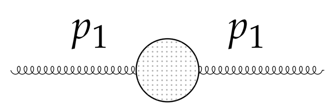
$$S_G[U] = \frac{2}{g^2} \sum_n \sum_{\mu<\nu} \text{Re}\,\text{Tr}\left(\mathbb{1} - U_{\mu\nu}(n)\right) = \frac{a^4}{2g^2} \sum_n \sum_{\mu,\nu} \text{Tr}\left(F_{\mu\nu}(n)^2\right) + \mathcal{O}\left(a^2\right) \tag{5}$$

$$U_\mu(n) = e^{iaA_\mu(n+a\hat{\mu}/2)} + \mathcal{O}(a). \tag{6}$$

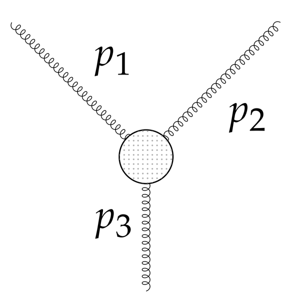
Other than making possible to compute observables for QCD, the lattice formulation makes the theory mathematically well defined since it provides a **natural regularization scheme**. The non-perturbative approach using LQCD permits the study of the low momenta region of the interactions between quarks and gluons and addresses, for example, the problem of why are the fundamental particles of QCD confined within hadrons.

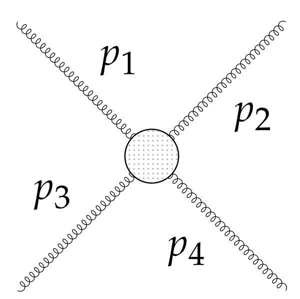
The focus of this work falls on the gluon propagator and self-interaction vertices - 2, 3 and 4 gluon correlation functions:

$$\left\langle A^{a_1}_{\mu_1}(p_1) A^{a_2}_{\mu_2}(p_2) \right\rangle$$



$$\left\langle A^{a_1}_{\mu_1}(p_1) A^{a_2}_{\mu_2}(p_2) A^{a_3}_{\mu_3}(p_3) \right\rangle \qquad \left\langle A^{a_1}_{\mu_1}(p_1) A^{a_2}_{\mu_2}(p_2) A^{a_3}_{\mu_3}(p_3) A^{a_4}_{\mu_4}(p_4) \right\rangle$$





These objects depend on the gauge (we use the **Landau gauge**, $p_\mu A^\mu(p) = 0$), and carry important information about the theory:

- **Building blocks** for a quantum field theory;
- Description on **confinement**;
- Calculation of **bound states** - particles.

The main interest are the 'pure', **amputated vertices** Γ, related to the correlation function by the external full propagators. For the three-gluon interaction the relation is

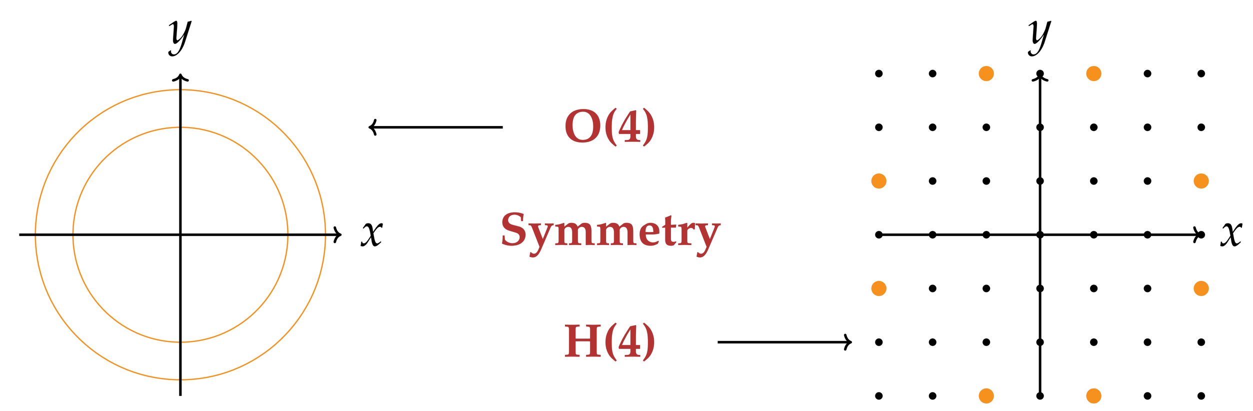
$$\left\langle A^{a_1}_{\mu_1}(p_1) A^{a_2}_{\mu_2}(p_2) A^{a_3}_{\mu_3}(p_3) \right\rangle \propto D_{\mu_1\nu_1}(p_1) D_{\mu_2\nu_2}(p_2) D_{\mu_3\nu_3}(p_3) \Gamma^{a_1a_2a_3}_{\nu_1\nu_2\nu_3}(p_1, p_2, p_3). \tag{7}$$

Method and Lattice Artifacts

The main goal is to obtain the momentum dependence for the correlation functions, being assembled by bosons and respecting gauge symmetry have various symmetry constraints. These constraints reduce the dimension of the tensor basis used to describe the objects through the use of **form factors**.

The description of lattice correlation functions using continuum tensors is not completely suited since these respect continuum symmetries which are broken on the lattice - importance of considering **discretization corrections**.

First we increase the statistics for each momentum configuration by averaging over the complete discrete group symmetry - H(4) - **Z4 Averaging**:



To lessen the systematic errors from the discretization we consider various possible schemes:

Lattice tensors

Continuum tensors are not suited for the description of the correlation functions. We should look for different tensor representations. There are some hints from lattice perturbation theory that allows to substitute the normal momentum p_μ with the lattice version [3],

$$\tilde{p}_\mu = \frac{2}{a} \sin(p_\mu/2). \tag{8}$$

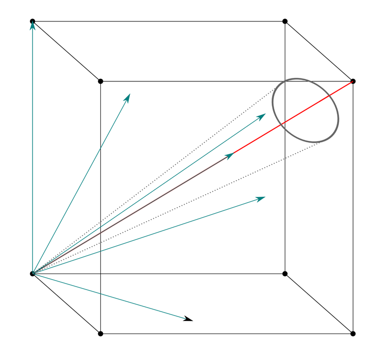
Structures that respect the $H(4)$ symmetry are also considered to construct a full lattice basis

$$p_\mu \to v^n_\nu = p^{2n+1}_\nu, n \in \mathbb{N}$$

$$\delta_{\mu\nu}, \quad p_\mu p_\nu \to d^{n,m}_{\mu\nu} = p^{2n+1}_\mu p^{2m+1}_\nu, n, m \in \mathbb{N}$$

Momentum Cuts

Momenta closer to the diagonal is preferred to reproduce the continuum results - discretization errors are suppressed in the diagonal of the lattice [3]. This method eliminates all data for which the momentum configuration is far from the diagonal. Easily applied to configurations with a single independent momentum scale.



H4 Method

This method considers explicitly the symmetry group invariants which label the different 'orbits' to which each O(4) orbit splits into [2, 1]. The concept of the correction is purely related to the breaking of spherical symmetry. Generally, after the Z4 averaging there are still various values for each momentum p. One can obtain a single, corrected value using the expansion in the group invariants ($p^{[n]}$, $n = 2, 4, 6, 8$),

$$F\left(p^{[n]}\right) \approx F\left(p^2\right) + p^{[4]} \frac{\partial F}{\partial p^{[4]}} \left(p^2, 0\right) + \mathcal{O}\left(a^2\right). \tag{9}$$

Using a linear regression on $p^{[4]}$ it is possible to estimate an $\mathcal{O}\left(a^2\right)$ correction to the discretization.

Results

To evaluate the completeness of a basis, a **reconstruction** procedure is considered

$$\mathcal{R} = \frac{\sum_{\mu,\nu} |D^{\text{orig}}_{\mu\nu}|}{\sum_{\mu,\nu} |D^{\text{rec}}_{\mu\nu}|} \tag{10}$$

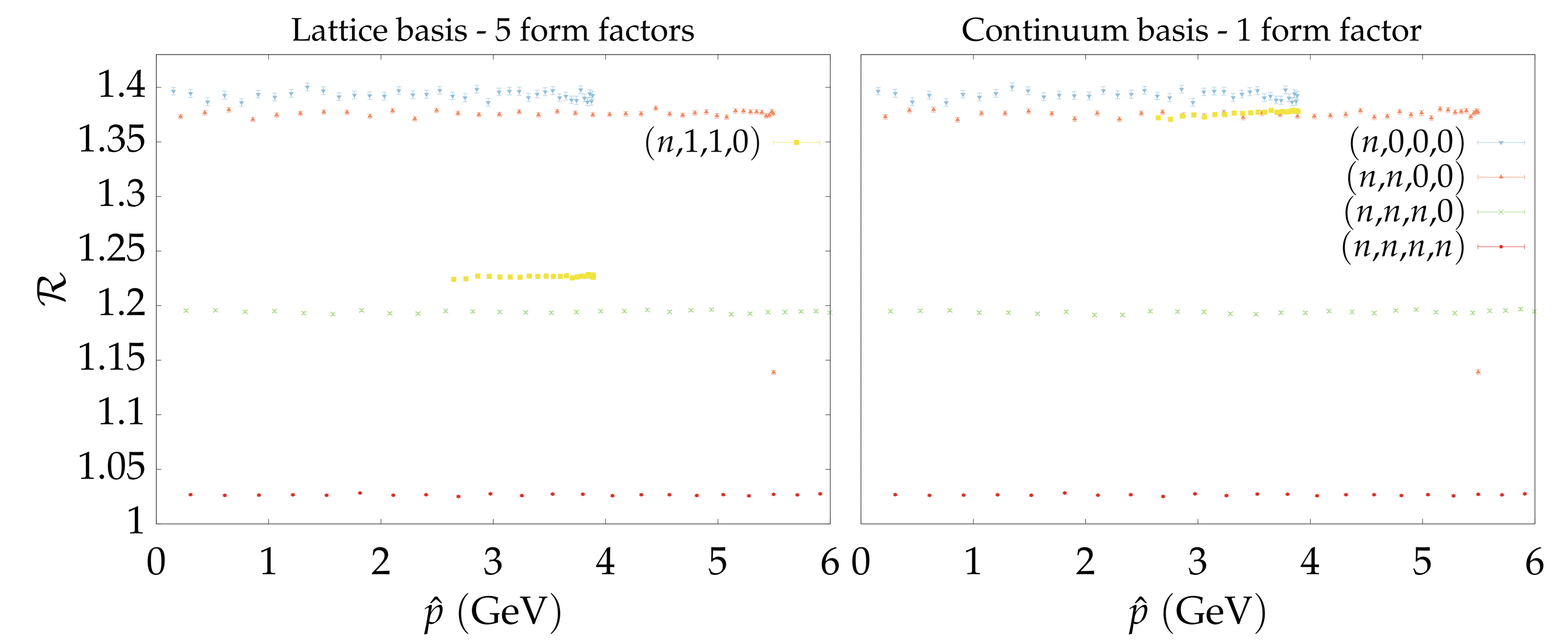


Figure 1: Reconstruction ratio $\mathcal{R}$ for the lattice basis (left) and for the usual continuum basis (right). A kinematic configuration out of the diagonal is also shown for comparison.

The IR behaviour for the propagator and the **three-gluon** 1-PI should be affected by the masslessness of the ghost. For the vertex, the configuration $(p, 0, -p)$ allows to extract a single form factor $\Gamma(p^2)$. The ghost dynamics is reflected in a logarithmic divergence and a *zero-crossing* around ~ 0.2 GeV - see Fig. 2.

For the **Four-gluon** vertex, it is important to select the kinematics that simplify the tensor structure and also remove the disconnected parts of the correlation function.

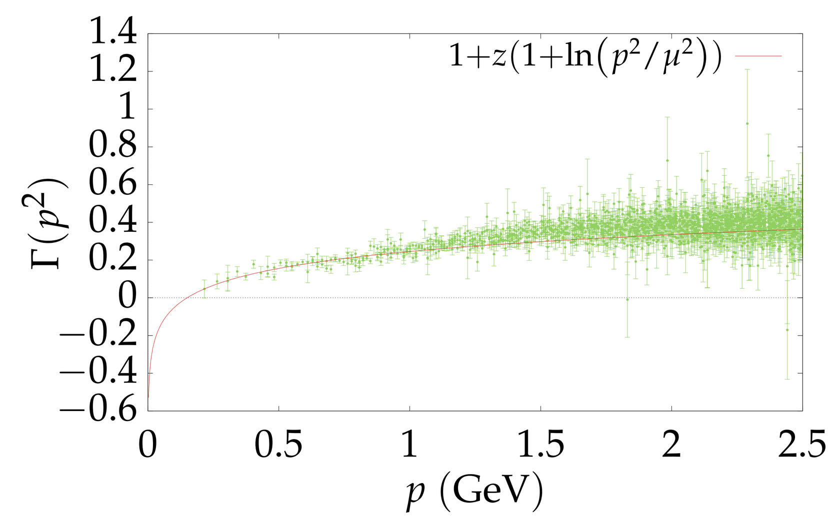
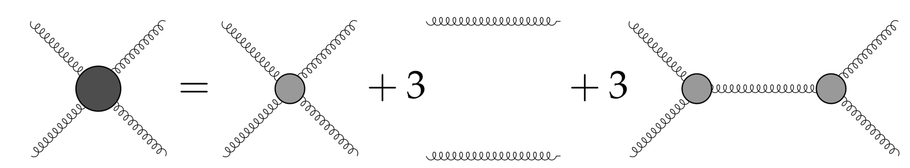


Figure 2: Three-gluon 1-PI function for low momentum. The expected logarithmic divergence is shown at red.



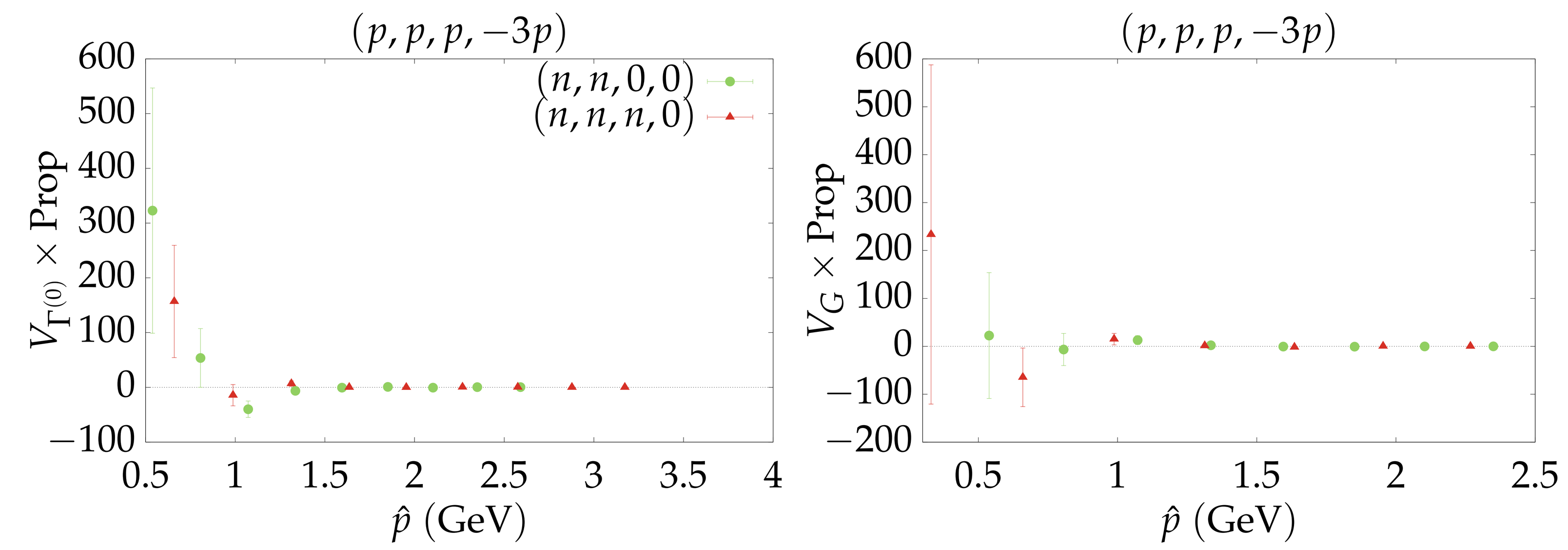


Figure 3: Four gluon form factors corresponding to the tree-level structure (left) and to the tensor $G^{a_1a_2a_3a_4}_{\mu_1\mu_2\mu_3\mu_4}$ (right) with external propagators.

References

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