Analytic Structure of the Gluon Propagator

Alexandre Falcão†, Orlando Oliveira†, Paulo Silva†
†Universidade de Coimbra

Abstract

In a quantum filed theory, the determination of the analytic structure of the propagators, i.e., the position of poles and branch cuts for complex momenta, can be obtained within the perturbative solution of the theory. The analytic structure has information on the properties of the associated quanta and, particurarly, if they are or not confined particles. In Quantum Chromodynamics the computation of the complete propagators resorts in numerical calculations and, typically, only a limited range of momenta is obtained. Herein, we use Padé approximates to explore the analytic structure of the gluon propagator as computed in lattice QCD.

Inroduction

The Quantum Field Theory, gives us all the tools to calculate the propagator of the force carrier quanta of the Quantum Chromodynamics, the gluon. However, we can only solve the equations and have an analytic solution in the perturbation theory. Typically, the complete propagator $D(p^2)$ is known on a discrete set of momuentum and it is difficult to extract its analytic structure directly.

A Padé approximant is a very useful tool to investigate the analytic structure of a function. In our case, we approximate

$$D(p^2) \approx P_N^M(x) = \frac{Q_M(x)}{R_N(x)}, \tag{1}$$

where $Q_M(x)$ and $R_N(x)$ are polynomials of order M and N, respectively. The poles of P_N^M are identified with the roots of R_N.

Testing the Padé approximants

To make sure the complete gluon propagator is well described by a Padé approximant, we first test for the perturbative result

$$D(p^2) = \frac{1}{p^2}\left[\frac{11 N_f \alpha_s}{12\pi}\log\left(\frac{p^2}{\Lambda^2}\right)+1\right]^{-\gamma}, \tag{2}$$

using $\alpha_s = 0.3837$, $\Lambda = 0.425$ GeV and $\gamma = 13/22$. This function has a branch cut on the negative part of the real axis of the Argand plane (due to the logarithm), and a simple pole at $p^2 = 0$ (Figure 2).

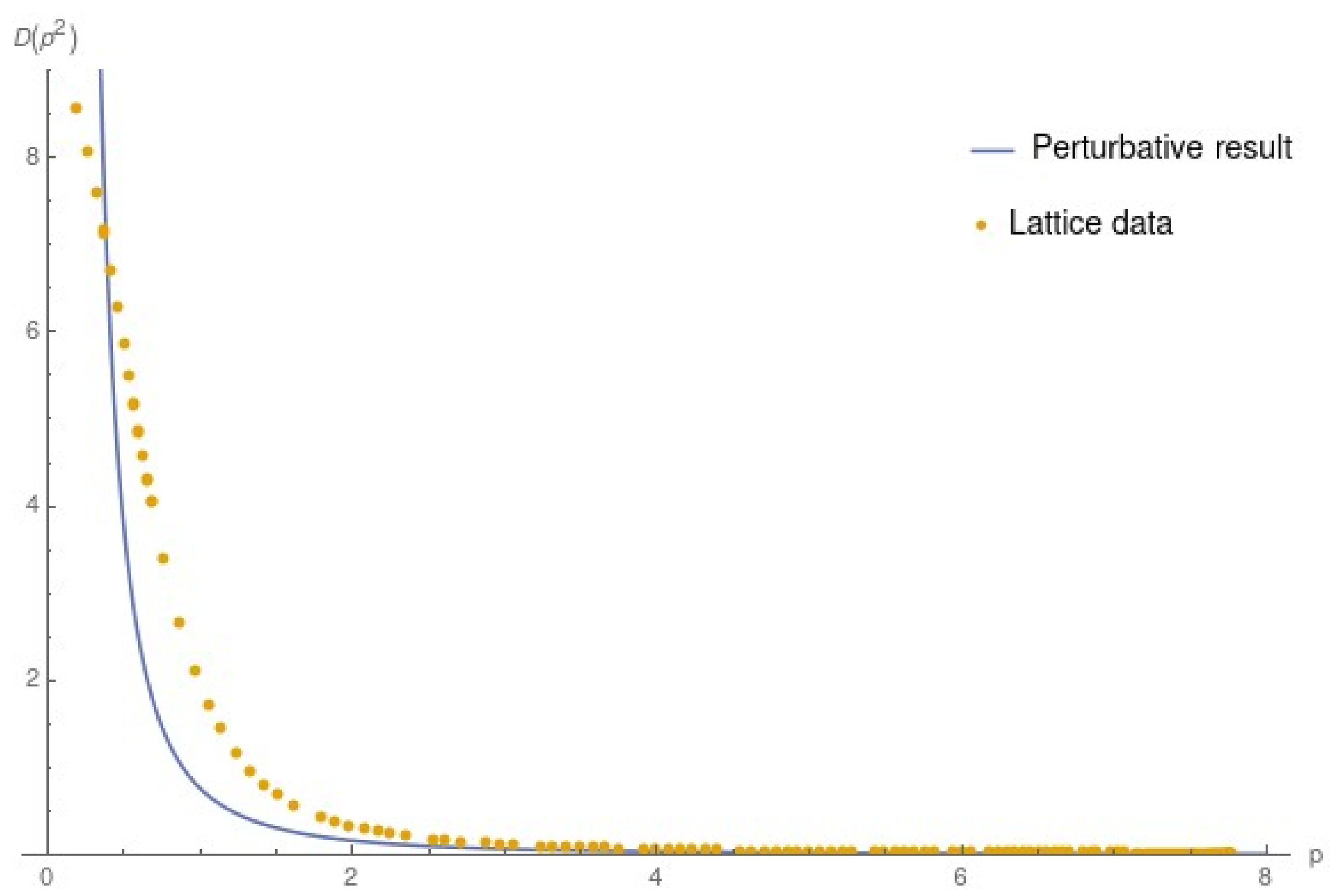

Figure 1: Representation of the gluon propagator obtained from lattice QCD and its perturbative result.

The Padé approximant P_N^M can be constructed from the first $M+N+1$ coefficients of the Taylor series of $D(p^2)$. Once P_N^M is determined, the denominator's roots are calculated, and the important poles are sifted from the rest (based on their residues). The computed analytic structure is then compared to the original one. In Figure 2 we can see that the analytic structure is well reproduced.

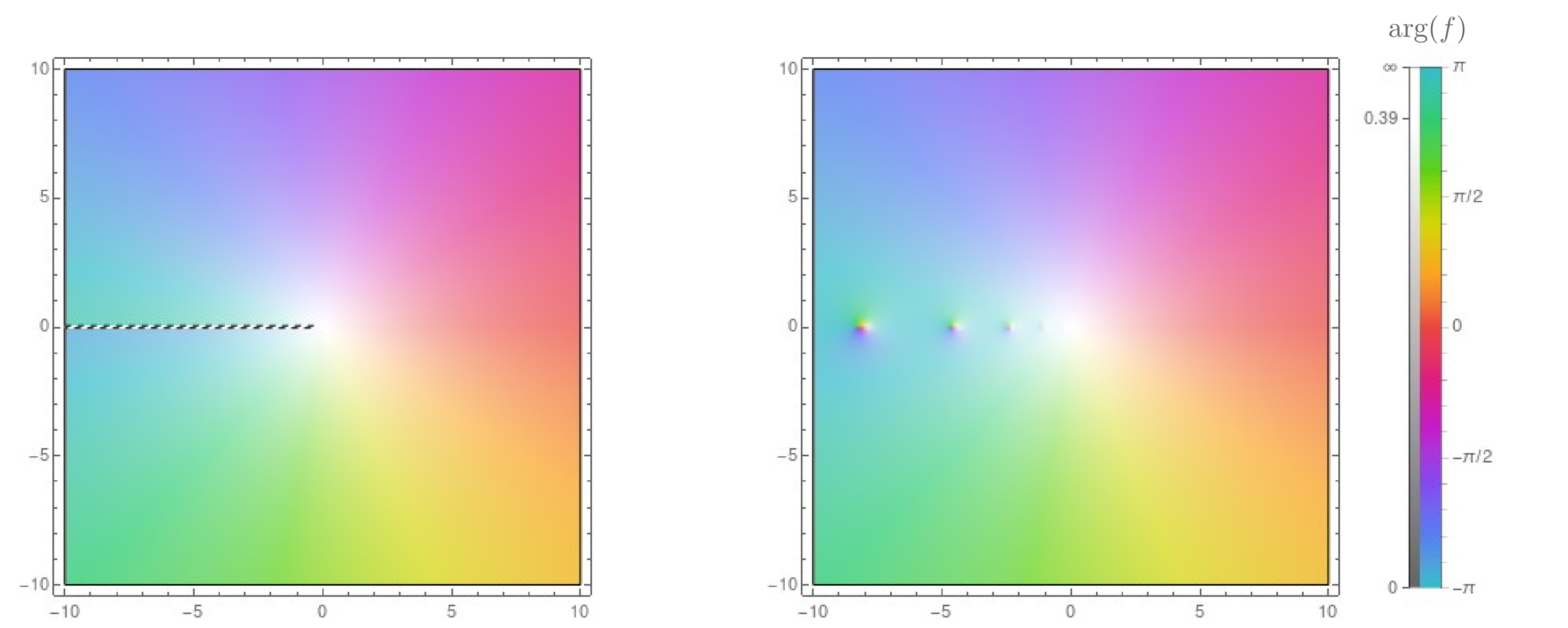

Figure 2: *Left:* Analytic structure of the perturbative result for the gluon propagator. *Right:* The analytic structure of P_{51}^{50}.

Looking at the lattice data

For a discrete set of data points, it is not possible to construct P_N^M from a Taylor series. One way to build a Padé approximant is to minimize a suitable reduced chi squared ($\tilde{\chi}^2$) that measures the deviation of the lattice data to the Padé approximant. In the minimization we use the Differential Evolution method, that gives the results in Figure 3.

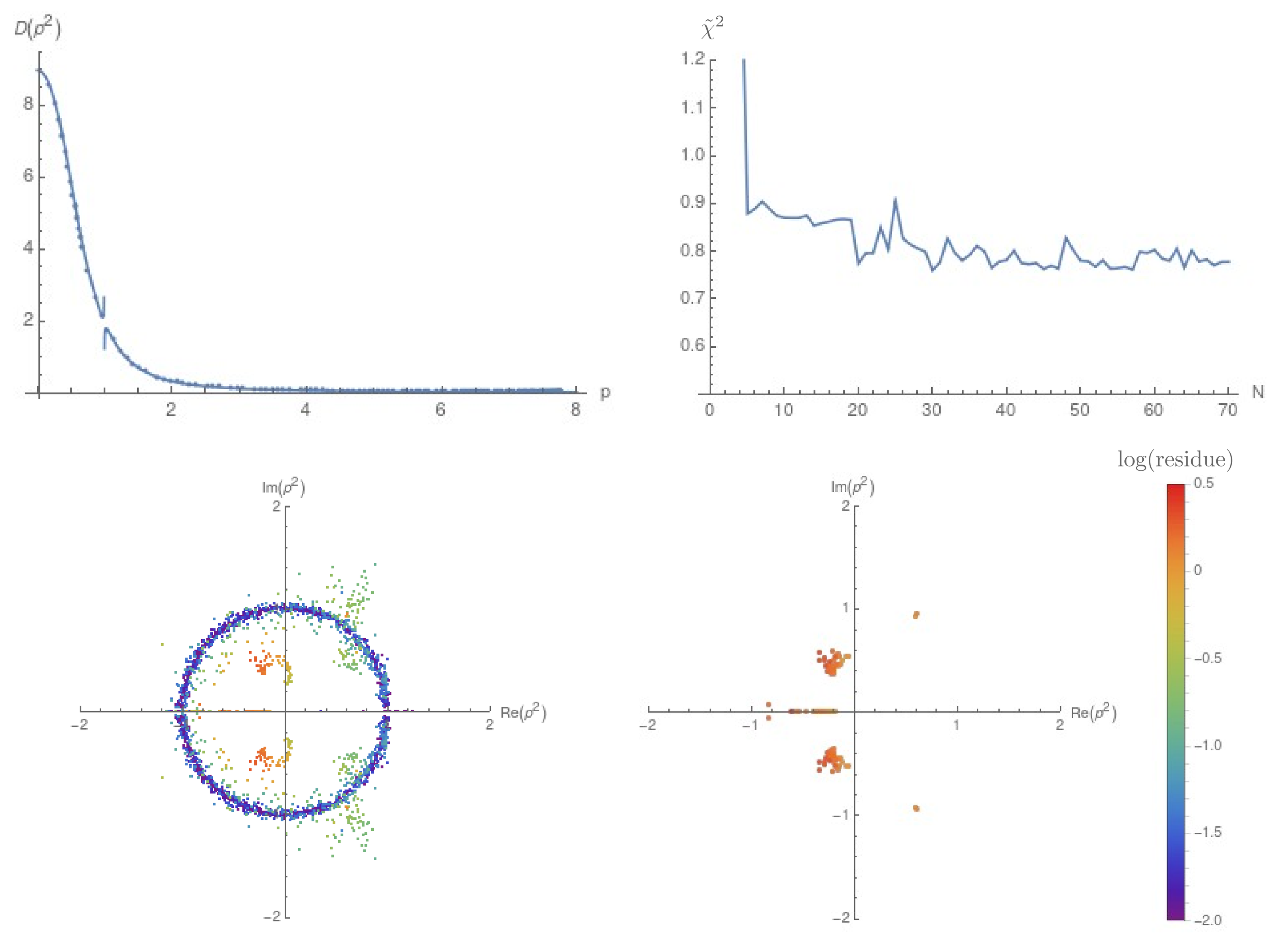

Figure 3: *Top left:* Representation of the best Padé approximant of the form P_{41}^{40} found for the lattice data. *Top right:* Evolution of the $\tilde{\chi}^2$ obtained for the minimizations of P_N^{N-1} with N from 1 to 70. *Bottom left and right:* Overlap of the structure of poles for all P_N^{N-1}, from $N=1$ to $N=70$. The color code shows that the bluish ring is an artifact of the method. By representing just the poles with a residue higher than 1, we can identify a possible branch cut as well as two poles at the left side of the complex plane.

Conclusion

Despite its simplicity, the method above has shown to be surprisingly good in the reproduction of the analytic structure. Nevertheless, other methods must be tried in order to verify the stability of the analytic structure of the gluon propagator. We also intend to investigate other propagators, such as the ghost propagator.

Bibliography

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