Semilinear wave model for critical collapse

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grit gravitation in técnico

The story begins

Weak Cosmic Censorship

Critical phenomena in General Relativity (GR) [1]

What are the critical phenomena?

Universal

Self-similar behaviour

Power law behaviour near the threshold

[1] Phys. Rev. Lett. 70,

What do we want?

Structural requirements on the non linearities

The extent to which nearby threshold solutions might display critical behaviour

How do we do it?

Construct *simple* models that qualitatively capture this behaviour near threshold blow-up solutions

How to obtain models

$$\Box\varphi = 0 \qquad \phi = D[\varphi]$$

$$\Box\phi - \frac{\partial_\varphi^2 D}{(\partial_\varphi D)^2}\nabla_a\phi\nabla^a\phi = 0$$

Model example

$$\Box\phi + \frac{\phi - \sqrt{1-\phi^2}}{1-\phi^2}\nabla_a\phi\nabla^a\phi = 0$$

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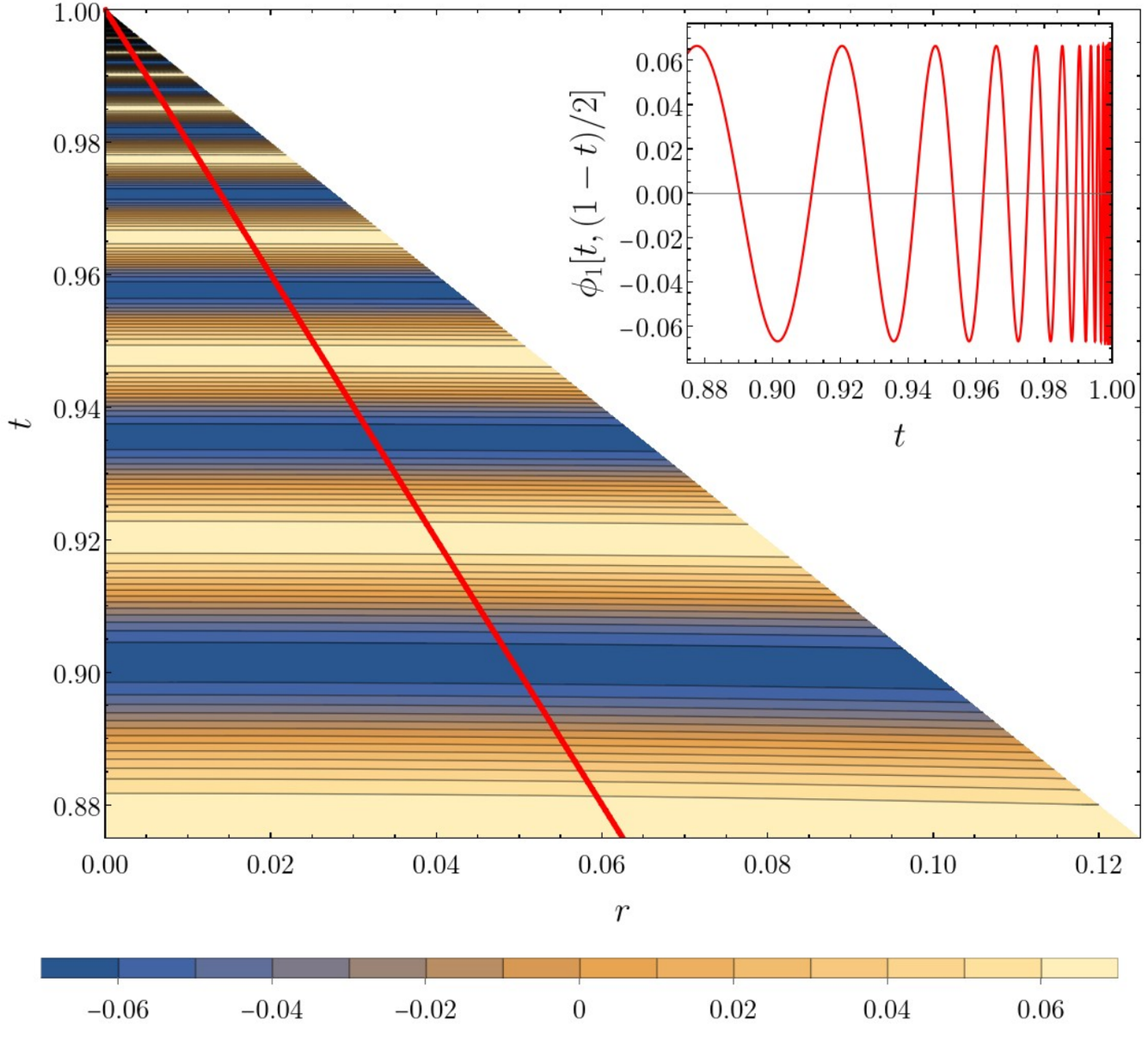
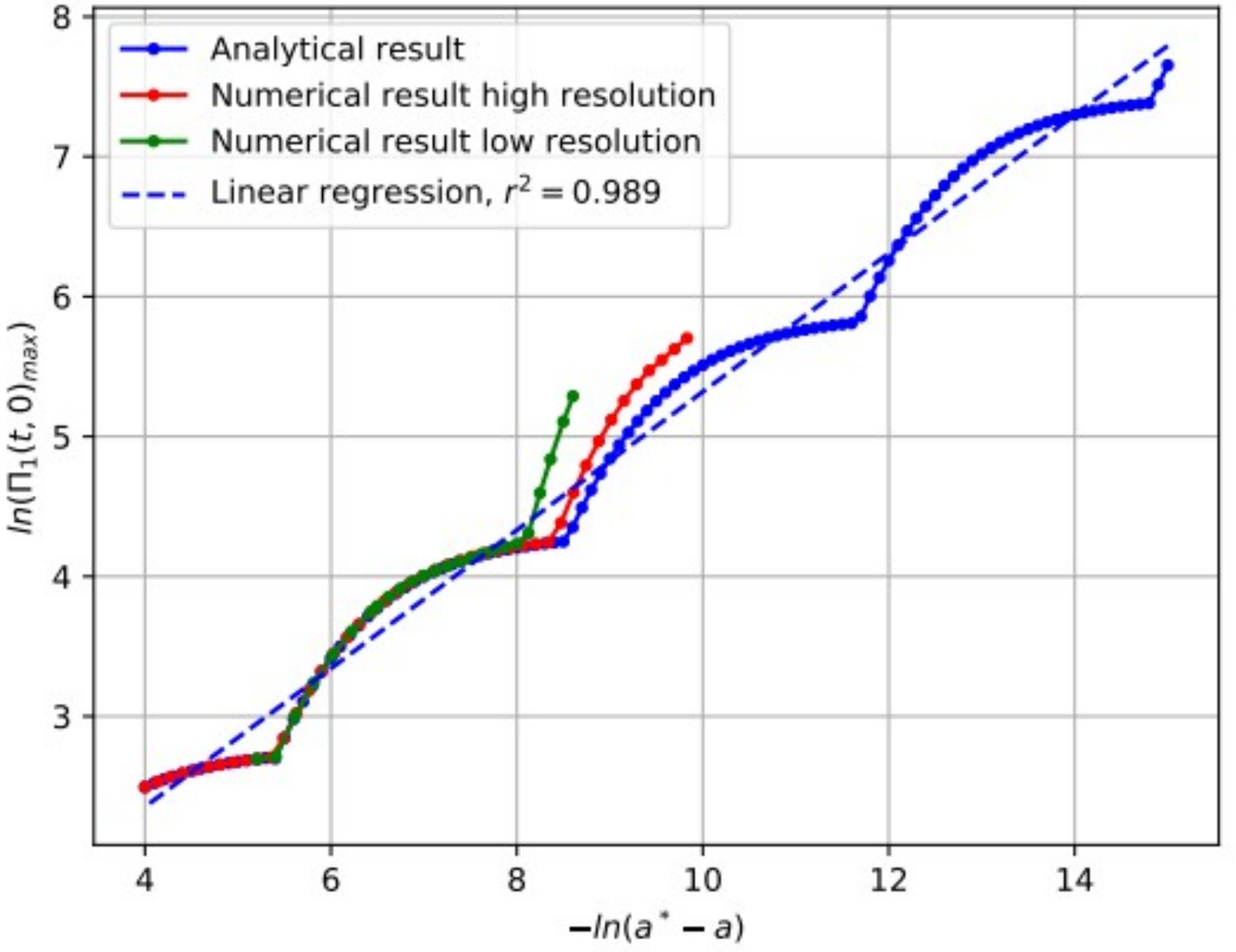


$$\Box\phi + \frac{\phi - \sqrt{1-\phi^2}}{1-\phi^2}\nabla_a\phi\nabla^a\phi = 0$$



What happens if we deviate from spherical symmetry?

$$\Box\varphi = -\partial_t^2\varphi + \partial_r^2\varphi + \frac{2}{r}\partial_r\varphi + \Delta\varphi = 0$$

$$\varphi = \sum_{l=0}^{\infty}\sum_{m=-l}^{l} \varphi_{lm}(t,r)Y_{lm}(\theta^A)$$

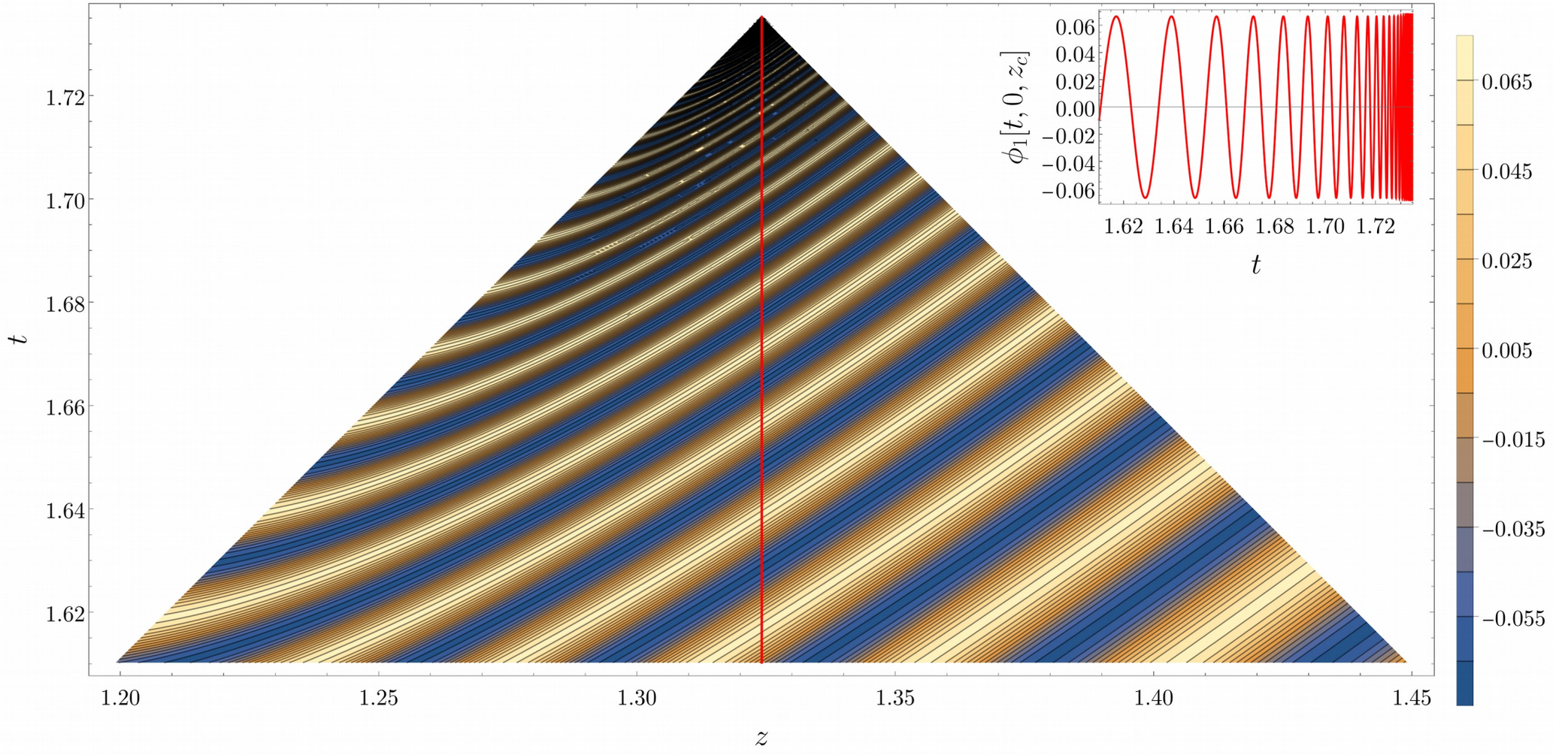
Partial waves

$$-\partial_t^2 \varphi_{lm} + \partial_r^2 \varphi_{lm} + \frac{2}{r}\varphi_{lm} - \frac{l(l+1)}{r^2}\varphi_{lm} = 0$$

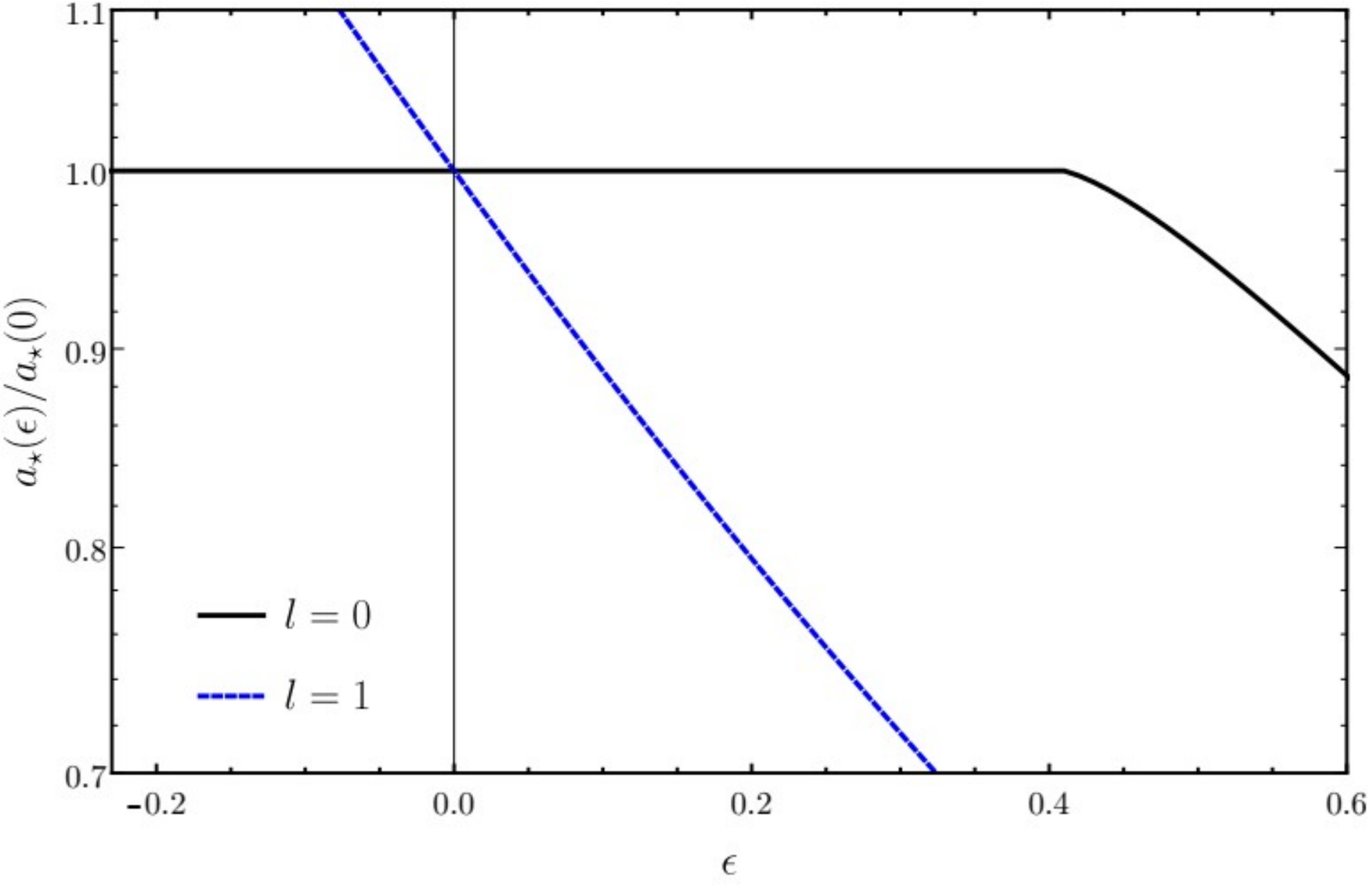
$$\sum_{k=0}^{l} \frac{(k+l)!}{2^k k!(l-k)!} \frac{1}{r^{k+1}} [F^{l-k}(u) - (-1)^{l-k} F^{l-k}(v)]$$

$$u = t - r$$
$$v = t + r$$

$$l = 1, m = 0, \varphi_{10}$$


1.72
1.70
1.68
1.66
1.64
1.62
t
1.20
1.25
1.30
1.35
1.40
1.45
z
0.06
0.04
0.02
0.00
−0.02
−0.04
−0.06
$\phi_1[t, 0, z_c]$
1.62 1.64 1.66 1.68 1.70 1.72
t
0.065
0.045
0.025
0.005
−0.015
−0.035
−0.055

$$\phi_a(t,r) = D[a\,\varphi_\star(t,r)] \qquad \tilde{\phi}_a = D[a\,\varphi_\star + \epsilon\,\tilde{\varphi}]$$



Conclusions

First known simple model that shows critical phenomena (including DSS)

Spherical symmetry: Model that captures all the characteristics of critical phenomena

Code: We have a tool to study nonlinear models with non analytic solution

Aspherical deformations: Away from spherical symmetry we would loose universality

Predictions for GR, that are compatible up to now (Work in progress)