Calculations of the nuclear matrix elements for double-β decay: an overview of the present status

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Double-Beta Decay: the Road to Normal Hierarchy Sensitivity

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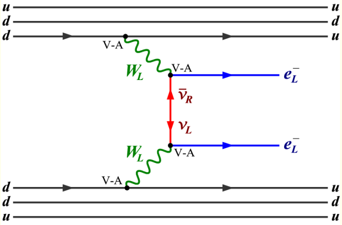
The neutrinoless double β-decay

The detection of the $0\nu\beta\beta$ decay is nowadays one of the main targets in many laboratories all around the world, since its detection would correspond to a violation of the conservation of the leptonic number, and may provide more informations on the nature of the neutrinos and its effective mass



- The inverse of the $0\nu\beta\beta$-decay half-life is proportional to the squared nuclear matrix element $M^{0\nu}$, which relates the parent and grand-daughter wave functions via the decay operator.

$$\left[T_{1/2}^{0\nu}\right]^{-1} = G^{0\nu}\left|M^{0\nu}\right|^2\left|g_A^2\frac{\langle m_\nu\rangle}{m_e}\right|^2$$

- The calculation of $M^{0\nu}$ links $\left[T_{1/2}^{0\nu}\right]^{-1}$ to the neutrino effective mass $\langle m_\nu\rangle = \mid \sum_k m_k U_{ek}^2 \mid$ (light-neutrino exchange)

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The calculation of $M^{0\nu}$

The matrix elements $M^{0\nu}_{\alpha}$ are defined as follows:

$$M^{0\nu}_{\alpha} = \sum_{k} \sum_{j_p j_{p'} j_n j_{n'}} \langle f | a^{\dagger}_{p} a_n | k \rangle \langle k | a^{\dagger}_{p'} a_{n'} | i \rangle \left\langle j_p j_{p'} \mid \tau^{-}_{1} \tau^{-}_{2} \Theta^{k}_{\alpha} \mid j_n j_{n'} \right\rangle$$

with $\alpha = (GT,\ F,\ T)$

$$\begin{aligned} \Theta^{GT}_{12} &= \vec{\sigma}_1 \cdot \vec{\sigma}_2 H_{GT}(r) \\ \Theta^{F}_{12} &= H_F(r) \\ \Theta^{T}_{12} &= [3 (\vec{\sigma}_1 \cdot \hat{r}) (\vec{\sigma}_1 \cdot \hat{r}) \\ &\quad - \vec{\sigma}_1 \cdot \vec{\sigma}_2] H_T(r) \end{aligned}$$

H_{α} depends on the energy of the initial, final, and intermediate states:

$$H_{\alpha}(r) = \frac{2R}{\pi} \int_0^{\infty} \frac{j_{\alpha}(qr) h_{\alpha}(q^2) q dq}{q + E_k - (E_i + E_f)/2}$$

Actually, because of the computational complexity, the energies of the intermediate states are replaced by an average value:

$$E_k - (E_i + E_f)/2 \rightarrow \langle E \rangle$$

$$\sum_{k} \langle f | a^{\dagger}_{p} a_n | k \rangle \langle k | a^{\dagger}_{p'} a_{n'} | i \rangle = \langle f | a^{\dagger}_{p} a_n a^{\dagger}_{p'} a_{n'} | i \rangle$$

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The closure approximation

Consequently, the expression of the neutrino potentials becomes:

$$H_\alpha(r) = \frac{2R}{\pi} \int_0^\infty \frac{j_\alpha(qr) h_\alpha(q^2) q dq}{q + < E >}$$

The matrix elements $M_\alpha^{0\nu}$ are then defined, within the closure approximation, as follows:

$$M_\alpha^{0\nu} = \sum_{j_n j_{n'} j_p j_{p'}} \langle f | a_p^\dagger a_n a_{p'}^\dagger a_{n'} | i \rangle \langle j_p j_{p'} \mid \tau_1^- \tau_2^- \Theta_\alpha \mid j_n j_{n'} \rangle$$

The matrix elements $\langle f | a_p^\dagger a_n a_{p'}^\dagger a_{n'} | i \rangle$ are the two-body transition-density matrix elements, and the Gamow-Teller (GT), Fermi (F), and tensor (T) operators:

The closure approximation works since $q \approx$ 100-200 MeV, while model-space excitation energies $E_{exc} \approx 10$ MeV

Sen'kov and Horoi *(Phys. Rev. C* **88**, *064312 (2013))* have evaluated the non-closure *vs* closure approximation within 10%

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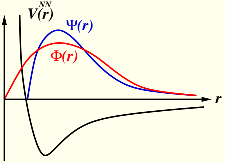
$0\nu\beta\beta$ decay: short-range correlations

Nuclear structure calculations provide non-correlated wave functions Φ, namely Slater determinants of unperturbed basis states, which do not vanish in the short-range region of the nuclear potential



The short-range repulsion of V^{NN} makes the "real" correlated wave function Ψ to approach to zero as the internucleon distance diminishes

Short-range correlations (SRC) are then introduced to soften the $0\nu\beta\beta$ decay operator, consistently with the renormalization of V^{NN}

- Calculation of the defect wave functions (*G*-matrix renormalization)
- Jastrow-type functions (the most popular ones)
- Unitary Correlation Operator Method (UCOM)
- $V_{\mathrm{low-k}}$ unitary transformation

$0\nu\beta\beta$ decay: LO contact transition operator

Within the framework of ChEFT, there is the need to introduce a LO short-range operator, which is missing in standard calculations of $M^{0\nu}$s, to renormalize the operator and make it independent of the ultraviolet regulator

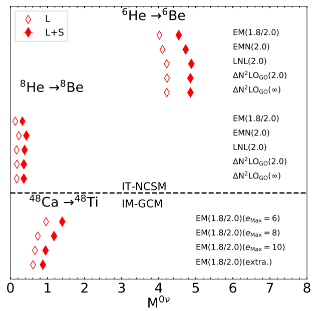
V. Cirigliano et al., Phys. Rev. Lett. **120** *202001 (2020)*

$$M^{0\nu}_{sr} = \frac{1.2 A^{1/3}\,\mathrm{fm}}{g_A^2} \langle 0_f^+ | \sum_{n,m} \tau_m^- \tau_n^- \mathbf{1} \left[\frac{4 g_\nu^{NN}}{\pi} \int j_0(qr)\, f_S(p/\Lambda_S)\, q^2 dq \right] | 0_i^+ \rangle$$

The open question is the determination of the low-energy constant g_ν^{NN}

A recent attempt to fit g_ν^{NN} by computing the transition amplitude of the $nn \rightarrow ppe^- e^-$ process using nuclear NN and NNN interactions has shown that $M^{0\nu}_{sr}$ enlarges the $M^{0\nu}$ for ^{48}Ca $0\nu\beta\beta$ decay

R. Wirth et al., Phys. Rev. Lett. **127** *242502 (2021)*



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Theoretical nuclear structure calculations

The study of the many-body Schrödinger equation, for system with $A > 4$, needs the introduction of truncations and approximations, and follows two main approaches:

Mean-field and collective models

- Energy Density Functional (EDF)
- Quasiparticle Random-Phase Approximation (QRPA)
- Interacting Boson Model IBM

Microscopic approaches

- *ab initio* methods
  - No-Core Shell Model (NCSM)
  - Coupled-Cluster Method (CCM)
  - In-Medium Similarity Renormalization Group (IMSRG)
  - Self-Consistent Green's Function approach (SCGF)
- Nuclear Shell Model

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The many-body problem

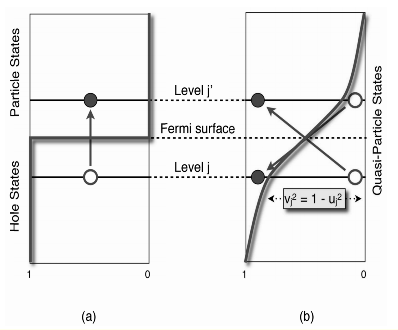
- Mean-field and collective models operate a drastic cut of the nuclear degrees of freedom, the computational problem is alleviated
- Their effective Hamiltonian $H_{\rm eff}$ cannot be derived from realistic nuclear forces and depend from parameters fitted to reproduce a selection of observables
- This reduces the predictive power, that is crucial to search "new physics"

- The degrees of freedom of *ab initio* methods and SM Hamiltonians are the microscopic ones of the single nucleons (very demanding calculations)
- Consequently, they may operate starting from realistic nuclear forces
- These features enhance the predictiveness and the calculated wave functions are more reliable

The QRPA

The quasiparticle random-phase approximation is based on the concept of "pairing" among the nucleons.

Particles are substituted with "quasiparticles".



- Advantage → The dimension of the hamiltonian does not scale rapidly with the mass number A as with the shell model.
- Shortcoming → Results are strongly dependent on the choice of the free renormalization-parameter g_{pp} (g_{ph} is determined from experiment), that is fixed to reproduce both spectroscopy and GT transitions

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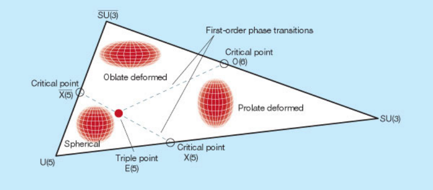
The IBM

In the interacting boson model identical nucleons are paired so to generate bosons:

- $L = 0 \rightarrow s$-boson
- $L = 2 \rightarrow d$-boson

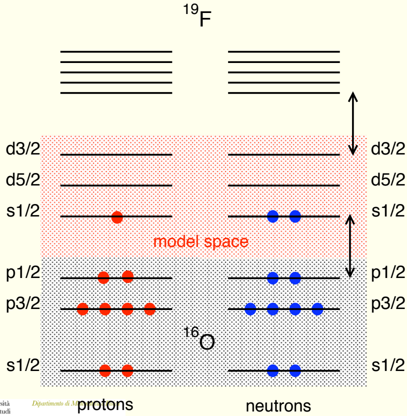


- Advantage $\rightarrow$ The computational complexity is drastically simplified
- Shortcoming $\rightarrow$ The configuration space is strongly reduced

The nuclear shell model

The nucleons are subject to the action of a mean field, that takes into account most of the interaction of the nucleus constituents.

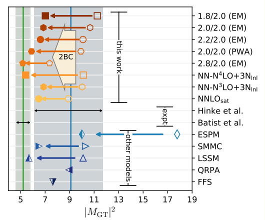
Only valence nucleons interact by way of a residual two-body potential, within a reduced model space.



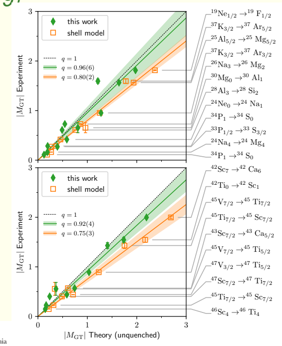
- Advantage → It is a microscopic and flexible model, the degrees of freedom of the valence nucleons are explicitly taken into account.
- Shortcoming → High-degree computational complexity.

Coupled-cluster method CCM and in-medium SRG (IMRSG) calculations have recently performed to overcome the quenching problem g_A to reproduce β-decay observables in heavier systems

P. Gysbers et al., Nat. Phys. **15** *428 (2019)*



Coupled-Cluster Method



In-Medium SRG

- **Advantage** → The degrees of freedom of *all* constituent nucleons are included, the number of correlations among nucleons is enormous
- **Shortcoming** → Highest-degree of computational complexity, the comparison with spectroscopic data is not yet satisfactory

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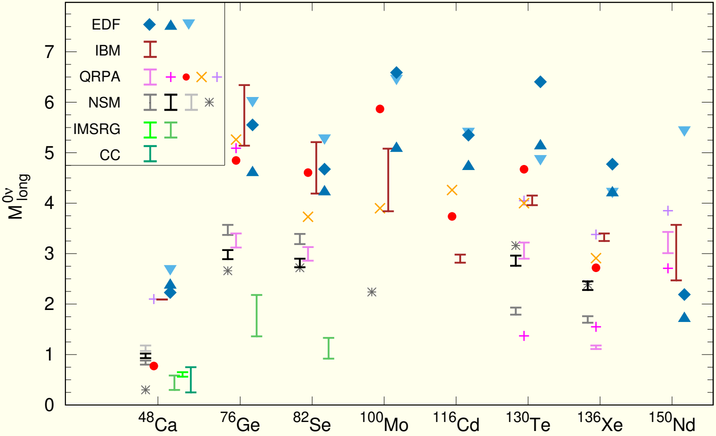
Nuclear structure calculations

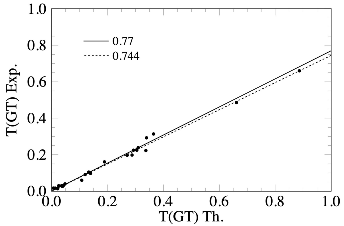


- The spread of nuclear structure calculations evidences inconsistencies among results obtained with different models

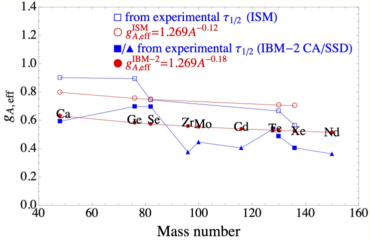
Figure courtesy from Javier Menéndez

The quenching of g_A

A major issue in the calculation of quantities related to GT transitions, is the need to quench the axial coupling constant g_A by a factor q in order to reproduce data.



G. Martínez Pinedo et al., Phys. Rev. C **53**, *R2602 (1996)*



J. Barea, J. Kotila, and F. Iachello, Phys. Rev. C **91**, *034304 (2015)*

The introduction of quenching factor may largely affect the value of the half-life $T_{1/2}^{0\nu}$, since the latter would be enlarged by a factor q^{-4}.

That is why experimentalists are deeply concerned about q, its value has a strong impact on the sensitivity of the experimental apparatus.

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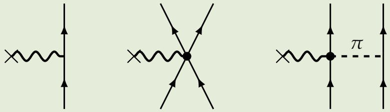
The quenching of g_A

The two main sources of the need of a quenching factor q may be identified as:

Nucleon internal degrees of freedom

Nucleons are not point-like particles ⇒ contributions to the free value of g_A come from two-body meson exchange currents:



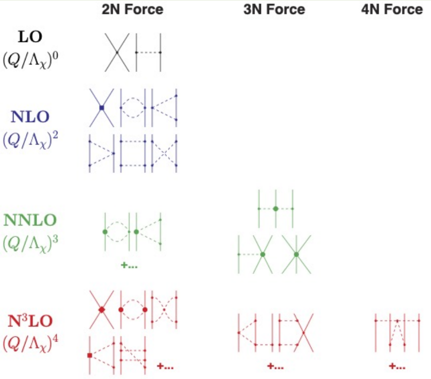
Truncation of the nuclear configurations

Nuclear models operate a cut of the nuclear degrees of freedom in order to diagonalize the nuclear Hamiltonian ⇒ effective Hamiltonians and decay operators must be considered to account for the neglected configurations in the nuclear wave function

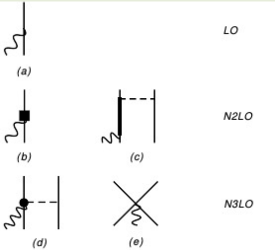
Two-body meson exchange currents

A powerful approach to the derivation of two-body currents (2BC) is to resort to effective field theories (EFT) of quantum chromodynamics.
In such a way, both nuclear potentials and 2BC may be consistently constructed, since in the EFT approach they appear as subleading corrections to the one-body Gamow-Teller (GT) operator $\sigma\tau^+$.

Nuclear Hamiltonian



Two-body currents



The impact of 2BC on the calculated β-decay properties has been investigated in terms of *ab initio* methods

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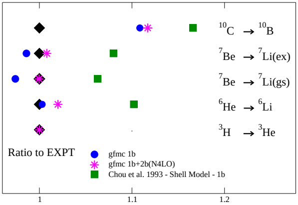
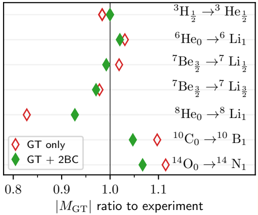
β-decay in light nuclei

GT nuclear matrix elements of the β-decay of p-shell nuclei have been calculated with Green's function Monte Carlo (GFMC) and no-core shell model (NCSM) methods, including contributions from 2BC

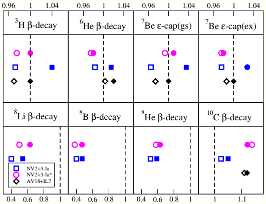


S. Pastore et al., Phys. Rev. C **97** 022501(R) (2018)



P. Gysbers et al., Nat. Phys. **15** 428 (2019)

The contribution of 2BC improves systematically the agreement between theory and experiment



G. B. King et al., Phys. Rev. C **102** 025501 (2020)

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Effective operators for nuclear models

The truncation of the Hilbert space of nuclear configuration implies the need of effective Hamiltonians and decay operators $H_{\rm eff},\Theta_{\rm eff}$
$H_{\rm eff},\Theta_{\rm eff}$ must take into account all the degrees of freedom not explicitly considered in the truncated model space

Two alternative approaches

- phenomenological: the parameters defining $H_{\rm eff},\Theta_{\rm eff}$ are fitted to data (SP energies, TBME, quenching factors, effective charges, etc.)
- microscopic:

$V_{NN}\ \ (+V_{NNN}) \Rightarrow$ many-body theory $\Rightarrow H_{\rm eff},\Theta_{\rm eff}$

Definition

The eigenvalues and $\Theta_{\rm eff}$ matrix elements of model-space wave functions should be equal to those of the original H,Θ of the full Hilbert-space wave functions, respectively.
This may be provided by a similarity transformation Ω of the full Hilbert-space hamiltonian H onto the effective one $H_{\rm eff}$

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The effective operators for decay amplitudes

- Ψ_α eigenstates of the full Hamiltonian H with eigenvalues E_α
- Φ_α eigenvectors obtained diagonalizing $H_{\rm eff}$ in the model space P and corresponding to the same eigenvalues E_α

$$\Rightarrow |\Phi_\alpha\rangle = P\,|\Psi_\alpha\rangle$$

Obviously, for any decay-operator Θ:

$$\langle\Phi_\alpha|\Theta|\Phi_\beta\rangle \neq \langle\Psi_\alpha|\Theta|\Psi_\beta\rangle$$

We then require an effective operator $\Theta_{\rm eff}$ defined as follows

$$\Theta_{\rm eff} = \sum_{\alpha\beta} |\Phi_\alpha\rangle\,\langle\Psi_\alpha|\Theta|\Psi_\beta\rangle\,\langle\Phi_\beta|$$

Consequently, the matrix elements of $\Theta_{\rm eff}$ are

$$\langle\Phi_\alpha|\Theta_{\rm eff}|\Phi_\beta\rangle = \langle\Psi_\alpha|\Theta|\Psi_\beta\rangle$$

This means that the parameters characterizing $\Theta_{\rm eff}$ are renormalized with respect to $\Theta \Rightarrow g_A^{\rm eff} = q \cdot g_A \neq g_A$

Realistic shell-model

The nuclear shell model is a microscopic one, then it is possible to construct, within the many-body theory, effective Hamiltonians and decay operators starting from realistic nuclear potentials

Realistic shell model (RSM)

1. Choose a realistic *NN* potential (*NNN*)
2. Renormalize its short range correlations
3. Identify the model space better tailored to study the physics problem
4. Derive the effective shell-model Hamiltonian and consistently effective shell-model operators for decay amplitudes, by way of the many-body perturbation theory
5. Calculate the observables (energies, e.m. transition probabilities, β-decay amplitudes...), using only theoretical SP energies, two-body matrix elements, and effective SM operators.

The effective shell-model Hamiltonian

We start from the many-body Hamiltonian H defined in the full Hilbert space:

$$H = H_0 + H_1 = \sum_{i=1}^{A}(T_i + U_i) + \sum_{i<j}(V_{ij}^{NN} - U_i)$$

$$\left(\begin{array}{c|c} PHP & PHQ \\ \hline QHP & QHQ \end{array}\right) \quad \overset{\mathcal{H} = \Omega^{-1} H \Omega}{\underset{Q\mathcal{H}P = 0}{\Longrightarrow}} \quad \left(\begin{array}{c|c} P\mathcal{H}P & P\mathcal{H}Q \\ \hline 0 & Q\mathcal{H}Q \end{array}\right)$$

$$H_{\mathrm{eff}} = P\mathcal{H}P$$

Suzuki & Lee $\Rightarrow \Omega = e^{\omega}$ with $\omega = \left(\begin{array}{c|c} 0 & 0 \\ \hline Q\omega P & 0 \end{array}\right)$

$$H_1^{\mathrm{eff}}(\omega) = PH_1P + PH_1Q\frac{1}{\epsilon - QHQ}QH_1P -$$
$$- PH_1Q\frac{1}{\epsilon - QHQ}\omega H_1^{\mathrm{eff}}(\omega)$$

The perturbative approach to the shell-model $H^{\rm eff}$

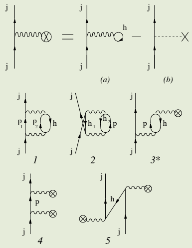
The $\hat{Q}$-box vertex function

$$\hat{Q}(\epsilon) = PH_1P + PH_1Q\frac{1}{\epsilon - QHQ}QH_1P$$

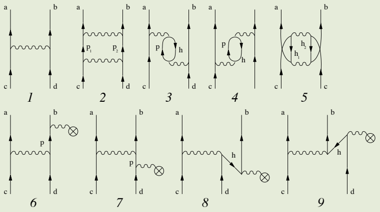
Exact calculation of the $\hat{Q}$-box is computationally prohibitive for many-body system $\Rightarrow$ we perform a perturbative expansion

$$\frac{1}{\epsilon - QHQ} = \sum_{n=0}^{\infty} \frac{(QH_1Q)^n}{(\epsilon - QH_0Q)^{n+1}}$$

$\hat{Q}$-box: 1st- & 2nd-order 1-b diagrams



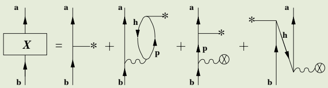
$\hat{Q}$-box: 1st- & 2nd-order 2-b diagrams



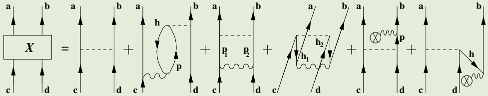
The effective SM operators for decay amplitudes

Diagrams up 2nd order of the perturbative expansion of the effective decay operator $\Theta_{\rm eff}$:

One-body operator



Two-body operator



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- *H. Q. Song, H. F. Wu, T. T. S. Kuo, Phys. Rev. C* **40***, 2260 (1989)*
- *J. D. Holt and J. Engel, Phys. Rev. C* **87***, 064315 (2013).*
- *LC, L. De Angelis, T. Fukui, A. Gargano, and N. Itaco, Phys. Rev. C* **95***,*
- *LC, L. De Angelis, T. Fukui, A. Gargano, N. Itaco, and F. Nowacki, Phys. Rev. C* **100***, 014316 (2019).*

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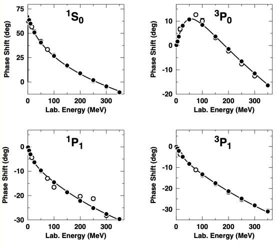
Our recipe for realistic shell model

- Input V_{NN}: $V_{\mathrm{low-k}}$ derived from the high-precision *NN* CD-Bonn potential with a cutoff: $\Lambda = 2.6\ \mathrm{fm}^{-1}$.



- H_{eff} obtained calculating the $\hat{Q}$ box up to the 3rd order in perturbation theory.
- Effective operators are consistently derived by way of the the MBPT
- Short-range correlations of the nuclear wave functions for the calculation of the nuclear matrix elements for $0\nu\beta\beta$ decay are obtained from the $V_{\mathrm{low}\text{-}k}$ renormalization procedure

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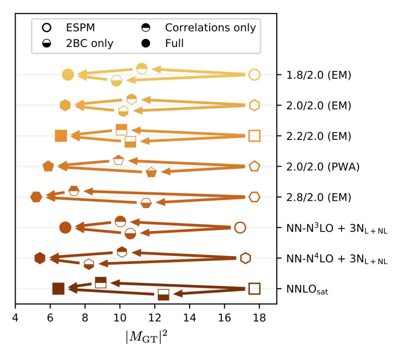
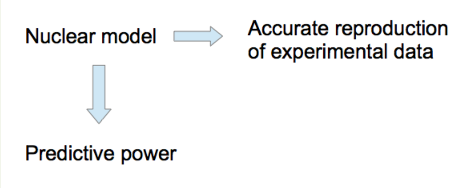
The choice of the *NN* potential



P. Gysbers et al., arXiv:1903.00047 (2019)

A $V_{\text{low-}k}$ potential from the CD-Bonn *NN* with a "hard cutoff" $\Lambda = 2.6$ fm^{-1} has been chosen to reduce the impact of GT two-body currents

Nuclear models and predictive power



RSM calculations, starting from the CD-Bonn potential for spectroscopic, spin-, and spin-isospin dependent observables of ^{48}Ca, ^{76}Ge, ^{82}Se, ^{100}Mo, ^{130}Te, and ^{136}Xe

$\Downarrow$

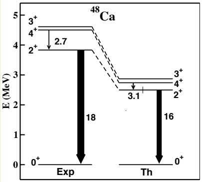
Check RSM approach calculating GT strengths and $2\nu\beta\beta$-decay

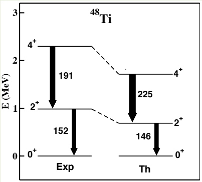
$$\left[T^{2\nu}_{1/2}\right]^{-1} = G^{2\nu}\left|M^{2\nu}_{\mathrm{GT}}\right|^2$$

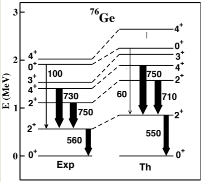
where

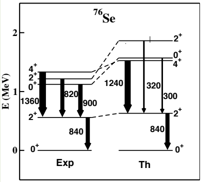
$$M^{\mathrm{GT}}_{2\nu} = \sum_n \frac{\langle 0^+_f || \vec{\sigma}\tau^- || 1^+_n \rangle \langle 1^+_n || \vec{\sigma}\tau^- || 0^+_i \rangle}{E_n + E_0}$$

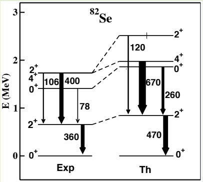
Spectroscopic properties

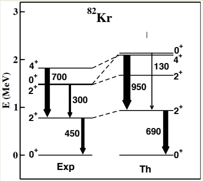












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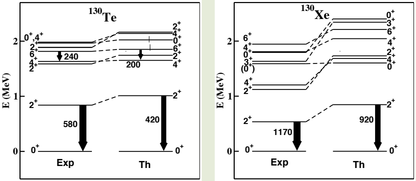
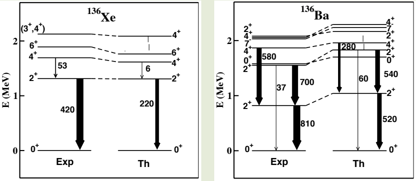
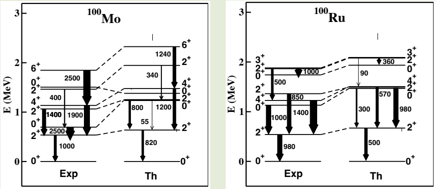
Spectroscopic properties







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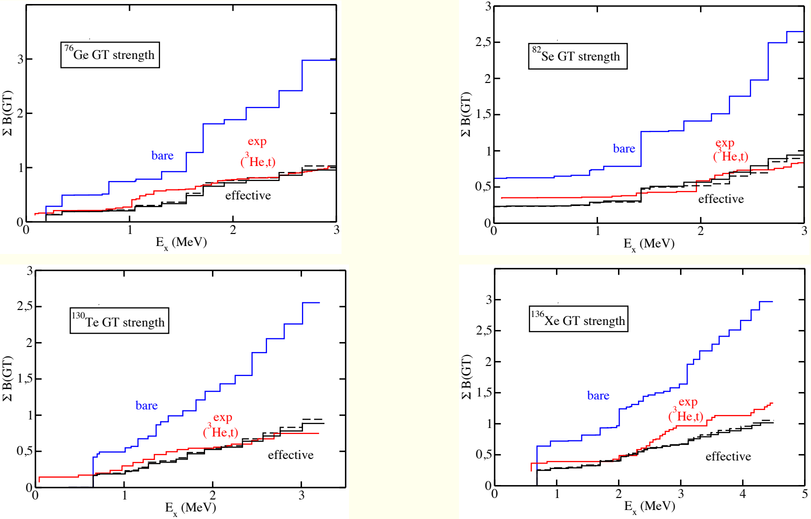
GT$^-$ running sums



$$B(p,n) = \frac{\left| \langle \Phi_f || \sum_j \vec{\sigma}_j \tau_j^- || \Phi_i \rangle \right|^2}{2J_i + 1} \; , \quad (1)$$

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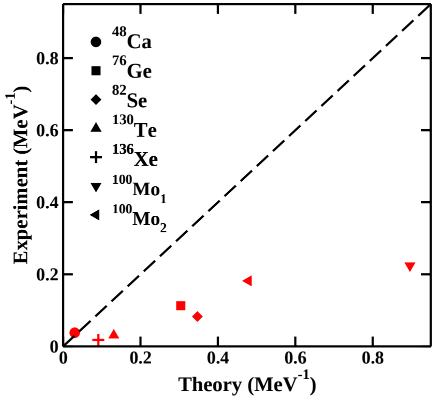
$2\nu\beta\beta$ nuclear matrix elements

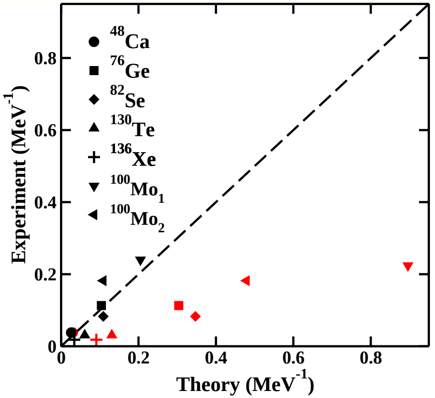


Red symbols: bare GT operator

Decay	Expt.	Bare
$^{48}\mathrm{Ca}_1 \rightarrow {}^{48}\mathrm{Ti}_1$	0.038 ± 0.003	0.030
$^{76}\mathrm{Ge}_1 \rightarrow {}^{76}\mathrm{Se}_1$	0.113 ± 0.006	0.304
$^{82}\mathrm{Se}_1 \rightarrow {}^{82}\mathrm{Kr}_1$	0.083 ± 0.004	0.347
$^{130}\mathrm{Te}_1 \rightarrow {}^{130}\mathrm{Xe}_1$	0.031 ± 0.004	0.131
$^{136}\mathrm{Xe}_1 \rightarrow {}^{136}\mathrm{Ba}_1$	0.0181 ± 0.0007	0.0910
$^{100}\mathrm{Mo}_1 \rightarrow {}^{100}\mathrm{Ru}_1$	0.224 ± 0.002	0.896
$^{100}\mathrm{Mo}_1 \rightarrow {}^{100}\mathrm{Ru}_2$	0.182 ± 0.006	0.479

Experimental data from *Thies et al, Phys. Rev. C* **86**, *044309 (2012); A. S. Barabash, Universe* **6**, *(2020)*

$2\nu\beta\beta$ nuclear matrix elements



Red symbols: bare GT operator
Black symbols: effective GT operator

Decay	Expt.	Eff.
^{48}Ca$_1$ $\rightarrow$ ^{48}Ti$_1$	0.038 $\pm$ 0.003	0.026
^{76}Ge$_1$ $\rightarrow$ ^{76}Se$_1$	0.113 $\pm$ 0.006	0.104
^{82}Se$_1$ $\rightarrow$ ^{82}Kr$_1$	0.083 $\pm$ 0.004	0.109
^{130}Te$_1$ $\rightarrow$ ^{130}Xe$_1$	0.031 $\pm$ 0.004	0.061
^{136}Xe$_1$ $\rightarrow$ ^{136}Ba$_1$	0.0181 $\pm$ 0.0007	0.0341
^{100}Mo$_1$ $\rightarrow$ ^{100}Ru$_1$	0.224 $\pm$ 0.002	0.205
^{100}Mo$_1$ $\rightarrow$ ^{100}Ru$_2$	0.182 $\pm$ 0.006	0.109

Experimental data from *Thies et al, Phys. Rev. C* **86**, *044309 (2012); A. S. Barabash, Universe* **6**, *(2020)*

Decay	q
^{48}Ca $\rightarrow$ ^{48}Ti	0.83
^{76}Ge $\rightarrow$ ^{76}Se	0.58
^{82}Se $\rightarrow$ ^{82}Kr	0.56
^{100}Mo $\rightarrow$ ^{100}Ru	0.48
^{130}Te $\rightarrow$ ^{130}Xe	0.68
^{136}Xe $\rightarrow$ ^{136}Ba	0.61

- *LC, L. De Angelis, T. Fukui, A. Gargano, and N. Itaco, Phys. Rev. C* **95**, *064324 (2017).*
- *LC, L. De Angelis, T. Fukui, A. Gargano, N. Itaco, and F. Nowacki, Phys. Rev. C* **100**, *014316 (2019).*

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RSM calculations of $M^{0\nu}$

Earliest RSM calculation of $M^{0\nu}$ performed by Kuo and coworkers for ^{48}Ca decay *(Phys. Lett. B* **162** *227 (1985))*

- SM effective TBMEs and decay operator from Paris and Reid potential
- Brueckner *G*-matrix and 2nd-order MBPT
- SRC derived through the calculation of the defect function

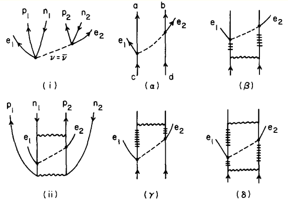


Fig. 1. Diagrams of $H^{\text{eff}}_{\beta\beta}$; virtual Majorana neutrinos are represented by dotted lines, nucleon *G*-matrix interaction by wavy lines, and nucleons outside the model space P by railed lines.

$M^{0\nu}$ for ^{76}Ge, ^{82}Se, and ^{48}Ca decay by Holt and Engel *(Phys. Rev. C* **87** *064315 (2013), Phys. Rev. C* **88** *045502 (2014))*

- Wave functions calculated with GCN28.50, JUN45, and GXPF1A Hamiltonians
- SM decay operator from N^{3}LO (EM) potential, $V_{\text{low}-k}$ renormalization, and 3rd-order MBPT
- Jastrow-type short-range correlations

$M^{0\nu}$ for ^{48}Ca, ^{76}Ge, ^{82}Se, ^{100}Mo, ^{130}Te, and ^{136}Xe decay by our collaboration *(Phys. Rev. C* **101** *044315 (2020); Phys. Rev. C* **105** *034312 (2022))*

- H_{eff} and effective decay operators from CD-Bonn potential, $V_{\text{low}-k}$ renormalization, and 3rd-order MBPT
- $V_{\text{low}-k}$-transformation SRC
- Three-body correlations contributions included to account for the Pauli-blocking effect

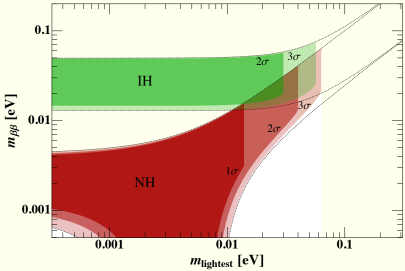
RSM calculations of $M^{0\nu}$: results

Decay	bare operator	$\Theta_{\rm eff}$	
^{48}Ca $\rightarrow$ ^{48}Ti	0.53	0.30	−40%
^{76}Ge $\rightarrow$ ^{76}Se	3.35	2.66	−20%
^{82}Se $\rightarrow$ ^{82}Kr	3.30	2.72	−20%
^{100}Mo $\rightarrow$ ^{100}Ru	3.96	2.24	−40%
^{130}Te $\rightarrow$ ^{130}Xe	3.27	3.16	−3%
^{136}Xe $\rightarrow$ ^{136}Ba	2.47	2.39	−3%

- Results obtained with the effective shell-model operator are relatively reduced with respect those with bare operator: quenching effect is much smaller than the two-neutrino double-β decay

Decay	q	bare operator	quenched operator	
^{48}Ca $\rightarrow$ ^{48}Ti	0.83	0.53	0.40	−20%
^{76}Ge $\rightarrow$ ^{76}Se	0.58	3.35	1.41	−60%
^{82}Se $\rightarrow$ ^{82}Kr	0.56	3.30	1.32	−60%
^{100}Mo $\rightarrow$ ^{100}Ru	0.48	3.96	1.33	−70%
^{130}Te $\rightarrow$ ^{130}Xe	0.68	3.27	1.78	−50%
^{136}Xe $\rightarrow$ ^{136}Ba	0.61	2.47	1.15	−50%

The calculation of $M^{0\nu}$: results

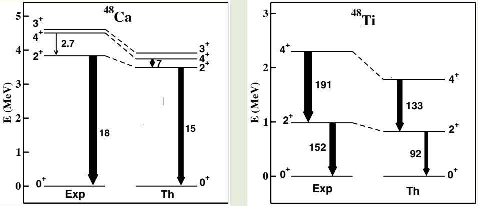


To rule out the Inverted Hierarchy of neutrino mass spectra, the upper bound of neutrino effective mass should be $\langle m_{\beta\beta} \rangle < 1.84 \pm 1.3$ meV.

We could then evaluate the lower bound of the half lives of the decay processes, accordingly to our calculated $M^{0\nu}$

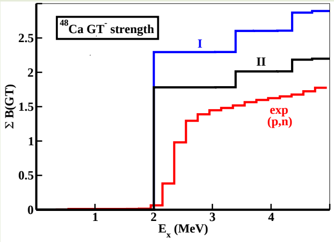
	$^{76}Ge \rightarrow ^{76}Se$	$^{82}Se \rightarrow ^{82}Kr$	$^{100}Mo \rightarrow ^{100}Ru$	$^{130}Te \rightarrow ^{130}Xe$	$^{136}Xe \rightarrow ^{136}Ba$
$T^{0\nu}_{1/2}$ (in yr)	$> 2 \times 10^{28}$	$> 4 \times 10^{27}$	$> 4 \times 10^{27}$	$> 2 \times 10^{27}$	$> 4 \times 10^{28}$

^{48}Ca $2\nu\beta\beta$-decay with chiral 2- and 3-body potentials



Preliminary results for the $2\nu\beta\beta$-decay of ^{48}Ca, employing Entem-Machleidt N^{3}LO 2NF plus N^{2}LO 3NF

Decay	Expt.	Bare	Eff.
$^{48}\mathrm{Ca} \rightarrow {}^{48}\mathrm{Ti}$	0.038 ± 0.003	0.057	0.047



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Conclusions and Outlook

- Microscopic approaches to the nuclear many-body problem may get rid of the so-called "quenching puzzle" in the study of β decay, with and without the emission of neutrinos
- This goal may be achieved by focusing theoretical efforts on two main issues:
  - a) improving our knowledge of nuclear forces;
  - b) estimation of the theoretical error from the application of many-body methods.
- Benchmark calculations with different approaches to the calculation of $0\nu\beta\beta$ nuclear matrix elements should lead to narrowing the spread among the theoretical results

Backup slides

The effective SM operators for decay amplitudes

Any shell-model effective operator may be derived consistently with the $\hat{Q}$-box-plus-folded-diagram approach to $H_{\rm eff}$

It has been demonstrated that, for any bare operator Θ, a non-Hermitian effective operator $\Theta_{\rm eff}$ can be written in the following form:

$$\Theta_{\rm eff} = (P + \hat{Q}_1 + \hat{Q}_1\hat{Q}_1 + \hat{Q}_2\hat{Q} + \hat{Q}\hat{Q}_2 + \cdots)(\chi_0 + \\ + \chi_1 + \chi_2 + \cdots) \ ,$$

where

$$\hat{Q}_m = \frac{1}{m!}\frac{d^m\hat{Q}(\epsilon)}{d\epsilon^m}\bigg|_{\epsilon=\epsilon_0} \ ,$$

ϵ_0 being the model-space eigenvalue of the unperturbed Hamiltonian H_0

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K. Suzuki and R. Okamoto, Prog. Theor. Phys. **93** *, 905 (1995)*

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The effective SM operators for decay amplitudes

The χ_n operators are defined as follows:

$$
\begin{aligned}
\chi_0 &= (\hat{\Theta}_0 + h.c.) + \Theta_{00} \quad , \\
\chi_1 &= (\hat{\Theta}_1 \hat{Q} + h.c.) + (\hat{\Theta}_{01} \hat{Q} + h.c.) \quad , \\
\chi_2 &= (\hat{\Theta}_1 \hat{Q}_1 \hat{Q} + h.c.) + (\hat{\Theta}_2 \hat{Q}\hat{Q} + h.c.) + \\
& \quad (\hat{\Theta}_{02} \hat{Q}\hat{Q} + h.c.) + \hat{Q}\hat{\Theta}_{11} \hat{Q} \quad , \\
& \quad \ldots
\end{aligned}
$$

and

$$
\hat{\Theta}(\epsilon) = P\Theta P + P\Theta Q \frac{1}{\epsilon - QHQ} QH_1 P
$$

$$
\hat{\Theta}(\epsilon_1;\epsilon_2) = PH_1 Q \frac{1}{\epsilon_1 - QHQ} \times Q\Theta Q \frac{1}{\epsilon_2 - QHQ} QH_1 P
$$

$$
\hat{\Theta}_m = \frac{1}{m!} \frac{d^m \hat{\Theta}(\epsilon)}{d\epsilon^m} \bigg|_{\epsilon=\epsilon_0}
$$

$$
\hat{\Theta}_{nm} = \frac{1}{n!m!} \frac{d^n}{d\epsilon_1^n} \frac{d^m}{d\epsilon_2^m} \hat{\Theta}(\epsilon_1;\epsilon_2) \bigg|_{\epsilon_{1,2}=\epsilon_0}
$$

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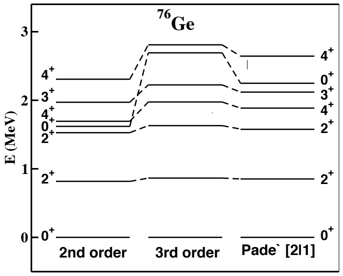
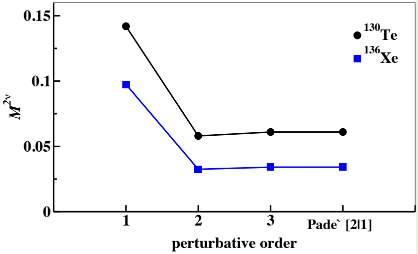
Perturbative properties

Order-by-order convergence





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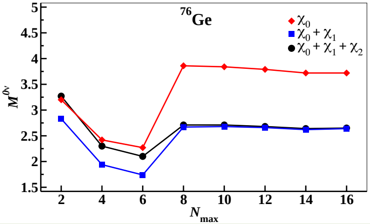
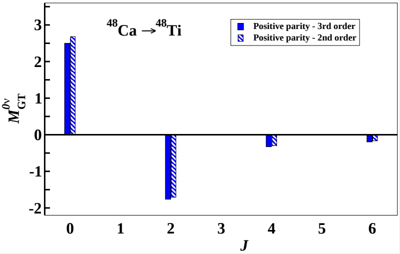
Perturbative properties of the 0ν effective operator

Convergence with respect the number of intermediate states

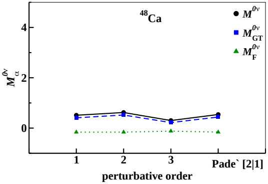


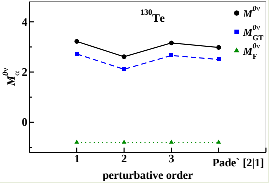
Order-by-order convergence

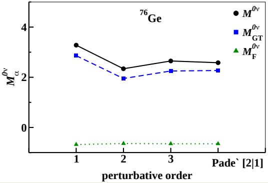


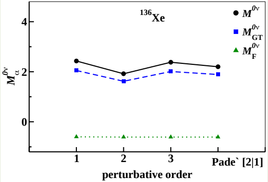
Contributions from pairs of decaying neutrons with given J^{π} to $M^{0\nu}_{GT}$ for ^{48}Ca $0\nu\beta\beta$ decay. The bars filled in blue corresponds to the results obtained with Θ_{eff} calculated a 3rd order in perturbation theory, those in dashed blue are calculated at 2nd order

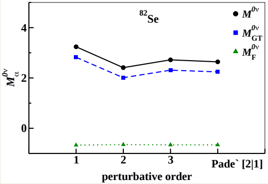
Perturbative behavior of the effective $0\nu\beta\beta$ operator









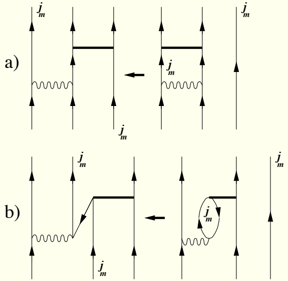


The perturbative behavior is dominated by the GT component, the renormalization procedure does not affect significantly $M^{0\nu}_{\mathrm{F}}$

The perturbative expansion of the effective $0\nu\beta\beta$ operator is less satisfactory than the single-β decay operator

The blocking effect

Blocking (Pauli) effect: as for the one-body operators, the filling of the model-space orbitals by the valence nucleons affects the effective $0\nu\beta\beta$ operator:



Present shell model codes cannot manage the contributions of these three-body correlations diagrams to the effective $0\nu\beta\beta$-decay operator

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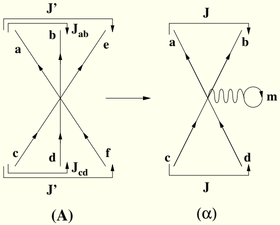
The blocking effect

Many-body correlations are then taken into account by calculating three-body correlations diagrams and summing over one of the incoming/outcoming nucleons

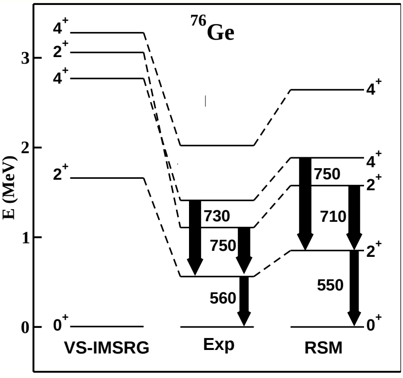


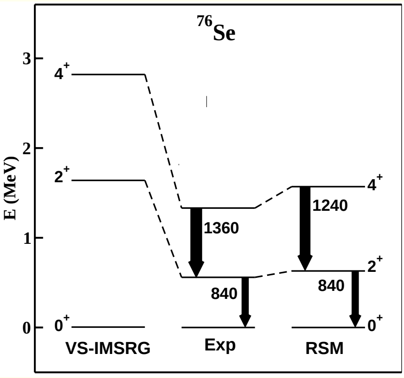
⇓

We obtain a density-dependent two-body $0\nu\beta\beta$ effective operator

$$\langle (j_a j_b)_J | O^\alpha | (j_c j_d)_J \rangle = \sum_{m,J'} \rho_m \frac{\hat{J'}^2}{\hat{J}^2} \langle [(j_a j_b)_J, j_m]_{J'} | O^A | [(j_c j_d)_J, j_m]_{J'} \rangle$$

Ab initio vs NSM calculations





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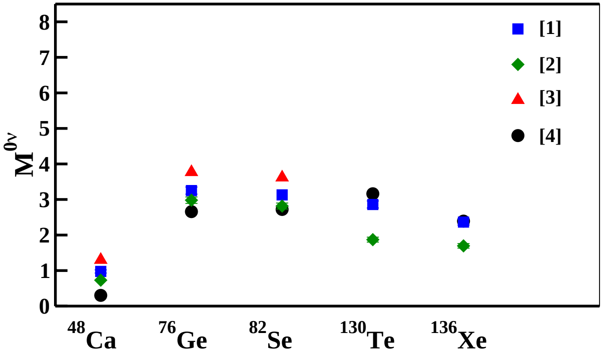
Shell model calculations of $M^{0\nu}$



1 *J. Menéndez, J. Phys. G* **45**, *014003 (2018).*

2 *A. Neacsu and M. Horoi, Phys. Rev. C* **91**, *024309 (2015), R. A. Sen'kov and M. Horoi, Phys. Rev. C* **88**, *064312 (2013), R. A. Sen'kov, M. Horoi, and B. A. Brown, Phys. Rev. C* **89**, *054304 (2014).*

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4 *LC, A. Gargano, N. Itaco, R. Mancino, and F. Nowacki, Phys. Rev. C* **101** *044315 (2020).*