Accessing the initial conditions of ultrarelativistic heavy-ion collisions

12th MPI at LHC

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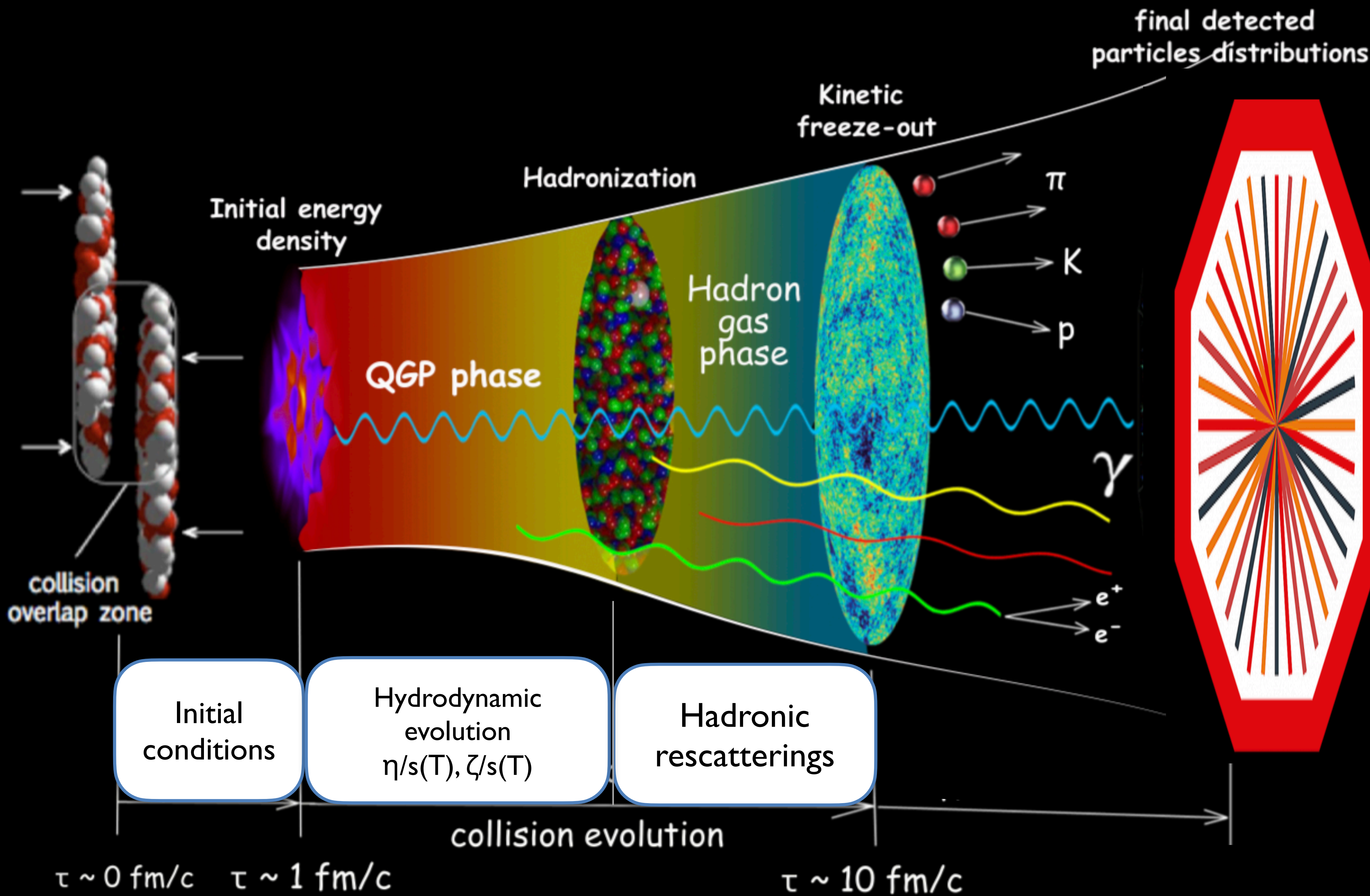
THE VELUX FOUNDATIONS

VILLUM FONDEN VELUX FONDEN

Evolution in the *Little Bang*



Current status of initial state models

Credits: G. Giacalone @ IS2021

THERE ARE CURRENTLY THREE CATEGORIES OF MODELS.

– **"sharp" models: IP-GLASMA and TRENTo 2016 (v-USPhydro)**

[Schenke, Shen, Tribedy **2005.14682**]

[Bass, Bernhard, Moreland **1605.03954**]

Nucleons have a width of ~0.5fm (trento), 3 sub-nucleons with size ~0.3fm (IP-Glasma). Trento is used for the entropy density at the beginning of hydro. <u>IP-Glasma is the only model which incorporates a realistic pre-equilibrium evolution with longitudinal cooling.</u>

– **"fat" models: TRENTo 2019 and JETSCAPE**

[Bass, Bernhard, Moreland **Nature Phys. 15 (2019)**]

[JETSCAPE Collaboration **2011.01430**, **2010.03928**]

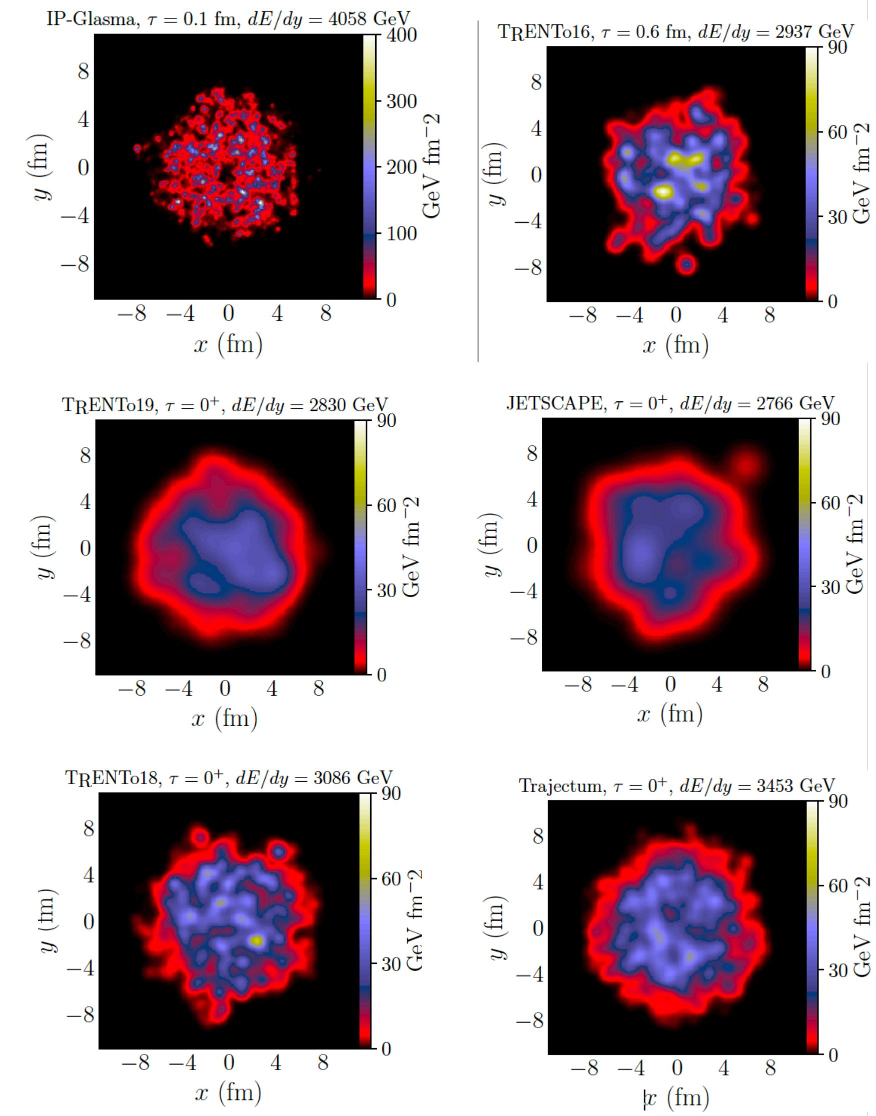
The Trento parametrization is now used for the energy density at tau=0+. There is no substructure. The nucleon width is now ~1fm. Very smooth profiles.

– **"lumpy fat" models: TRENTo 2018 and Trajectum**

[Bass, Bernhard, Moreland **1808.02106**]

[Nijs, van der Schee, Gürsoy, Snellings **2010.15130**, **2010.15134**]

The Trento parametrization is the energy density at tau=0+. Substructure is included: 4-6 constituents with width ~0.5fm. Profiles with some 'old school' lumpiness.



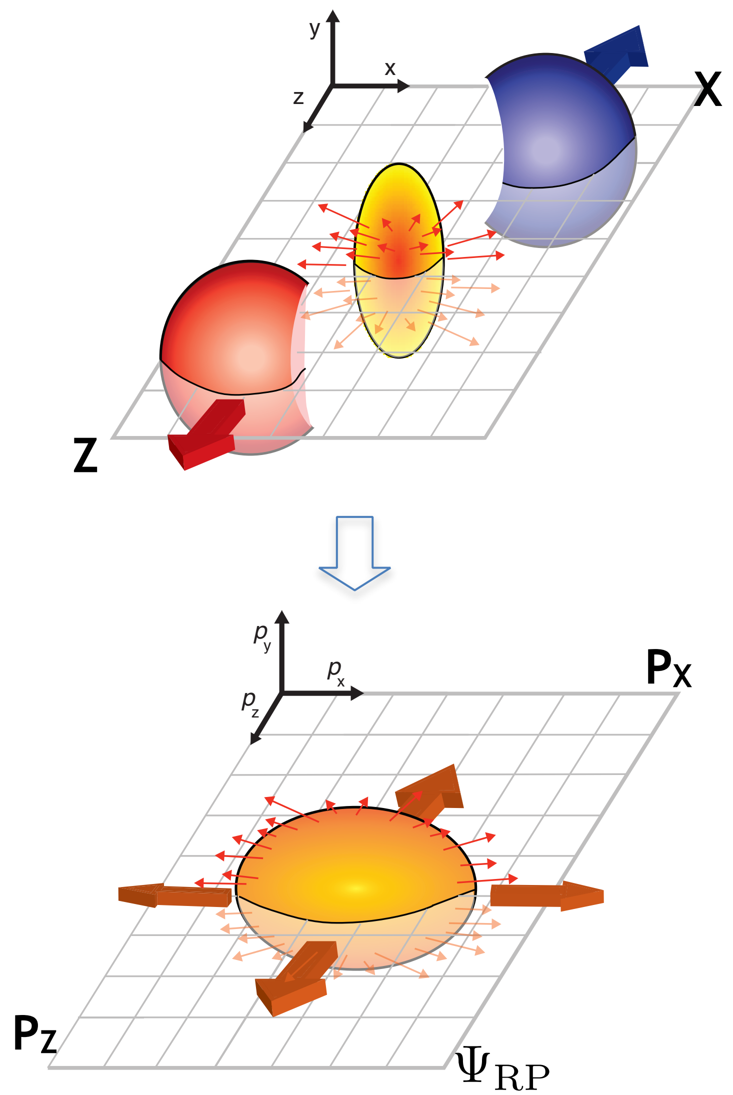
How can we access the initial conditions in EXP ?

Studying QGP with flow

❖Spatial eccentricity in the initial state converted to momentum anisotropic particle distributions

- known as ***elliptic flow***
- reflect initial **eccentricity** and **transport properties** of QGP

J.Y. Ollitrault, PRD 46 (1992) 229



system expansion

Initial state

$$\varepsilon_2 = \left\langle \frac{y^2 - x^2}{y^2 + x^2} \right\rangle$$

Initial spatial Eccentricity

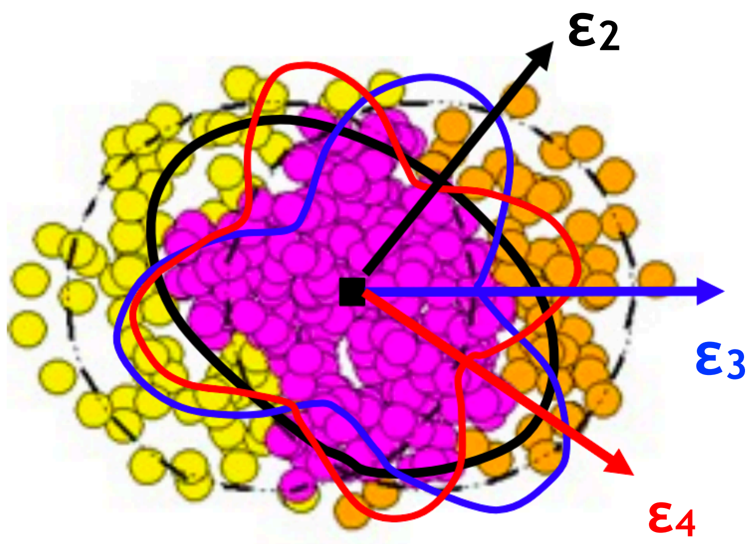
Final state

$$v_2 = \langle \cos 2\left(\varphi - \Psi_{\rm RP}\right)\rangle$$

momentum space Elliptic Flow

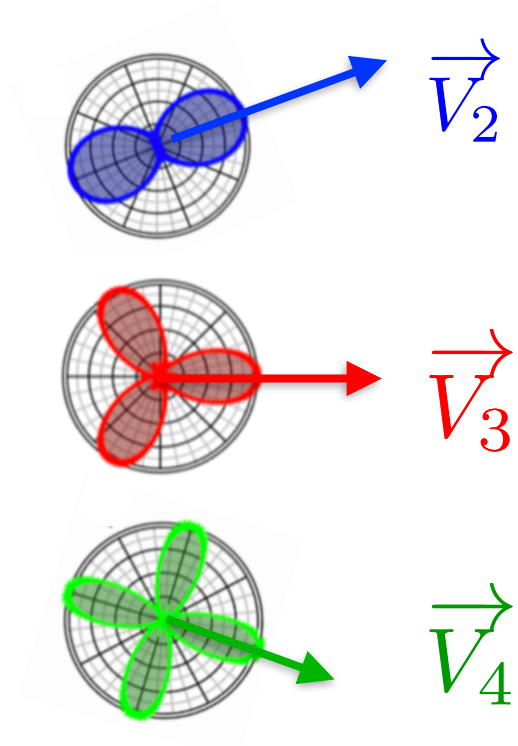
From initial anisotropy to anisotropic flow

Initial state



System expansion

Final state



$$\vec{V}_n = v_n \, e^{-in\Psi_n}$$

- v_n : Anisotropic flow
- ψ_n : Flow symmetry plane

$$P(\varepsilon_m, \varepsilon_n, \varepsilon_k, ..., \Phi_m, \Phi_n, \Phi_k, ...) \Rightarrow P(v_m, v_n, v_k, ..., \Psi_m, \Psi_n, \Psi_k, ...)$$

How does v_n fluctuate

ALICE, JHEP 07 (2018) 103

How does ψ_n fluctuate

ALICE, JHEP 09 (2017) 032

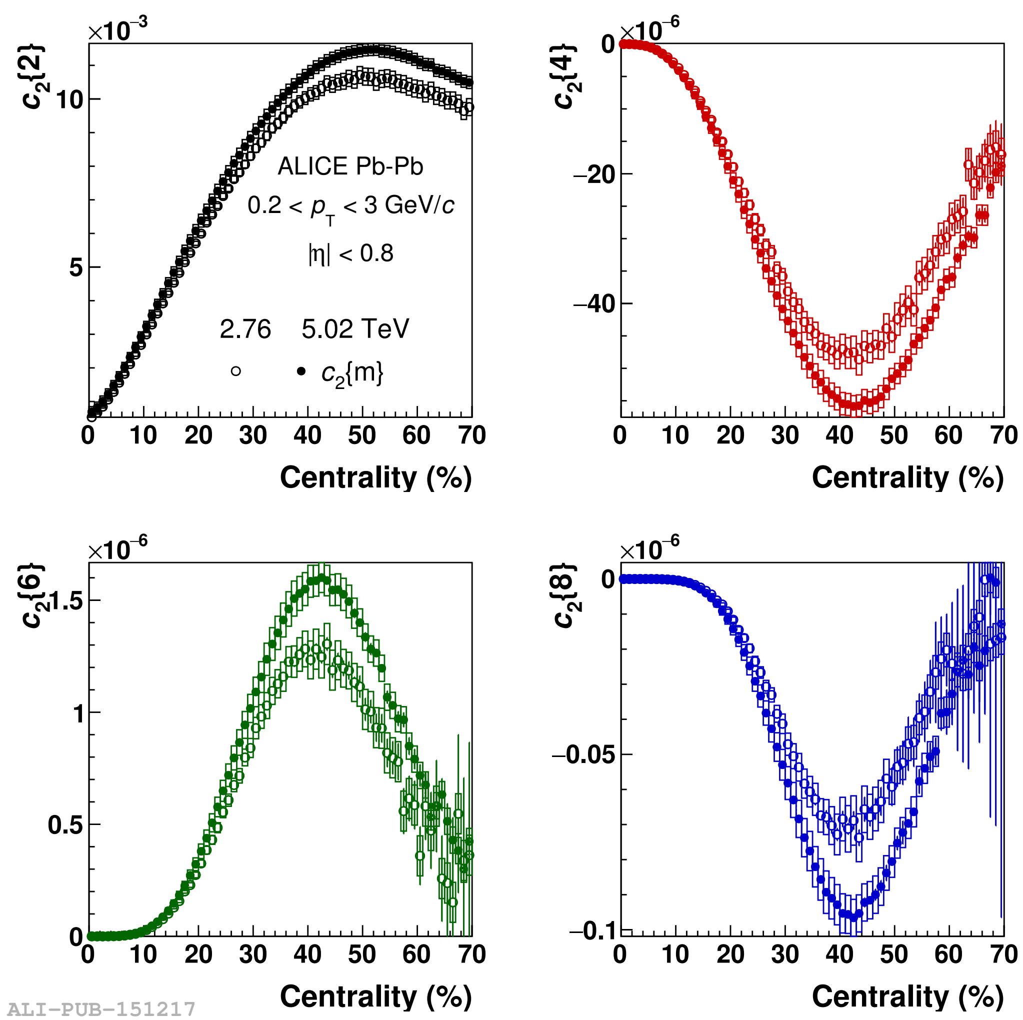
How do v_n and v_m correlate

ALICE, PRL117, (2016) 182301
PRC97, (2018) 024906
PLB818 (2021) 136354

How do ψ_n and ψ_m correlate

ALICE, JHEP05 (2020) 085
JHEP06 (2020) 147

P(v_n) from multi-particle cumulants of v_n



p(ε_n) — System Expansion → p(v_n)

$v_n\{2\}$, $v_n\{4\}$, $v_n\{6\}$, $v_n\{8\}$, $v_n\{10\}$, $v_n\{12\}$ ...

Multi-particle ***correlations*** of single harmonic v_n

$$\langle\langle \cos(n\phi_1 - n\phi_2 + n\phi_3 - n\phi_4)\rangle\rangle = \left\langle v_n^4 \cos(n\Phi_n - n\Phi_n + n\Phi_n - n\Phi_n)\right\rangle = \left\langle v_n^4\right\rangle$$

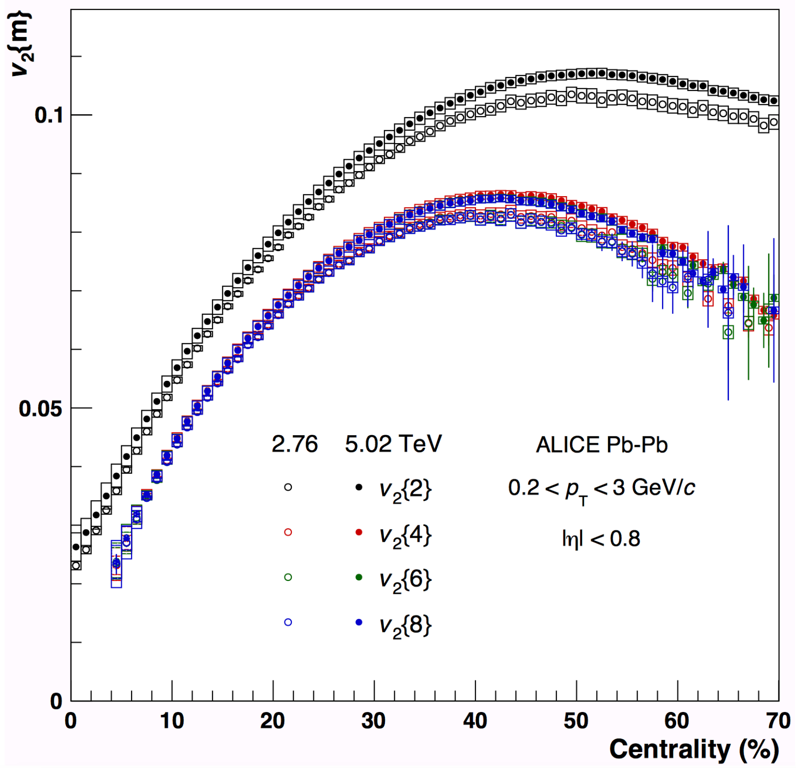
Multi-particle ***cumulants*** of single harmonic v_n

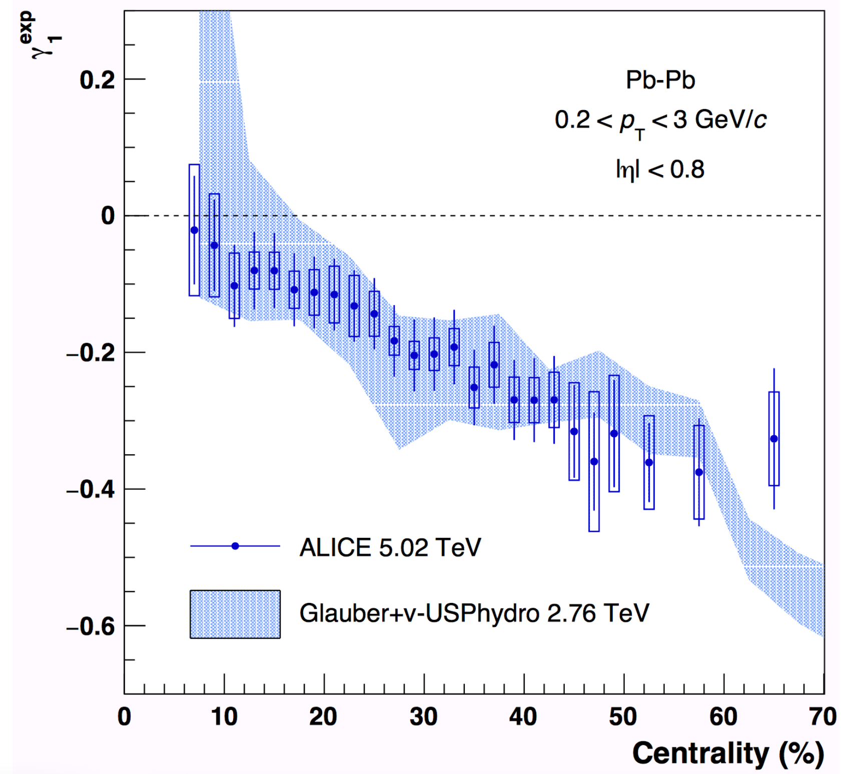
$$\langle\langle \cos(n\phi_1 - n\phi_2 + n\phi_3 - n\phi_4)\rangle\rangle_c = \langle \cos(n\phi_1 - n\phi_2 + n\phi_3 - n\phi_4)\rangle$$
$$- \langle\langle \cos(n\phi_1 - n\phi_2)\rangle\rangle \langle\langle \cos(n\phi_3 - n\phi_4)\rangle\rangle$$
$$- \langle\langle \cos(n\phi_1 - n\phi_4)\rangle\rangle \langle\langle \cos(n\phi_2 - n\phi_3)\rangle\rangle$$
$$= \left\langle v_n^4\right\rangle - 2\left\langle v_n^2\right\rangle^2$$

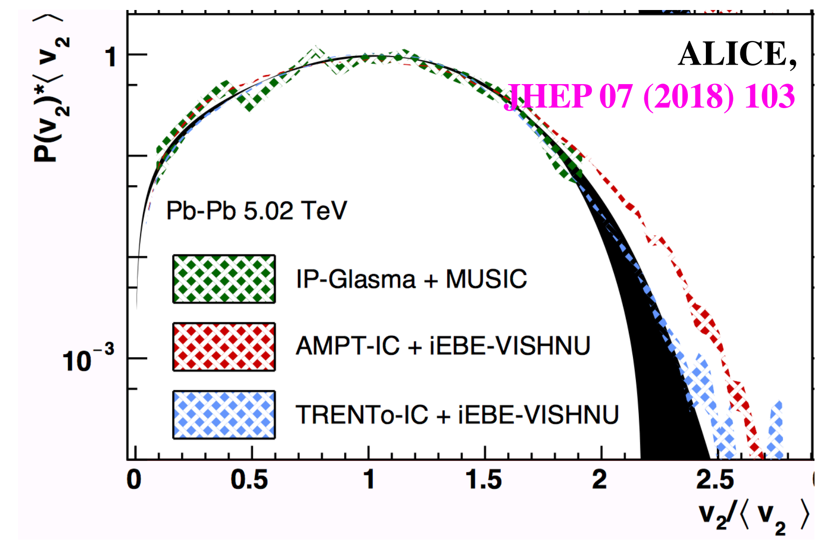
$$v_n\{2\} = \sqrt[2]{\langle v_n^2\rangle},$$
$$v_n\{4\} = \sqrt[4]{2\langle v_n^2\rangle^2 - \langle v_n^4\rangle},$$
$$v_n\{6\} = \sqrt[6]{\langle v_n^6\rangle - 9\langle v_n^2\rangle\langle v_n^4\rangle + 12\langle v_n^2\rangle^3},$$
$$v_n\{8\} = \sqrt[8]{\langle v_n^8\rangle - 16\langle v_n^2\rangle\langle v_n^6\rangle - 18\langle v_n^4\rangle^2 + 144\langle v_n^2\rangle^2\langle v_n^4\rangle - 144\langle v_n^2\rangle^4}.$$

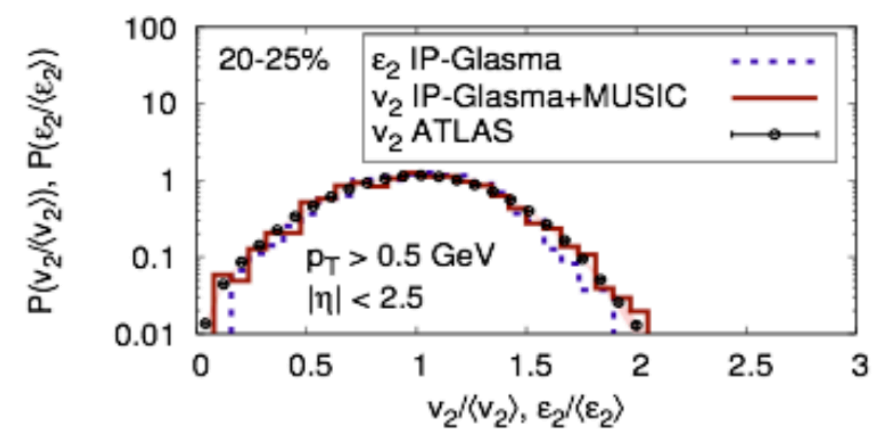
P(v_n) and P(ε_n)

$v_n\{m\}$ → Moments → $p(v_n)$ → $p(\varepsilon_n)$









$$v_n\{2\} = \sqrt[2]{\langle v_n^2 \rangle},$$
$$v_n\{4\} = \sqrt[4]{2\langle v_n^2 \rangle^2 - \langle v_n^4 \rangle},$$
$$v_n\{6\} = \sqrt[6]{\langle v_n^6 \rangle - 9\langle v_n^2 \rangle\langle v_n^4 \rangle + 12\langle v_n^2 \rangle^3},$$
$$v_n\{8\} = \sqrt[8]{\langle v_n^8 \rangle - 16\langle v_n^2 \rangle\langle v_n^6 \rangle - 18\langle v_n^4 \rangle^2 + 144\langle v_n^2 \rangle^2\langle v_n^4 \rangle - 144\langle v_n^2 \rangle^4}.$$

$$\gamma_1^{\rm exp} = -6\sqrt{2}\, v_2\{4\}^2 \frac{v_2\{4\} - v_2\{6\}}{(v_2\{2\}^2 - v_2\{4\}^2)^{3/2}}$$

$$\gamma_2 \simeq \gamma_2^{\rm expt} \equiv -\frac{3}{2}\frac{v_2\{4\}^4 - 12 v_2\{6\}^4 + 11 v_2\{8\}^4}{(v_2\{2\}^2 - v_2\{4\}^2)^2}$$

❖ Investigating p(v_2) with multi-particle cumulants

- First three moments done at LHC RUN1 & 2, Kurtosis at RUN3
- Ultra-higher order cumulants e.g. $v_2\{10\}\{12\}\{14\}\{16\}$ is implemented for HL-LHC

(Normalized) Symmetric Cumulant

Symmetric cumulants:

$$SC(m,n) = \langle v_m^2\, v_n^2 \rangle - \langle v_m^2 \rangle \langle v_n^2 \rangle$$

PHYSICAL REVIEW C **89**, 064904 (2014)

Generic framework for anisotropic flow analyses with multiparticle azimuthal correlations

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[2]*Nikhef, Science Park 105, 1098 XG Amsterdam, The Netherlands*
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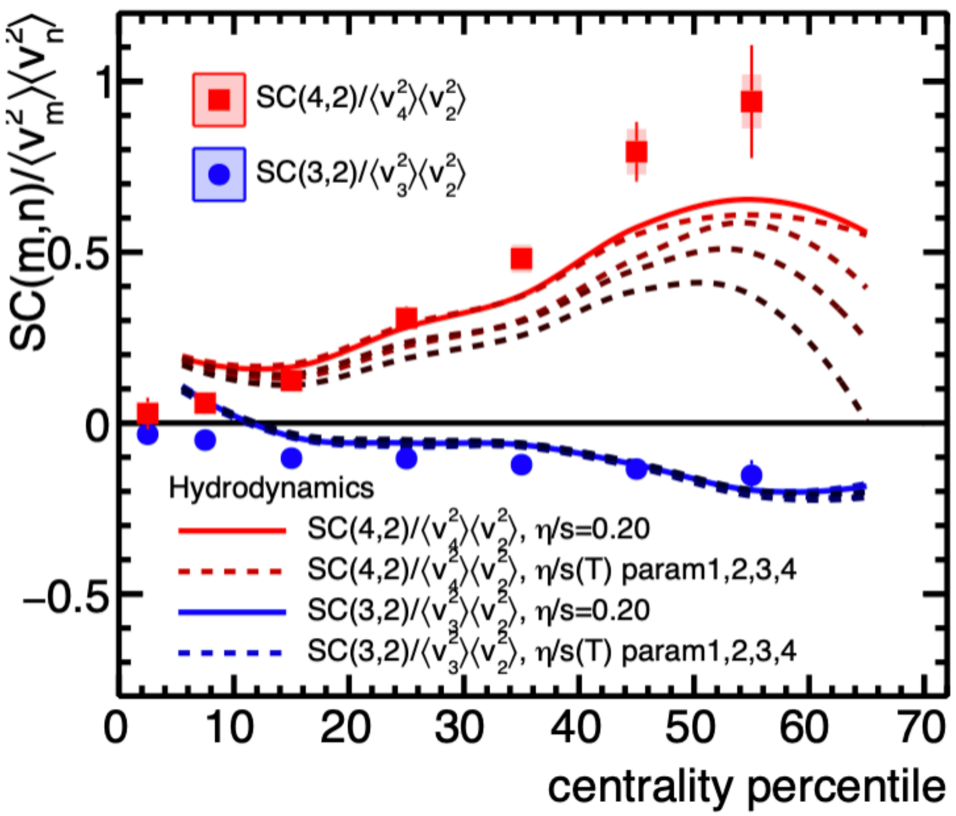
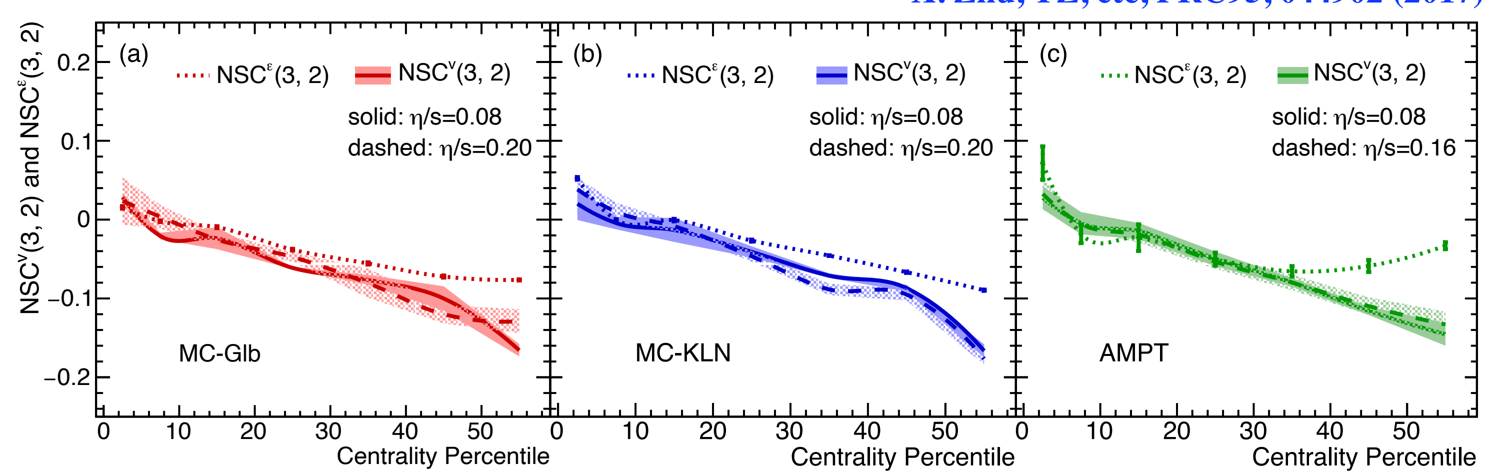
ALICE, PRL117, 182301 (2016)



X. Zhu, YZ, etc, PRC95, 044902 (2017)



$$v_2 \propto \varepsilon_2 \qquad v_3 \propto \varepsilon_3 \quad \Rightarrow \quad \frac{\langle v_3^2\, v_2^2\rangle}{\langle v_3^2\rangle \langle v_2^2\rangle} \approx \frac{\langle \varepsilon_3^2\, \varepsilon_2^2\rangle}{\langle \varepsilon_3^2\rangle \langle \varepsilon_2^2\rangle}$$

NSC^v(3,2) NSC$^\varepsilon$(3,2)

- Comparison of SC and Normalized SC (NSC) to hydrodynamic calculations
  - Although hydro describes v_n fairly well, there is not a single centrality for which a given η/s parameterization describes simultaneously SC and NSC -> tighter constraints!
  - NSC(3,2) measurements provide direct access into the initial conditions (despite details of systems evolution)
  - what is the general correlation between any order of v_n^k and v_m^p and the correlations among multiple flow coefficients

$P(v_m, v_n, v_k, \ldots)$

PHYSICAL REVIEW C **103**, 024913 (2021)

Generic algorithm for multiparticle cumulants of azimuthal correlations in high energy nucleus collisions

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❖ Multi-particle mixed harmonic cumulants
- correlation between v_m^k, v_n^l and v_p^q
- correlation between v_m^k and v_n^l

Mixed harmonic cumulants with 4-particles

$$\mathrm{MHC}(v_m^2, v_n^2) = \mathrm{SC}(m,n) = \langle v_m^2\, v_n^2 \rangle - \langle v_m^2 \rangle \langle v_n^2 \rangle$$

Mixed harmonic cumulants with 6-particles

$$\begin{aligned}\mathrm{MHC}(v_2^4, v_3^2) &= \langle\langle e^{i(2\varphi_1+2\varphi_2+3\varphi_3-2\varphi_4-2\varphi_5-3\varphi_6)}\rangle\rangle_c \\ &= \langle v_2^4\, v_3^2\rangle - 4\langle v_2^2\, v_3^2\rangle\langle v_2^2\rangle - \langle v_2^4\rangle\langle v_3^2\rangle \\ &\quad + 4\langle v_2^2\rangle^2\langle v_3^2\rangle.\end{aligned}$$

$$\begin{aligned}\mathrm{MHC}(v_2^2, v_3^4) &= \langle\langle e^{i(2\varphi_1+3\varphi_2+3\varphi_3-2\varphi_4-3\varphi_5-3\varphi_6)}\rangle\rangle_c \\ &= \langle v_2^2\, v_3^4\rangle - 4\langle v_2^2\, v_3^2\rangle\langle v_3^2\rangle - \langle v_2^2\rangle\langle v_3^4\rangle \\ &\quad + 4\langle v_2^2\rangle\langle v_3^2\rangle^2.\end{aligned}$$

$$\begin{aligned}\mathrm{MHC}(v_2^2, v_3^2, v_4^2) &= \langle\langle e^{i(2\varphi_1+3\varphi_2+4\varphi_3-2\varphi_4-3\varphi_5-4\varphi_6)}\rangle\rangle_c \\ &= \langle v_2^2\, v_3^2\, v_4^2\rangle - \langle v_2^2\, v_3^2\rangle\langle v_4^2\rangle - \langle v_2^2\, v_4^2\rangle\langle v_3^2\rangle \\ &\quad - \langle v_3^2\, v_4^2\rangle\langle v_2^2\rangle + 2\langle v_2^2\rangle\langle v_3^2\rangle\langle v_4^2\rangle.\end{aligned}$$

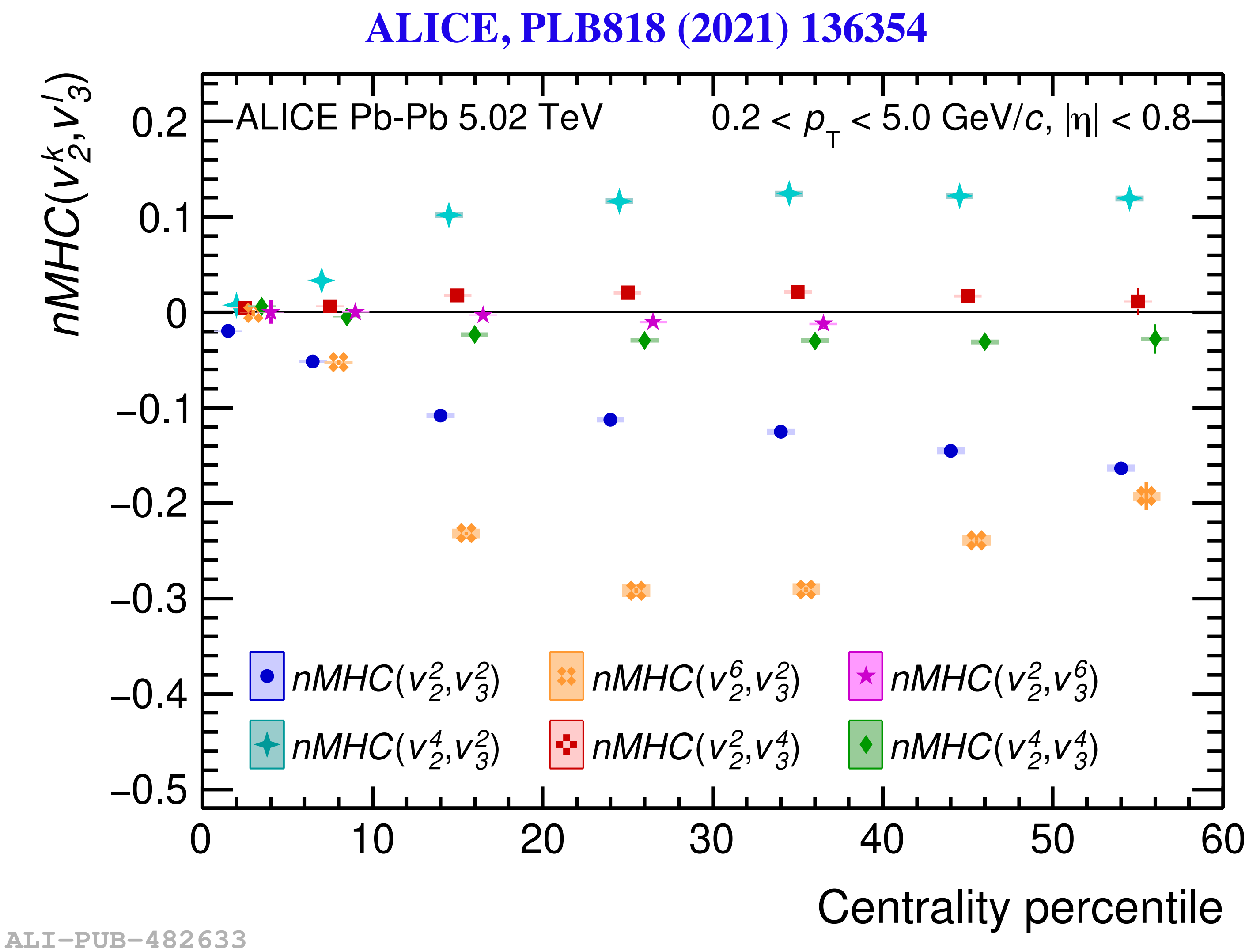
Mixed harmonic cumulants with 8-particles

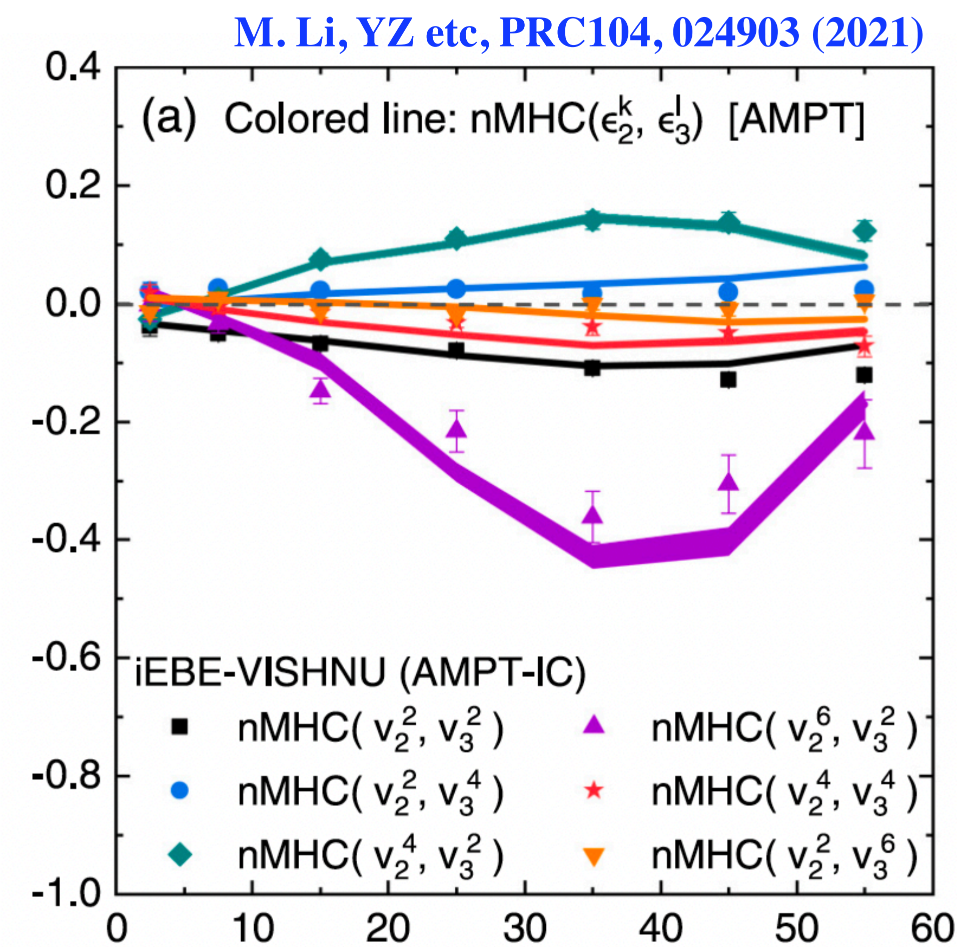
$$\begin{aligned}\mathrm{MHC}(v_2^6, v_3^2) &= \langle\langle e^{i(2\varphi_1+2\varphi_2+2\varphi_3+3\varphi_4-2\varphi_5-2\varphi_6-2\varphi_7-3\varphi_8)}\rangle\rangle_c \\ &= \langle v_2^6\, v_3^2\rangle - 9\langle v_2^4\, v_3^2\rangle\langle v_2^2\rangle - \langle v_2^6\rangle\langle v_3^2\rangle \\ &\quad - 9\langle v_2^4\rangle\langle v_2^2\, v_3^2\rangle - 36\langle v_2^2\rangle^3\langle v_3^2\rangle \\ &\quad + 18\langle v_2^2\rangle\langle v_3^2\rangle\langle v_2^4\rangle + 36\langle v_2^2\rangle^2\langle v_2^2\, v_3^2\rangle.\end{aligned}$$

$$\begin{aligned}\mathrm{MHC}(v_2^4, v_3^4) &= \langle\langle e^{i(2\varphi_1+2\varphi_2+3\varphi_3+3\varphi_4-2\varphi_5-2\varphi_6-3\varphi_7-3\varphi_8)}\rangle\rangle_c \\ &= \langle v_2^4\, v_3^4\rangle - 4\langle v_2^4\, v_3^2\rangle\langle v_3^2\rangle \\ &\quad - 4\langle v_2^2\, v_3^4\rangle\langle v_2^2\rangle - \langle v_2^4\rangle\langle v_3^4\rangle \\ &\quad - 8\langle v_2^2\, v_3^2\rangle^2 - 24\langle v_2^2\rangle^2\langle v_3^2\rangle^2 \\ &\quad + 4\langle v_2^2\rangle^2\langle v_3^4\rangle + 4\langle v_2^4\rangle\langle v_3^2\rangle^2 \\ &\quad + 32\langle v_2^2\rangle\langle v_3^2\rangle\langle v_2^2\, v_3^2\rangle.\end{aligned}$$

$$\begin{aligned}\mathrm{MHC}(v_2^2, v_3^6) &= \langle\langle e^{i(2\varphi_1+3\varphi_2+3\varphi_3+3\varphi_4-2\varphi_5-3\varphi_6-3\varphi_7-3\varphi_8)}\rangle\rangle_c \\ &= \langle v_2^2\, v_3^6\rangle - 9\langle v_2^2\, v_3^4\rangle\langle v_3^2\rangle - \langle v_3^6\rangle\langle v_2^2\rangle \\ &\quad - 9\langle v_3^4\rangle\langle v_2^2\, v_3^2\rangle - 36\langle v_2^2\rangle\langle v_3^2\rangle^3 \\ &\quad + 18\langle v_2^2\rangle\langle v_3^2\rangle\langle v_3^4\rangle + 36\langle v_3^2\rangle^2\langle v_2^2\, v_3^2\rangle.\end{aligned}$$

Correlations between v_2^k and v_3^p



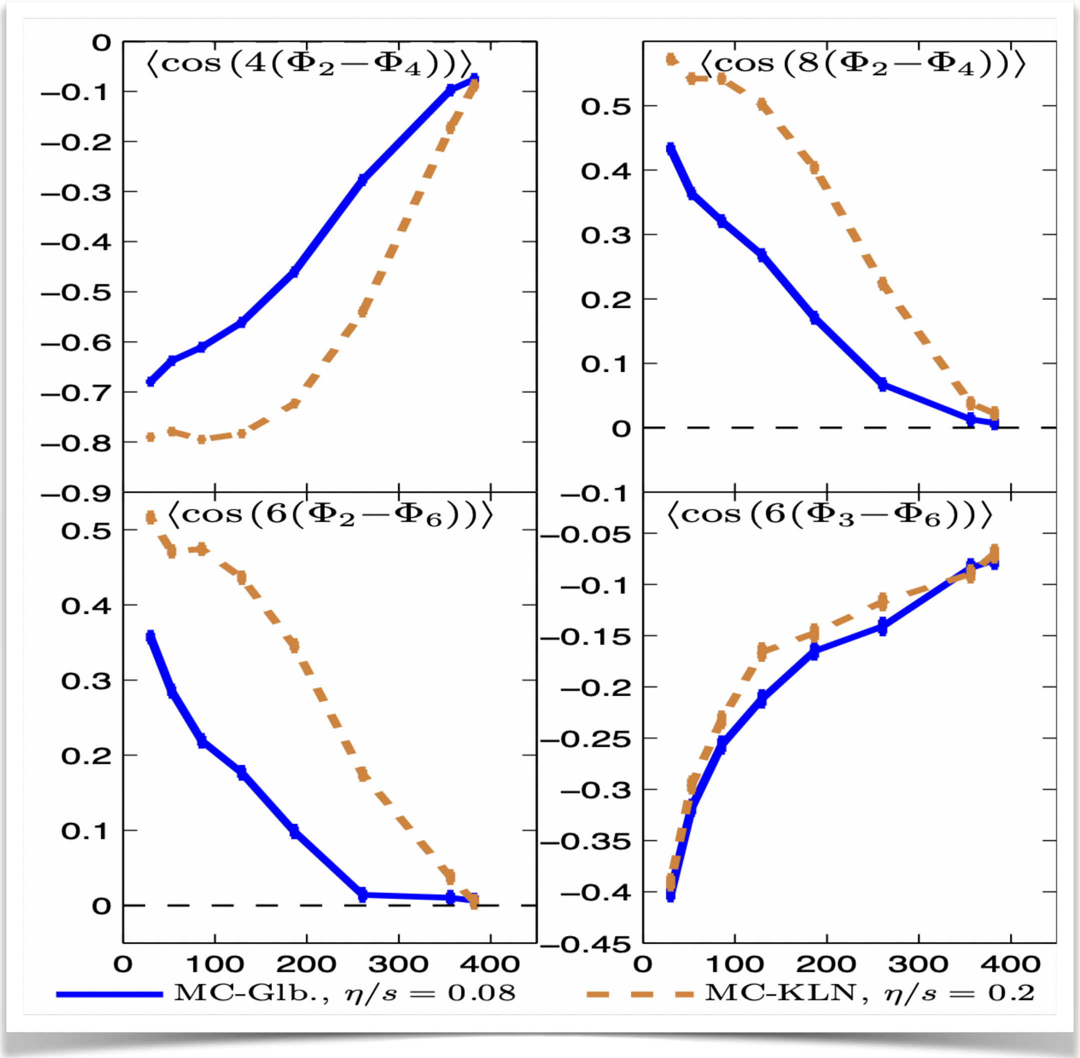


❖ First measurement of correlations between higher order moments of v_2 and v_3

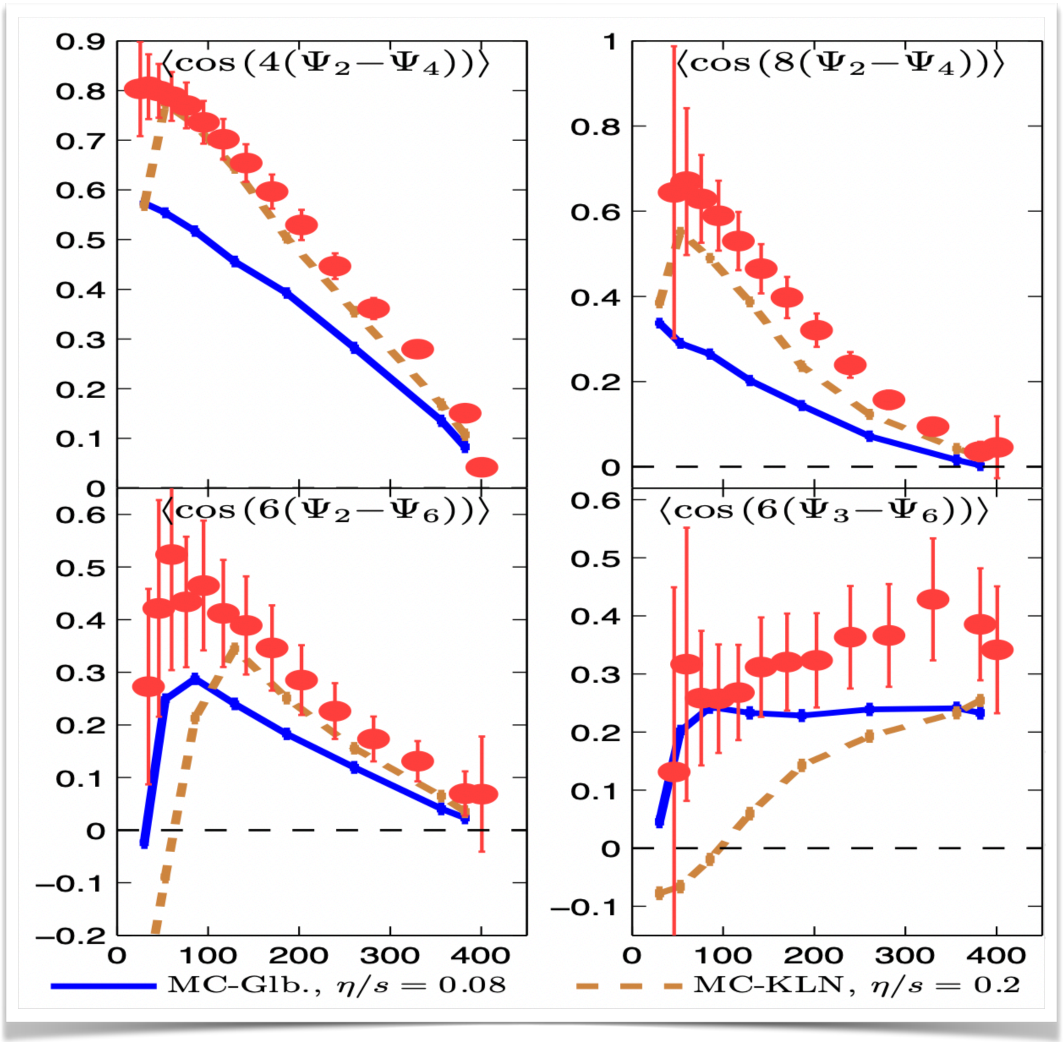
- characteristic +, -, + signs observed for 4-, 6- and 8-particle cumulants of *mixed harmonic*
- Final state results quantitatively reproduced by the initial state correlations
- Experimental data provides direct constraints on the correlations of higher order moments of eccentricity coefficients

Ψ_n correlations: $P(\Psi_m, \Psi_n, \Psi_k)$

$P(\Phi_m, \Phi_n, \Phi_k)$



$P(\Psi_m, \Psi_n, \Psi_k)$

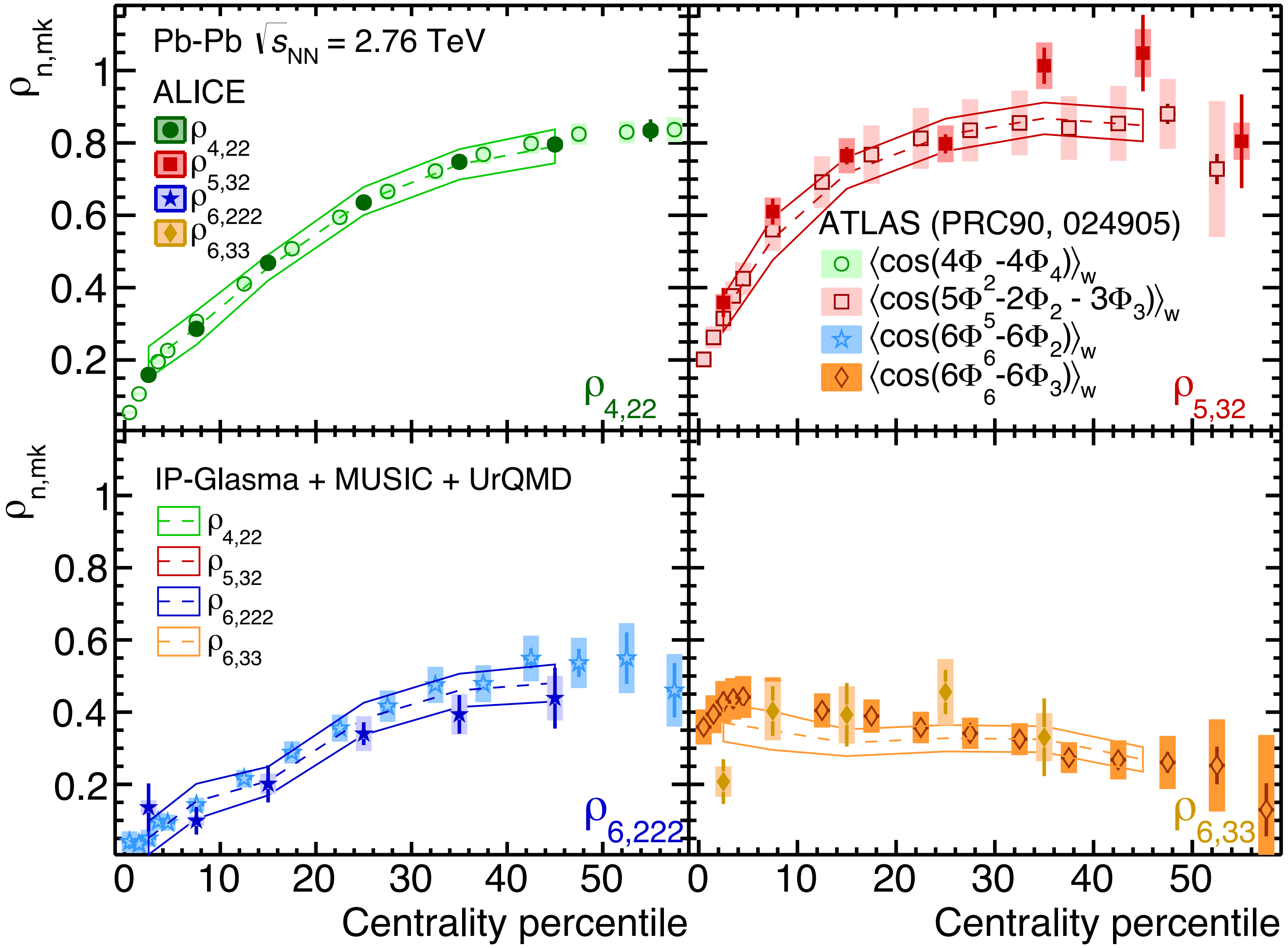


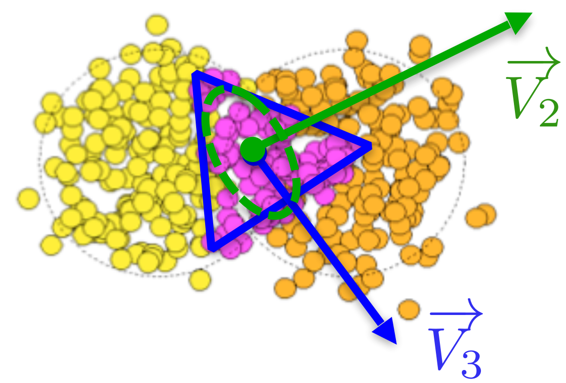
Z. Qiu etc, *PLB* 707 (2012) 151

- Stronger initial symmetry plane correlations results in stronger final state flow symmetry plane correlations
  - MC-KLN tends to generate stronger symmetry plane correlations

Ψ_n correlations: $P(\Psi_m, \Psi_n, \Psi_k)$

ALICE, PLB773 (2017) 68 S. McDonald et al., PRC95, 064913 (2017)





$\rho_{422} \approx \langle\cos(4\Psi_4 - 4\Psi_2)\rangle$

$\rho_{532} \approx \langle\cos(5\Psi_5 - 3\Psi_3 - 2\Psi_2)\rangle$

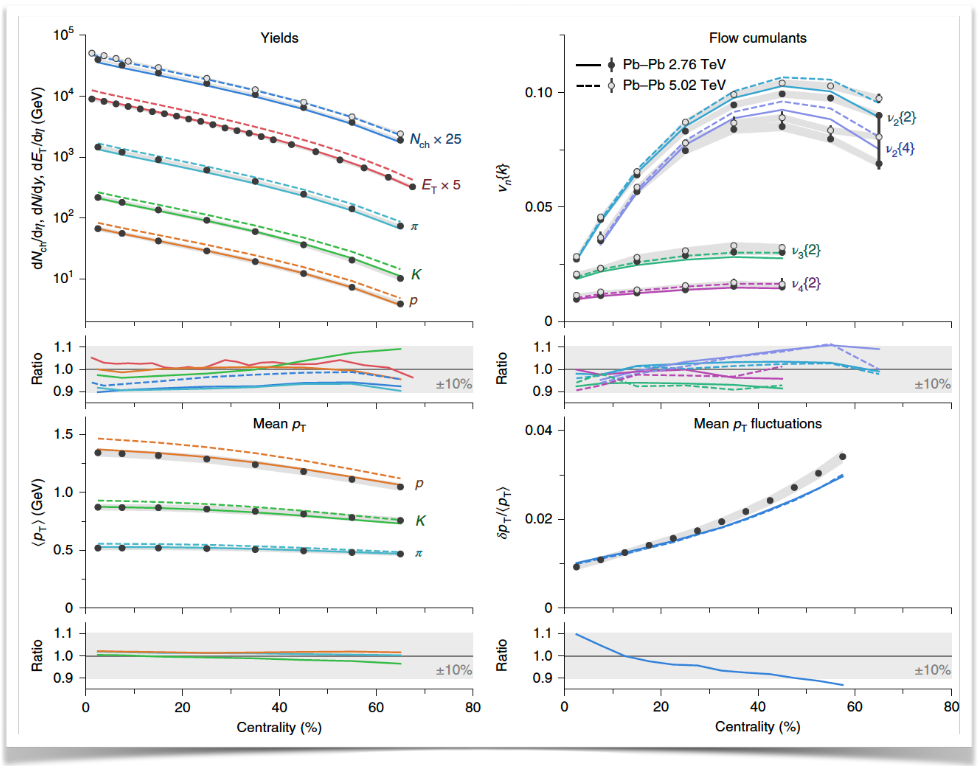
$\rho_{6222} \approx \langle\cos(6\Psi_6 - 6\Psi_2)\rangle$

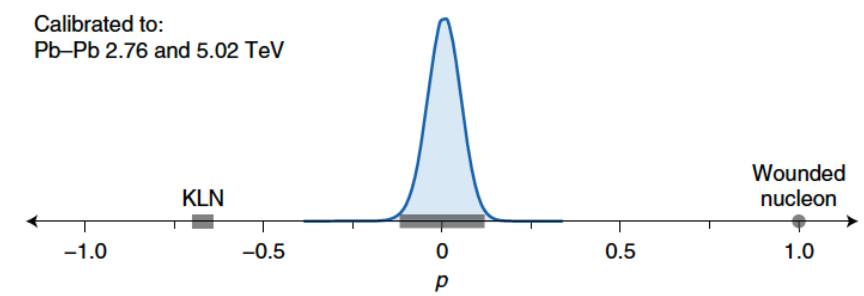
$\rho_{633} \approx \langle\cos(6\Psi_6 - 6\Psi_3)\rangle$

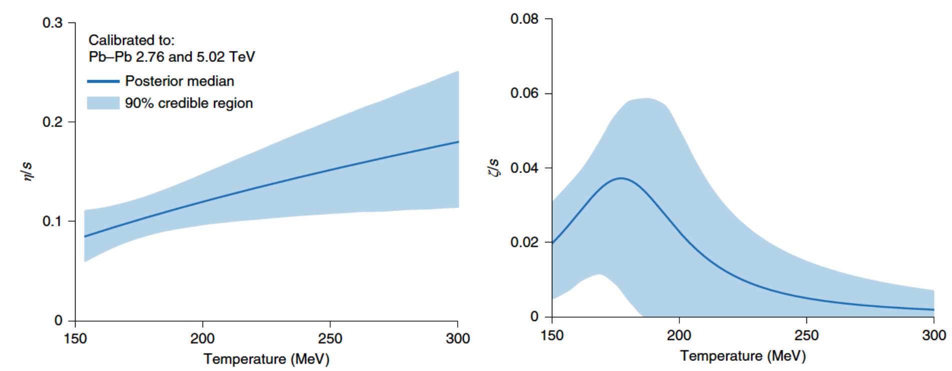
- ρ_{mn} (probes the symmetry plane correlations)
  - Agreement between ALICE and ATLAS (different eta coverage)
  - *Results are compatible with hydrodynamic calculations using IP-Glasma & $\eta/s=0.095$*,
  - calculations using other initial conditions have difficulties to quantitatively describe the data.
- The next: test $\rho_{5432} = \rho_{532} * \rho_{422}$

With Bayesian analyses — End Game?

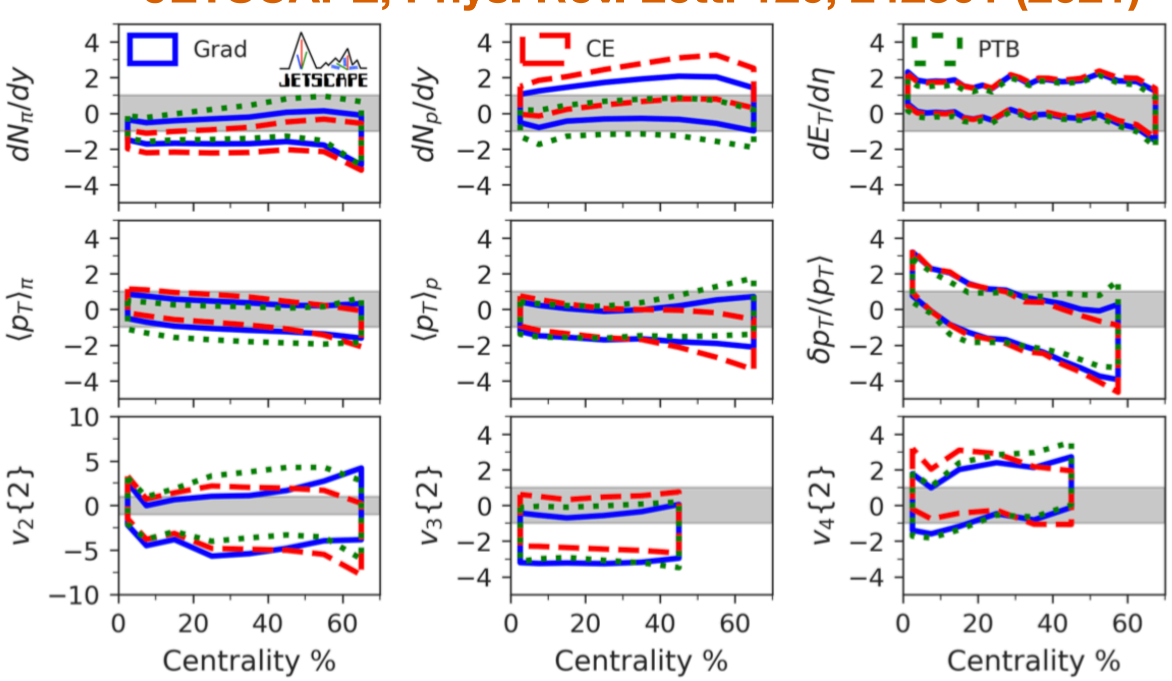
J.E. Bernhard etc, Nature Physics,15, 1113 (2019)

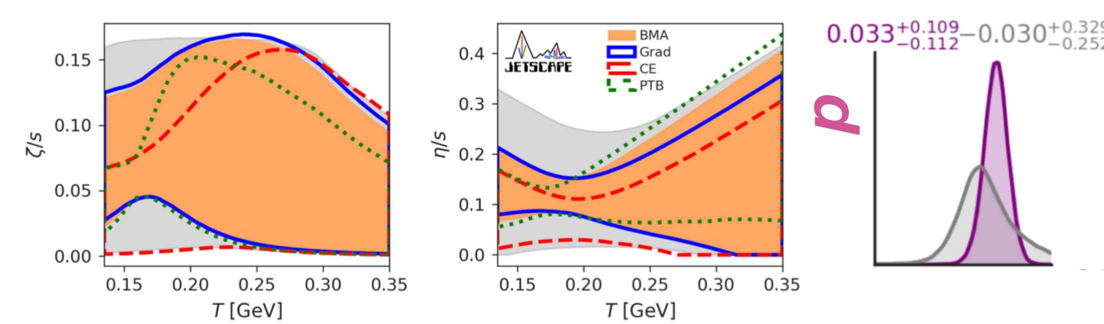


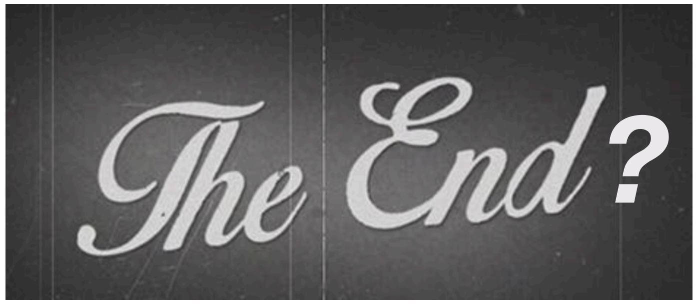




JETSCAPE, Phys. Rev. Lett. 126, 242301 (2021)







$\langle p_T \rangle$ - v_n correlations

❖Shape of the fireball: Anisotropic flow

❖Size of the fireball: radial flow, $[p_T]$

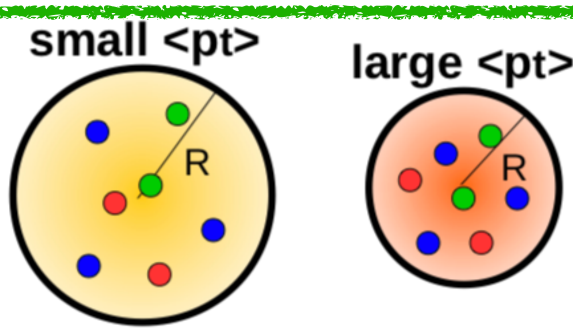
❖Initial geometry and fluctuations of shape and size

❖Final state: correlation between v_n and p_T

$$V_n \propto \mathcal{E}_n$$

$$\frac{d\langle p_t\rangle}{\langle p_t\rangle} \propto \frac{dE}{E}$$

$$\frac{d\langle p_t\rangle}{\langle p_t\rangle} \propto -\frac{dR}{R}$$



$$\rho(v_n^2, [p_T]) = \frac{cov(v_n^2, [p_T])}{\sqrt{var(v_n^2)}\sqrt{var([p_T])}}$$

P. Bozek etc, PRC96 (2017) 014904

☆ $cov(v_n^2, [p_T])$: 3-particle correlation (2 azimuthal, 1 [pT])

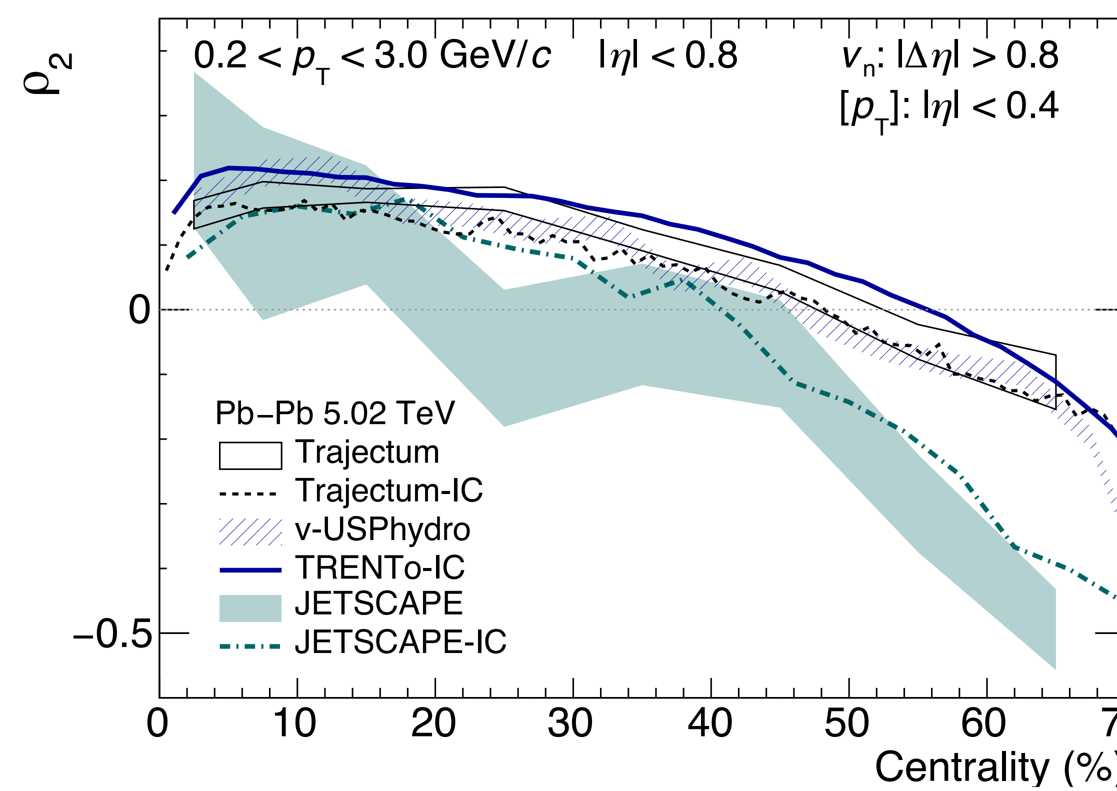
$$\left\langle \frac{\sum_{i\neq j\neq k} w_i w_j w_k e^{in\phi_i} e^{-in\phi_j} (p_{T,k} - \langle\langle p_T\rangle\rangle)}{\sum_{i\neq j\neq k} w_i w_j w_k} \right\rangle_{\rm evt}$$

☆ $\sqrt{var(v_n^2)}$: 2 and 4-particle azimuthal correlations

$$= v_n\{2\}^4 - v_n\{4\}^4$$

☆ $\sqrt{var([p_T])}$: 2-particle [pT] correlations

$$\left\langle \frac{\sum_{i\neq j} w_i w_j (p_{T,i} - \langle\langle p_T\rangle\rangle)(p_{T,j} - \langle\langle p_T\rangle\rangle)}{\sum_{i\neq j} w_i w_j} \right\rangle_{\rm evt}$$



ρ_2 in Pb-Pb

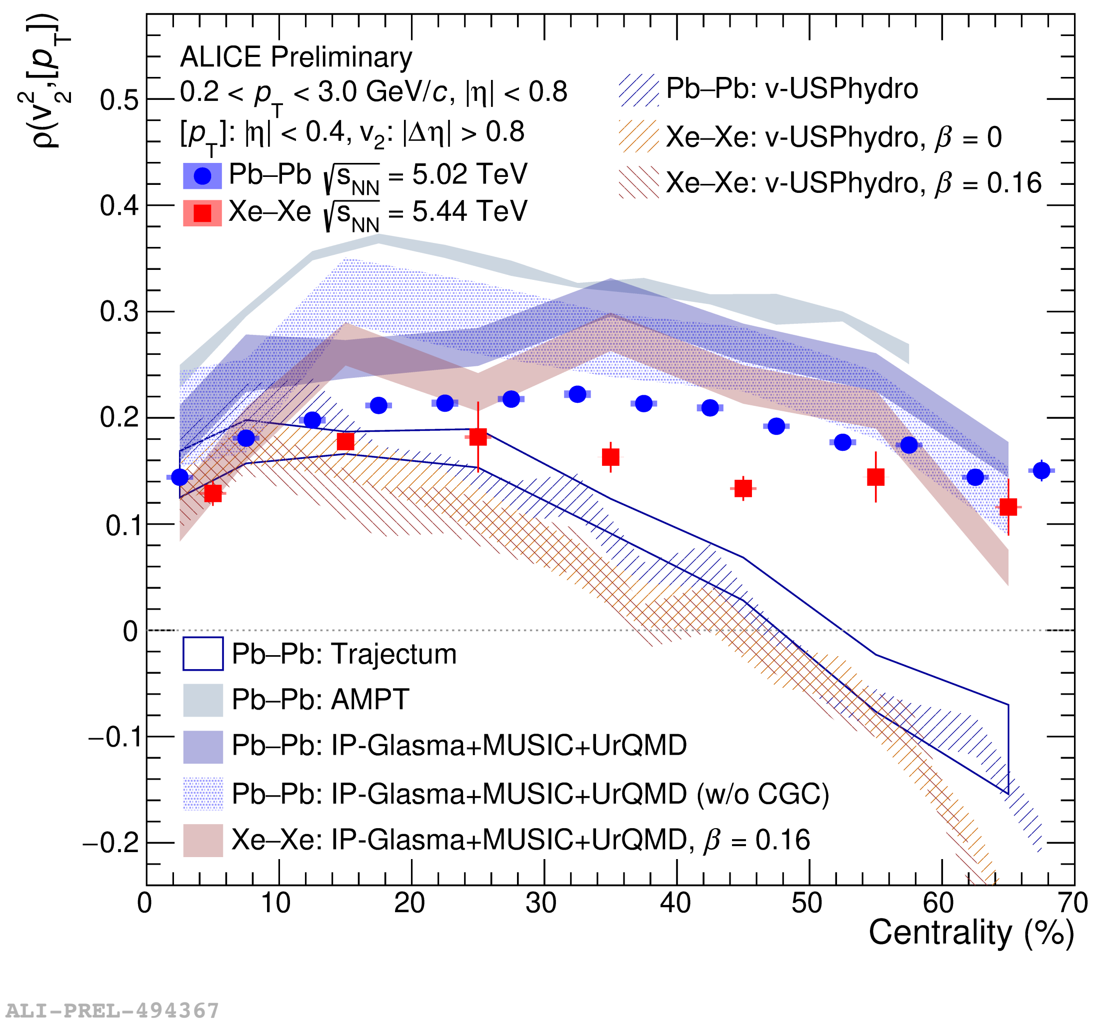
v-USPhydro, PRC103 (2021) 2, 024909

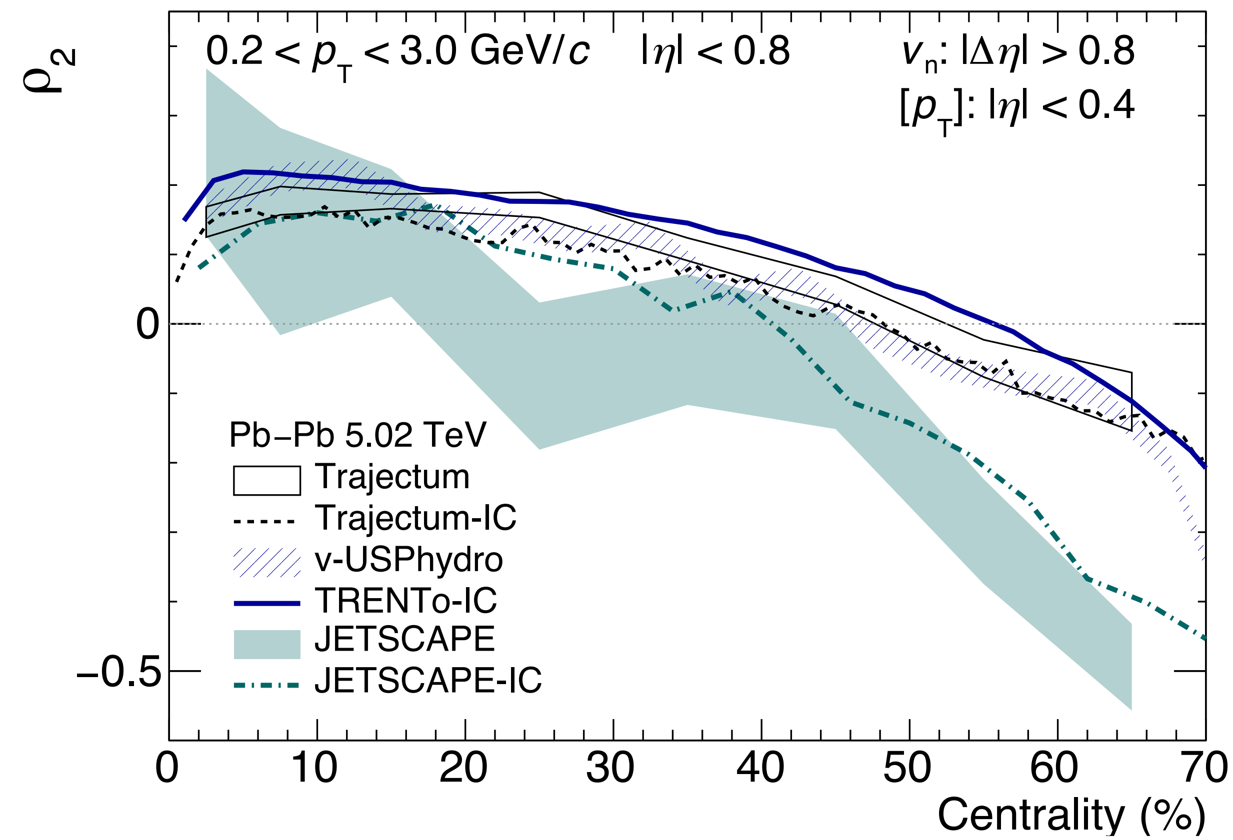
Trajectum, PRL126, 202301 (2021)

Privation communication

JETSCAPE, PRL126, 242301 (2021)

Privation communication

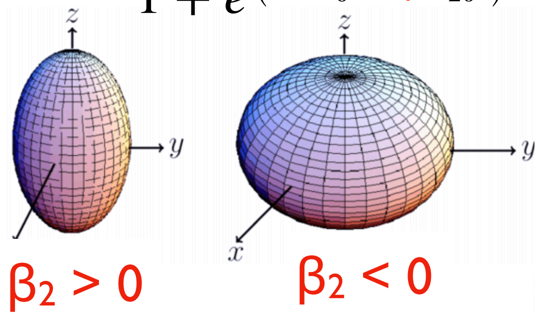




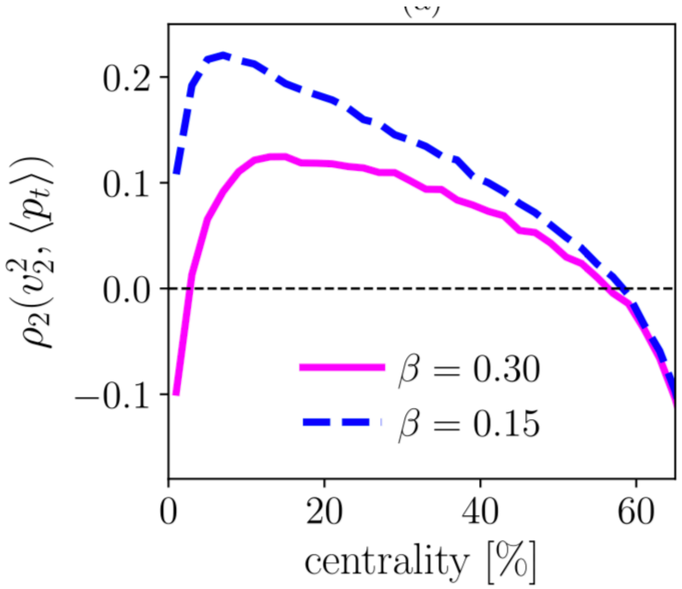
- ❖ IP-Glasma-IC: IP-Glasma + MUSIC + UrQMD works well for Pb-Pb
- ❖ TRENTo-IC based calculations show strong centrality dependence, negative values for centrality >40%
  - v-USPhydro, Trajectum, JETSCAPE
- ❖ The difference is from the initial stage: geometric effects or initial momentum anisotropy (CGC)?
  - No significant difference between the "full IP-Glasma" and "FSE only" for the presented centralities
  - Difference not from initial momentum anisotropy and confirm the different geometric effects

ρ_2 in Xe-Xe

$$D_{WS} = \frac{D_0}{1 + e^{(r - R_0(1+\beta Y_{20}))/a}}$$

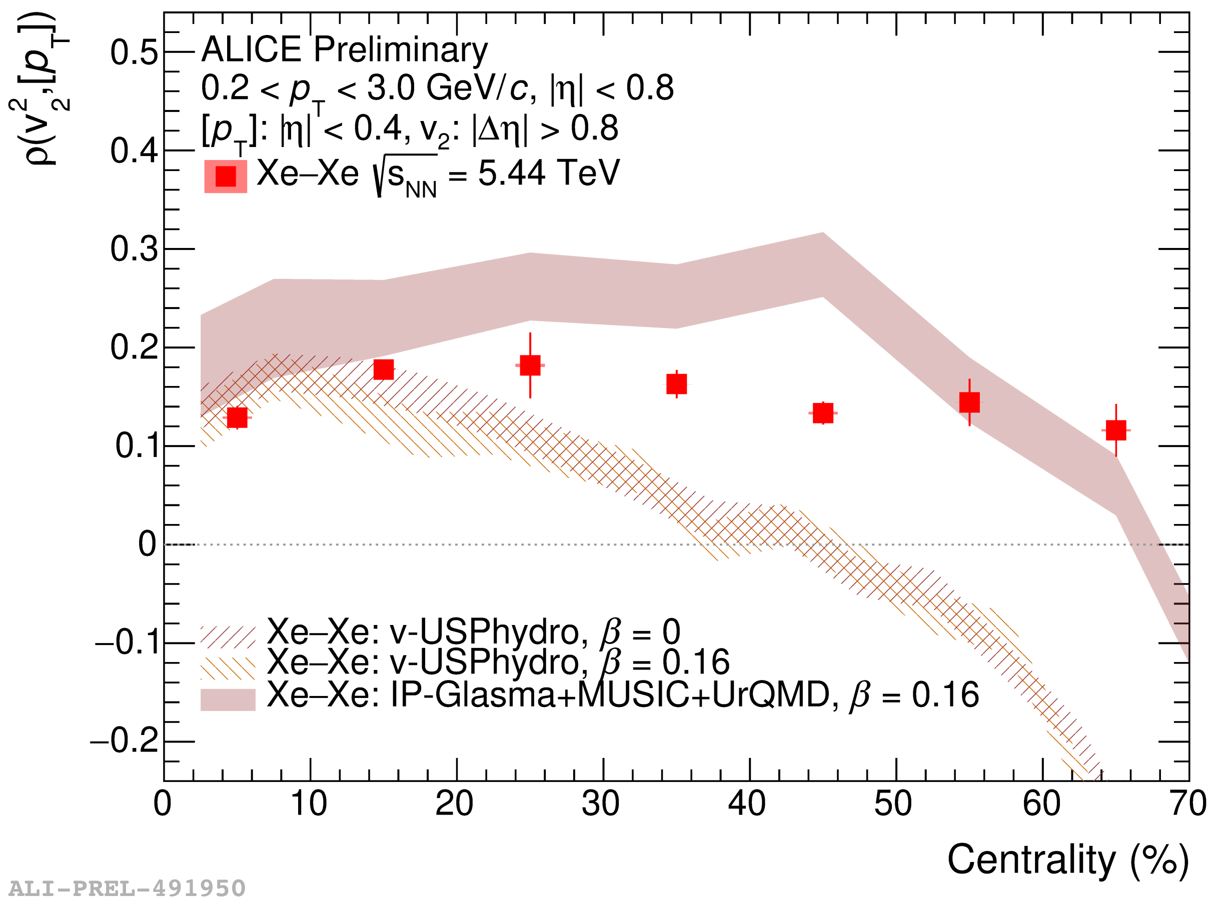


Pb—Pb: $\beta \approx 0$
Xe—Xe: $\beta \approx 0.16$



G.Giacalone, PRC 102 024901 (2020)

v-USPhydro, PRC103 (2021) 2, 024909

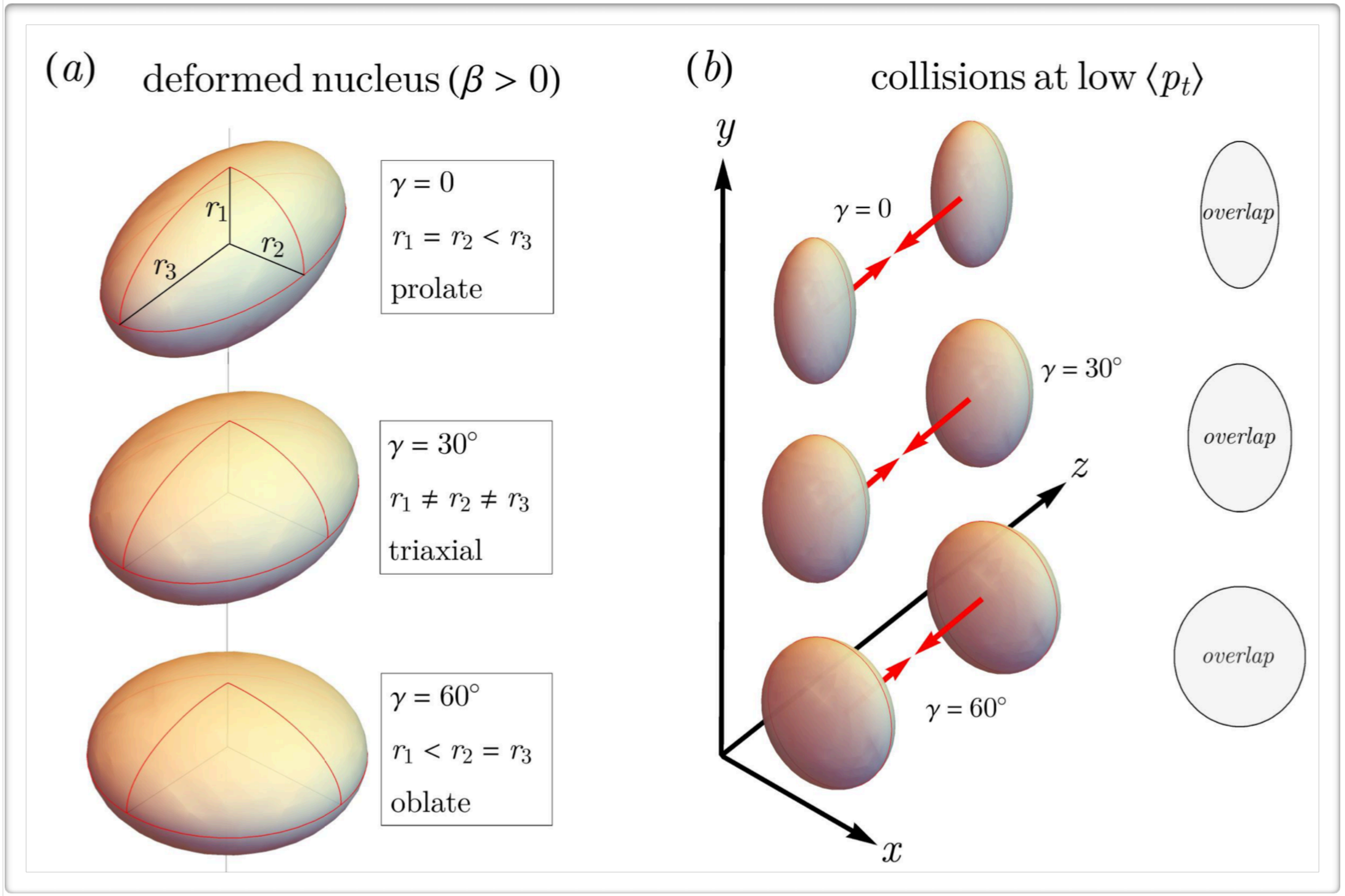


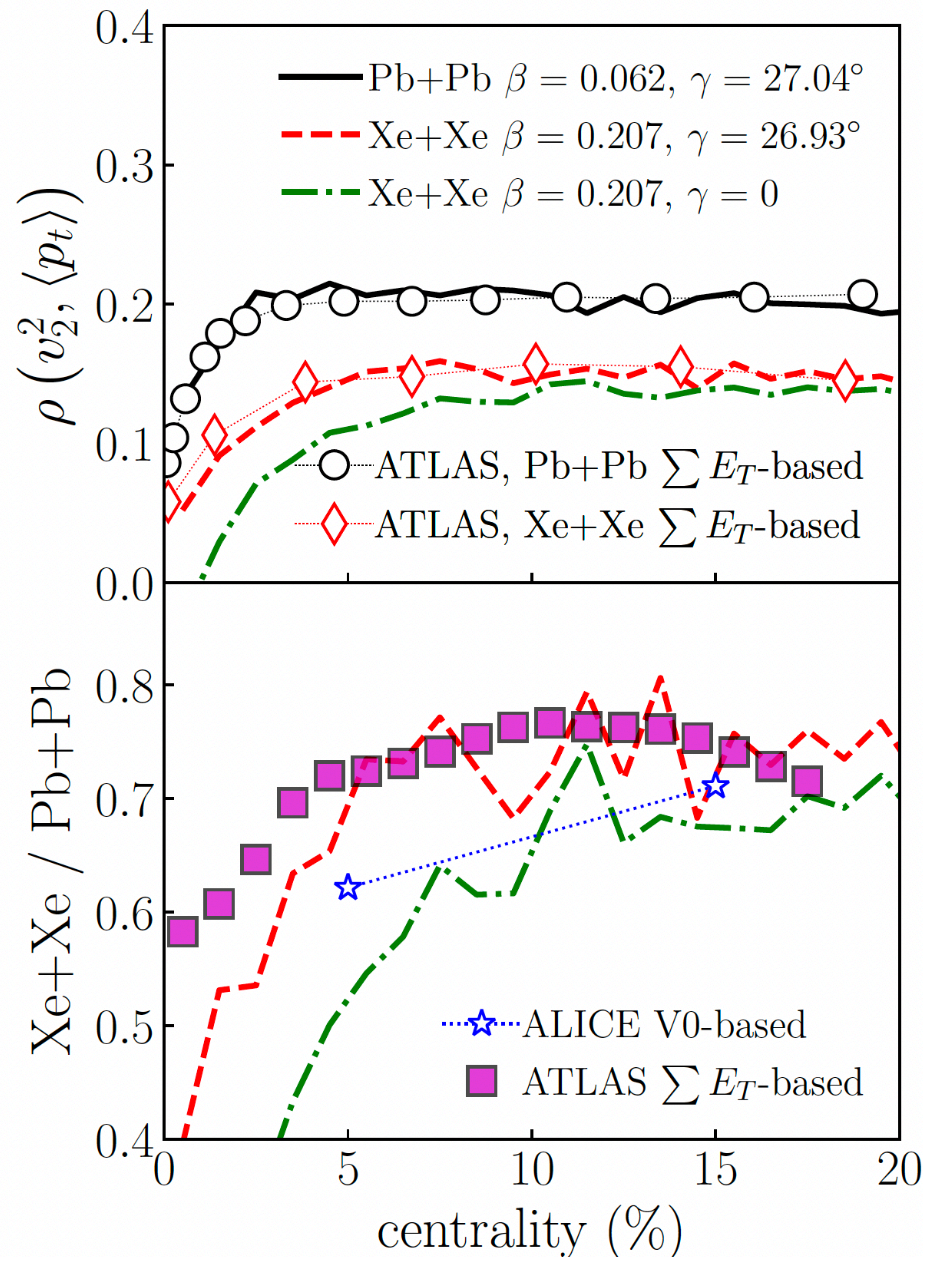
❖ Significant differences of initial state calculations using different deformation parameter in central Xe-Xe collisions

- ρ_2 is sensitivties to β_2
- The uncertainty of current v-USPhydro calculations is too large to draw a confirm conclusions
- Experimental data (in Xe-Xe@LHC and U-U@RHIC) open a new window to study nucleon deformation.

Probe triaxial structure of Xe

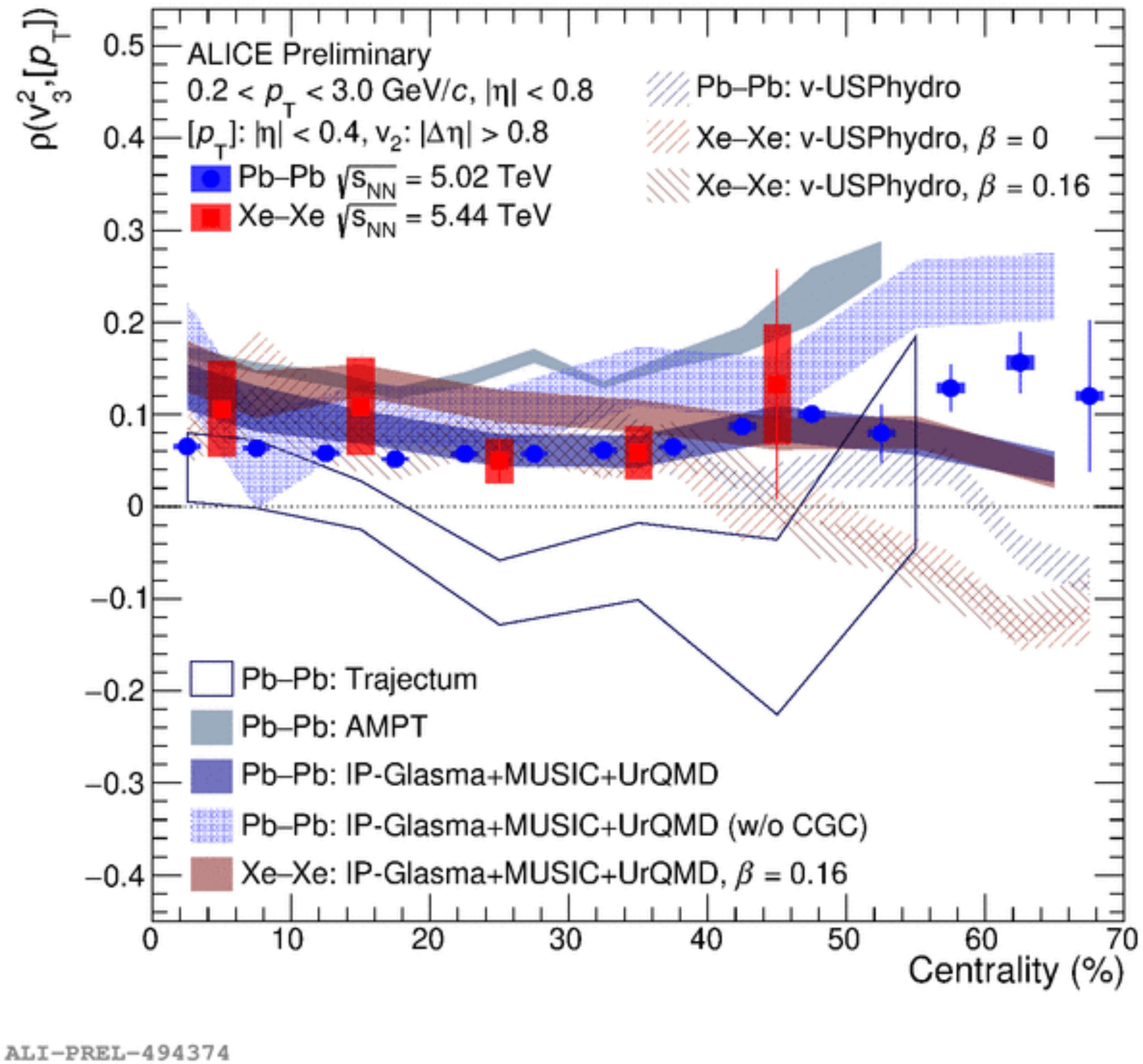
B. Bally etc, arXiv:2108.09578





- Better agreement between LHC data and calculations with $\gamma = 26.93°$
  - Indication of triaxial structure of Xe at high energy
  - New connection of high-energy heavy-ion physics to low-energy nuclear (structure) physics

ρ_3 in Pb-Pb and Xe-Xe



ALICE, in preparation

Trajectum, PRL126, 202301 (2021)
Privation communication

v-USPhydro, PRC103 (2021) 2, 024909

JETSCAPE, PRL126, 242301 (2021)
Privation communication

- ❖ ρ_3 in Pb–Pb is compatible with Xe–Xe for the presented centralities, qualitatively predicted by hydrodynamic calculations

- ❖ ρ_3 values:
  - positive
  - have a modest centrality dependence for the presented centralities,
  - better described by IP-Glasma,
  - TRENTo predicts negative ρ_3 , getting worse for Trajectum and JETSCAPE calculations

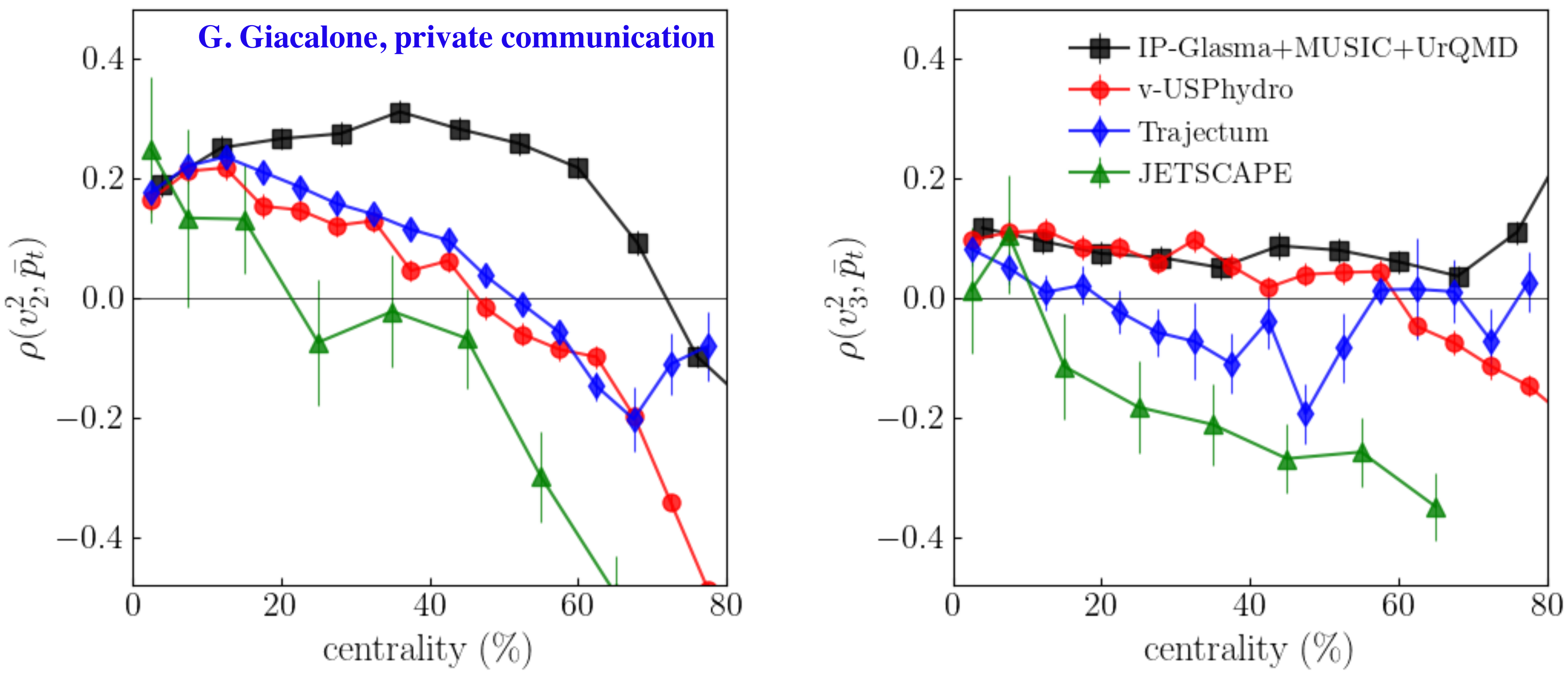
- ❖ model shows that ρ_3 is not sensitive to β_2

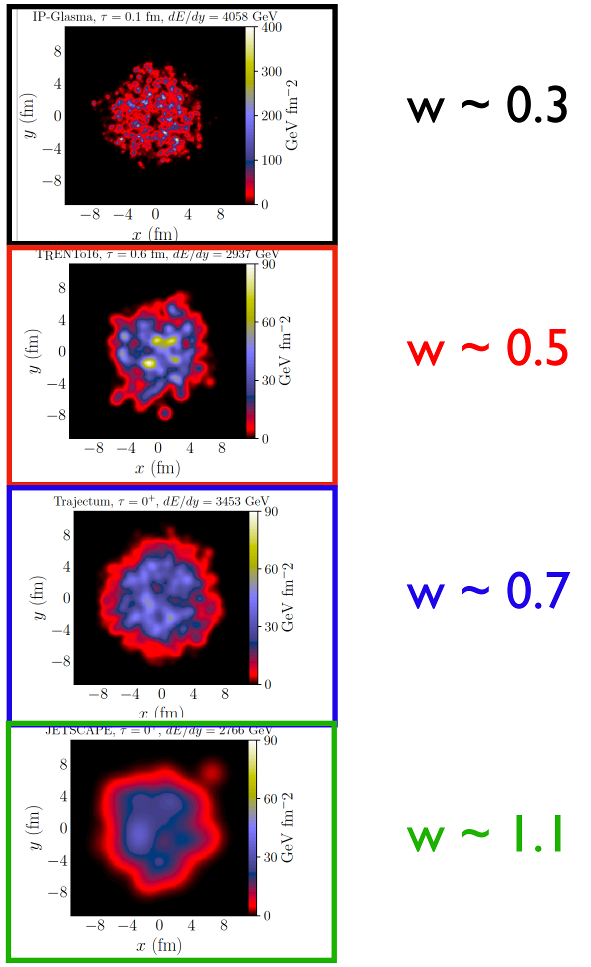
- ❖ Difference of full IP-Glasma and FSE only, indication of potential contributions from IMA in peripheral?

Difference in IP-Glasma and TRENTo: potential explanations

❖ Sensitive to the nucleon width parameter (size of nucleon)

- IP-Glasma ~ 0.3; v-USPhydro ~ 0.5; Trajectum~0.7; JETSCAPE (TRENTo) ~ 1.1
- w(IP-Glasma) < w(v-USPhydro)<w(Trajectum) < w(JETSCAPE)





❖ Different types of thickness functions

- TRENTo $\left(\frac{T_A^p + T_B^p}{2}\right)^{1/p}$ with p≈0 $\sqrt{T_A T_B}$, IP-Glasma $T_A T_B$ type

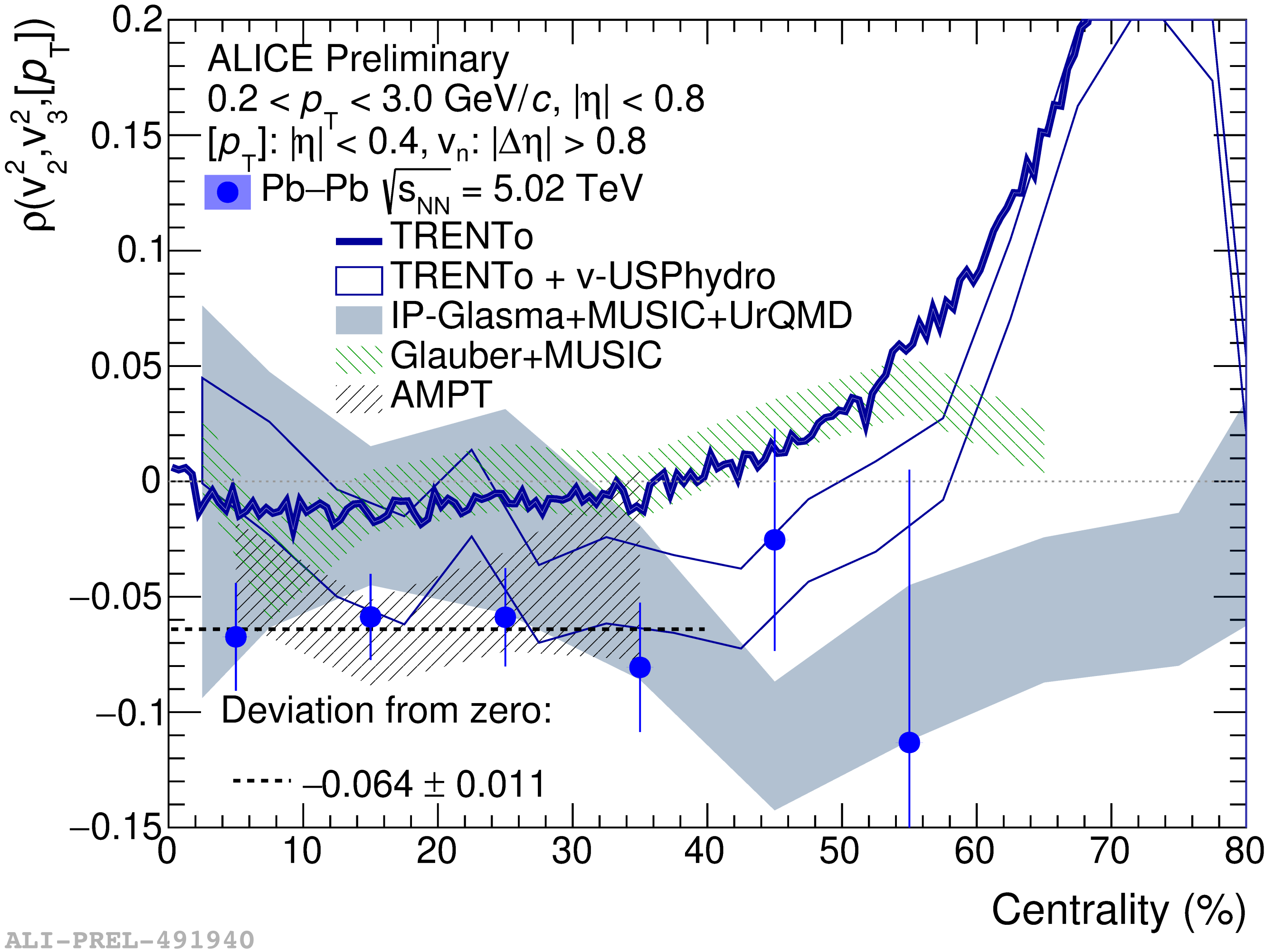
❖ Different contributions from pre-hydrodynamic phase (free streaming) and sub-nucleon structure

Higher-order correlations

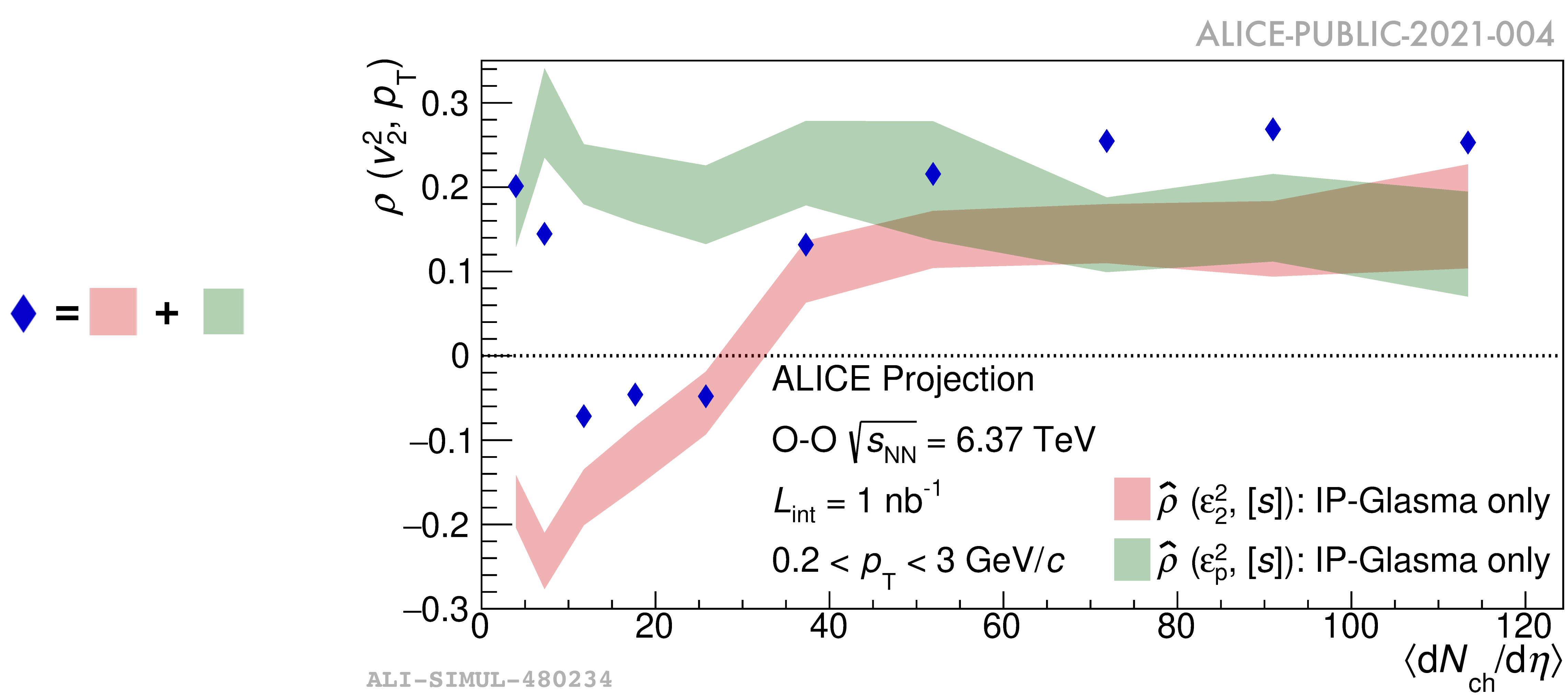
❖ The **first** measurement of higher-order [pT], v_2 and v_3 correlations P. Bozek etc, PRC104 (2021) 1, 014905

$$\rho(v_m^2, v_n^2, [p_T]) = \frac{C(v_m^2, v_n^2, [p_T])}{\sqrt{\mathrm{Var}(v_m^2)}\sqrt{\mathrm{Var}(v_n^2)}\sqrt{c_k}} - \frac{\langle v_m^2\rangle}{\sqrt{\mathrm{Var}(v_m^2)}} \cdot \rho_n - \frac{\langle v_n^2\rangle}{\sqrt{\mathrm{Var}(v_n^2)}} \cdot \rho_m - \frac{\langle [p_T]\rangle}{\sqrt{c_k}} \cdot \frac{SC(m,n)}{\sqrt{\mathrm{Var}(v_m^2)}\sqrt{\mathrm{Var}(v_n^2)}}$$

- ❖ the first ρ_{23} measurement is non-zero
  - negative for the presented centrality
  - anti-correlations between two flow coefficients and [pT]
- ❖ ρ_{23} from IP-Glasma and v-USPhydro are different for centrality >40%
- ❖ Not conclusive on which model works better due to sizeable uncertainties.



More results in smaller colliding systems



❖ Search for the initial momentum anisotropy (IMA) in smaller colliding systems
- Signature of IMA:
  - once sign change in Pb-Pb and pp collisions
  - twice sign changes in O-O collisions
  - New possibility to probe IMA at LHC-Run3

Summary

Characterizing the initial conditions in heavy-ion collisions

☆ **Initial geometry:**

- The "standard" flow analyses give the access to the $P(\varepsilon_n, \varepsilon_m, \varepsilon_k, \ldots)$ from the initial conditions
- For the v_n and $[p_T]$ correlation, for first time we see completely different behaviours using IP-Glasma and TRENTo initial state models
  - Sensitivities to nucleon width, thickness function, number of constituent quarks…

☆ **Initial momentum anisotropy:**

- The observed differences from different models are not originated from initial momentum anisotropy (IMA)
- Potential signal for ρ_3 in peripheral collisions and ρ_n in small systems, not yet conclusive in experiments.

☆ **Nucleon structure**

- results in Xe-Xe collisions (and also in U-U and isobar runs @ RHIC) open a new window to constrain deformation parameter and explore the triaxial structure of nucleon.

Thanks for your attention!

Backup

Generic algorithm (2021)

❖ 2021, Generic algorithm method, the most *efficient*, *precise* and *reliable* method

arXiv: 2005.07974

PHYSICAL REVIEW C **103**, 024913 (2021)

Generic algorithm for multiparticle cumulants of azimuthal correlations in high energy nucleus collisions

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(Received 23 November 2020; accepted 11 February 2021; published 26 February 2021)

```
complex Correlator(int* harmonic, int n, int mult = 1, int skip = 0)
{
  int har_sum = 0;
  for (int i = 0; i<mult; ++i) har_sum += harmonic[n-1+i];
  complex c(Q(har_sum, mult));
  if (n == 1) return c;
  c *= Correlator(harmonic, n-1);
  if (n == 1+skip) return c;

  complex c2 = 0;
  int h_hold = harmonic[n-2];
  for (int counter = 0; counter <= n-2-skip; ++counter)
  {
    harmonic[n-2] = harmonic[counter];
    harmonic[counter] = h_hold;
    c2 += Correlator(harmonic, n-1, mult+1, n-2-counter);
    harmonic[counter] = harmonic[n-2];
  }
  harmonic[n-2] = h_hold;
  return c-mult*c2;
}
```

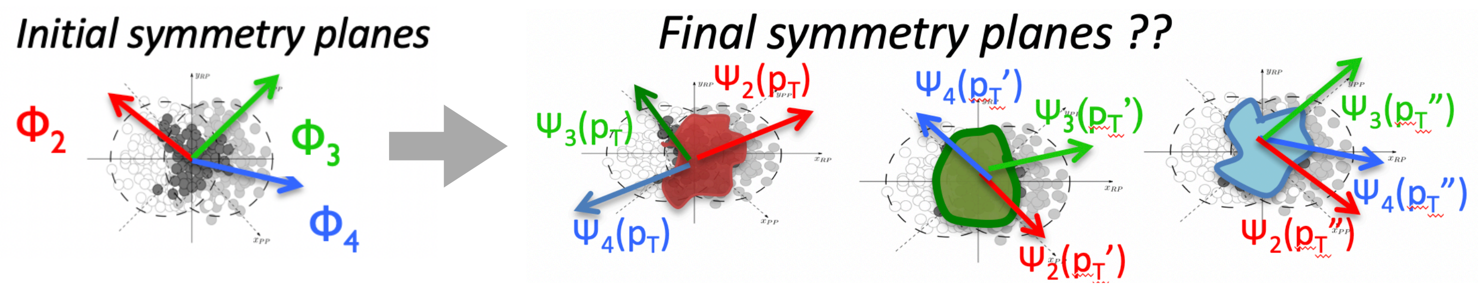
- Few lines of code, for **any** multi-particle correlations
- Much faster than generic framework (much shorter CPU times)

Feel free to contact you.zhou@cern.ch
if you have any technical question

Jiangyong Jia, J.Phys.G 41 (2014) 12

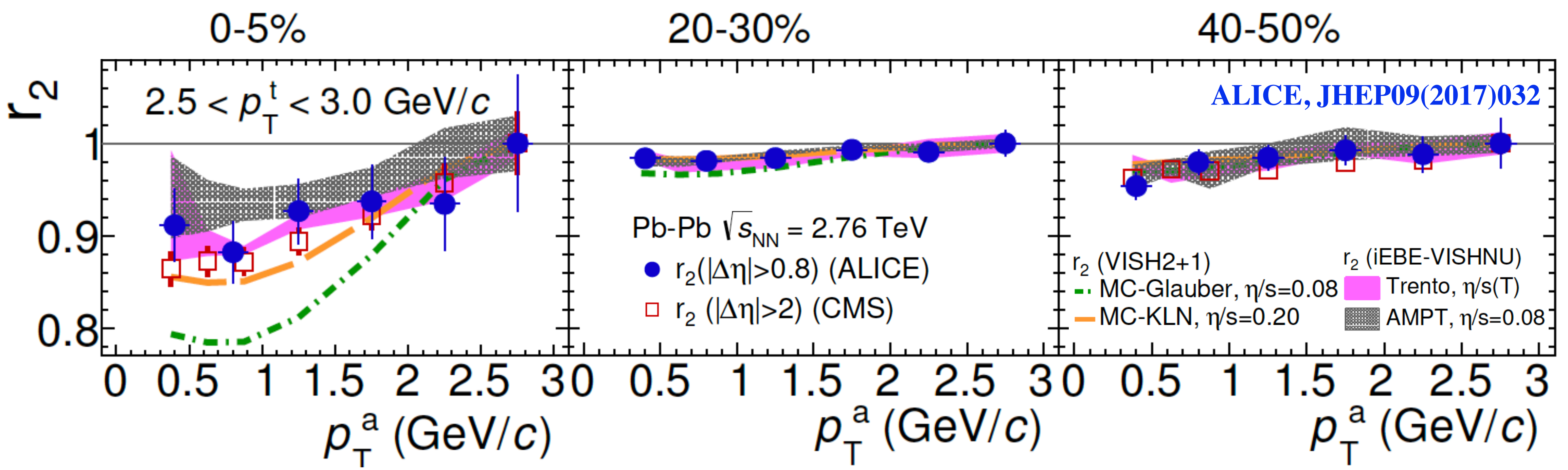
	pdfs	cumulants
Flow-amplitudes	$p(v_n)$	$v_n\{2k\}$, $k = 1,2,...$
	$p(v_n, v_m)$	$\langle v_n^2 v_m^2\rangle - \langle v_n^2\rangle\langle v_m^2\rangle$, $n \neq m$ ...
	$p(v_n, v_m, v_l)$	$\langle v_n^2 v_m^2 v_l^2\rangle + 2\langle v_n^2\rangle\langle v_m^2\rangle\langle v_l^2\rangle -$ $\langle v_n^2 v_m^2\rangle\langle v_l^2\rangle - \langle v_m^2 v_l^2\rangle\langle v_n^2\rangle - \langle v_l^2 v_n^2\rangle\langle v_m^2\rangle$ $n \neq m \neq l$ ...
	...	Obtained recursively as above
EP-correlation	$p(\Phi_n, \Phi_m, ...)$	$\langle v_n^{\lvert c_n\rvert} v_m^{\lvert c_m\rvert} ... \cos(c_n n\Phi_n + c_m m\Phi_m + ...)\rangle$ $\sum_k kc_k = 0$
Mixed-correlation	$p(v_l, \Phi_n, \Phi_m, ...)$	$\langle v_l^2 v_n^{\lvert c_n\rvert} v_m^{\lvert c_m\rvert} ... \cos(c_n n\Phi_n + c_m m\Phi_m + ...)\rangle -$ $\langle v_l^2\rangle\langle v_n^{\lvert c_n\rvert} v_m^{\lvert c_m\rvert} ... \cos(c_n n\Phi_n + c_m m\Phi_m + ...)\rangle$ $\sum_k kc_k = 0$, $n \neq m \neq l ...$

ψ_n fluctuations $P(\psi_n)$



$$r_n = \frac{V_{n\Delta}(p_T^a, p_T^b)}{\sqrt{V_{n\Delta}(p_T^a, p_T^a) \cdot V_{n\Delta}(p_T^b, p_T^b)}}$$

- r_n probes < a, b > ➡ < a, a > & < b, b >
- r_n < 1, Factorization broken



❖ Breakdown of factorization more pronounced in central collisions.

❖ Hydrodynamic reproduce the factorization broken

- Indication of p_T dependent flow angle (and magnitude) fluctuations

❖ Using novel multi-particle correlations, both flow-angle and flow magnitude fluctuations are observed in experiments (see backup for more details)

$P(v_n) \to P(\varepsilon_n)$

ATLAS, JHEP11, 183 (2013)

❖ Elliptic-power function:

$$P(v_2) = \frac{d\varepsilon_2}{dv_2} P(\varepsilon_2) = \frac{1}{k_2} P\left(\frac{v_2}{k_2}\right) = \frac{2\alpha v_2}{\pi k_2^2} (1-\varepsilon_0^2)^{\alpha+1/2} \int_0^{\pi} \frac{(1-v_2^2/k_2^2)^{\alpha-1}}{(1-v_2\varepsilon_0\cos\varphi/k_2)^{2\alpha+1}} d\varphi$$

$$\begin{aligned}
c_2\{2\} =& k_2^2 \, (1-f_1)\,, \\
c_2\{4\} =& -k_2^4 \, (1 - 2f_1 + 2f_1^2 - f_2)\,, \\
c_2\{6\} =& k_2^6 \, (4 + 18f_1^2 - 12f_1^3 + 12f_1(3f_2-1) - 6f_2 - f_3)\,, \\
c_2\{8\} =& -k_2^8 \, (33 - 288f_1^3 + 144f_1^4 - 66f_2 + 18f_2^2 - 24f_1^2(-11+6f_2) \\
& - 12f_3 + 4f_1(-33+42f_2+4f_3) - f_4)
\end{aligned}$$

$$f_k \equiv \langle (1-\varepsilon_n^2)^k \rangle = \frac{\alpha}{\alpha+k}(1-\varepsilon_0^2)^k \, {}_2F_1\left(k+\frac{1}{2}, k; \alpha+k+1, \varepsilon_0^2\right)$$