A Monte-Carlo Simulation of Double Parton Scattering

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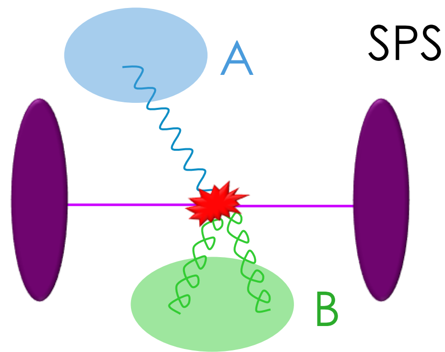
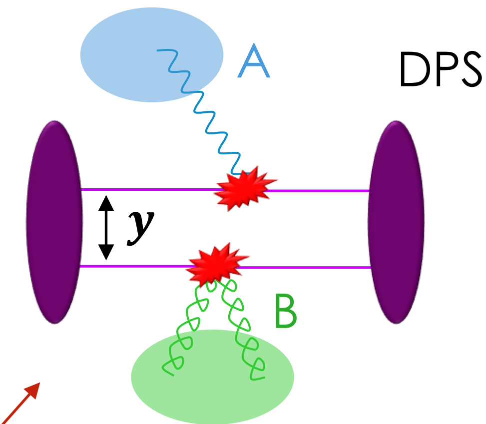
Based on JHEP 11 (2019) 061, JHEP 10 (2020) 012 with Baptiste Cabouat and Kiran Ostrolenk

Taming the accuracy of event generators, 27th August 2021

DOUBLE PARTON SCATTERING

Double parton scattering (DPS) is where we have **two separate hard scatters** in one collision





$$\sigma_S = f(x_1) \otimes \hat{\sigma}_{AB} \otimes f(x_1')$$

Single parton distributions (PDFs)

$$\sigma_D = \int d^2\boldsymbol{y}\, F(x_1, x_2, \boldsymbol{y}) \otimes \hat{\sigma}_A \; \hat{\sigma}_B \otimes F(x_1', x_2', \boldsymbol{y})$$

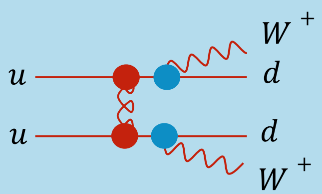
Double parton densities (DPDs)

Paver, Treleani, Nuovo Cim. A70 (1982) 215
Mekhfi, Phys.Rev. D32 (1985) 2371
Diehl, Ostermeier, Schafer, JHEP 1203 (2012) 089

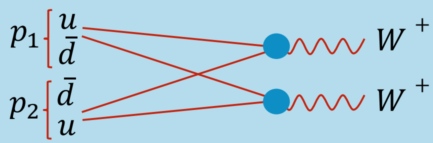
WHY STUDY DPS?

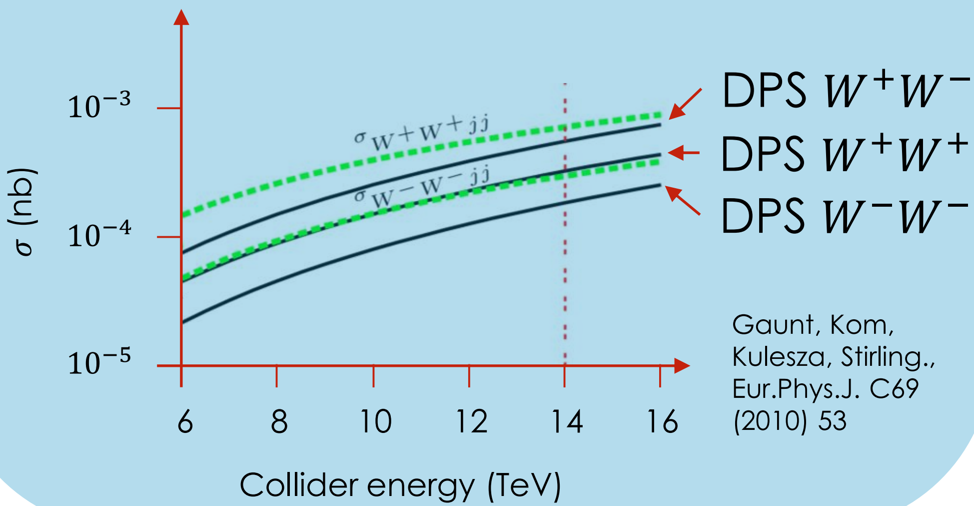
DPS can give a significant contribution to processes where SPS is suppressed by small/multiple coupling constants:

SPS:



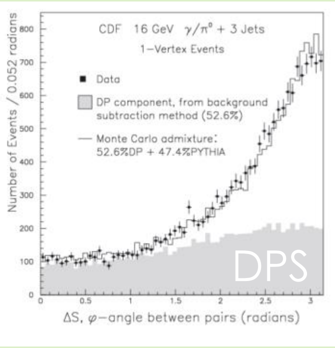
DPS:





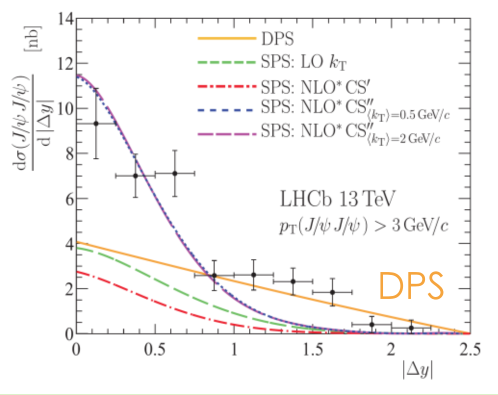
Gaunt, Kom, Kulesza, Stirling., Eur.Phys.J. C69 (2010) 53

…or in certain phase space regions



CDF, $\gamma + 3j$, Phys.Rev. D56 (1997) 3811-3832

LHCb, double J/ψ, JHEP 06, 047, (2017)



Intrinsically interesting: tells us about **correlations** between partons!

THE DPS POCKET FORMULA

How do we make predictions for DPS cross sections and event shapes?

Most common approach: **ignore correlations** between partons, assume $(\boldsymbol{y})$ and (x_1, x_2) dependence in DPD factorises

$$\boxed{\sigma_D^{(A,B)} = \frac{\sigma_S^{(A)} \sigma_S^{(B)}}{\sigma_{\text{eff}}}}$$

‘**Pocket formula**’

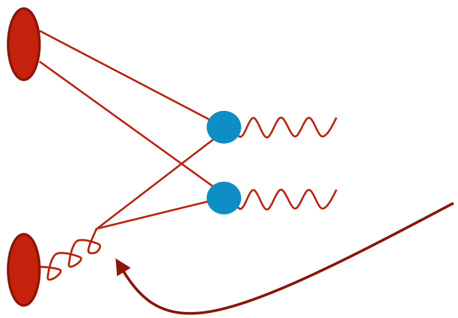
~ proton transverse area

Current Monte Carlo models of DPS (and more general multiple parton interactions) based on pocket formula picture.
[Although Pythia model has some notable improvements: account of the fact that each interaction takes momentum and potentially flavour out of the proton]

Sjöstrand, van Zijl, Phys.Rev. D36 (1987) 2019, Sjöstrand, Skands, JHEP 0403 (2004) 053 Eur.Phys.J. C39 (2005) 129-154

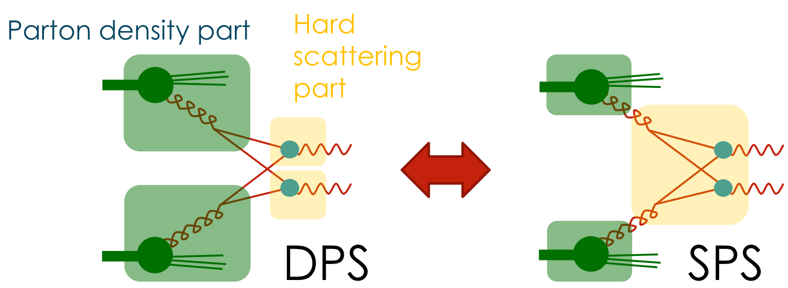
DPS IN QCD

The pocket formula can't be the full picture. Lots of progress towards the **full description of DPS in QCD.** JG, Stirling, Diehl, Ostermeier, Plößl, Nagar, Schäfer, Blok, Dokshitzer, Strikman, Frankfurt, Manohar, Waalewijn, Ryskin, Snigirev…



In QCD – parton pair from one/both protons can have arisen from a **perturbative '1 → 2' splitting**. Induces correlations, nontrivial y behaviour.

Contributions with '1 → 2' splittings in both protons **overlap with loop corrections to SPS**. Framework to include 1 → 2 splittings & avoid double counting developed.



Diehl, JG, Schoenwald, JHEP 06 (2017) 083

DPDs must obey **momentum and number sum rules.** JG, Stirling, JHEP 1003 (2010) 005, Blok, Dokshitzer, Frankfurt, Strikman, Eur.Phys.J. C74 (2014) 2926, Ceccopieri, Phys.Lett. B734 (2014) 79-85, Diehl, Plößl, Schäfer, Eur.Phys.J. C79 (2019) no.3, 253

Also: possibility of **correlations between partons in spin, colour, flavour**…

Mekhfi, Phys. Rev. D32 (1985) 2380, Diehl, Ostermeier and Schafer (JHEP 1203 (2012)), Manohar, Waalewijn, Phys.Rev. D85 (2012) 114009

A MONTE CARLO SIMULATION OF DPS

Theory now developed → **need pheno tools** to translate this into predictions.

Would be particularly useful to have a Monte Carlo generator for DPS based on full theory picture.

(For example), DPS extractions often involve computing DPS/SPS shape in multiple observables

11 variables in CMS same-sign WW:

$$p_T^{l_1}, p_T^{l_2}, p_T^{miss}, \eta_1\eta_2, |\eta_1 + \eta_2|, m_{T(l_1, p_T^{miss})}, m_{T(l_1, l_2)}, \left|\Delta\phi_{(l_1, l_2)}\right|, \left|\Delta\phi_{(l_2, p_T^{miss})}\right|, \left|\Delta\phi_{(ll, l_2)}\right|, m_{T2}^{ll}$$

CMS, Eur. Phys. J. C 80 (2020) 41

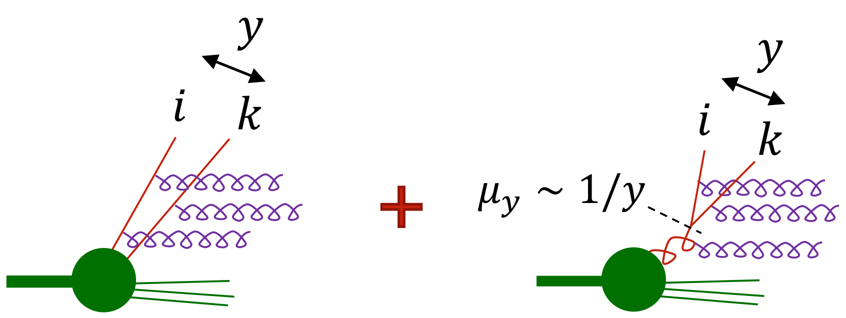
→ A Monte Carlo simulation of DPS, dShower

Cabouat, JG, Ostrolenk, JHEP 1911 (2019) 061

A NEW DPS SHOWER

Part I: MC description of DPS:

- With account of $\boldsymbol{y}$ **dependence**
- Including **1→2 splitting** effects
- With possibility to incorporate **parton correlations**



[Cabouat, JG, Ostrolenk JHEP 11 (2019) 061]

Part II: Combination of DPS and SPS in MC without double counting

- Implement double counting approach of DGS, but now at **differential level**, with showers attached to all terms

[Cabouat, JG, JHEP 10 (2020) 012]

STEP I: THE DPS SHOWER

DSHOWER ALGORITHM

Summary of algorithm:

(i) Select x_i of initiating partons and y using DPS formula:

$$\sigma^{\mathrm{DPS}}_{(A,B)}(s) = \frac{1}{1+\delta_{AB}} \sum_{i,j,k,l} \int \mathrm{d}\tau_A \, \mathrm{d}Y_A \, \mathrm{d}\hat{t}_A \, \mathrm{d}\tau_B \, \mathrm{d}Y_B \, \mathrm{d}\hat{t}_B \, \frac{\mathrm{d}\hat{\sigma}_{ij\to A}}{\mathrm{d}\hat{t}_A} \, \frac{\mathrm{d}\hat{\sigma}_{kl\to B}}{\mathrm{d}\hat{t}_B}$$
$$\times \int 2\pi \, y \, \mathrm{d}y \, \Phi^2(y\nu) \, F_{ik}(x_1, x_3, \boldsymbol{y}, \mu^2) \, F_{jl}(x_2, x_4, \boldsymbol{y}, \mu^2)$$

Cut-off of DPS for y values $\lesssim 1/\nu \sim 1/Q$

Some external DPD set – correlations fed into shower

Diehl, JG, Schoenwald, JHEP 06 (2017) 083

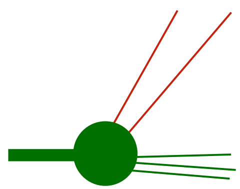
Small y region is the domain of SPS

[Dependence on cut-off parameter ν?]

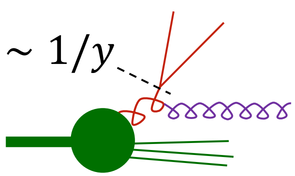
DPDS FOR DSHOWER

Use of any DPDs possible
provided they satisfy:

(1) Two components:



'INTRINSIC'



'SPLITTING'

(2) $yF_{\text{int}}(y) \to 0$ as $y \to 0$

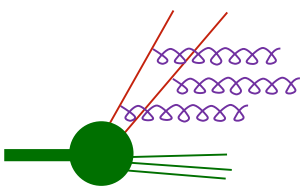
$F_{\text{spl}}(y, \mu \sim {}^{1}/_{y}) \to \frac{1}{\pi y^2} \frac{f_k(x_1+x_2, \mu^2)}{x_1+x_2} \frac{\alpha_s(\mu^2)}{2\pi} P_{k\to ij}\left(\frac{x_1}{x_1+x_2}\right)$ for perturbative y

Up to small corrections, DPD given by perturbative splitting at small y
(required theoretically)

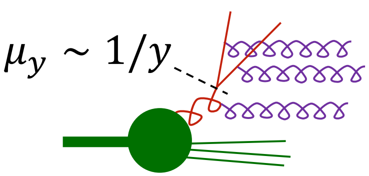
DPDS FOR DSHOWER

Use of any DPDs possible provided they satisfy:

(3) DPDs evolve with scale μ according to homogeneous double DGLAP equation



'INTRINSIC'



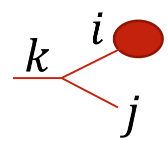
'SPLITTING'

(4) DPDs obey number and momentum sum rule constraints, at least approximately.

DSHOWER ALGORITHM

(ii) Generate QCD emissions, going backwards from hard process

Evolution variable as in Herwig:



For ISR: $$Q^2 = \tilde{q}^2_{ISR} = -\frac{(p_i^2 - m_i^2)}{(1-z)} \approx E_k^2 \theta_j^2$$ Angular ordering

Probability that partons ij survive from Q_h to Q, and then at Q there is an emission from one leg:

$$\mathrm{d}\mathcal{P}^{\mathrm{ISR}}_{ij} = \mathrm{d}\mathcal{P}_{ij} \underbrace{\exp\left(-\int_{Q^2}^{Q_h^2} \mathrm{d}\mathcal{P}_{ij}\right)}_{\text{'Sudakov factor'}}$$

Emission probability

Emission from leg 1

$$\mathrm{d}\mathcal{P}_{ij} = \frac{\mathrm{d}Q^2}{Q^2}\left(\sum_{i'}\int_{x_1}^{1-x_2}\frac{\mathrm{d}x_1'}{x_1'}\frac{\alpha_{\mathrm{s}}(p_\perp^2)}{2\pi}P_{i'\to i}\left(\frac{x_1}{x_1'}\right)\frac{F_{i'j}(x_1',x_2,\boldsymbol{y},Q^2)}{F_{ij}(x_1,x_2,\boldsymbol{y},Q^2)}\right.$$
$$\left.+\sum_{j'}\int_{x_2}^{1-x_1}\frac{\mathrm{d}x_2'}{x_2'}\frac{\alpha_{\mathrm{s}}(p_\perp^2)}{2\pi}P_{j'\to j}\left(\frac{x_2}{x_2'}\right)\frac{F_{ij'}(x_1,x_2',\boldsymbol{y},Q^2)}{F_{ij}(x_1,x_2,\boldsymbol{y},Q^2)}\right)$$

Emission from leg 2

Use 'competing veto algorithm' to decide which leg emits

Kleiss, Verheyen, Eur. Phys. J. C76 (2016) 359

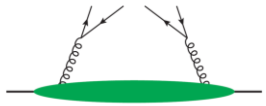
DSHOWER ALGORITHM

(iii) At scale $\mu_y \sim 1/y$, decide whether to merge partons i and j. Merging is done with a probability given by:

$$p_{Mrg} = F_{ij}^{spl}(x_1, x_2, y, \mu_y^2)/F_{ij}^{tot}(x_1, x_2, y, \mu_y^2)$$

Total DPD

$$F_{ij}^{spl}(x_1, x_2, y, \mu_y^2) = \frac{1}{\pi y^2} \frac{f_k(x_1+x_2, \mu_y^2)}{x_1+x_2} \frac{\alpha_s(\mu_y^2)}{2\pi} P_{k\to ij}\left(\frac{x_1}{x_1+x_2}\right) \times \text{large } \boldsymbol{y} \text{ suppression}$$



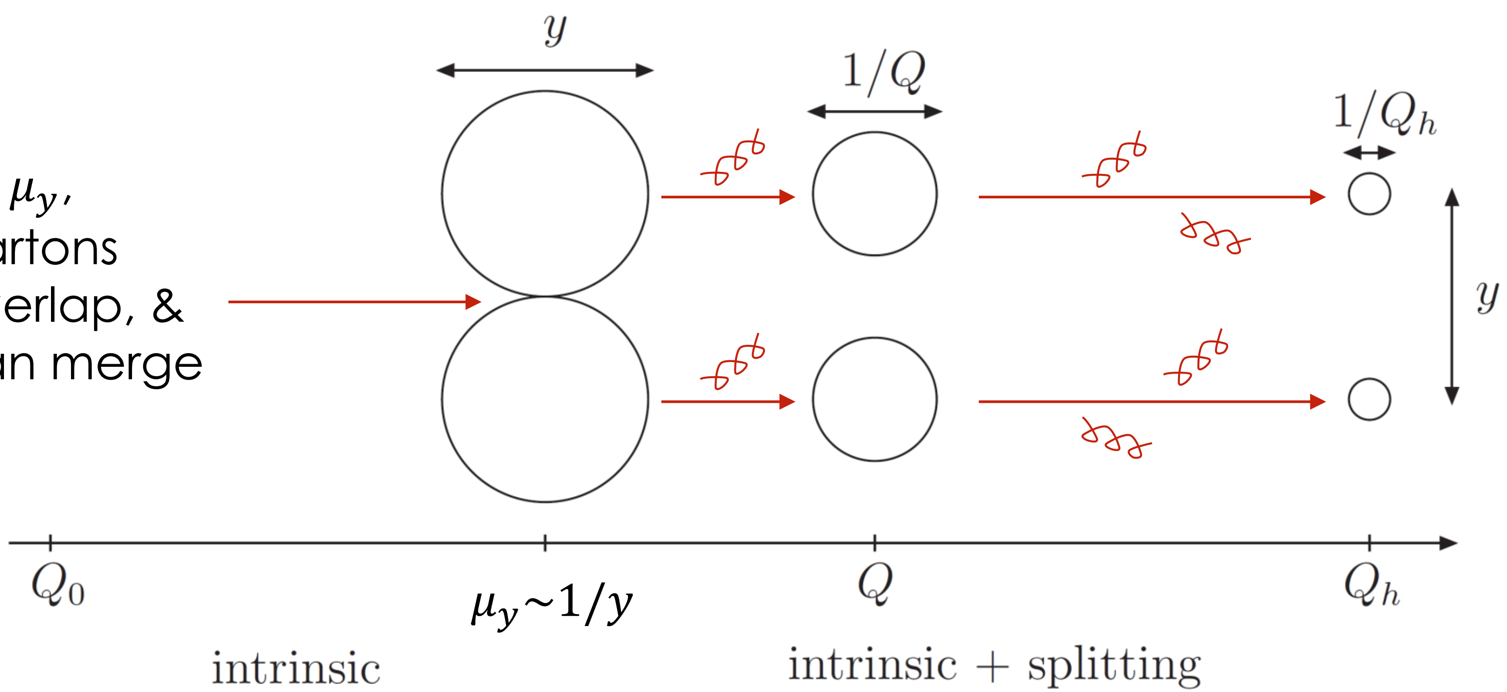
If no merging: continue with two parton branching algorithm from (2), using only 'intrinsic' DPDs.

If merging: shower single parton a la Herwig.

MERGING: SCHEMATIC PICTURE

Intuitive picture:

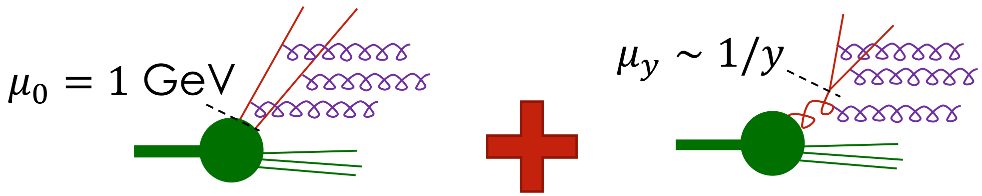
At μ_y, partons overlap, & can merge



SOME FIRST NUMERICS

DPDs from JHEP 1706 (2017) 083 (Diehl, JG, Schönwald):

DPD =



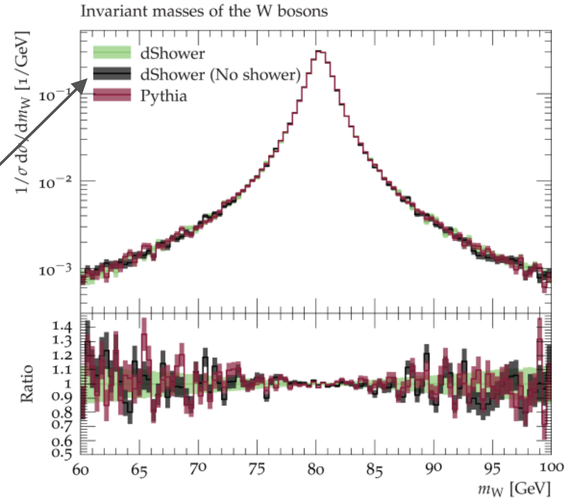
'INTRINSIC' 'SPLITTING'

3 flavours (u, d, s) only. Inputs adjusted such that DPDs approximately satisfy number and momentum sum rule constraints.

DPS process studied: $pp \to W^+W^+ \to e^+\nu_e\mu^+\nu_\mu$

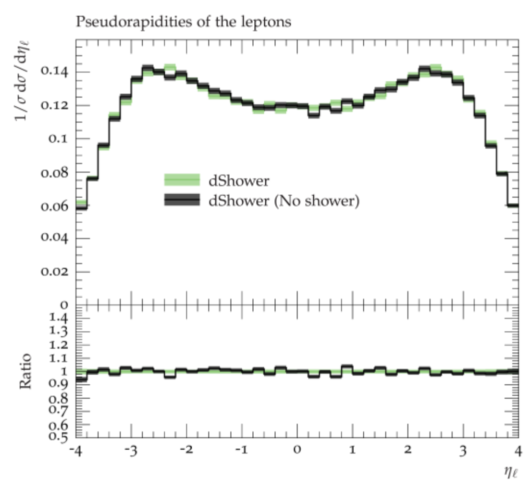
VALIDATION OF DSHOWER

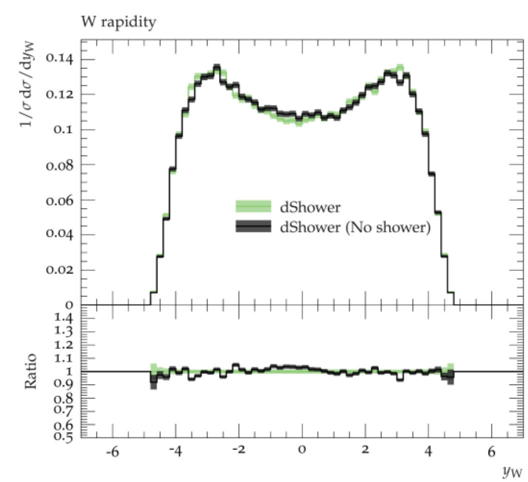
DPS cross section formula



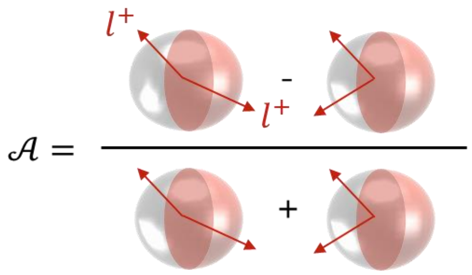
dShower preserves invariant mass spectrum of Ws

Rapidity distributions of leptons and Ws preserved





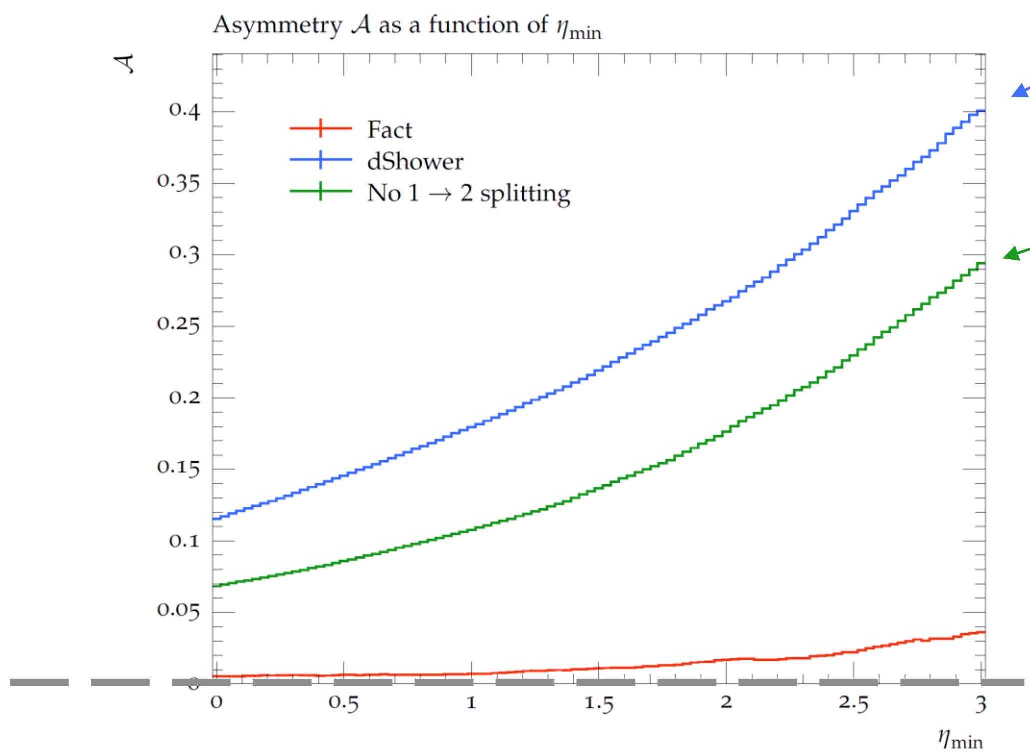
RESULTS: ASYMMETRY



Key indicator of correlations - $\mathcal{A} \neq 0$ implies parton correlations

CMS studies indicate $\mathcal{A} \simeq 0.1$ will be measurable at hi-lumi LHC. CMS-TDR-016

JG, Kom, Kulesza, Stirling, Eur. Phys. J. C69 (2010) 53-65, 2010



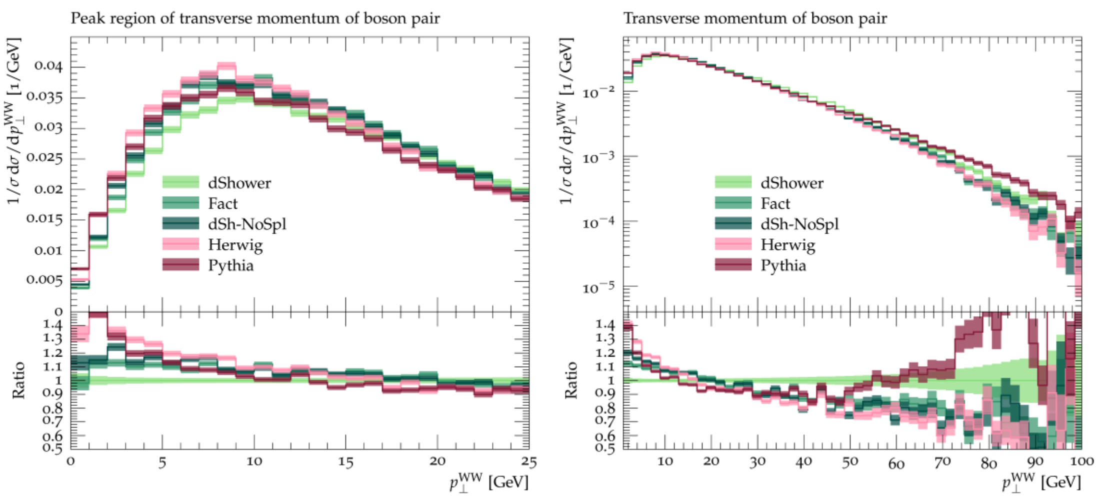
Includes 1→2 splittings + valence number effects

Simple valence number effects

No parton-parton correlations

RESULTS: WW TRANSVERSE MOMENTUM

WW p_T spectrum: dShower result skewed more towards larger p_T



(a) (b)

Explanation: larger qg distributions when including 1→2 splitting effects, leads to greater chance of $\tilde{q}g \to \tilde{q}q + \bar{q}$ and finite p_T of the $\tilde{q}q$ system.

STEP II: COMBINING SPS AND DPS IN THE SHOWER

COMBINING SPS AND DPS

DPS and SPS **overlap**: in general **must simulate both together**, removing overlap. DPS cross section not a physical quantity on its own.

[Sometimes overlap is not so significant – e.g. same-sign WW, processes involving small x. Then DPS cross section alone has physical meaning.]

We have augmented dShower to be able to simulate the combination of SPS+DPS without double counting

Cabouat, JG, JHEP 10 (2020) 012

Observable

$$\frac{d\sigma^{tot}_{A+B}}{dO} = \underbrace{\boldsymbol{S}_1(t_1) \otimes \left[\frac{d\sigma^{SPS}_{A+B}}{dO}\right.}_{\text{Usual SPS shower}} \left. - \frac{d\sigma^{sub}_{(A,B)}}{dO}\right] + \int d^2\boldsymbol{y}\, \boldsymbol{S}_2(t_2) \otimes \frac{d\sigma^{DPS}_{(A,B)}}{dO\, d^2\boldsymbol{y}}$$

Single parton shower

Double parton shower

Subtraction term (removes overlap)

THE SUBTRACTION TERM

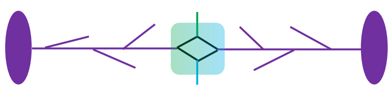
Subtraction term should be designed such that:

Cancel

At **small** $y \sim \frac{1}{Q}$:

$$\frac{d\sigma^{tot}_{A+B}}{dO} = \boldsymbol{S}_1(t_1) \otimes \left[\frac{d\sigma^{SPS}_{A+B}}{dO} - \cancel{\frac{d\sigma^{sub}_{(A,B)}}{dO}}\right] + \int d^2\boldsymbol{y}\, \boldsymbol{S}_2(t_2) \otimes \cancel{\frac{d\sigma^{DPS}_{(A,B)}}{dO\, d^2\boldsymbol{y}}}$$

Recover **SPS** description

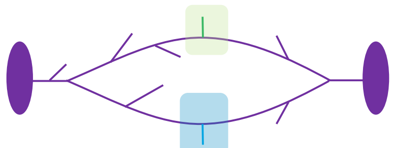


Smooth transition

Cancel

At **large** $y \gg \frac{1}{Q}$:

$$\frac{d\sigma^{tot}_{A+B}}{dO} = \boldsymbol{S}_1(t_1) \otimes \left[\cancel{\frac{d\sigma^{SPS}_{A+B}}{dO}} - \cancel{\frac{d\sigma^{sub}_{(A,B)}}{dO}}\right] + \int d^2\boldsymbol{y}\, \boldsymbol{S}_2(t_2) \otimes \frac{d\sigma^{DPS}_{(A,B)}}{dO\, d^2\boldsymbol{y}}$$



Recover **DPS** description

THE SUBTRACTION TERM

$$\frac{d\sigma_{A+B}^{tot}}{dO} = \boldsymbol{S}_1(t_1) \otimes \left[\frac{d\sigma_{A+B}^{SPS}}{dO} - \frac{d\sigma_{(A,B)}^{sub}}{dO}\right] + \int d^2\boldsymbol{y}\, \boldsymbol{S}_2(t_2) \otimes \frac{d\sigma_{(A,B)}^{DPS}}{dO\, d^2\boldsymbol{y}}$$

Choose hard cross section in this term to be DPS shower expanded to $\mathcal{O}(\alpha_s^2)$, keeping only merging terms in each proton, integrated over y

Then DPS & subtraction cancel at small y by definition. (DPS term at small y is dominated by double merging).

Subtraction and DPS term both depend on small-y cutoff ν – **dependence cancels** between the two terms.

THE SUBTRACTION TERM

Want sub and SPS loop-induced term to cancel at large y (also differential in O).

Generally we don't have an actual expression for $d\sigma_{A+B}^{SPS}/dO$ differential in y.

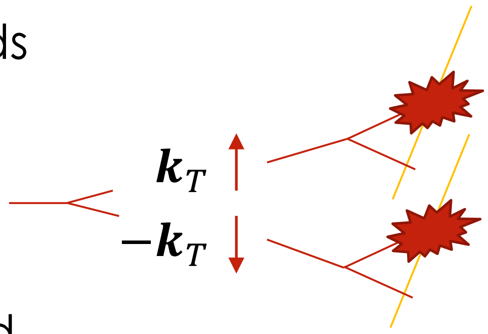
Consider on-shell diboson production (e.g. ZZ). Variables here are $\{Y_1, Y_2, p_{TZ}\}$.

Behaviour in Y_1 and Y_2 will be matched between SPS and subtraction at large y due to form of subtraction - at large y, dominant contribution to $d\sigma_{A+B}^{SPS}/dY_1 dY_2$ has the form of double-merging DPS expression.

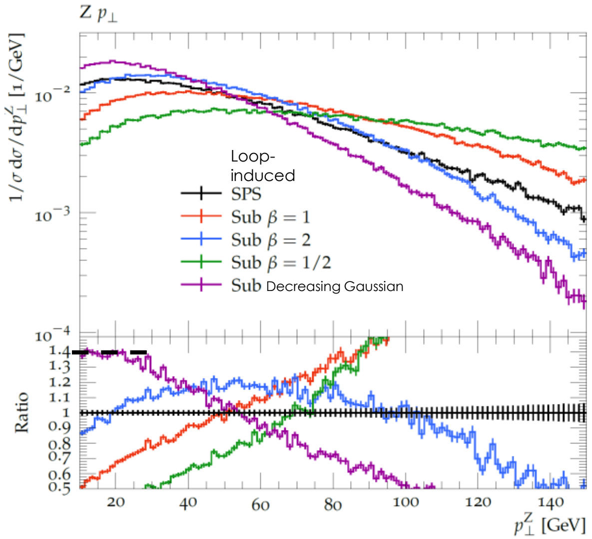
Diehl, JG, Schoenwald, JHEP 06 (2017) 083

THE SUBTRACTION TERM

What about p_{TZ} dependence? $d\sigma_{A+B}^{sub}/dp_{TZ}$ depends on $\boldsymbol{k}_T$ **profile given to daughter partons in splitting**, $g(\boldsymbol{k}_T, y)$.



Don't have $d\sigma_{A+B}^{SPS}/dp_{TZ}dy$, but it is known that small p_{TZ} dependence of loop-induced SPS is dominated by large y [JG, Stirling, JHEP 06 (2011) 048] ➔ try to choose $g(\boldsymbol{k}_T, y)$ such that LI SPS and sub match at small p_{TZ}



Options for $g(\boldsymbol{k}_T, y)$:

(a) Gaussian $g(\boldsymbol{k}_T, y) = \frac{\beta}{\pi} y^2 exp(-\beta y^2 k_T^2)$

(b) 'Decreasing Gaussian' (more realistic)

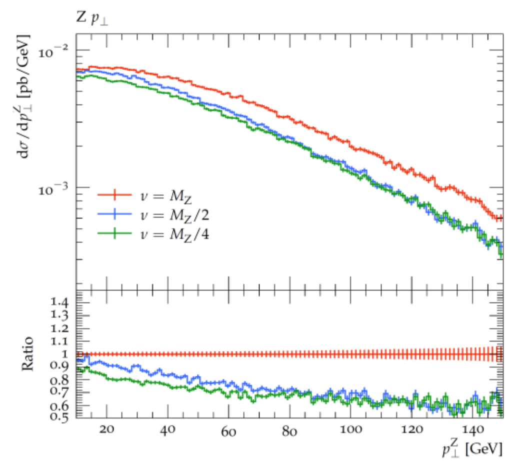
$$g(\boldsymbol{k}_T, y) = \frac{1}{\sqrt{2\pi}} \frac{y}{k_T} exp\left(-\frac{\pi}{2} y^2 k_T^2\right)$$

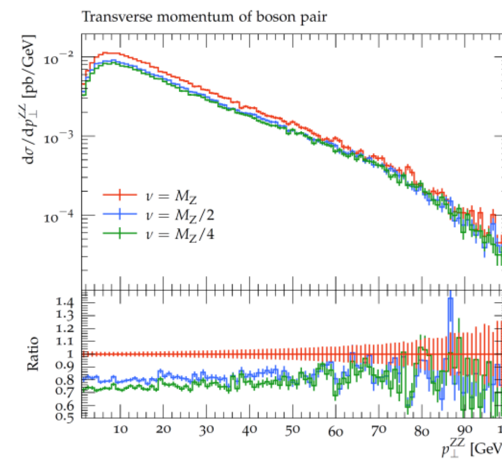
(Gives same dominant $\log^2(p_T/Q)$ term at small p_T as LI SPS)

VALIDATION: DPS & SUB AT SMALL Y

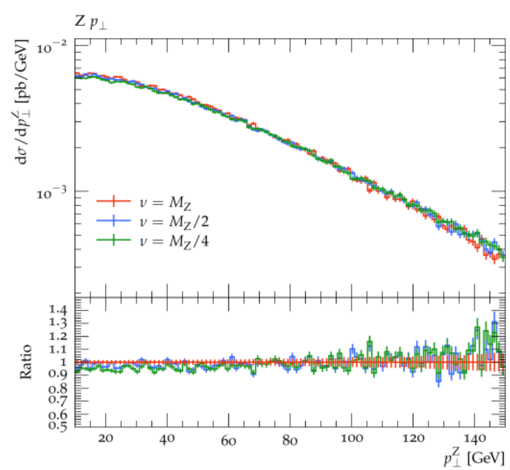
SPS is loop-induced only, divided by 10

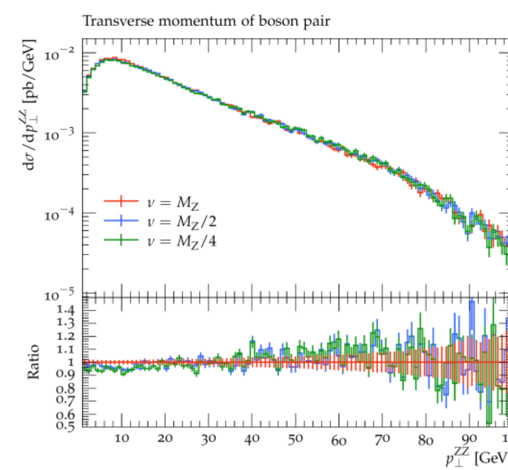
No subtraction:





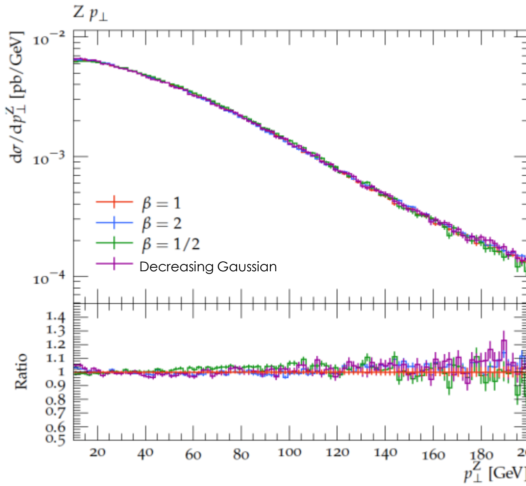
Subtraction included:

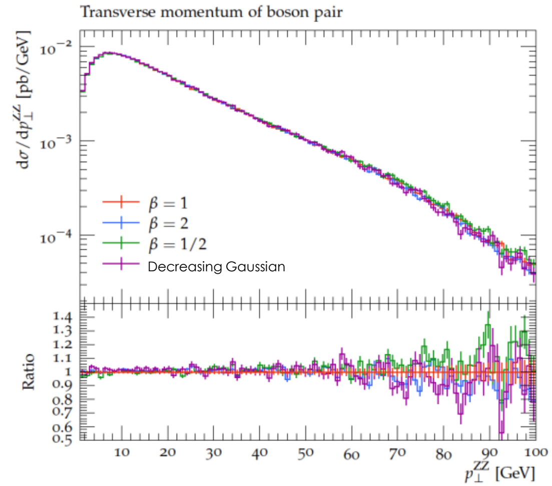


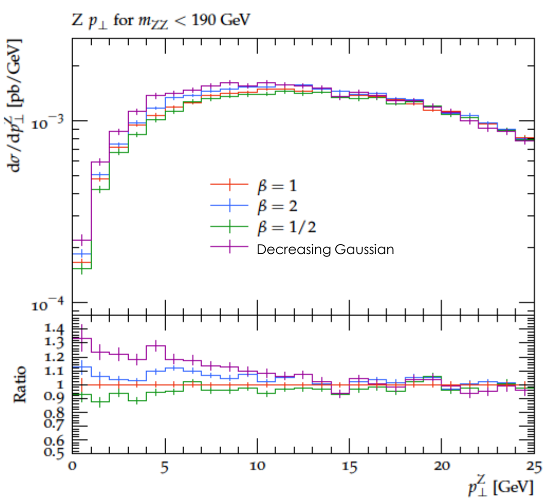


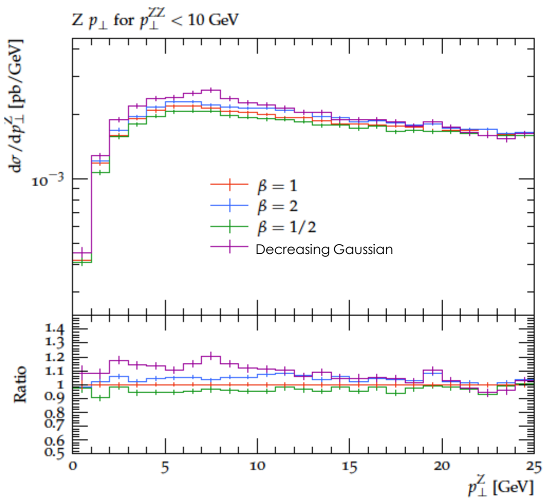
DIFFERENT PROFILES

Many distributions: ~ no difference







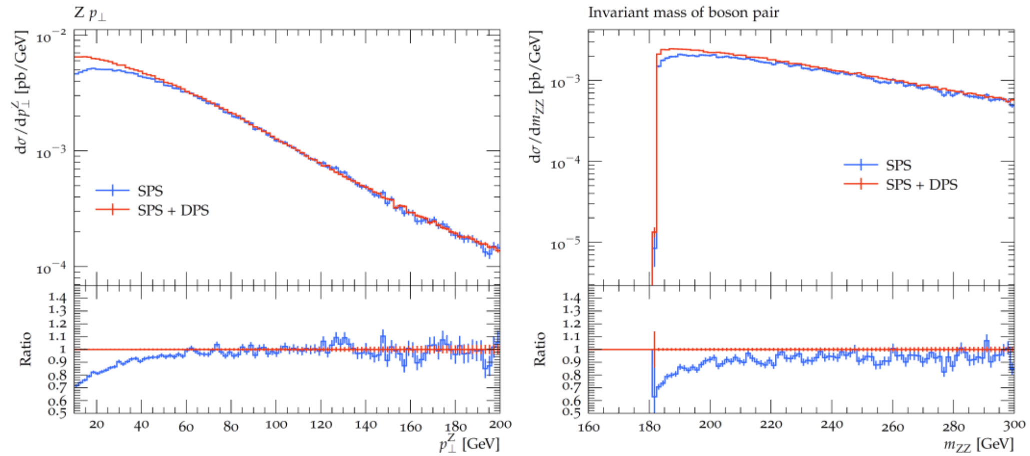


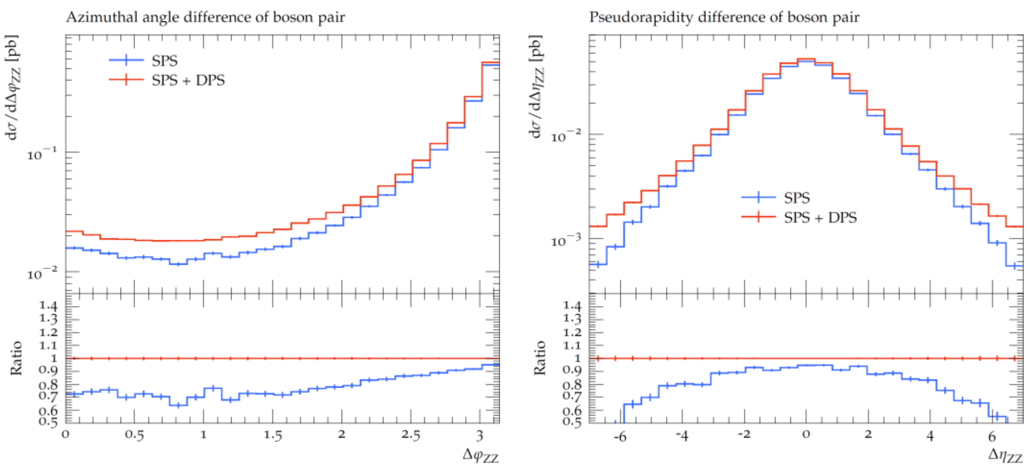
Can see some small differences focussing on region where p_Ts of both bosons are small

DISTINGUISHING SPS AND DPS IN ZZ

"Toy" study: SPS is loop induced only, divided by 10 (& 3 quark flavours)

Small p_T of bosons, small invariant mass of pair





Small(ish) angle between bosons, large rapidity separation

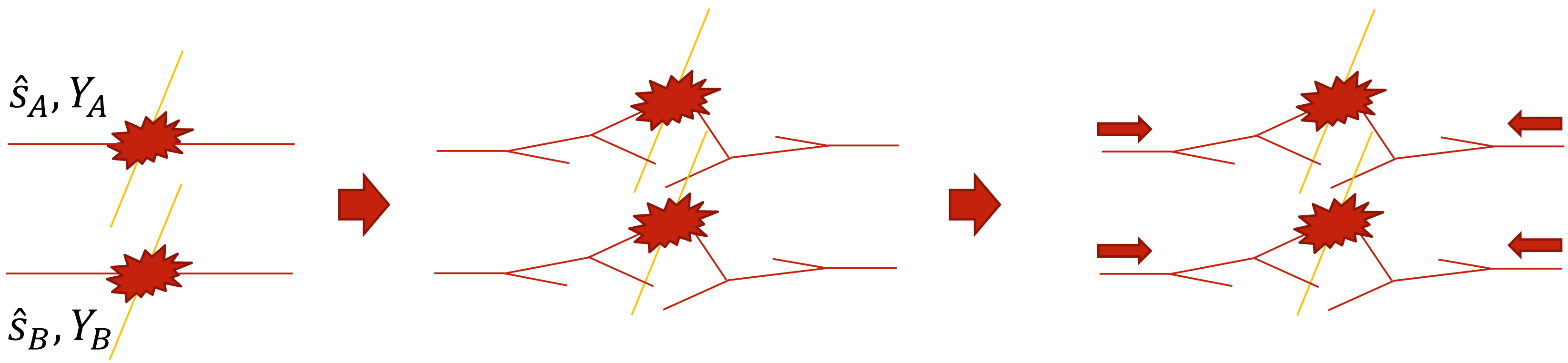
SUMMARY

- dShower is a new Monte Carlo shower for DPS that includes $\boldsymbol{y}$ dependence and the effects of $1 \to 2$ splittings in a consistent way.
  - Guided by DPDs → correlations encoded by these DPDs are fed into the shower. Any DPDs can be used, subject to a few sensible constraints.
  - Angular ordered shower, as for Herwig.
- Mechanism for incorporating both SPS and DPS in the shower without double counting has been developed.
- More developments to come – e.g. hadronization, unequal scales, spin correlations,…

BACKUP SLIDES

KINEMATICS: NO MERGING

No merging:



Generate hard process using DPS σ

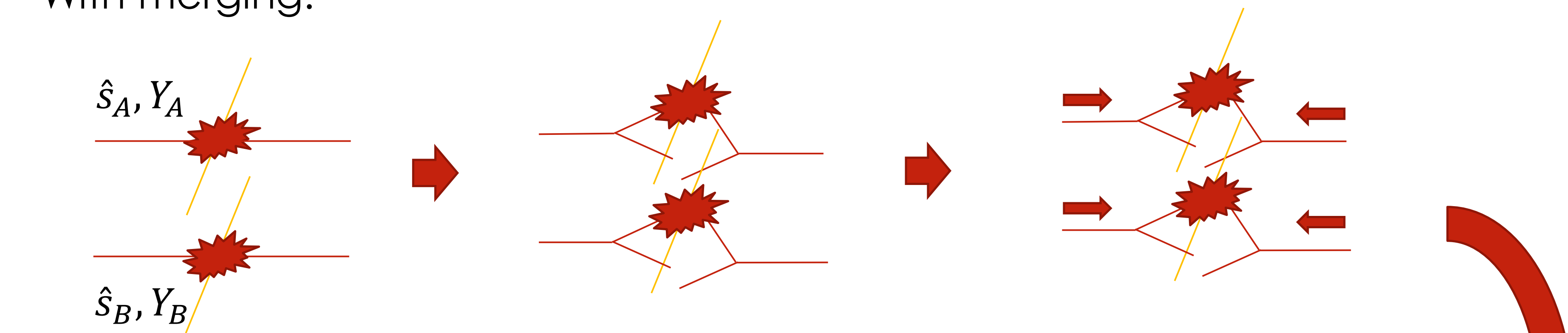
Add shower, kinematics of hard processes altered

Boost initiator partons to restore $\hat{s}_A, Y_A,\ \hat{s}_B, Y_B$

Works as 4 variables (boosts) and 4 constraints! What about if there is a merging? 2/3 initiator partons → overconstrained system!

KINEMATICS: MERGING

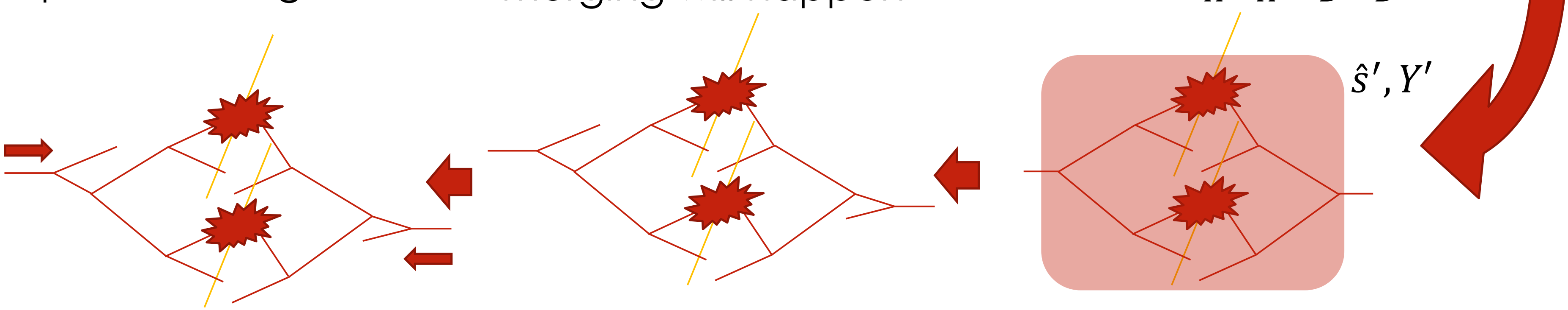
With merging:



Generate hard process using DPS σ

At μ_y, decided merging will happen

Boost initiator partons to restore $\hat{s}_A, Y_A, \hat{s}_B, Y_B$



Boost initiator partons to restore $\hat{s}', Y'$

Continue shower

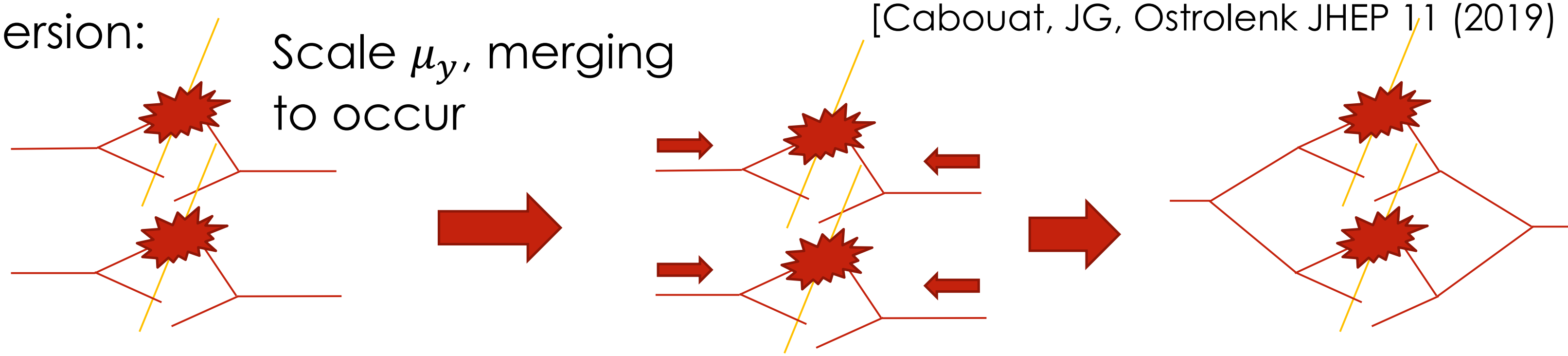
Merge.
Define new hard system.

KINEMATICS: MERGING

Transverse momentum of the partons in the merging?

Old version:

[Cabouat, JG, Ostrolenk JHEP 11 (2019) 061]

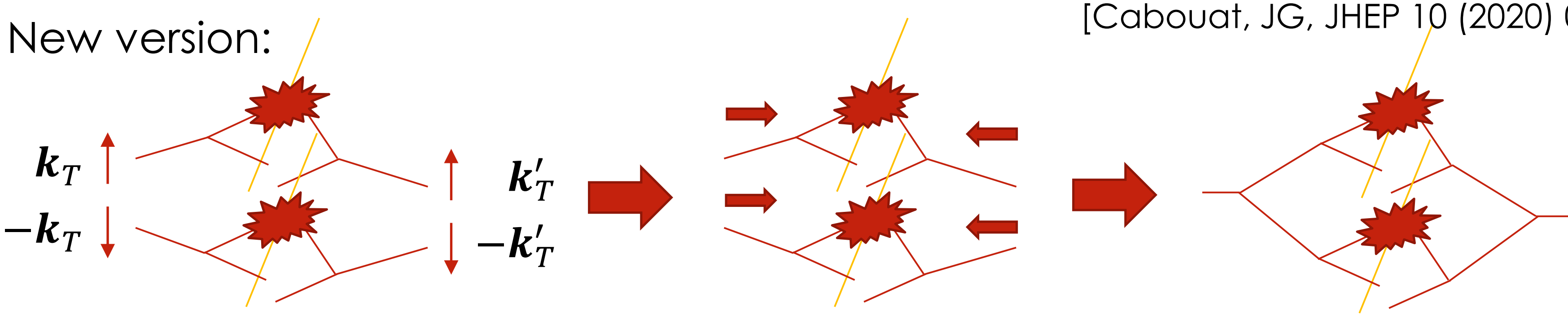


Set partons along beamline — Longitudinal boosts — Merge with no p_T

Not very realistic. Expect merging partons to have $p_T \sim \mu_y \ldots$

New version:

[Cabouat, JG, JHEP 10 (2020) 012]



Partons given $p_T \sim \mu_y$ (and virtuality). Same LC fractions.

Boost. Virtualities chosen → sum on either side on-shell LC mtm

SOME FIRST NUMERICS

DPDs from JHEP 1706 (2017) 083 (Diehl, JG, Schönwald):

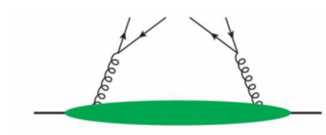
Initialise at low scale $\mu_0 = 1\ \mathrm{GeV}$

$$F^{ij}_{\mathrm{int}}(x_1, x_2, y, \mu_0) = \frac{1}{4\pi h_{ij}}\, e^{-\frac{y^2}{4h_{ij}}}\, f_i(x_1,\mu_0)\, f_j(x_2,\mu_0)(1-x_1-x_2)^2(1-x_1)^{-2}(1-x_2)^{-2}$$

Smooth transverse y profile, radius ~ R_p

'Usual' product of PDFs

Factor to suppress DPD near phase space limit $x_1 + x_2 = 1$



Initialise at scale $\mu_y \sim \frac{1}{y}$

$$F^{ij}_{\mathrm{spl}}(x_1, x_2, y, \mu_y) = e^{-\frac{y^2}{4h_{ij}}}\, \frac{1}{\pi y^2}\, \frac{\alpha_s(\mu_y)}{2\pi} \sum_k \frac{f_k(x_1+x_2,\mu_y)}{x_1+x_2}\, P_{k\to i}\left(\frac{x_1}{x_1+x_2}\right)$$

Gaussian suppression at large y

Perturbative splitting expression

with modifications to very approximately take account of number sum rule constraints [$uu \to uu - \frac{1}{2}u_v u_v$, $dd \to dd - d_v d_v$ in intrinsic]

Only 3 flavours (u, d, s) in this study.

IMPLEMENTATION

How do we implement this in practice?

$$\frac{d\sigma_{A+B}^{tot}}{dO} = \underbrace{\boldsymbol{S}_1(t_1) \otimes \left[\frac{d\sigma_{A+B}^{SPS}}{dO} - \frac{d\sigma_{(A,B)}^{sub}}{dO}\right]}_{\text{SPS-type events ('type 1')}} + \underbrace{\int d^2\boldsymbol{y}\, \boldsymbol{S}_2(t_2) \otimes \frac{d\sigma_{(A,B)}^{DPS}}{dO\, d^2\boldsymbol{y}}}_{\text{DPS-type events ('type 2')}}$$

Phase space for two pieces is different.
Consider e.g. on-shell diboson production (ZZ)

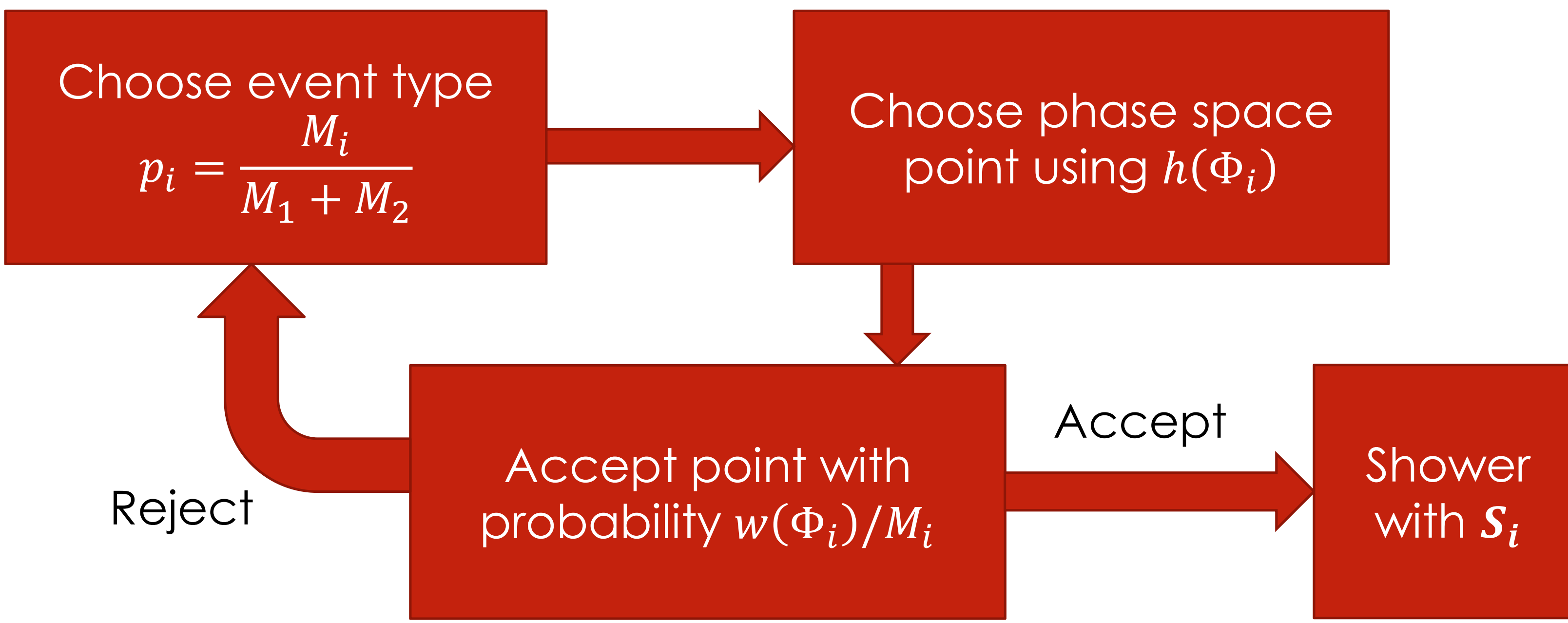
$$\Phi_1 = \{Y_1, Y_2, p_T\} \qquad \Phi_2 = \{Y_1, Y_2, y\}$$

IMPLEMENTATION

For each event type, define weight: $w(\Phi_i) = \dfrac{1}{h(\Phi_i)} \dfrac{d\sigma_i}{d\Phi_i}$ Dimension = $[\sigma]$

$$\int h(\Phi_i) d\Phi_i = 1$$

$$M_i = \max_{\Phi_i}[w(\Phi_i)]$$



IMPLEMENTATION

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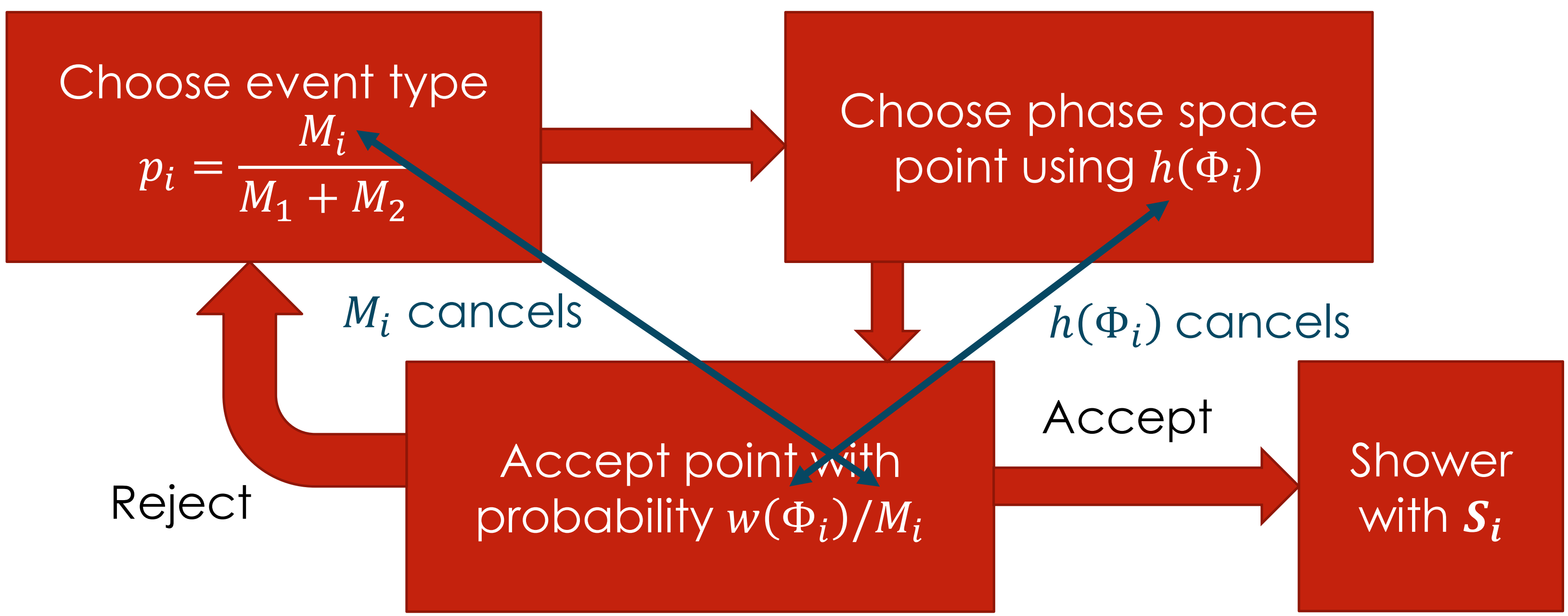
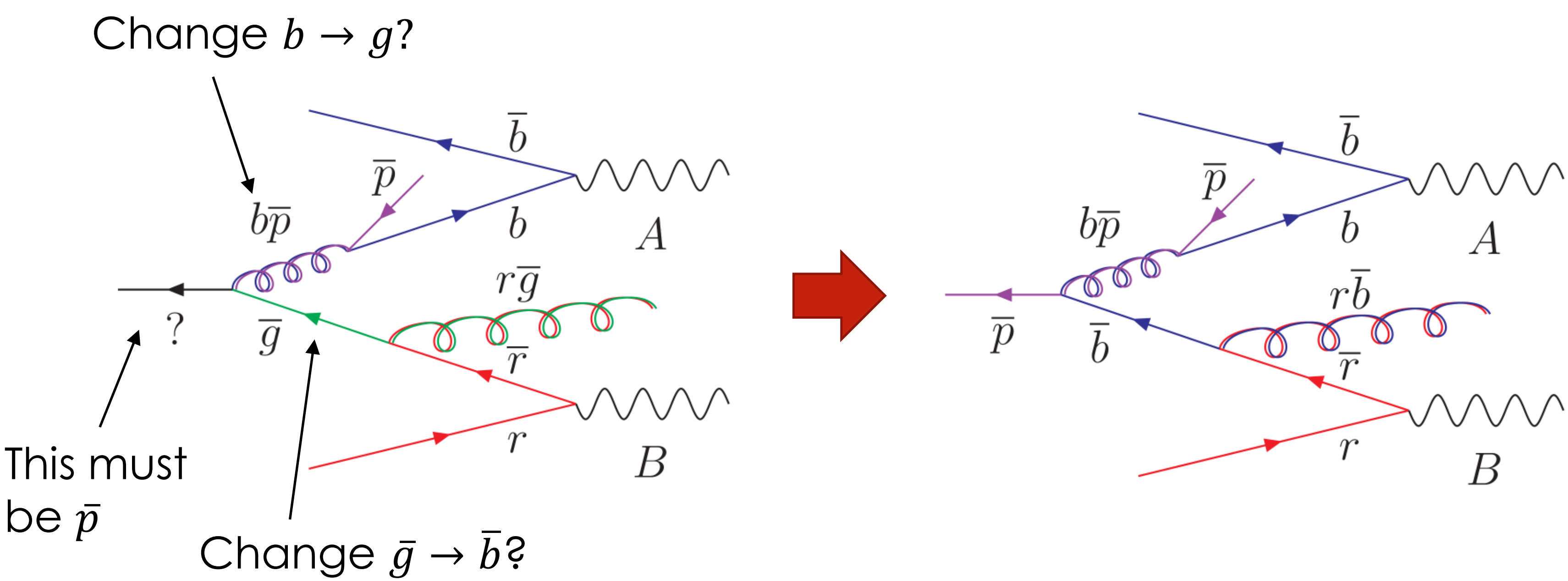


COLOUR WITH MERGING

Shower uses large N_c approximation. Each new emission → new colour. Independent showers before merging.

Mergings require some colour reshuffling. We impose minimal colour disruption.



Not so important for parton-level simulation, but could be important when we add hadronisation