Overview of DPS in perturbative QCD

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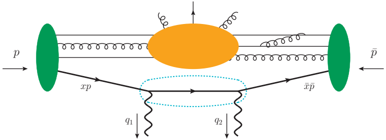
12th International workshop on Multiple Partonic Interactions at the LHC
Lisbon, Portugal, 11–15 October 2021

What is double parton scattering?

Definition

Double parton scattering (DPS) is a proton-proton scattering process in which two partons from each proton undergo **two separate hard interactions**.

SPS

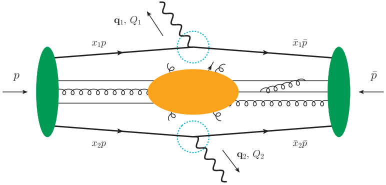


$$|\mathbf{q}_1^\perp + \mathbf{q}_2^\perp| \sim \Lambda \ll Q$$

First appearance in theory studies:
Politzer Nucl. Phys. B172 (1980) 349
Paver, Treleani Nuovo Cim. A70 (1982) 215
Mekhfi Phys. Rev. D32 (1985) 2371

Other ground-setting works:
Gaunt, Stirling JHEP 03 (2010) 005
Blok et al. Eur. Phys. J. C72 (2012) 1963
Diehl et al. JHEP 03 (2012) 089
Manohar, Waalewijn Phys. Rev. D85 (2012) 114009
Ryskin, Snigierev Phys. Rev. D86 (2012) 014018
...

DPS



$$|\mathbf{q}_1^\perp| \sim \Lambda \ll Q$$
$$|\mathbf{q}_2^\perp| \sim \Lambda \ll Q$$

hard scale is $Q \sim \min(Q_1, Q_2)$

transverse-momenta scale is Λ

with $\Lambda_{\mathrm{QCD}} \ll \Lambda \ll Q$

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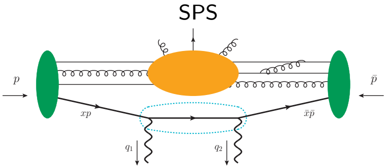
Size comparison to SPS

- integrated XS: $\frac{\sigma_{\text{DPS}}}{\sigma_{\text{SPS}}} \sim \mathcal{O}\left(\frac{\Lambda^2}{Q^2}\right)$
  $\implies$ phase-space suppressed
- differential XS:
  $\frac{d^2\sigma_{\text{SPS}}}{d^2q_1 d^2q_2} \sim \frac{d^2\sigma_{\text{DPS}}}{d^2q_1 d^2q_2}$
  $\implies$ same power counting!

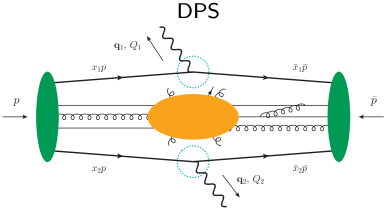
hard scale is $Q \sim \min(Q_1, Q_2)$

transverse-momenta scale is Λ

with $\Lambda_{\text{QCD}} \ll \Lambda \ll Q$



$|\mathbf{q}_1^\perp + \mathbf{q}_2^\perp| \sim \Lambda \ll Q$



$|\mathbf{q}_1^\perp| \sim \Lambda \ll Q$
$|\mathbf{q}_2^\perp| \sim \Lambda \ll Q$

When is DPS important?

Where DPS is enhanced

- generally, DPS relevance increases with **collision energy**
- competitive with SPS in regions of **small** $|\mathbf{q}_1^{\perp}|, |\mathbf{q}_2^{\perp}|$
  $\longrightarrow$ e.g. two pairs of back-to-back jets
- enhanced by parton luminosities at **small-x**, e.g. $F_{gg} \propto (f_g)^2$
- DPS dominant contribution for **coupling-suppressed processes** in SPS
  $\longrightarrow$ same-sign WW production at $\mathcal{O}(\alpha_s^2)$ in SPS, but $\mathcal{O}(1)$ in DPS

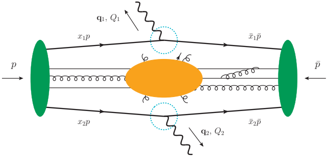
Some of the peculiarities of DPS

- equivalent of PDFs are **double parton distributions (DPDs)**: more complex, currently cannot be extracted from data
- **polarization** and **color non-singlet** combinations gain importance
- need to account for the **overlap of single and double parton scattering**

DPS cross section

For colorless final states, an analogous factorized form to the SPS case can be derived

- $\hat{\sigma}^{(i)}$ are regular partonic cross sections
- F_{ab} are double parton distributions (DPDs)
- y [GeV^{-1}] is inter-parton transverse separation



here neglecting color indices and $x_i, \bar{x}_i$ dependence in the functions

C is a symmetry factor

Transverse-momentum dependent (TMD) factorization:

$$\frac{d\sigma_{\text{DPS}}}{dq_1^{\perp}\, dq_2^{\perp}} = \frac{1}{C} \sum_{a_1 a_2 b_1 b_2} \hat{\sigma}^{(1)}_{a_1 b_1}\, \hat{\sigma}^{(2)}_{a_2 b_2}$$

$$\times \int d^2 y\, \frac{d^2 z_1}{2\pi^2}\, \frac{d^2 z_2}{2\pi^2}\, e^{-i q_1^{\perp} z_1 - i q_2^{\perp} z_2}\, F_{a_1 a_2}(z_1, z_2, y)\, F_{b_1 b_2}(z_1, z_2, y)$$

In TMD factorization, $F_{ab}(z_1, z_2, y)$ are the TMDDPDs in position space.

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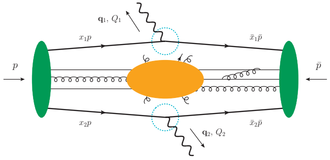

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C is a symmetry factor

Collinear factorization:

$$d\sigma_{\text{DPS}} = \frac{1}{C} \sum_{a_1 a_2 b_1 b_2} \hat{\sigma}^{(1)}_{a_1 b_1} \otimes \hat{\sigma}^{(2)}_{a_2 b_2} \otimes \int d^2 y \, F_{a_1 a_2}(y) \otimes F_{b_1 b_2}(y)$$

In collinear factorization, $F_{ab}(y)$ are the collinear DPDs in position space.

Assuming no inter-partonic correlations whatsoever, obtain convenient XS formula (the **DPS pocket formula**)

$$\sigma_{\text{DPS}} = \frac{1}{C} \frac{\sigma_1^{\text{SPS}} \sigma_2^{\text{SPS}}}{\sigma_{\text{eff}}}$$

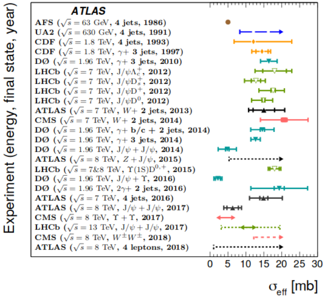
σ_{eff} used as a "measure" of DPS in exp's

Experimental searches

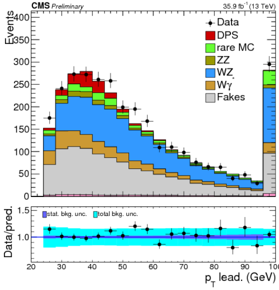
DPS observed since the '80s (4 jets, γ+3 jets, etc)

typical observables: WW, WJ/Ψ, $J/\Psi J/\Psi$, W+jets, ZZ, ...

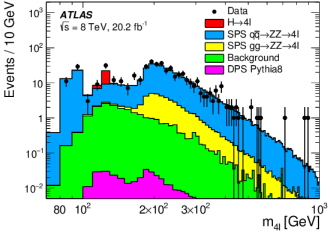
$\sigma_{\rm eff}$ measurements [CERN-EP-2018-274]



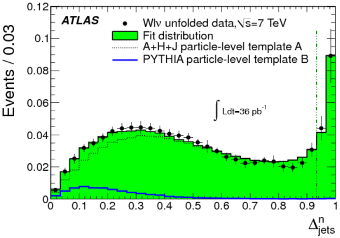
like-sign WW [CMS-PAS-FSQ-16-009]



4ℓ final state [CERN-EP-2018-274]

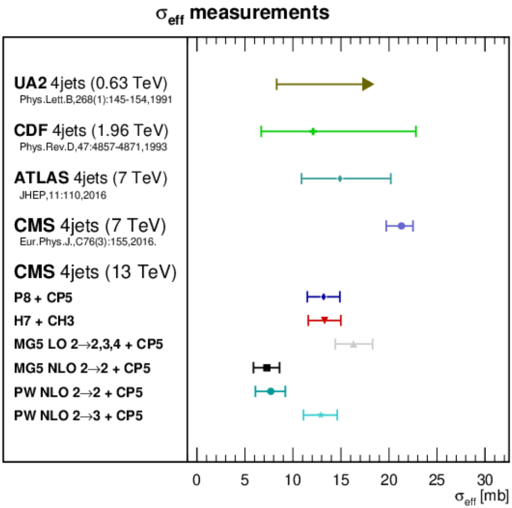


W + 2 jets [CERN-PH-EP-2012-355]



Latest extractions of σ_{eff}

latest measurements (CMS, 4-jets at 13 TeV) [CMS-PAS-SMP-20-007]



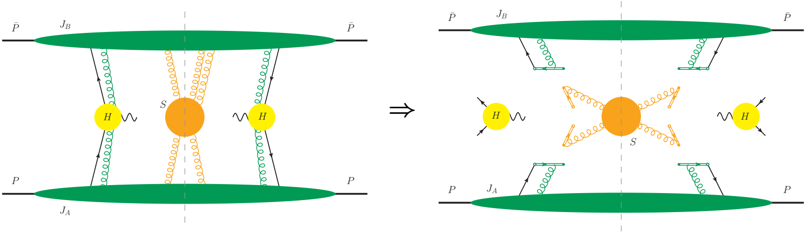
Status of factorization

A **formal all-order proof** of the factorization formulae in perturbative QCD **has been achieved for DPS** in the case of a **colorless final state**, both for the TMD and the collinear case. Current status is at the **same level as for the SPS** counterpart.

Diehl et al. JHEP 03 (2012) 089, JHEP 01 (2016) 076
Vladimirov JHEP 04 (2018) 045
Buffing et al. JHEP 01 (2018) 044
Diehl, RN JHEP 04 (2019) 124

The factorization procedure can be understood visually using cut diagrams:

SPS TMD factorization



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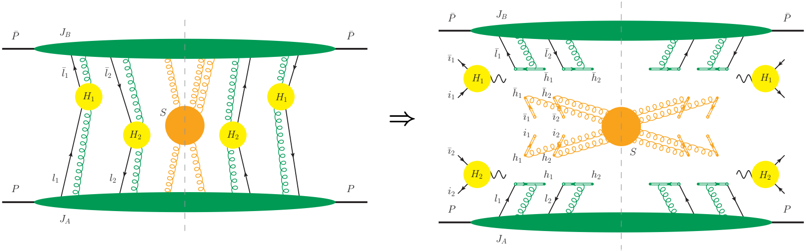
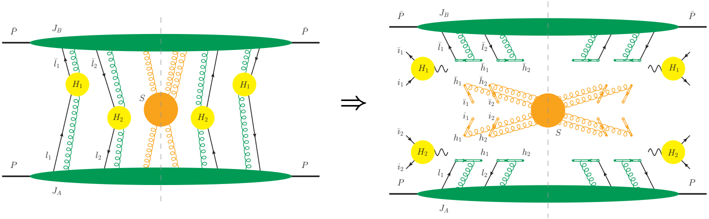


Structure of the proof

The all-order **factorization proof for DPS** generalizes the proofs by **Collins, Soper, Sterman** (CSS).

Sketch:

1. define a **power counting**, identify **leading regions** & introduce **kinematical approximations** — Diehl, Ostermeier, Schäfer JHEP 03 (2012) 089
2. loops have to be cut: establish a **subtraction** mechanism — Collins "Foundations of pQCD" (2011)
3. decouple **collinear gluons** from the hard interactions — CSS Nucl.Phys.B261 (1985) 104
4. show factorization of **Glauber gluons** — Diehl et al. JHEP 01 (2016) 076
5. factorize **soft gluons** from collinear graphs — Diehl, RN JHEP 04 (2019) 124
6. obtain renormalized **operator definitions** of soft & collinear factors — Diehl, Ostermeier, Schäfer JHEP 03 (2012) 089



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Factorized form (schematically):

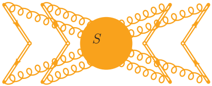
$$\frac{d\sigma_{\text{DPS}}}{dq_1^{\perp}\, dq_2^{\perp}} \propto H^{(1)}_{a_1 b_1}\, H^{(2)}_{a_2 b_2} \int [\text{F.T. on } \vec{\xi}_m] \int [\text{F.T. on } \vec{l}_m,\, l_m^-]\, S(\vec{\xi}_m) \times J_A(l_1, l_2)\Big|_{l_1^+ = q_1^+,\, l_2^+ = q_2^+} \times J_B(\bar{l}_1, \bar{l}_2)\Big|_{\bar{l}_1^- = q_1^-,\, \bar{l}_2^+ = q_2^-}$$

Soft factor

Buffing et al. JHEP 01 (2018) 044
Vladimirov JHEP 12 (2016) 038

The soft factor has a few properties:

- **hermitian in color space**: $S_{a_1 a_2} = S^\dagger_{a_1 a_2}$
- **hermitian in rep. space**: ${}^{RR'}S_{a_1 a_2} = ({}^{R'R}S_{a_1 a_2})^*$



TMD factorization

$$J_B{}^c\, S^{cd}\, J_A{}^d$$

In SPS and DPS, the soft factor depends on the color structure and on the Wilson-lines rapidity $y = \log \frac{v^+}{v^-}$

↓

Collins-Soper equation:

$$\frac{\partial}{\partial y} S(y) = K\, S(y)$$

collinear factorization

$$J_B{}^c\, S^{cd,ef}\, J_A{}^d\, H_1{}^e\, H_2{}^f$$

In SPS, integration over transverse momenta implies $S_q = 1$.

In DPS, ${}^{RR'}S_{a_1 a_2} \propto \delta_{RR'}$, color singlet ${}^{11}S_{qq} = 1$, but color non-singlets like ${}^{88}S_{qq}$ depend on rapidity (however Sudakov suppressed).

At NNLO, S can be expressed completely in terms of SPS TMD soft factor

Double parton distributions

"Bare" TMD DPDs

Definition of "bare" DPDs is similar to one of PDFs, obtained from $J_{A,B}$

$$F^{(0)}_{a_1 a_2}(x_1, x_2, z_1, z_2, y) \propto \langle p|\, \mathcal{O}_{a_1}(y, z_1)\, \mathcal{O}_{a_2}(0, z_2)\, |p\rangle \,\Big|_{z_i^+ = y_i^+ = 0}$$

in terms of operators $\mathcal{O}(y, z) \sim \bar{\psi}(y - \frac{1}{2}z)\, \Gamma\, \psi(y + \frac{1}{2}z)$.

- z_1 and z_2 analog to impact factor b of single TMDs
- y spacelike transverse separation between the two partons

"Bare" collinear DPDs

Collinear DPDs are obtained from TMD DPDs:

- position-space DPDs $F^{(0)}_{a_1 a_2}(x_1, x_2, y)$ by letting $z_1, z_2 \to 0$
  - ↪ appear in cross section
- momentum-space DPDs $F^{(0)}_{a_1 a_2}(x_1, x_2, \Delta)$ by Fourier transform
  - ↪ appear in DPD sum rules

DGLAP evolution for renormalized DPDs

Position space: double DGLAP evolution

Renormalizing the bare DPDs adds **one scale dependence for each parton**:

$$\frac{\mathrm{d}F_{a_1a_2}(x_i, y;\, \mu_1, \mu_2)}{\mathrm{d}\log \mu_1} = 2\left[P_{a_1c}(\mu_1) \underset{1}{\otimes} F_{cb_1}(y;\, \mu_1, \mu_2)\right](x_i)$$

$$\frac{\mathrm{d}F_{a_1a_2}(x_i, y;\, \mu_1, \mu_2)}{\mathrm{d}\log \mu_2} = 2\left[P_{ca_2}(\mu_2) \underset{2}{\otimes} F_{a_1c}(y;\, \mu_1, \mu_2)\right](x_i)$$

Momentum space: generalizedn DGLAP evolutio

Momentum-space dependent DPDs obey **inhomogeneous evolution equations**:

$$\frac{\mathrm{d}F_{a_1a_2}(x_i, \Delta;\, \mu, \mu)}{\mathrm{d}\log \mu} = 2\left[P_{a_1c}(\mu) \underset{1}{\otimes} F_{ca_2}(\Delta;\, \mu, \mu) + P_{ca_2}(\mu) \underset{1}{\otimes} F_{a_1c}(\Delta;\, \mu, \mu) + P_{s,\, a_1a_2,a_0}(\mu) \underset{12}{\otimes} f_{a_0}(\mu)\right](x_i)$$

where P_s is the $1 \to 2$ splitting function.

Numerical computation of DPD evolution

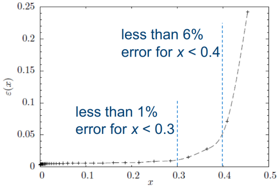
Double DGLAP evolution is a **non-trivial numerical task**, but it is also the main ingredient for DPS phenomenological studies.

Gaunt-Stirling JHEP 03 (2010) 005

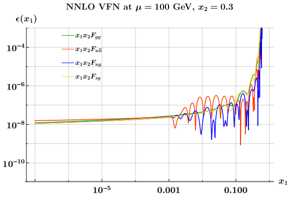
- LO DGLAP (both $\boldsymbol{y}$- and $\boldsymbol{\Delta}$-dependent)

Only publicly available set: `GS09` [gsdpdf.hepforge.org]

- based on products of `MSTW2008` PDFs
- $\boldsymbol{y}$-integrated DPDs



[J. Gaunt's talk @ MPI10]



ChiliPDF project to be released

- NNLO DGLAP, with NNLO flavor matching
- all polarizations included
- unequal-scale evolution ($\boldsymbol{\mu_1 \neq \mu_2}$)
- flexible input (numerical, analytical, ...)
- flexible $\boldsymbol{y}$-dependence Ansatz

[Diehl, RN, Tackmann, Plößl]

Private evolution codes have been developed by other groups e.g. Elias, Golec-Biernat, Staśto.

DPD sum rules

- integrated DPDs (i.e. momentum-space DPDs at $\Delta = 0$) obey **sum rules** analogous to the PDF ones, and expressed in terms of PDFs
- these can be used to constrain DPD models

Diehl et al. Eur.Phys.J.C 79 (2019) 3, 253, Eur.Phys.J.C 80 (2020) 5, 468

Momentum sum rule

$$\sum_{a_2} \int_0^{1-x_1} dx_2 \, x_2 \, F_{a_1 a_2}(x_1, x_2, \Delta = 0; \mu) = (1 - x_1) \, f_{a_1}(x_1; \mu)$$

Number sum rule

$$\int_0^{1-x_1} dx_2 \, [F_{a_1 a_2}(x_1, x_2, \Delta = 0; \mu) - F_{a_1 \bar{a}_2}(x_1, x_2, \Delta = 0; \mu)]$$
$$= (N_{a_2,\mathrm{v}} + \delta_{a_1 \bar{a}_2} - \delta_{a_1 a_2}) \, f_{a_1}(x; \mu)$$

where $N_{a,\mathrm{v}}$ is the number of valence partons of type a

sum rules for triple parton distributions in O. Fedkyevich's talk (Tuesday)

DPDs from perturbative splitting

At present, DPDs cannot be extracted from exp's $\rightarrow$ Ansatz necessary

A class of DPD Ansätze at small y

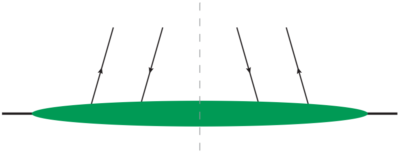
From OPE, at small y DPDs are sum of "**intrinsic**" and "**splitting**" piece

$$F(y) = F_{\text{int}}(y) + F_{\text{spl}}(y)$$

At large y DPDs can be modeled so that $\lim_{y\to\infty} F(y) = 0$.

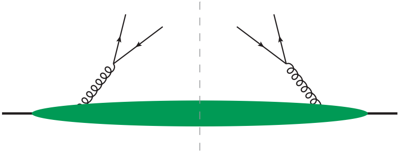
Perturbative splitting

- from MEs: $F_{\text{spl}}(y) \propto \dfrac{1}{y^2}$
- UV divergence in cross-section
$$\int d^2y\, F_1\, F_2 \sim \int \frac{d^2y}{y^4}$$
- reason: region of overlap between SPS and DPS



intrinsic (F_{int} or "2")

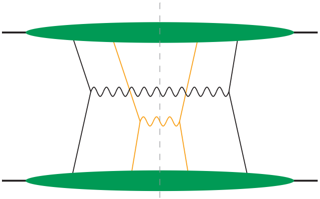
twist-4 distribution at small y, nonperturbative



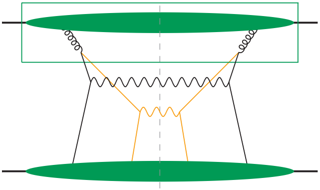
perturbative splitting (F_{spl} or "1")

- LO: $F_{ab} \propto P_{a_0 \to ab} \cdot f_{a_0}$
  Diehl et al. JHEP 03 (2012) 089
- NLO: computed, also color non-singlet
  Diehl et al. SciPost Phys. 7 (2019) 017
  Diehl et al. JHEP 08 (2021) 040
  see P. Plößl's talk

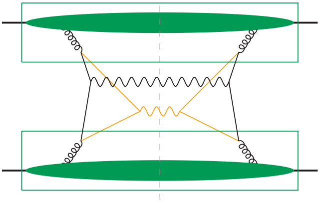
Interplay of splitting and intrinsic contributions



2v2 $\longrightarrow$ not divergent



2v1 $\longrightarrow$ divergence is $\dfrac{\mathbf{d}^2\boldsymbol{y}}{\boldsymbol{y}^2}$ $\longrightarrow$ $\mathbf{log}\,\boldsymbol{y}$ terms

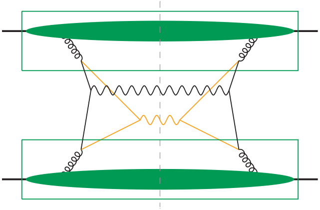


1v1 $\longrightarrow$ divergence is $\dfrac{\mathbf{d}^2\boldsymbol{y}}{\boldsymbol{y}^4}$, must be subtracted

Double-counting between SPS and DPS

The UV divergence in $\boldsymbol{y}$ is associated to the **double counting of SPS and DPS** contributions in the region where $\boldsymbol{y} \to \mathbf{0}$:

DPS interpretation (1v1)



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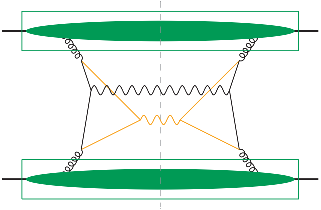

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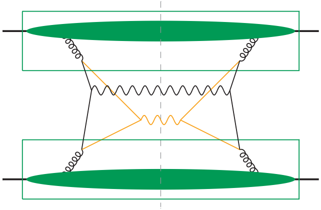
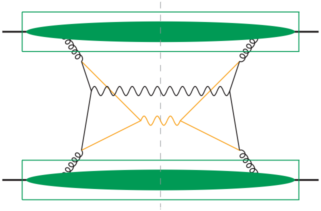


SPS interpretation



Solution: DGS scheme

The DGS subtraction scheme cancels the UV divergence at all orders:

$$\boxed{\sigma = \sigma_{\text{SPS}} + \sigma_{\text{DPS}} - \sigma_{\text{sub}}}\,, \qquad \sigma_{\text{sub}} = \sigma_{\text{DPS}} \text{ with } F_{1,2} = F_{\text{spl}}$$

where the DPS cross section is regularized introducing a cutoff $\nu \sim Q$

$$\sigma_{\text{DPS}} \propto \int \mathrm{d}^2 y\, F_1(y)\, F_2(y) \to \int \mathrm{d}^2 y\, \Phi^2(y\nu)\, F_1(y)\, F_2(y)$$

Simple cutoff regulator $\Phi(y\nu) = \Theta(y\nu - 2e^{-\gamma_E})$.

Diehl, Gaunt, Schönwald JHEP 06 (2017) 083

State of the art

Recent theory developments

- `dShower`: a **parton shower** combining SPS and DPS, implementing the "$\mathbf{1} \to \mathbf{2}$" splitting and treating the SPS-DPS double counting

  Cabouat, Gaunt JHEP 10 (2020) 012

  see Gaunt's talk (Thursday)
- **lattice QCD**: extracted moments of the pion DPD and of the proton DPD

  Bali et al. JHEP 02 (2021) 067, JHEP 09 (2021) 106
- a lot **more insight on DPDs**: evolution, sum rules, NLO splitting, color non-singlet distributions Diehl et al. SciPost Phys. 7 (2019) 2, 017, Eur.Phys.J.C 80 (2020) 5, 468, JHEP 08 (2021) 040, arXiv:2109.14304

  see Plößl's talk

DPS phenomenology

- DPD models

  ↪ constituent quark models (Rinaldi, Scopetta, Ceccopieri), "bag" model (Manohar, Waalewijn) valence quark models (Broniowski, Ruiz Arriola), KMR approach (Golec-Biernat, Staśto), . . .
- multitude of phenomenological studies that include DPS

  Blok, Dokshitzer, Frankfurt, Strikman, Maciuła, Szczurek, Kutak, van Hameren, Gaunt, Kom, Kulesza, Stirling, Fedkyevich, Kasemets, Myska, Cotogno, Lansberg, Yamanaka, Zhang, Shao, Ceccopieri, Rinaldi, Scopetta, ...

  talks by Fedkyevich, Rinaldi, Szczurek, Yamanaka this week
- DPS in pA collisions and TPS (triple parton scattering) D'Enterria, Snigirev, Blok

  see talks by Blok, D'Enterria

Summary

- **DPS** can be **comparable or even dominant** with SPS in several cases
- DPS **factorization proof** for color singlet is at the same level as for SPS
- **double-counting** of SPS and DPS in small-y region is understood
- double **DGLAP evolution** and flavor matching are under control thanks to developments of new tools
- perturbative **splitting** form of DPDs known **up to NLO**, also for color non-singlet distributions see next talk!
- numerous interesting contributions to DPS sessions at MPI@LHC21!

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Thank you for your attention!