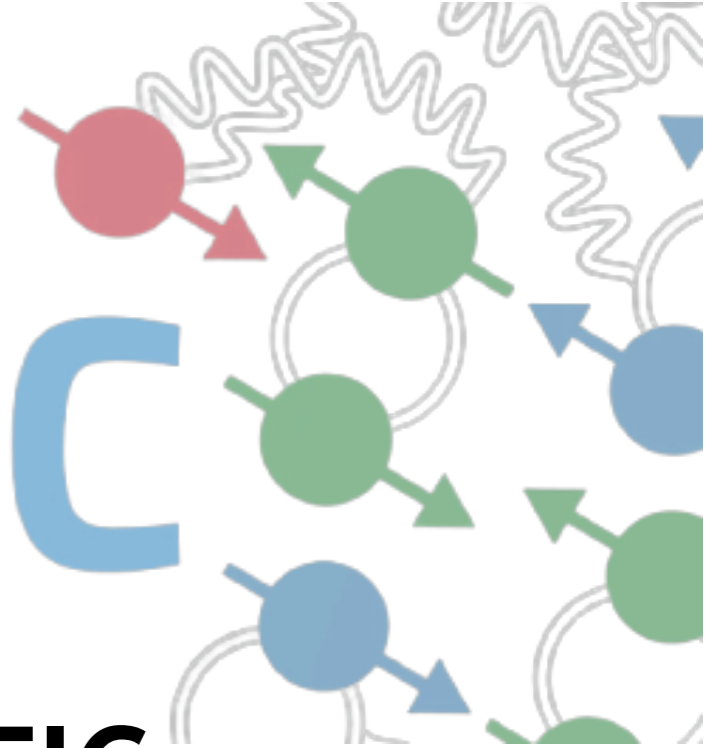


12th MPI at LHC



Forward dijets at the EIC beyond TMDs

Multi Partonic Interactions at the LHC Workshop
LIP, Lisbon
October 11th, 2021

Farid Salazar



Stony Brook University



Brookhaven[™]
National Laboratory



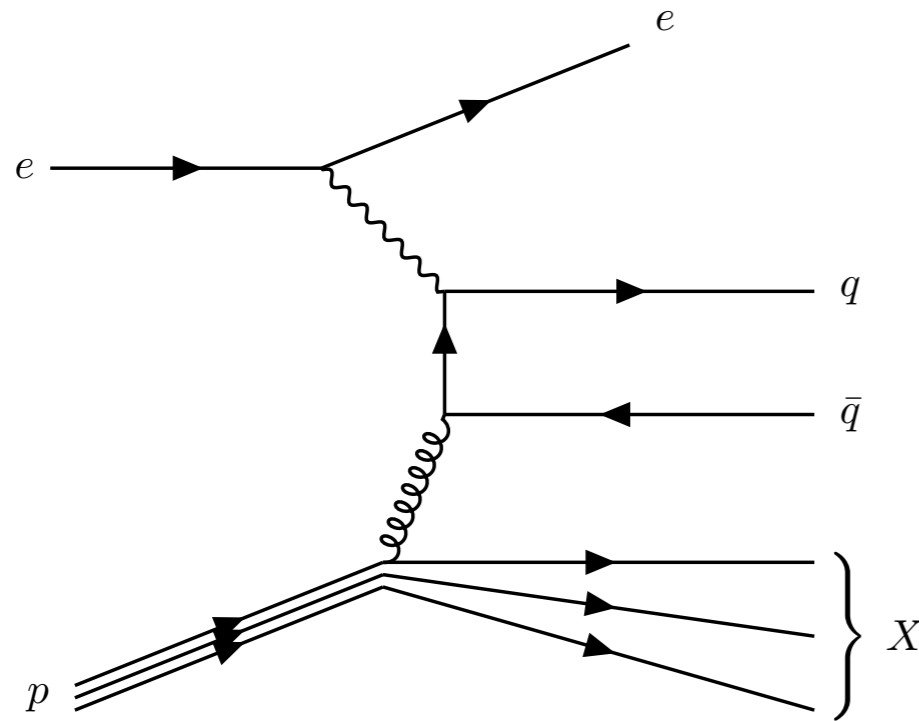
UCLA Mani L. Bhaumik Institute
for Theoretical Physics

Outline

- Dijet production in the TMD formalism
- Dijet production at EIC beyond TMDs: CGC formalism
- Dijet production at EIC in the CGC at NLO
- Connections to the LHC (very brief)

Dijet and dihadron production: TMD formalism

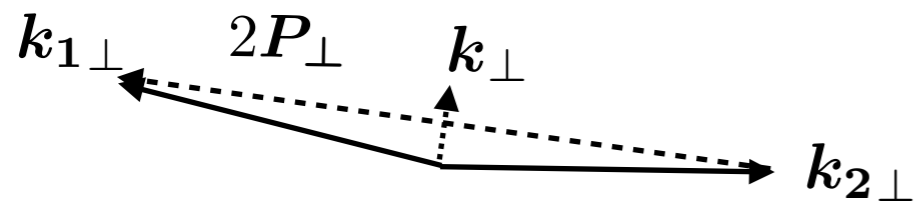
The Weizsäcker-Williams gluon TMD



LO diagram for $q\bar{q}$ production

Validity of TMD approach:

$$k_{\perp} \ll P_{\perp} \quad (\text{i.e. back-to-back configuration})$$



$$\mathbf{k}_{\perp} = \mathbf{k}_{1\perp} + \mathbf{k}_{2\perp}$$

$$\mathbf{P}_{\perp} = z_2 \mathbf{k}_{1\perp} - z_1 \mathbf{k}_{2\perp}$$

Bomhof, Mulders, Pijlman (2006)

Dominguez, Marquet, Xiao, Yuan (2011)

Dominguez, Qiu, Xiao, Yuan (2011)

$$d\sigma^{\gamma^* A \rightarrow q\bar{q}X} \sim \mathcal{H}_{\text{TMD}}^{ij}(\mathbf{P}_{\perp}) \alpha_s x G_{\text{WW}}^{ij}(x, \mathbf{k}_{\perp})$$

Perturbatively
calculable

WW gluon TMD

$$x G_{\text{WW}}^{ij}(x, \mathbf{k}_{\perp}) = \frac{1}{2} \delta^{ij} x G_{\text{WW}}^0(x, k_{\perp})$$

Unpolarized

$$+ \Pi^{ij}(\mathbf{k}_{\perp}) x h_{\text{WW}}^0(x, k_{\perp})$$

Linearly
polarized

$$\Pi^{ij}(\mathbf{k}_{\perp}) = \left(2 \frac{k_{\perp}^i k_{\perp}^j}{k_{\perp}^2} - \delta^{ij} \right)$$

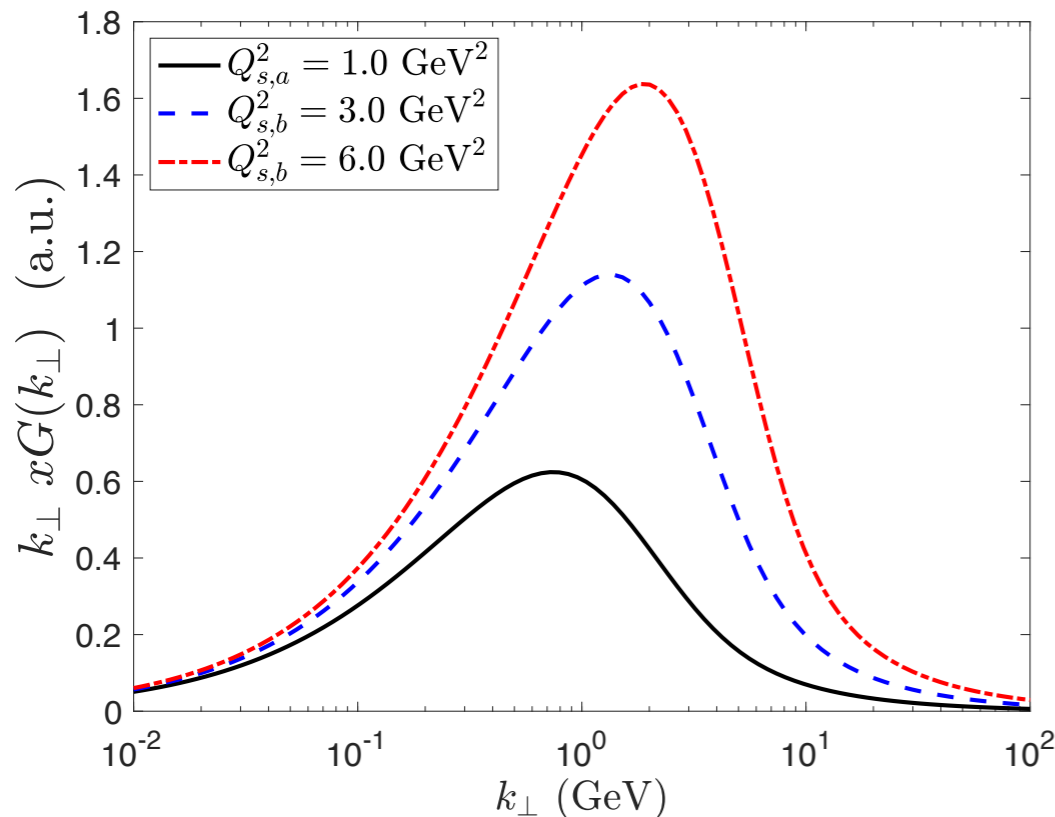
Dijet and dihadron production: TMD formalism

Forward dihadron azimuthal correlations and gluon saturation

Weizsäcker-Williams TMD
at small- x



Dihadron suppression
back-to-back peak

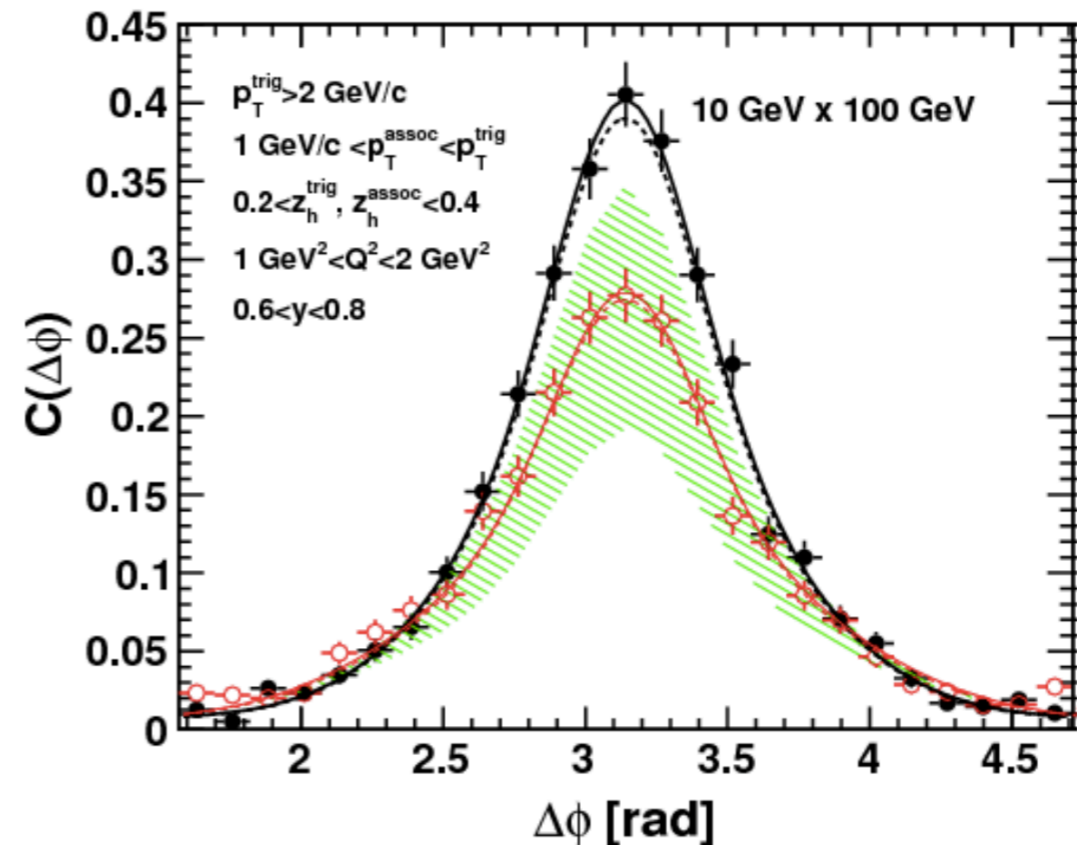


Typical momentum transfer from
hadron/nucleus to

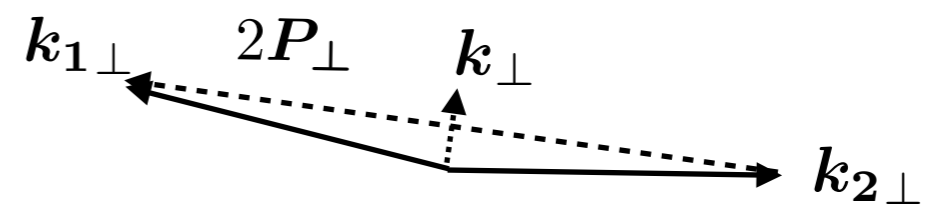
Momentum imbalance $\longrightarrow k_{\perp} \sim Q_s \longleftarrow$ Saturation scale

For lepton-jet correlations, see Ben's talk.

For more on EIC physics, see Heikki's talk on Wednesday.



Zheng, Aschenauer, Lee, Xiao (2014)

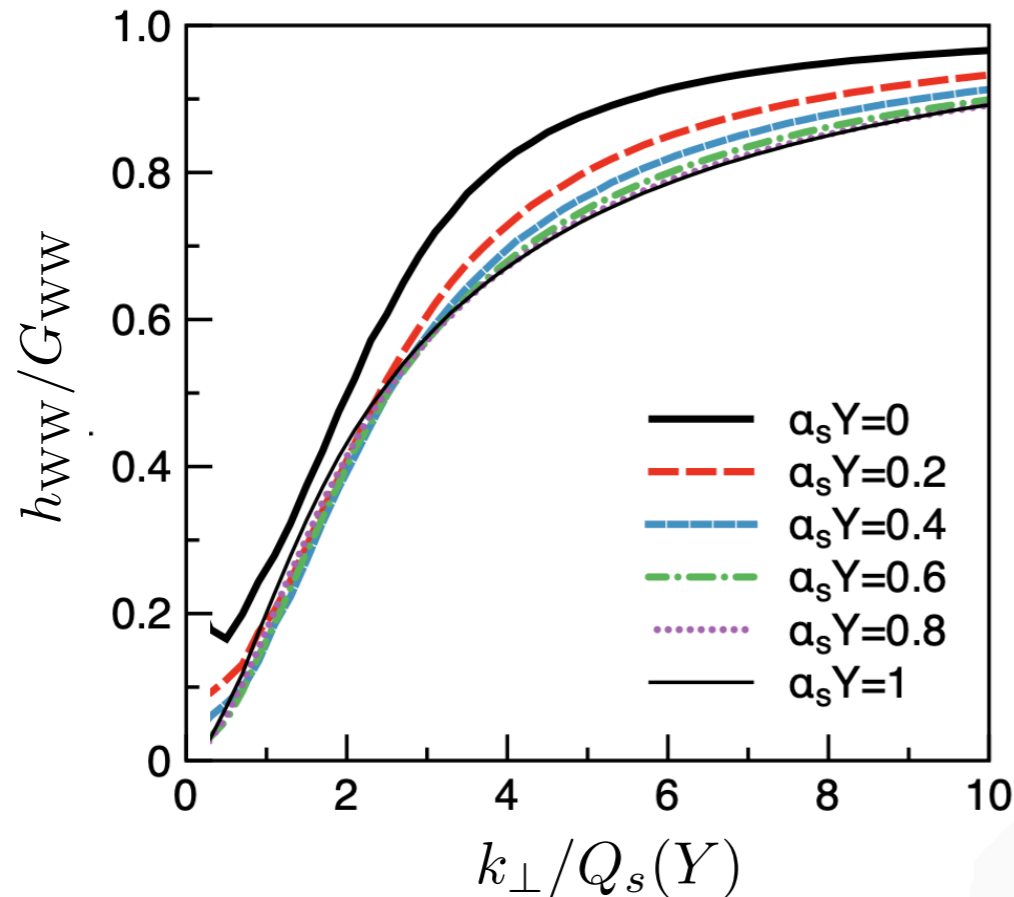


$\Delta\phi$ angle between $k_{1\perp}$ and $k_{2\perp}$

Dijet and dihadron production: TMD formalism

Forward dijet azimuthal asymmetries and linearly pol WW gluon TMD

Ratio of linearly polarized to unpolarized gluon WW



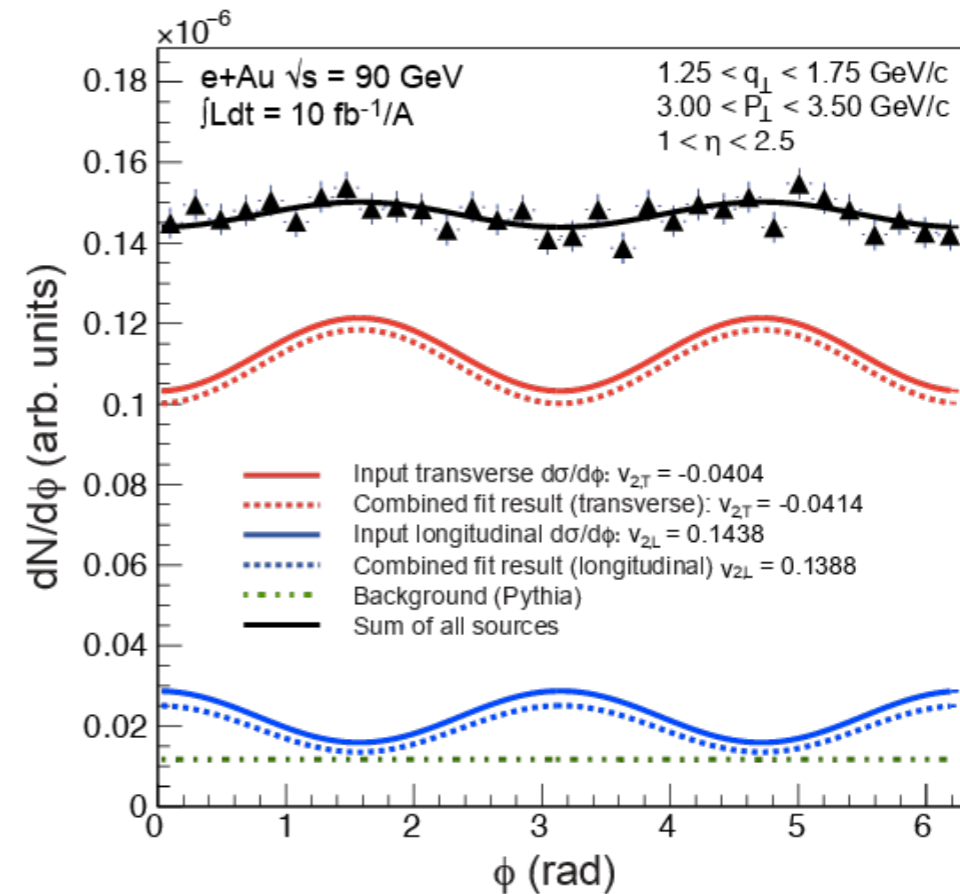
Dumitru, Lappi, Skokov (2015)

Accessed in azimuthal asymmetry of momentum imbalance in dijet pairs

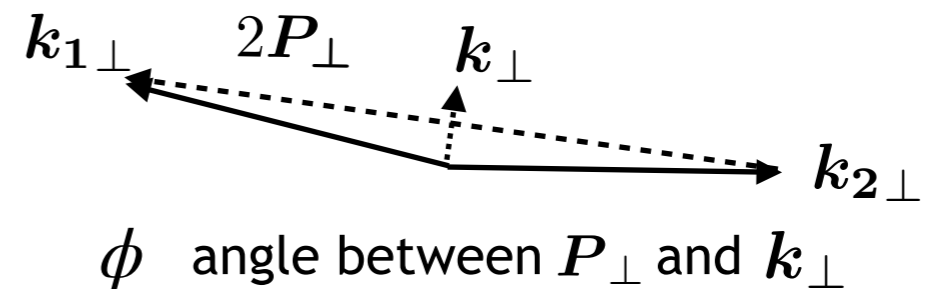
Dominguez, Qiu, Xiao, Yuan (2011)



Dijet azimuthal asymmetries in momentum imbalance

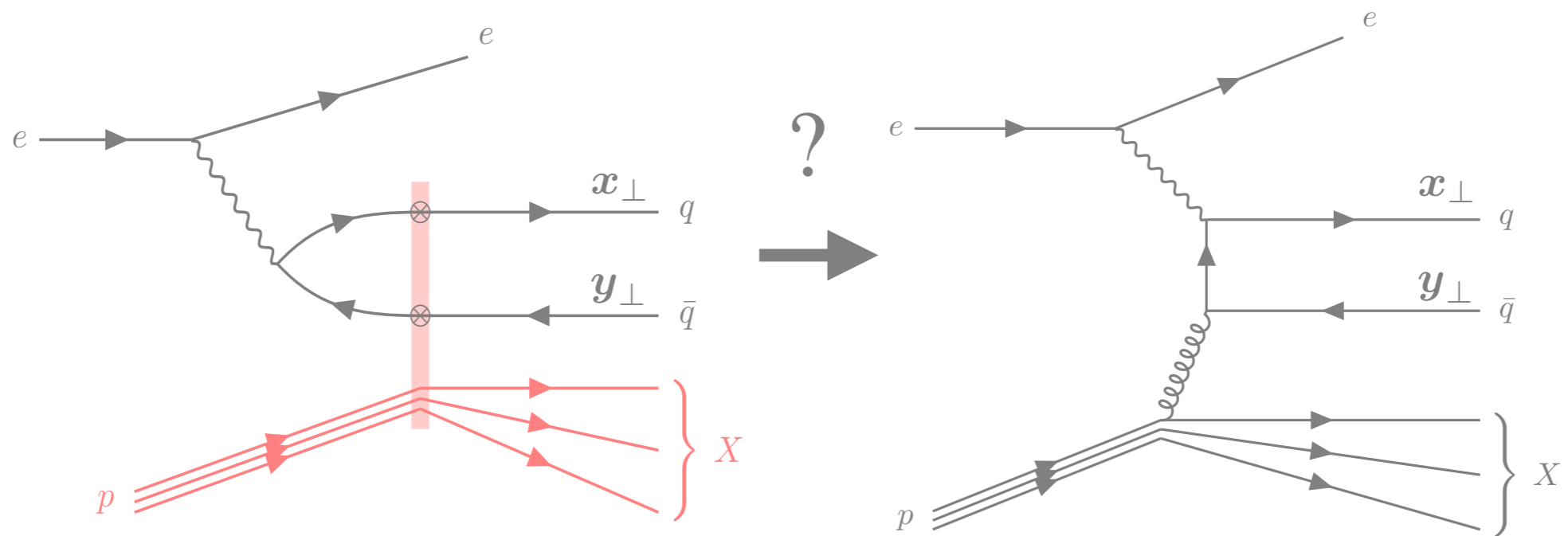


Dumitru, Skokov, Ullrich (2018)



$$|v_2| \sim \frac{x h_{WW}}{x G_{WW}}$$

Dijet production beyond TMDs



A comprehensive numerical study of the TMD/CGC correspondence

R. Boussarie, H. Mäntysaari, FS, and B. Schenke. [2106.11301](#) (JHEP09(2021)178)

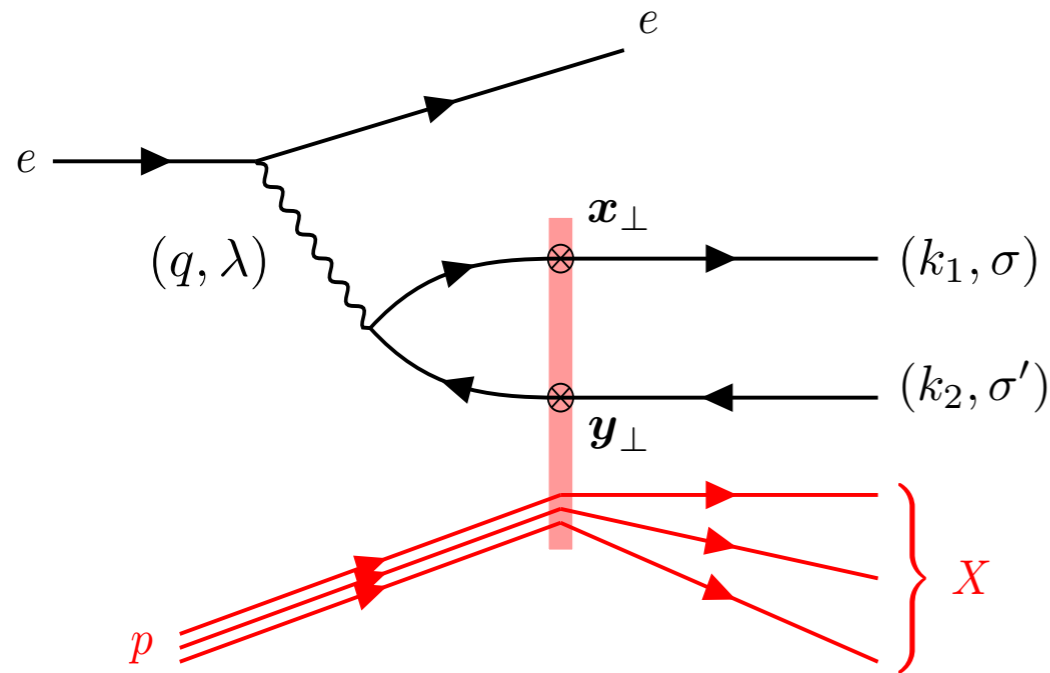


Dijet production beyond TMDs

Computation in the CGC: Wilson Lines

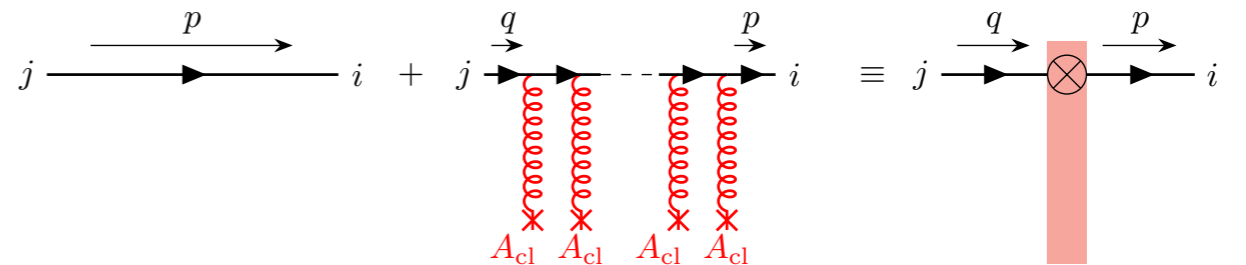
Dominguez, Marquet, Xiao, Yuan (2011)

For more on CGC see Alex's talk on Tuesday.



LO diagram for $q\bar{q}$ production in the CGC EFT

Dense gluon field $A_{cl} \sim 1/g$ needs resummation of multiple gluon interactions



$$V_{ij}(\mathbf{x}) = P \exp \left\{ ig \int dx^- A_{cl}^{+,a}(\mathbf{x}, x^-) t^a \right\}$$

Amplitude (modulo leptonic part):

$$\mathcal{M}_{LO}^{\lambda\sigma\sigma'} = \underbrace{\Psi \gamma_\lambda^* \rightarrow q\bar{q}(Q, \mathbf{r}_{xy}, z_q)}_{\text{perturbatively computable}} \otimes_{LO} \underbrace{[1 - V(\mathbf{x}_\perp) V^\dagger(\mathbf{y}_\perp)]}_{\text{non-perturbative}}$$

$$\otimes_{LO} \equiv \frac{ee_f q^-}{\pi} \int d^2 \mathbf{x}_\perp d^2 \mathbf{y}_\perp e^{-i\mathbf{k}_{1\perp} \cdot \mathbf{x}_\perp} e^{-i\mathbf{k}_{2\perp} \cdot \mathbf{y}_\perp}$$

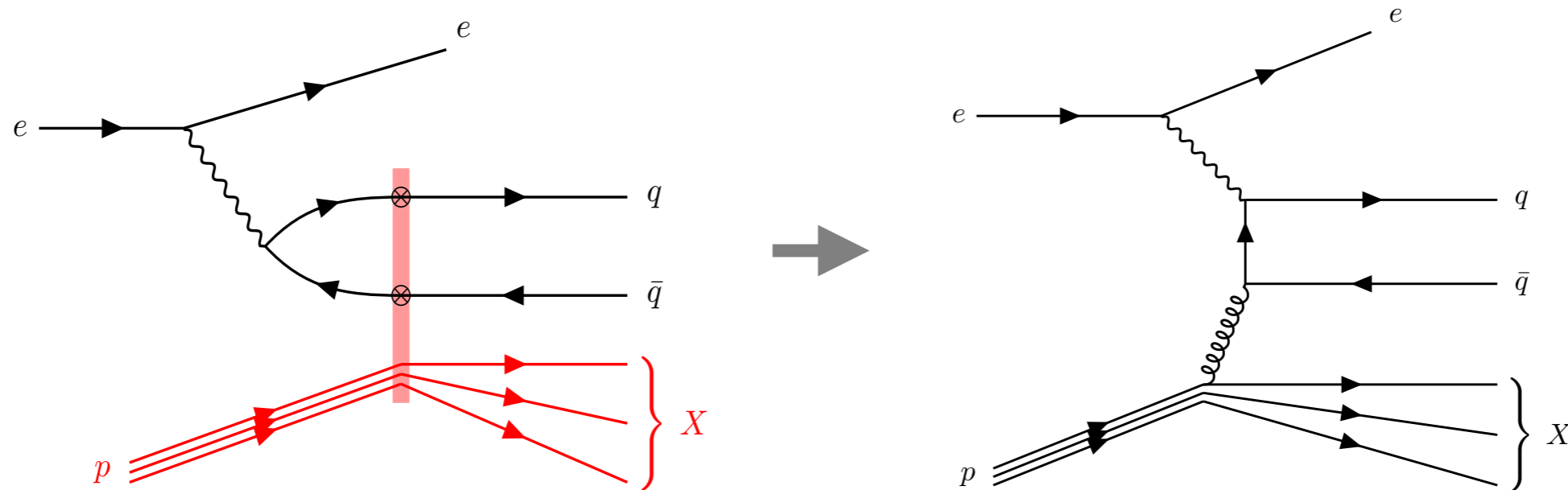
Dijet cross-section in the CGC will contain dipoles and quadrupole:

$$\frac{1}{N_c} \langle \text{Tr} [V(\mathbf{x}_\perp) V^\dagger(\mathbf{y}_\perp)] \rangle_Y$$

$$\frac{1}{N_c} \langle \text{Tr} [V(\mathbf{x}_\perp) V^\dagger(\mathbf{y}_\perp) V(\mathbf{y}'_\perp) V^\dagger(\mathbf{x}'_\perp)] \rangle_Y$$

Dijet production beyond TMDs

CGC, improved TMD and TMD frameworks



CGC

$$V(\mathbf{x}_\perp)V^\dagger(\mathbf{y}_\perp) = \mathcal{P} \exp \left[-ig \int_{\mathbf{y}_\perp}^{\mathbf{x}_\perp} d\mathbf{z}_\perp \cdot \tilde{\mathbf{A}}_\perp(\mathbf{z}_\perp) \right]$$

Boussarie, Mehtar-Tani (2020)

Improved TMD

$$= 1 - ig \int_{\mathbf{y}_\perp}^{\mathbf{x}_\perp} d\mathbf{z}_\perp \cdot \tilde{\mathbf{A}}_\perp(\mathbf{z}_\perp) + \dots$$

TMD

Altinoluk, Boussarie, Kotko (2019)

$$= 1 + ig \mathbf{r}_\perp \cdot \tilde{\mathbf{A}}_\perp(\mathbf{z}_\perp) + \dots$$

$$\mathbf{r}_\perp = \mathbf{x}_\perp - \mathbf{y}_\perp$$

Dominguez, Marquet, Xiao, Yuan (2011)

Dijet production beyond TMDs

The TMD and improved TMD limit as an expansion

$$d\sigma_{\text{CGC}} = \underbrace{d\sigma_{\text{TMD}}}_{\text{d}\sigma_{\text{ITMD}}} + \overbrace{\mathcal{O}\left(\frac{k_{\perp}}{Q_{\perp}}\right)}^{\text{kinematic}} + \overbrace{\mathcal{O}\left(\frac{Q_s}{Q_{\perp}}\right)}^{\text{genuine}}$$

Dominguez, Marquet, Xiao, Yuan (2011)

TMD valid $k_{\perp}, Q_s \ll Q_{\perp}$

back-to-back hadrons/jets
and transverse momenta larger
than sat scale

Hard factor

Weizsäcker-Williams gluon TMD

$$d\sigma^{\gamma^* A \rightarrow q\bar{q}X} \sim \mathcal{H}_{\text{TMD}}^{ij}(\mathbf{P}_{\perp}) \alpha_s x G_{\text{WW}}^{ij}(x, \mathbf{k}_{\perp}) + \mathcal{O}(k_{\perp}/P_{\perp}) + \mathcal{O}(Q_s/P_{\perp})$$

Altinoluk, Boussarie, Kotko (2019)

Improved TMD valid $Q_s \ll Q_{\perp}$

transverse momenta larger
than sat scale

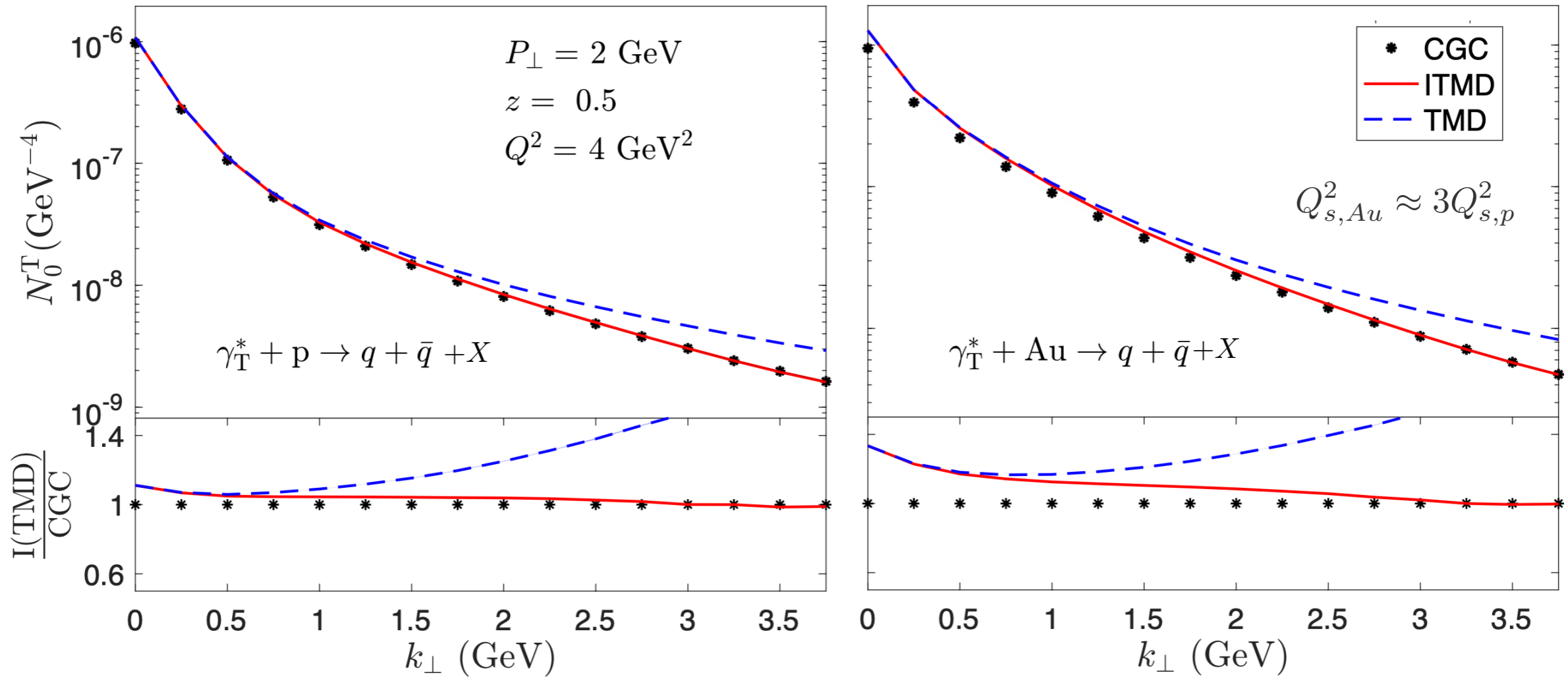
Hard factor resums
kinematic powers k_{\perp}/P_{\perp}

$$d\sigma^{\gamma_{\lambda}^* A \rightarrow q\bar{q}X} \sim \mathcal{H}_{\text{ITMD}}^{\lambda,ij}(\mathbf{P}_{\perp}, \mathbf{k}_{\perp}) \alpha_s x G_{\text{WW}}^{ij}(x, \mathbf{k}_{\perp}) + \mathcal{O}(Q_s/P_{\perp})$$

Dijet production beyond TMDs

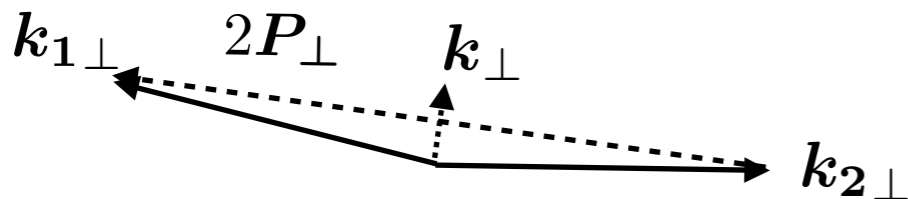
Differential yield: TMD, ITMD and CGC

R. Boussarie, H. Mäntysaari,
FS, B. Schenke (2021)



proton ~ smaller Q_s^2

Gold nucleus ~ larger Q_s^2



$$\frac{dN^{\gamma_\lambda^* + A \rightarrow q\bar{q} + X}}{d^2\mathbf{P}_\perp d^2\mathbf{k}_\perp d\eta_1 d\eta_2} = N_0^\lambda(P_\perp, k_\perp) \left[1 + 2 \sum_{k=1}^{\infty} v_{k,\lambda}(P_\perp, k_\perp) \cos(k\phi) \right]$$

$$\phi \equiv \phi_{\mathbf{k}_\perp} - \phi_{\mathbf{P}_\perp}$$

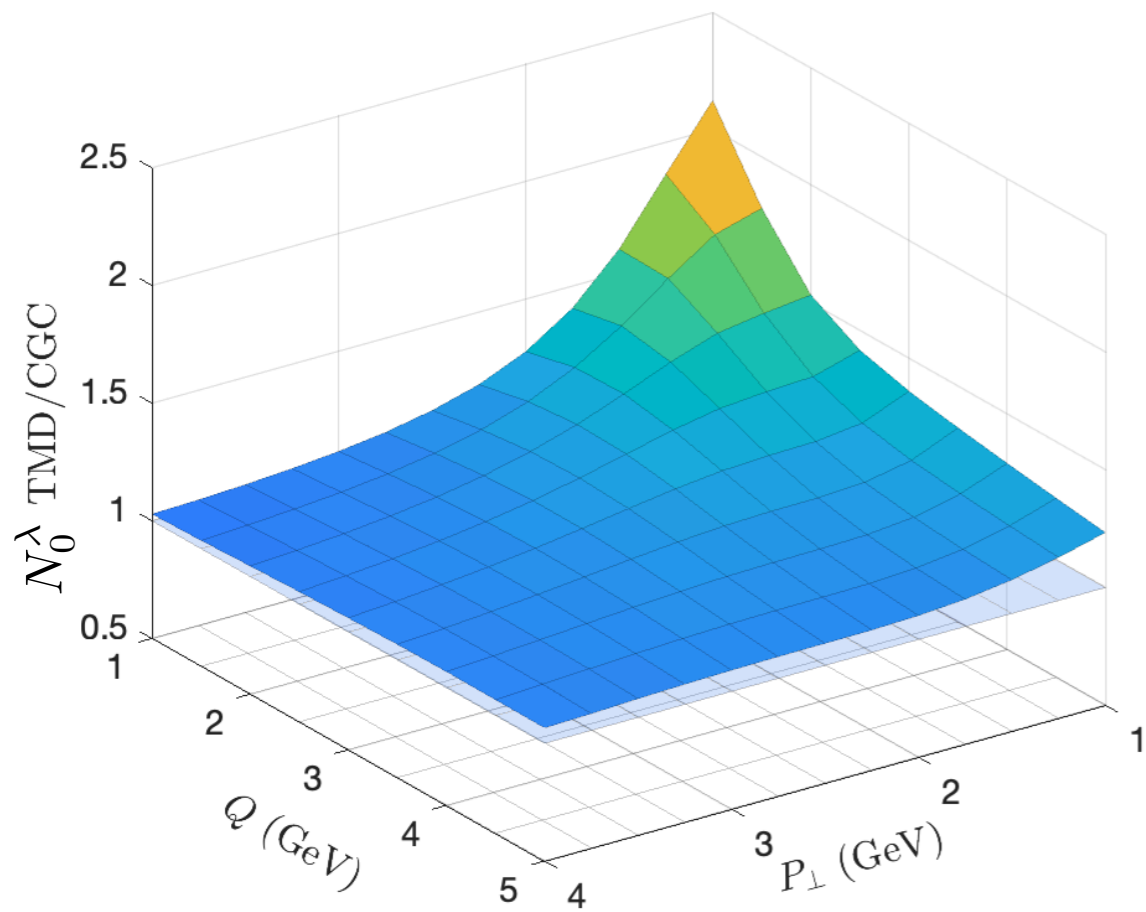
Dijet production beyond TMDs

Q^2 and P_\perp dependence of genuine saturation

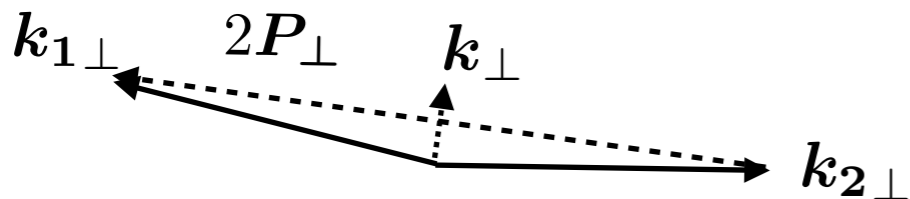
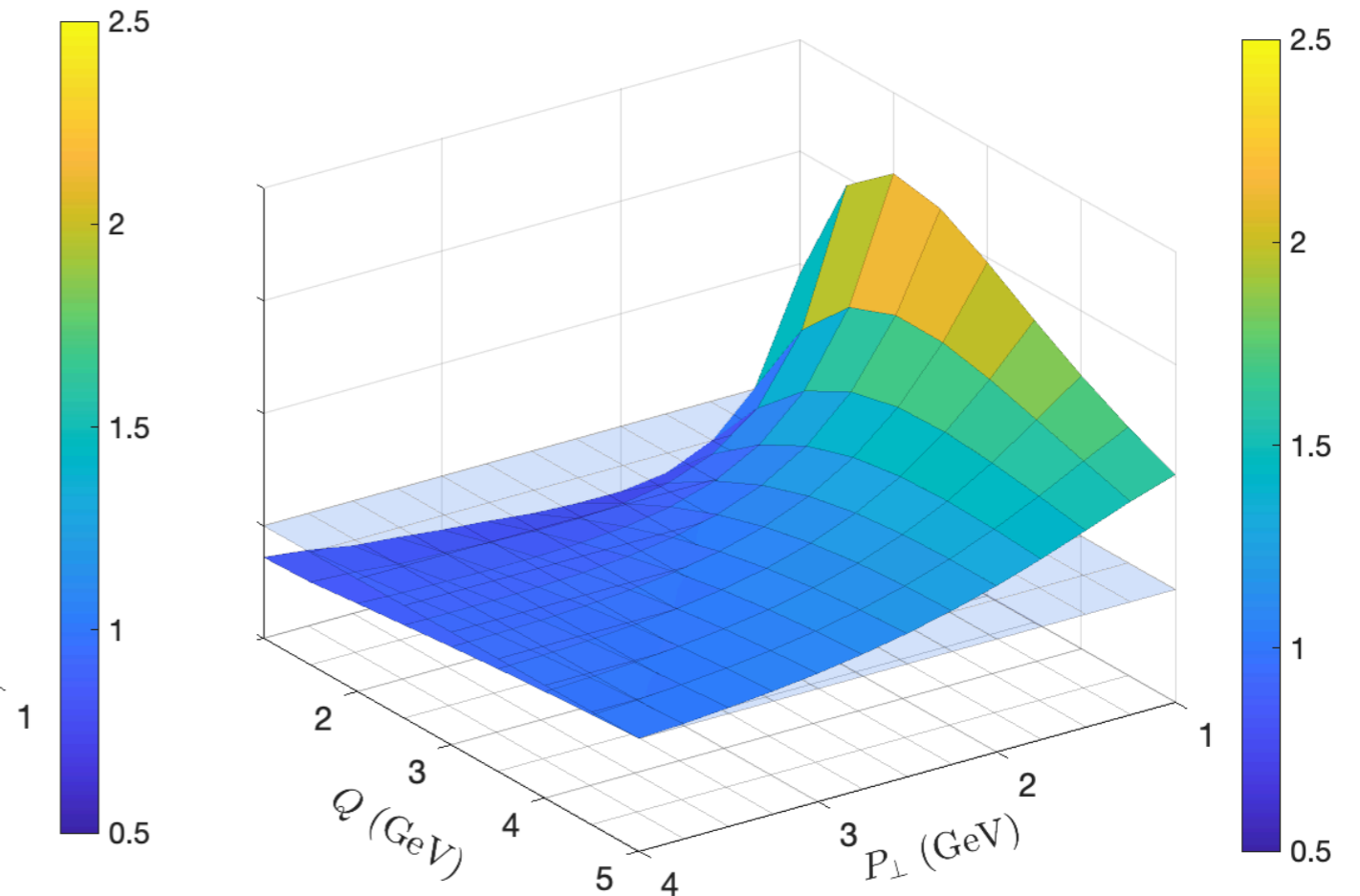
R. Boussarie, H. Mäntysaari,
FS, B. Schenke (2021)

At exactly back-to-back $k_\perp \approx 0$ the ratio of CGC/TMD is sensitive to genuine twists

$$\gamma_T^* + \text{Au} \rightarrow q + \bar{q} + X$$



$$\gamma_L^* + \text{Au} \rightarrow q + \bar{q} + X$$



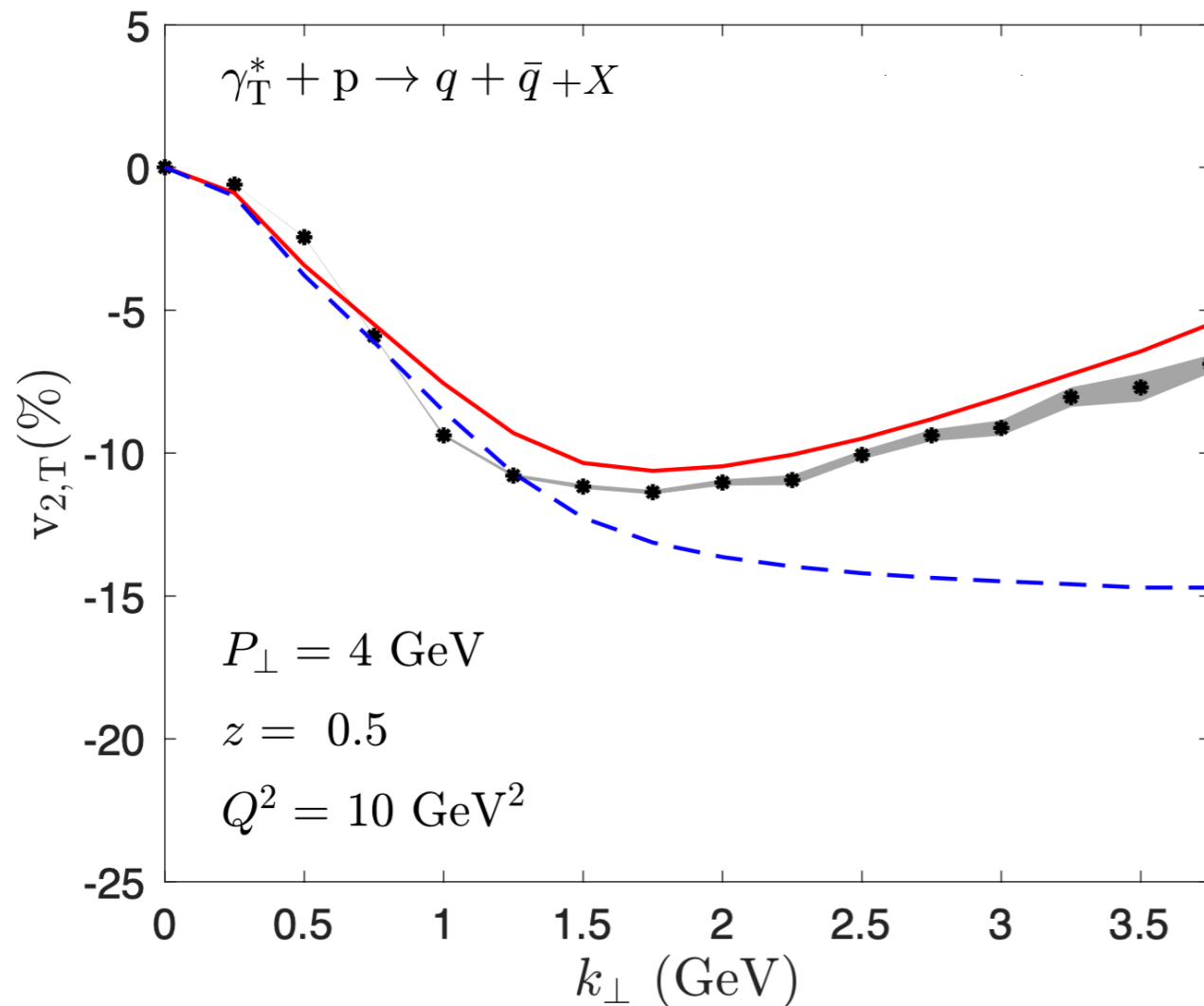
$$\frac{dN_{\gamma_\lambda^* + A \rightarrow q\bar{q} + X}}{d^2\mathbf{P}_\perp d^2\mathbf{k}_\perp d\eta_1 d\eta_2} = N_0^\lambda(P_\perp, k_\perp) \left[1 + 2 \sum_{k=1}^{\infty} v_{k,\lambda}(P_\perp, k_\perp) \cos(k\phi) \right]$$

$$\phi \equiv \phi_{\mathbf{k}_\perp} - \phi_{\mathbf{P}_\perp}$$

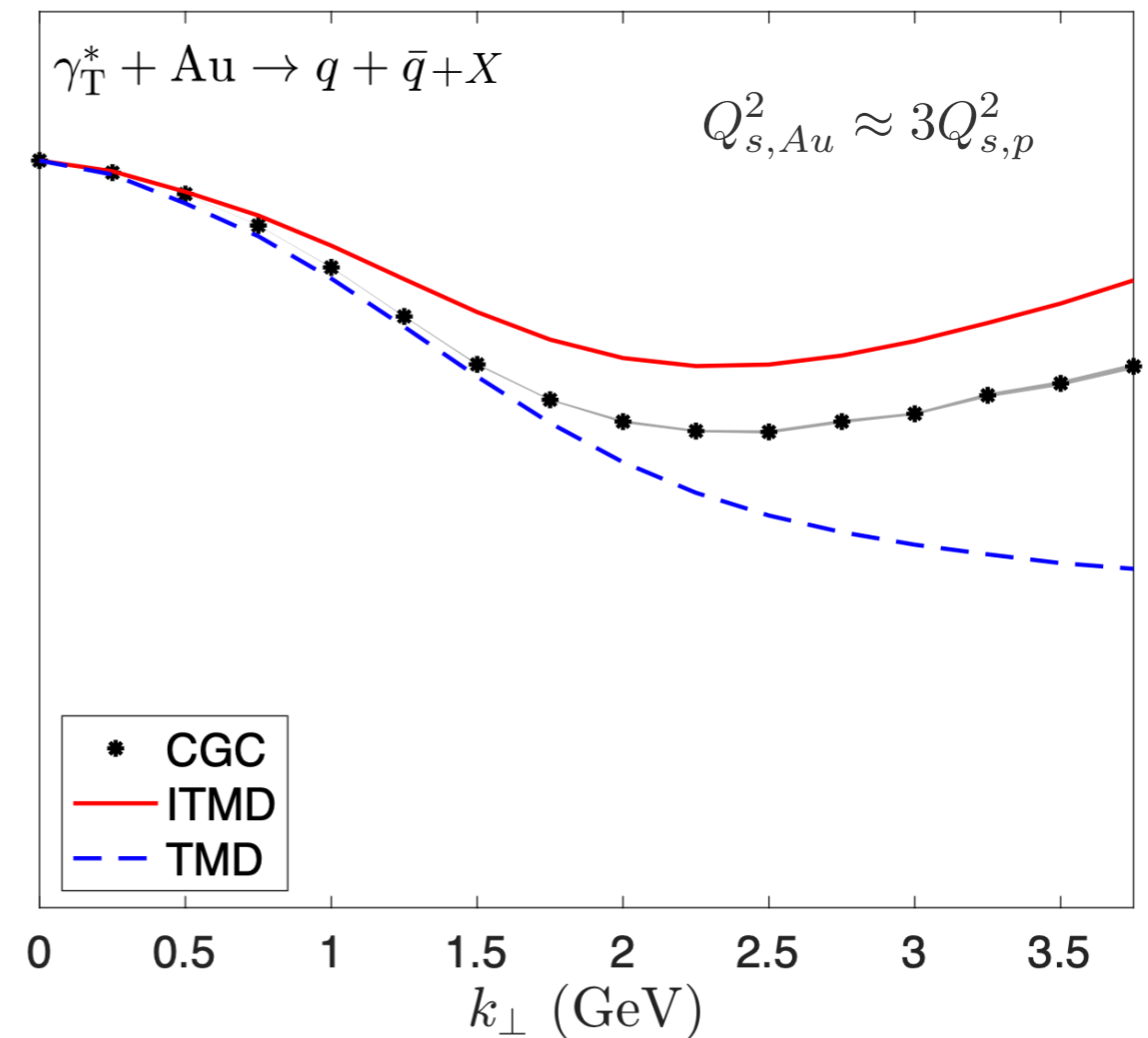
Dijet production beyond TMDs

Momentum imbalance elliptic anisotropies:
TMD vs ITMD vs CGC

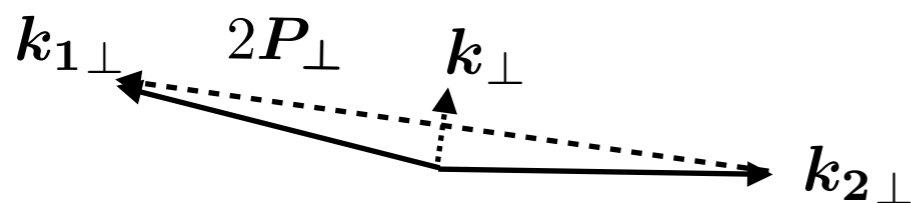
R. Boussarie, H. Mäntysaari,
FS, B. Schenke (2021)



proton ~ smaller Q_s^2



Gold nucleus ~ larger Q_s^2



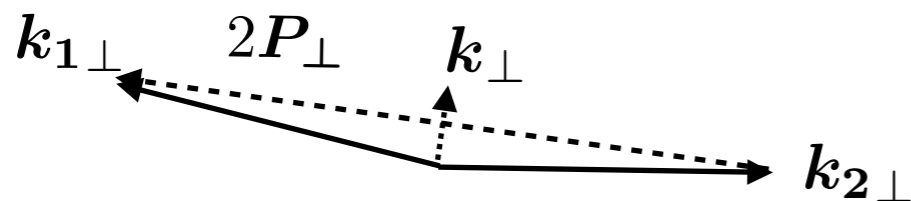
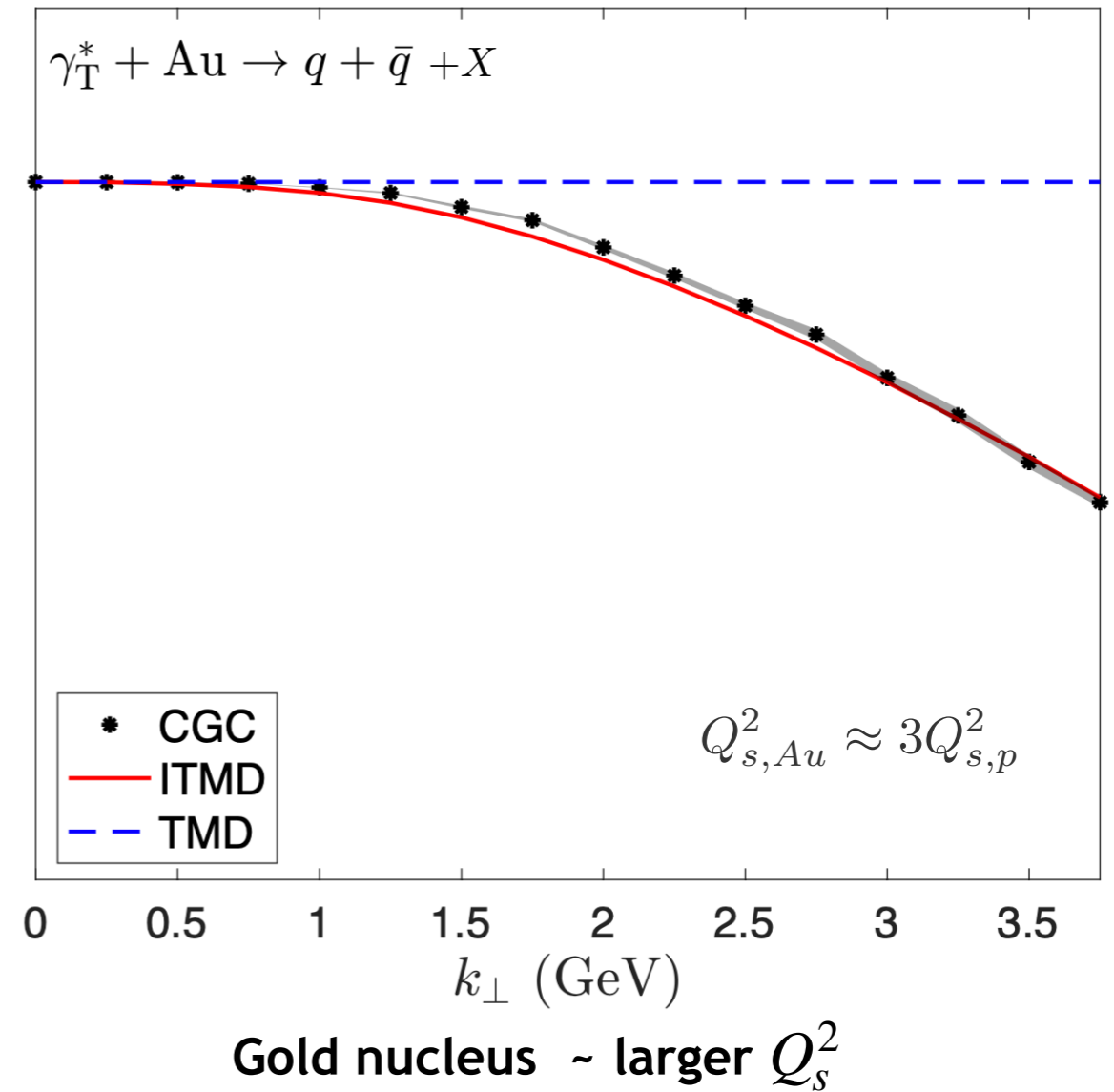
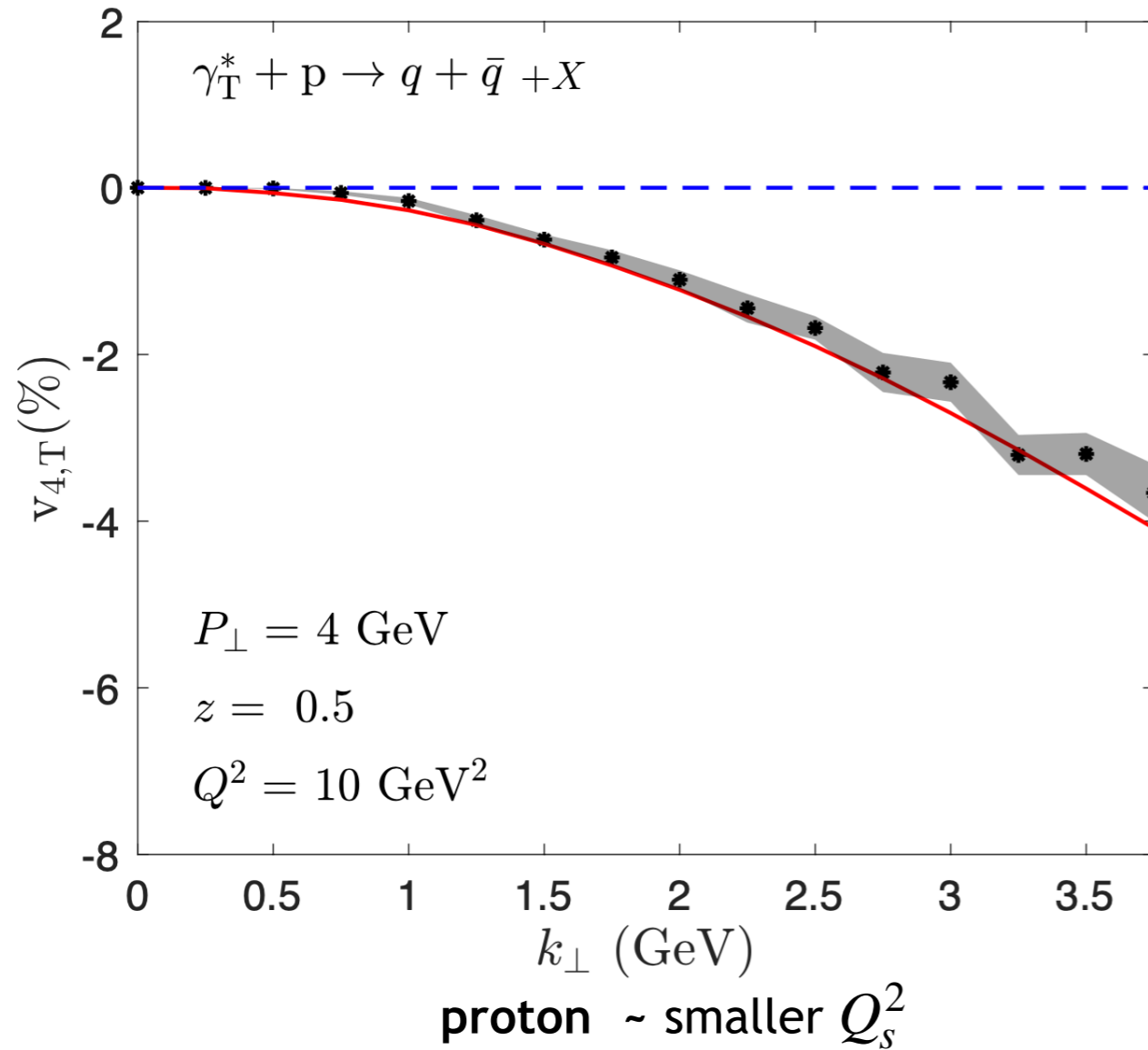
$$\frac{dN_{\gamma_\lambda^* + A \rightarrow q\bar{q} + X}}{d^2\mathbf{P}_\perp d^2\mathbf{k}_\perp d\eta_1 d\eta_2} = N_0^\lambda(P_\perp, k_\perp) \left[1 + 2 \sum_{k=1}^{\infty} v_{k,\lambda}(P_\perp, k_\perp) \cos(k\phi) \right]$$

$$\phi \equiv \phi_{\mathbf{k}_\perp} - \phi_{\mathbf{P}_\perp}$$

Dijet production beyond TMDs

Momentum imbalance quadrangular anisotropies:
TMD vs ITMD vs CGC

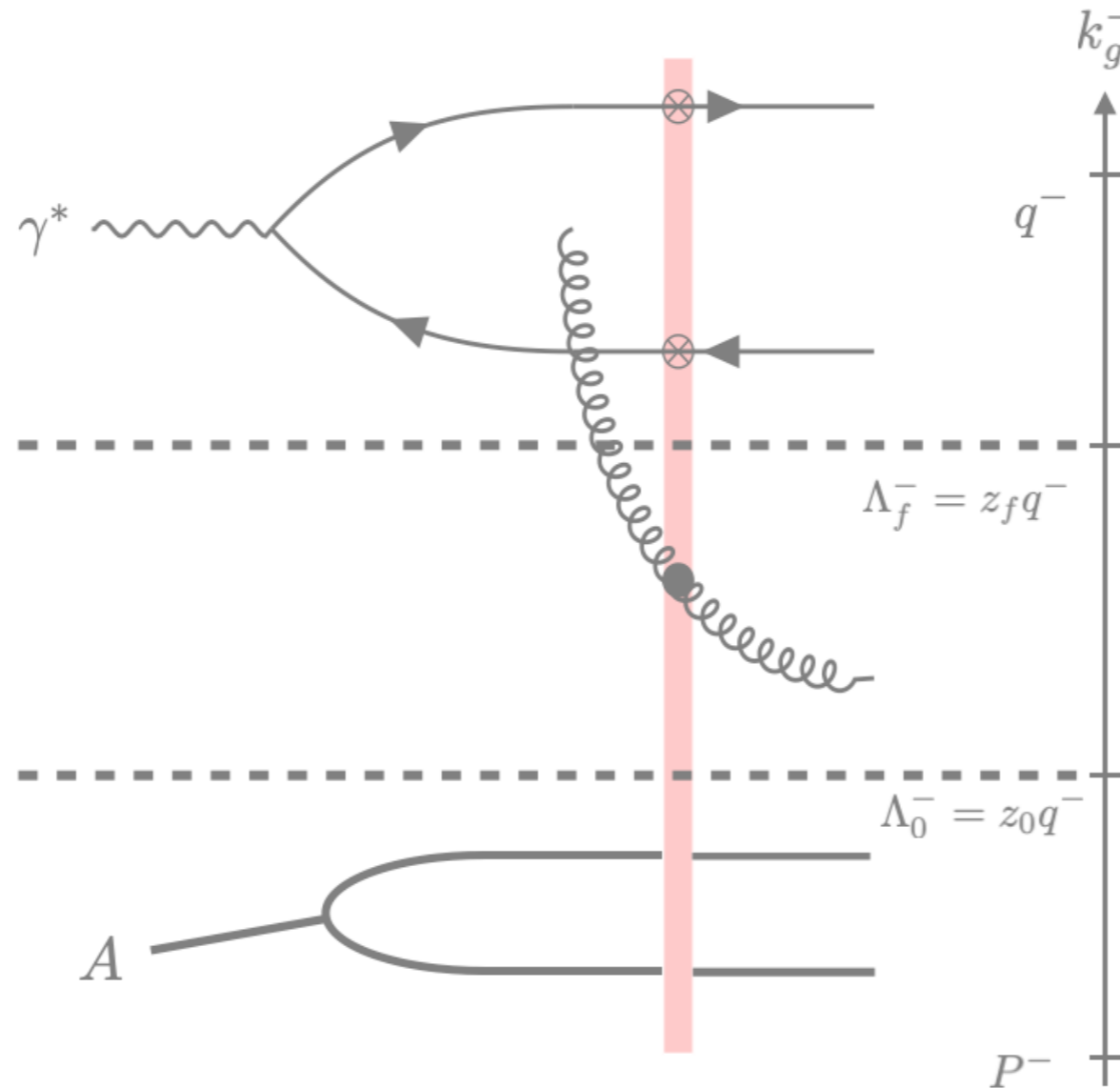
R. Boussarie, H. Mäntysaari,
FS, B. Schenke (2021)



$$\frac{dN^{\gamma_\lambda^* + A \rightarrow q\bar{q} + X}}{d^2\mathbf{P}_\perp d^2\mathbf{k}_\perp d\eta_1 d\eta_2} = N_0^\lambda(P_\perp, k_\perp) \left[1 + 2 \sum_{k=1}^{\infty} v_{k,\lambda}(P_\perp, k_\perp) \cos(k\phi) \right]$$

$$\phi \equiv \phi_{\mathbf{k}_\perp} - \phi_{\mathbf{P}_\perp}$$

Dijet production in the CGC at NLO



Rapidity factorization and NLO impact factor

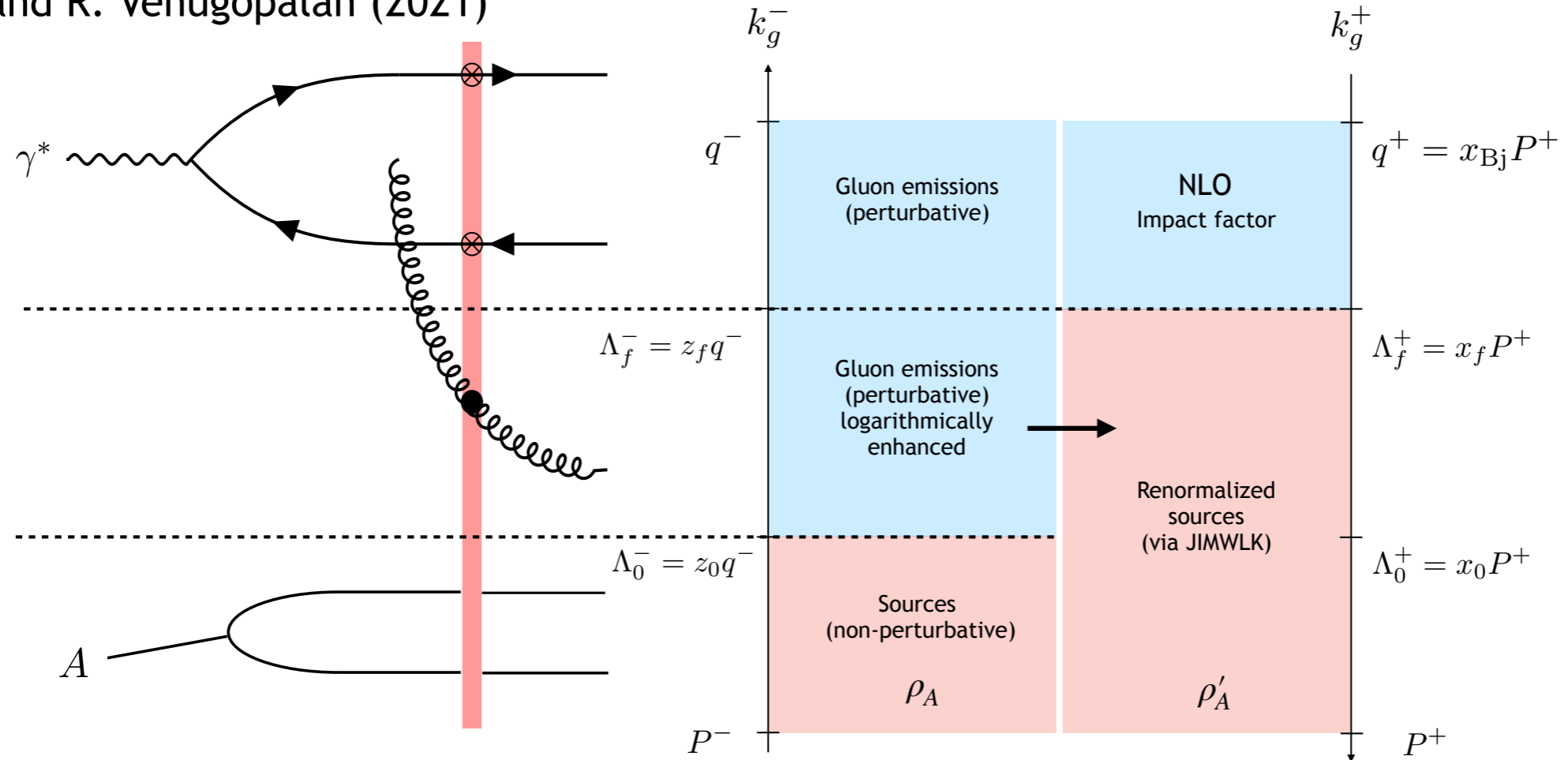
P. Caucal, FS, and R. Venugopalan. [2108.06347](#)
(Submitted to JHEP)



Dijet production in the CGC at NLO

Rapidity (slow gluon) divergences and JIMWLK factorization

P. Caucal, FS, and R. Venugopalan (2021)



- Terms proportional to $\alpha_s \ln^2 \left(\frac{z_f}{z_0} \right)$ cancel, only $\alpha_s \ln \left(\frac{z_f}{z_0} \right)$ survive and factorize:

$$d\sigma_{\text{NLO}} \sim \alpha_s \ln \left(\frac{z_f}{z_0} \right) \mathcal{H}_{\text{LL}} d\sigma_{\text{LO}} \quad \leftarrow \text{JIMWLK LL Hamiltonian}$$

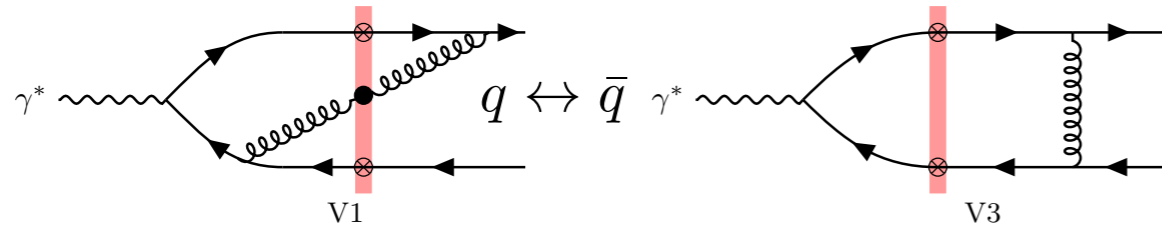
- Slow gluon radiation $\alpha_s \ln \left(\frac{z_f}{z_0} \right)$ can be large in resummed by redefining distribution of sources:

$$W_{x_0}[\rho_A] \rightarrow W_{x_f}[\rho'_A]$$

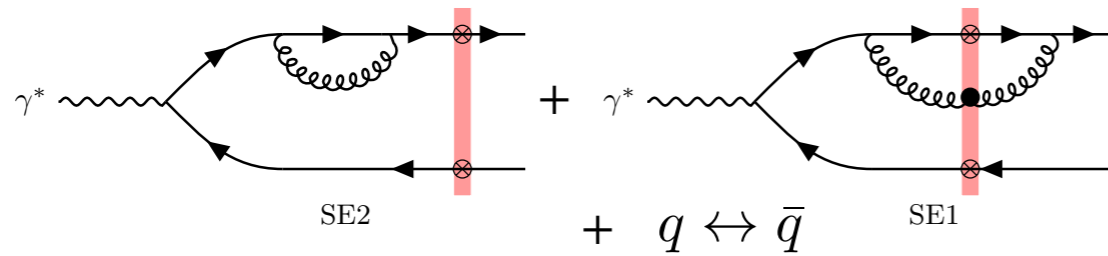
Dijet production in the CGC at NLO

Cancellation of divergences of UV divergences

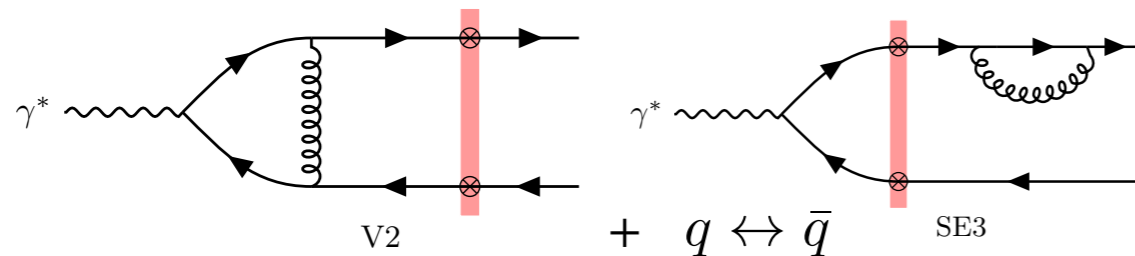
P. Caucal, FS, and R. Venugopalan (2021)



- UV finite diagrams



- UV divergences cancel among self energies contributions (before SW and crossing SW)



- UV divergence cancel in vertex contribution before SW and self energy contribution after SW

- UV finite, no need for counter-terms at this order in PT.
- Overall IR divergence is left in sum of virtual diagrams, and soft divergence left in V3. Both cancel with real emissions.

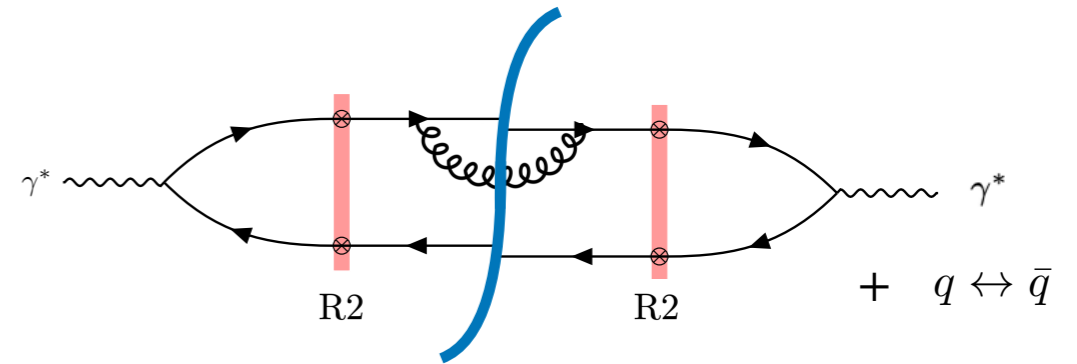
Dijet production in the CGC at NLO

Infrared and collinear safety

P. Caucal, FS, and R. Venugopalan (2021)

Collinear non-slow divergences

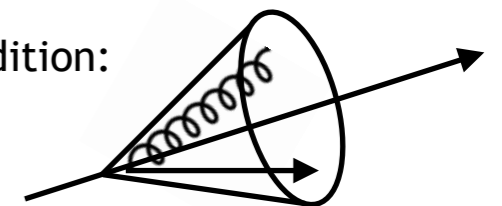
- Implement a jet algorithm* (small cone) excluding slow gluon divergence



Phase space for collinear non-slow gluon

$$\int_{z_f}^{z_j} \frac{dz_g}{z_g} \mu^\epsilon \int \frac{d^{2-\epsilon} \mathcal{C}_{qg,\perp}}{(2\pi)^{2-\epsilon}} \frac{1}{\mathcal{C}_{qg,\perp}^2}$$

Small-cone condition:

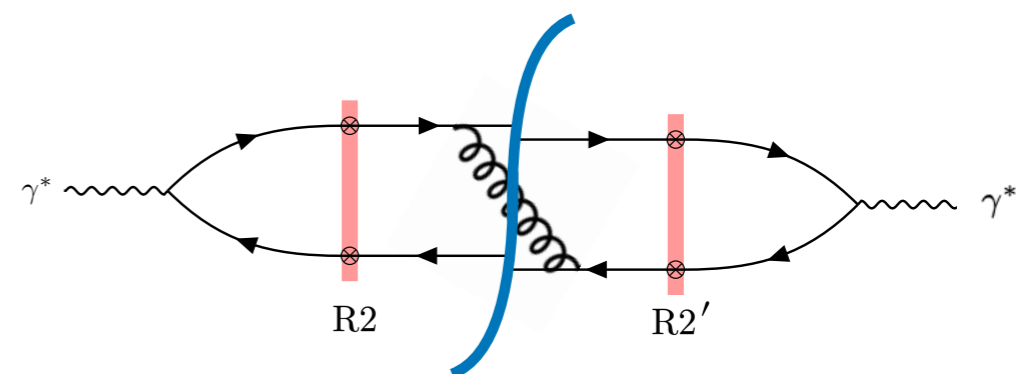
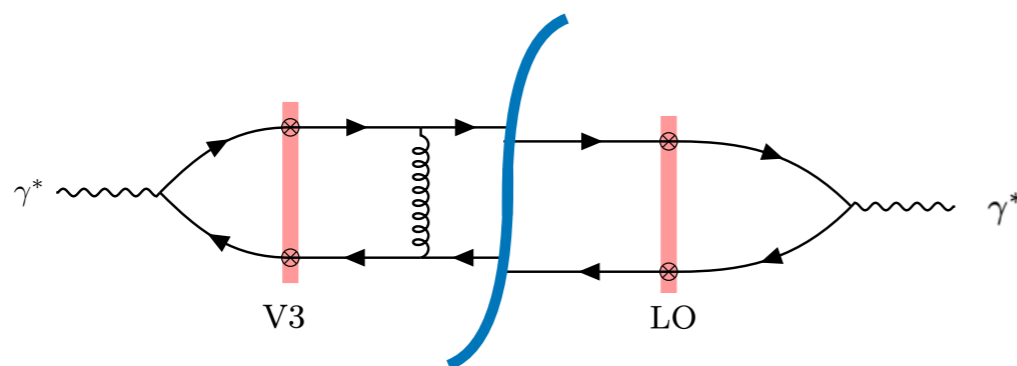


- Collinear divergence cancels against IR divergence left in virtual contributions

$$\mathcal{C}_{qg,\perp}^2 \leq \mathcal{C}_{qg,\perp}^2|_{\max} = R^2 p_j^2 \min \left(\frac{z_g^2}{z_j^2}, \frac{(z_j - z_g)^2}{z_j^2} \right)$$

Soft divergence

- Remaining soft divergence cancel between vertex correction after SW, and cross term real gluon emission after SW

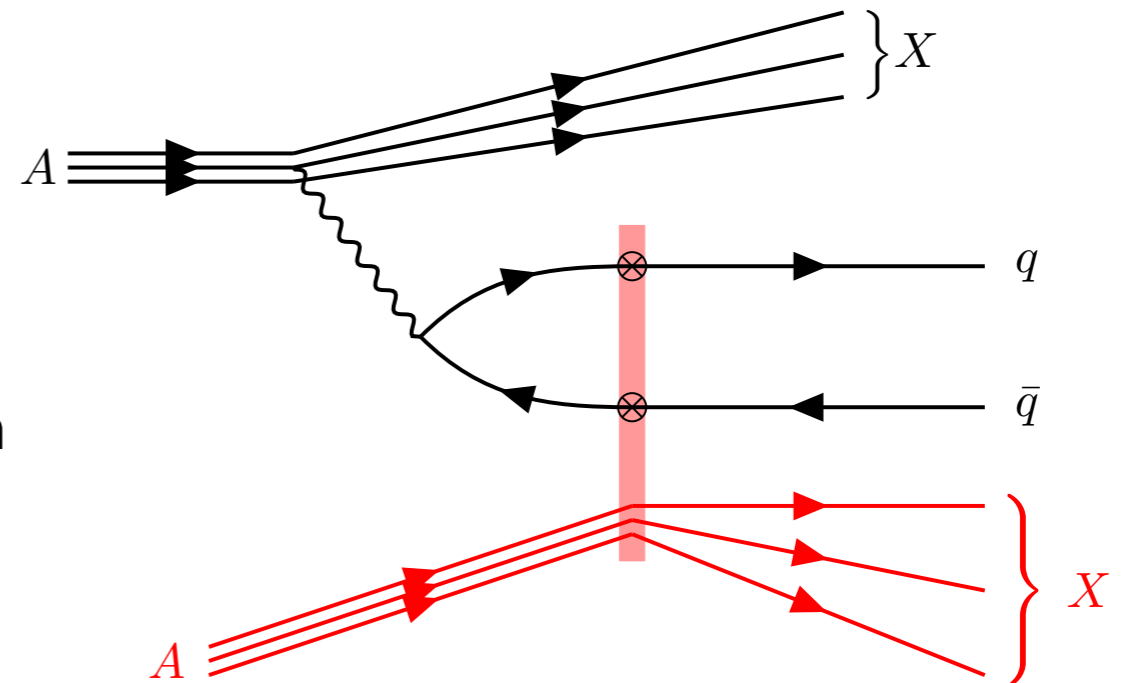


Connections to the LHC

Dijet production in ultra-peripheral collisions

$$A + A \rightarrow \gamma + A \rightarrow q\bar{q} + X$$

- Studied within the Improved TMD formalism in Kotko, Kutak, Sapeta, Stasto, Strikman (2017)



What are the size of genuine saturation corrections (not accounted in ITMD but included in CGC)?

How large are NLO corrections?

In principle...

Take real photon ($Q^2 \rightarrow 0$) limit in hadronic part of dijet production in DIS

In practice, this requires work!

Outlook

- Couple our partonic cross-sections to event generators

How much of the kinematic power and genuine saturation corrections survives in the actual observable?

- Investigate dijet production at NLO in the back-to-back limit

Match to TMD factorization at NLO

Xiao, Yuan, Zhou (2017)

del Castillo, Echevarria, Makris, Scimemi (2020)

Hentschinski (2021)

Is the ITMD framework valid at NLO?

- Numerical implementation of dijet production at NLO

Promoting saturation physics to a precision science

- Ultra-peripheral collisions at the LHC

Size of higher genuine saturation corrections

UPCs at NLO in the CGC

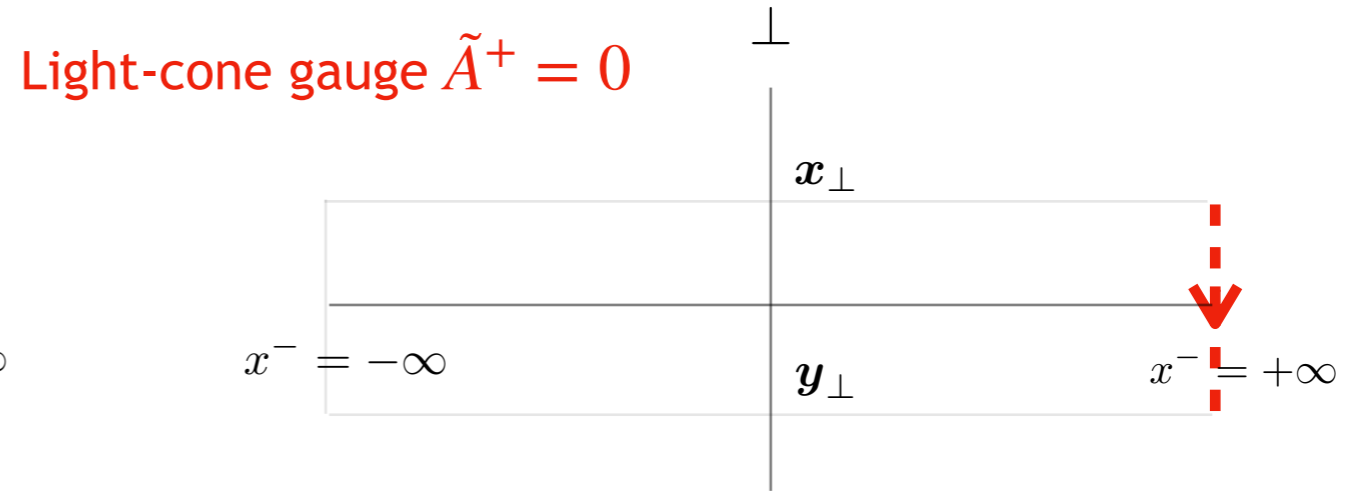
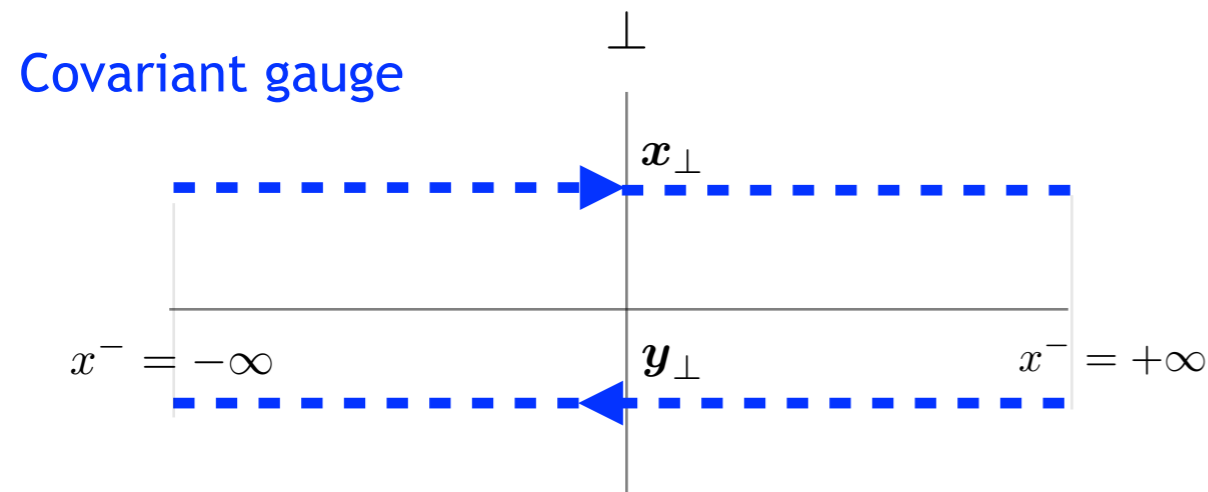
Back-up slides

Dijet production beyond TMDs

CGC, improved TMD and TMD frameworks

Boussarie, Mehtar-Tani (2020)

Pair of Wilson lines as **transverse gauge link**



$$V(\mathbf{x}_\perp)V^\dagger(\mathbf{y}_\perp) = \mathcal{P} \exp \left[-ig \int_{\mathbf{y}_\perp}^{\mathbf{x}_\perp} d\mathbf{z}_\perp \cdot \tilde{\mathbf{A}}_\perp(\mathbf{z}_\perp) \right]$$

gA expansion:

$$= 1 - ig \int_{\mathbf{y}_\perp}^{\mathbf{x}_\perp} d\mathbf{z}_\perp \cdot \tilde{\mathbf{A}}_\perp(\mathbf{z}_\perp) + \dots$$

Small dipole expansion:

$$= 1 + ig \mathbf{r}_\perp \cdot \tilde{\mathbf{A}}_\perp(\mathbf{z}_\perp) + \dots$$

Altinoluk, Boussarie, Kotko (2019)

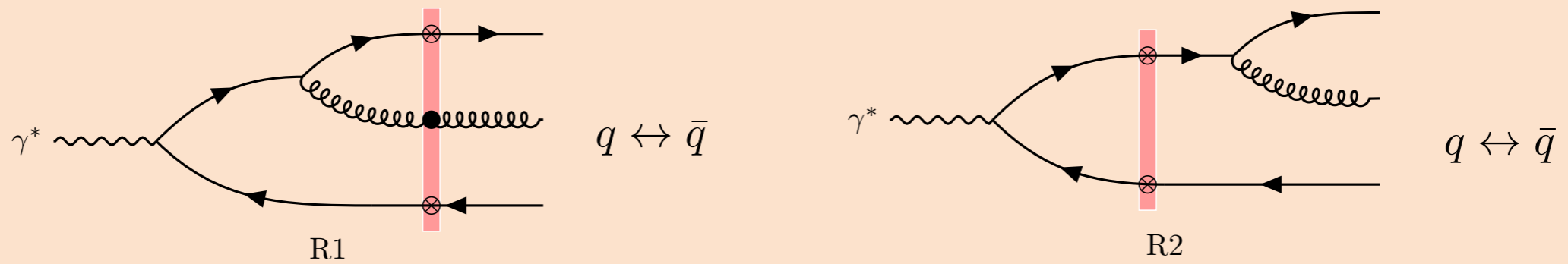
$$\mathbf{r}_\perp = \mathbf{x}_\perp - \mathbf{y}_\perp$$

Dominguez, Marquet, Xiao, Yuan (2011)

Dijet production in the CGC at NLO

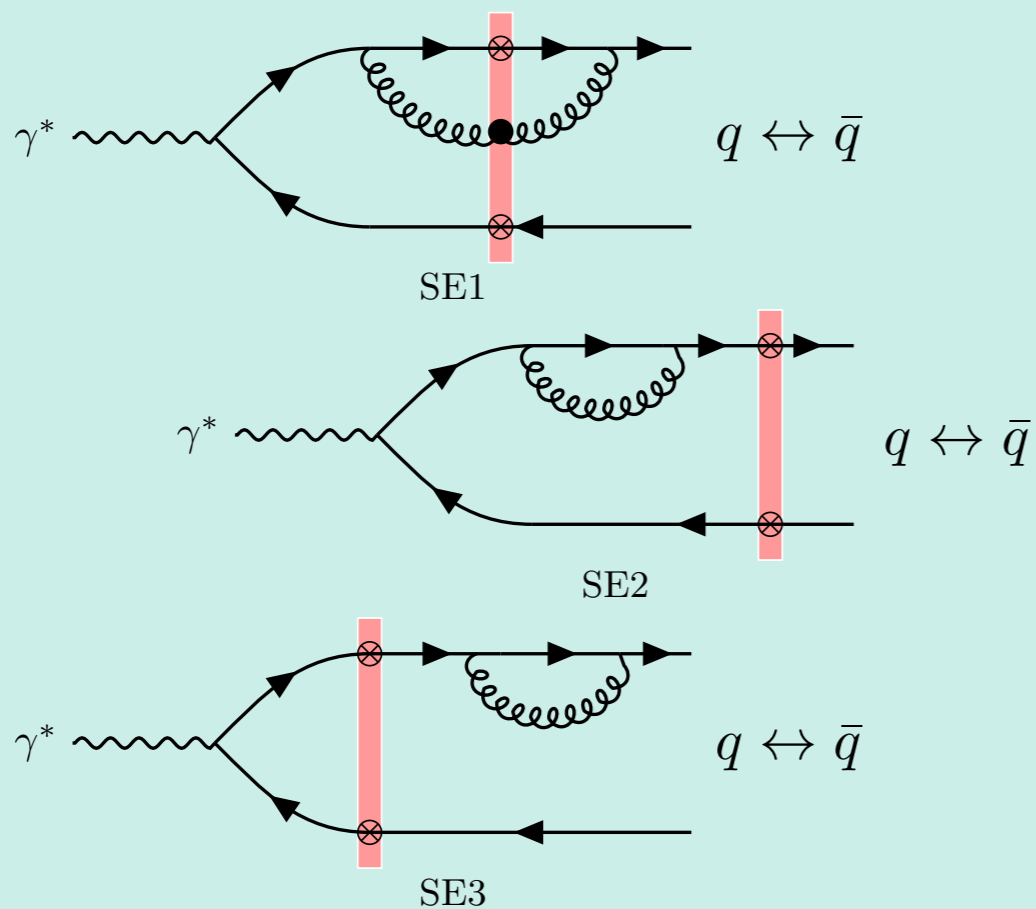
Real and virtual emissions

Real emission diagrams (loop opens in DA and closes in the CCA)

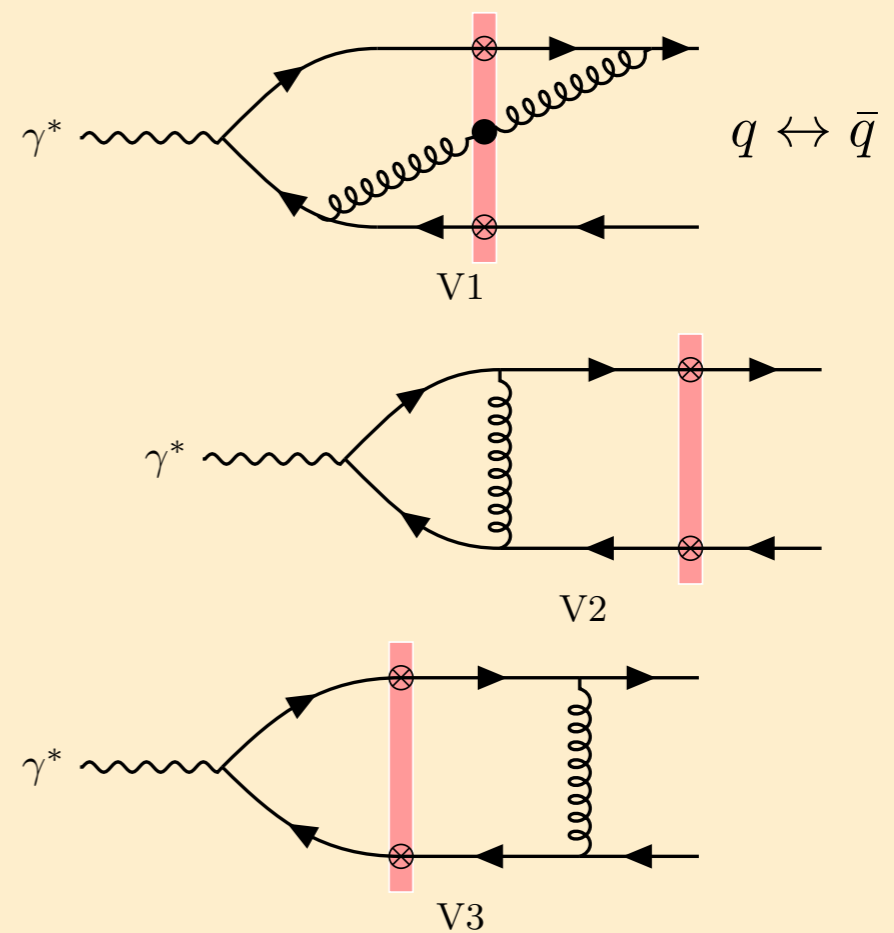


Virtual emission (loop open and closes in DA or CCA)

Self-energy contributions



Vertex contributions

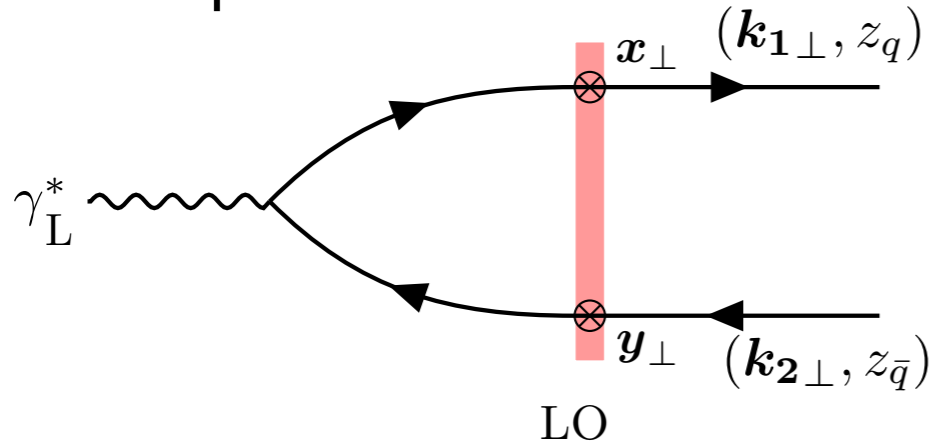


Dijet production in the CGC at NLO

An example of structure of LO vs NLO amplitudes

P. Caucal, FS, and R. Venugopalan (2021)

LO amplitude



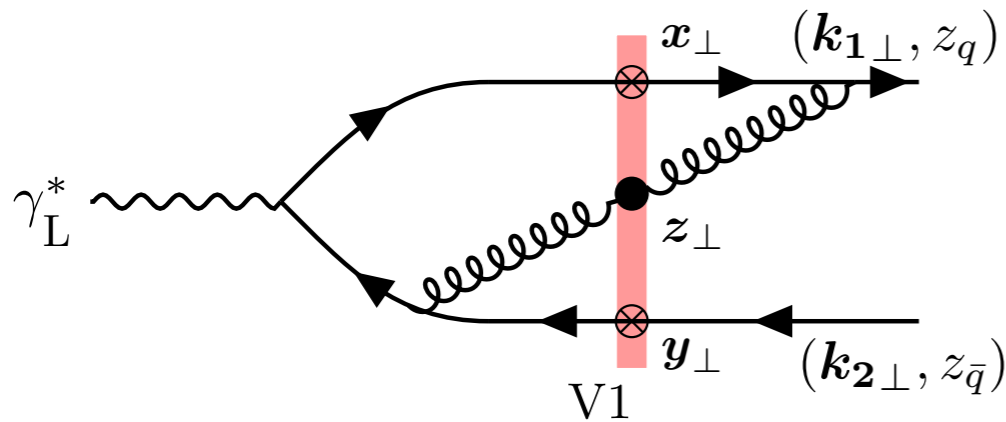
$$\otimes_{\text{LO}} \equiv \frac{eefq^-}{\pi} \int d^2\mathbf{x}_\perp d^2\mathbf{y}_\perp e^{-i\mathbf{k}_{1\perp} \cdot \mathbf{x}_\perp} e^{-i\mathbf{k}_{2\perp} \cdot \mathbf{y}_\perp}$$

$$X_{q\bar{q}}^2 = z_q z_{\bar{q}} r_{xy}^2 \quad \text{effective dipole size}$$

$$\mathcal{M}_{\text{LO}}^{L,\sigma\sigma'} = \left[1 - V(\mathbf{x}_\perp) V^\dagger(\mathbf{y}_\perp) \right] \otimes_{\text{LO}} 2(z_q z_{\bar{q}})^{3/2} Q K_0(Q X_{q\bar{q}}) \delta^{\sigma, -\sigma'}$$

non-perturbative perturbatively computable

Dressed vertex amplitude



$$\otimes_{\text{V}} \equiv \frac{eefq^-}{\pi} \int d^2\mathbf{x}_\perp d^2\mathbf{y}_\perp d^2\mathbf{z}_\perp e^{-i\mathbf{k}_{1\perp} \cdot \mathbf{x}_\perp} e^{-i\mathbf{k}_{2\perp} \cdot \mathbf{y}_\perp}$$

$$X_{q\bar{q}g}^2 = (z_q - z_g) z_{\bar{q}} r_{xy}^2 + (z_q - z_g) z_g r_{xz}^2 + z_g z_{\bar{q}} r_{zy}^2 \quad \text{effective dipole size}$$

$$\mathcal{M}_{\text{V1}}^{L,\sigma\sigma'} = \left[C_F \mathbb{1} - t^a V(\mathbf{x}_\perp) t^b V^\dagger(\mathbf{y}_\perp) U_{ab}(\mathbf{z}_\perp) \right] \otimes_{\text{V}} \frac{\alpha_s}{\pi^2} \int_{z_0}^{z_q} \frac{dz_g}{z_g} \frac{\mathbf{r}_{zx} \cdot \mathbf{r}_{zy}}{r_{zx}^2 r_{zy}^2} \left[\left(1 - \frac{z_g}{z_q}\right) \left(1 + \frac{z_g}{z_{\bar{q}}}\right) \left(1 - \frac{z_g}{2z_q} - \frac{z_g}{2(z_{\bar{q}} + z_g)}\right) + \dots \right] e^{-i\frac{z_g}{z_q} \mathbf{k}_\perp \cdot \mathbf{r}_{zx}}$$

non-perturbative perturbatively computable