A statistical approach to the study of blazar jet emission

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CONTENTS

AIM

Up to now blazars SED obtained by heuristic approach

→ NON RIGOROUS, BIASED

AIM

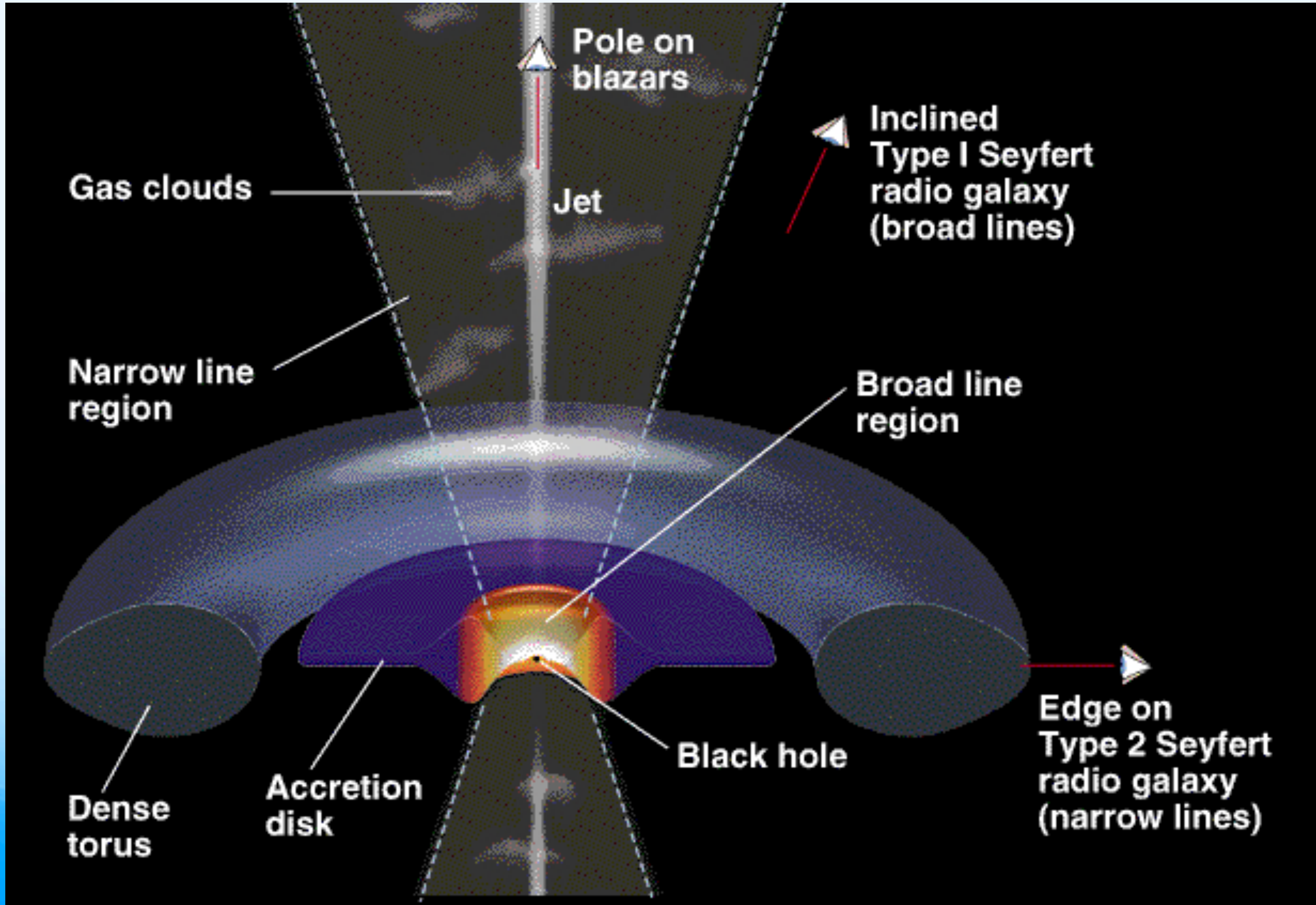
Provide statistical analysis to estimate most likely emission model parameters:

→ Non-linear least-square minimization

→ Goodness-of-fit test

ACTIVE GALACTIC NUCLEI

Morphology of an AGN



ACTIVE GALACTIC NUCLEI

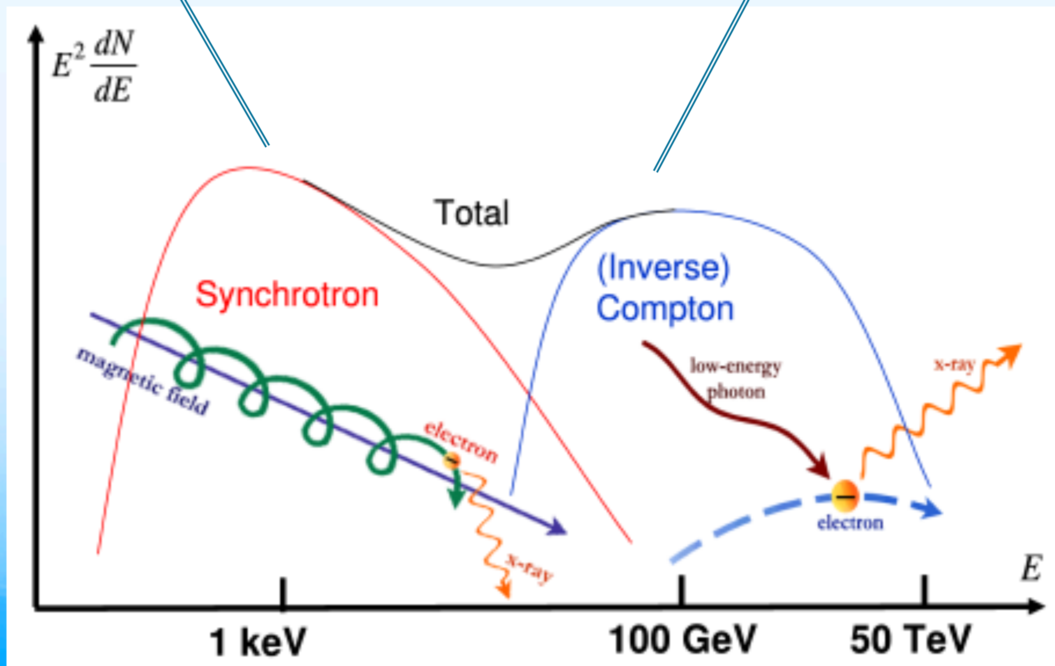
Spectral Energy Distribution of AGNs

$$j_s \propto nB^{\left(\frac{q+1}{2}\right)}$$

$$j_{CI} \propto n_e^2 B^{\left(\frac{q+1}{2}\right)}$$

Synchrotron emissivity

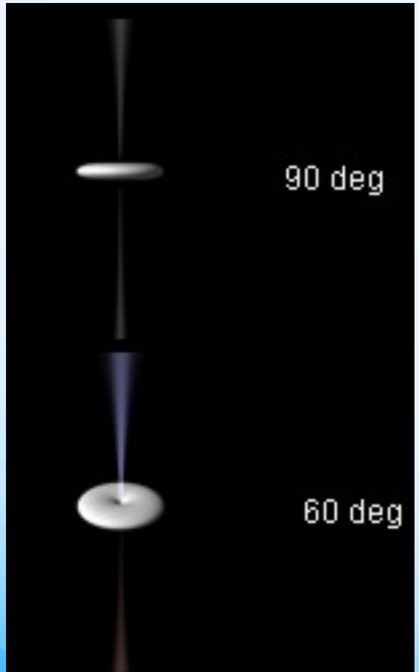
Inverse Compton emissivity



ACTIVE GALACTIC NUCLEI

Unified model of AGNs

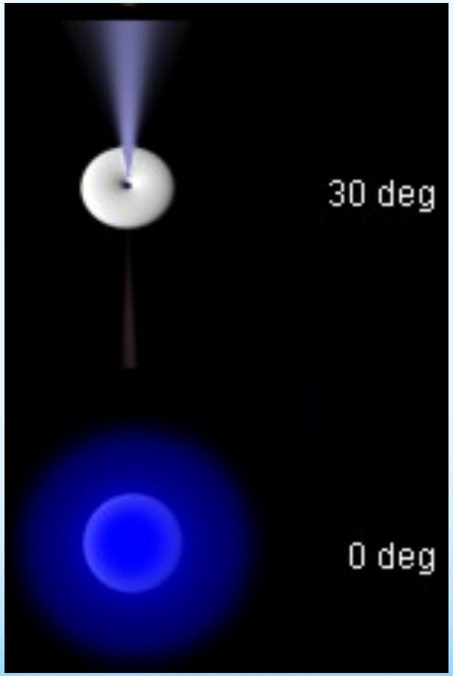
AGN Angle



SEYFERT II and RADIO GALAXIES

SEYFERT I and QUASARS

AGN Angle



SEYFERT I and QUASARS

BLAZARS

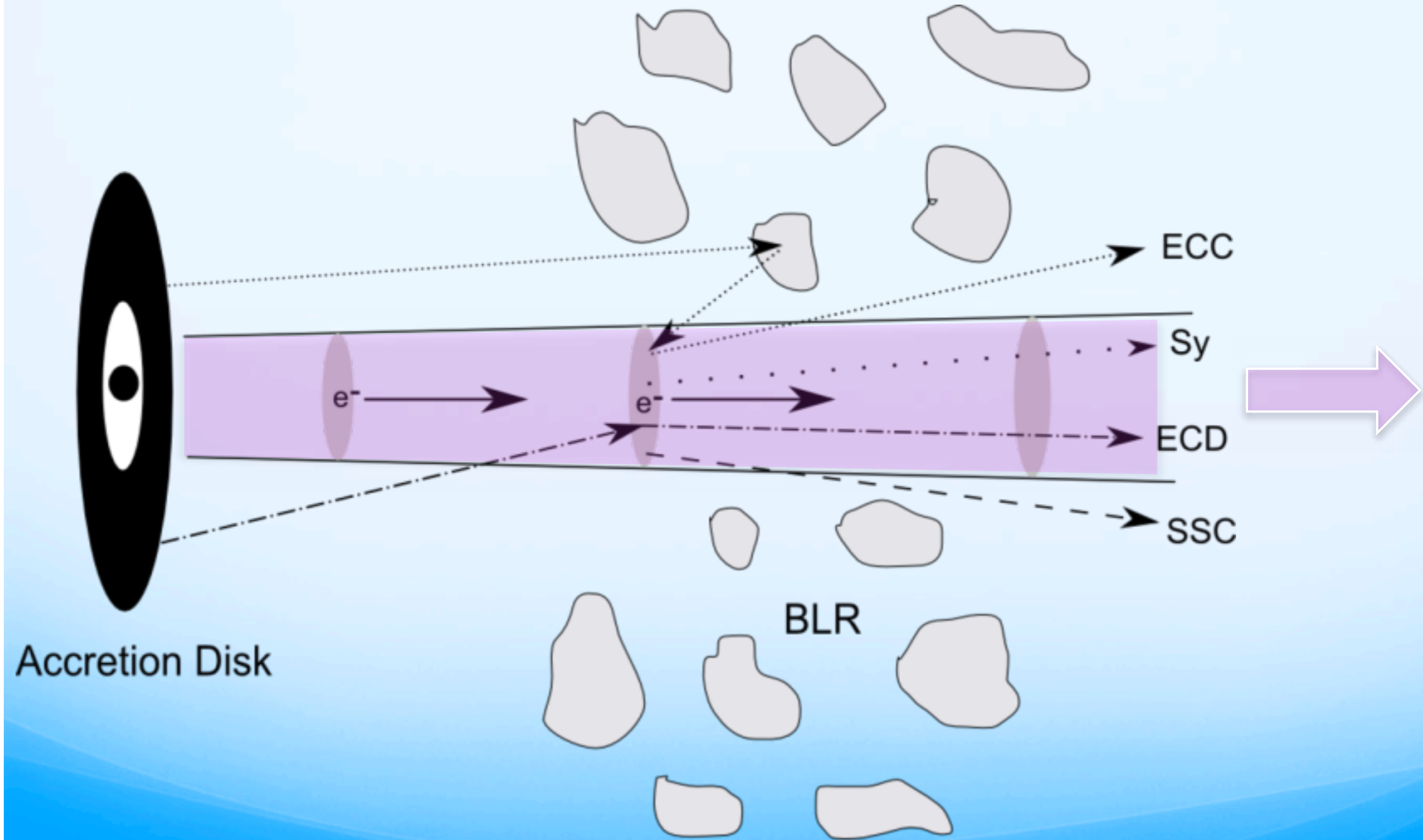
EMISSION MODELS

LEPTONIC EMISSION MODELS

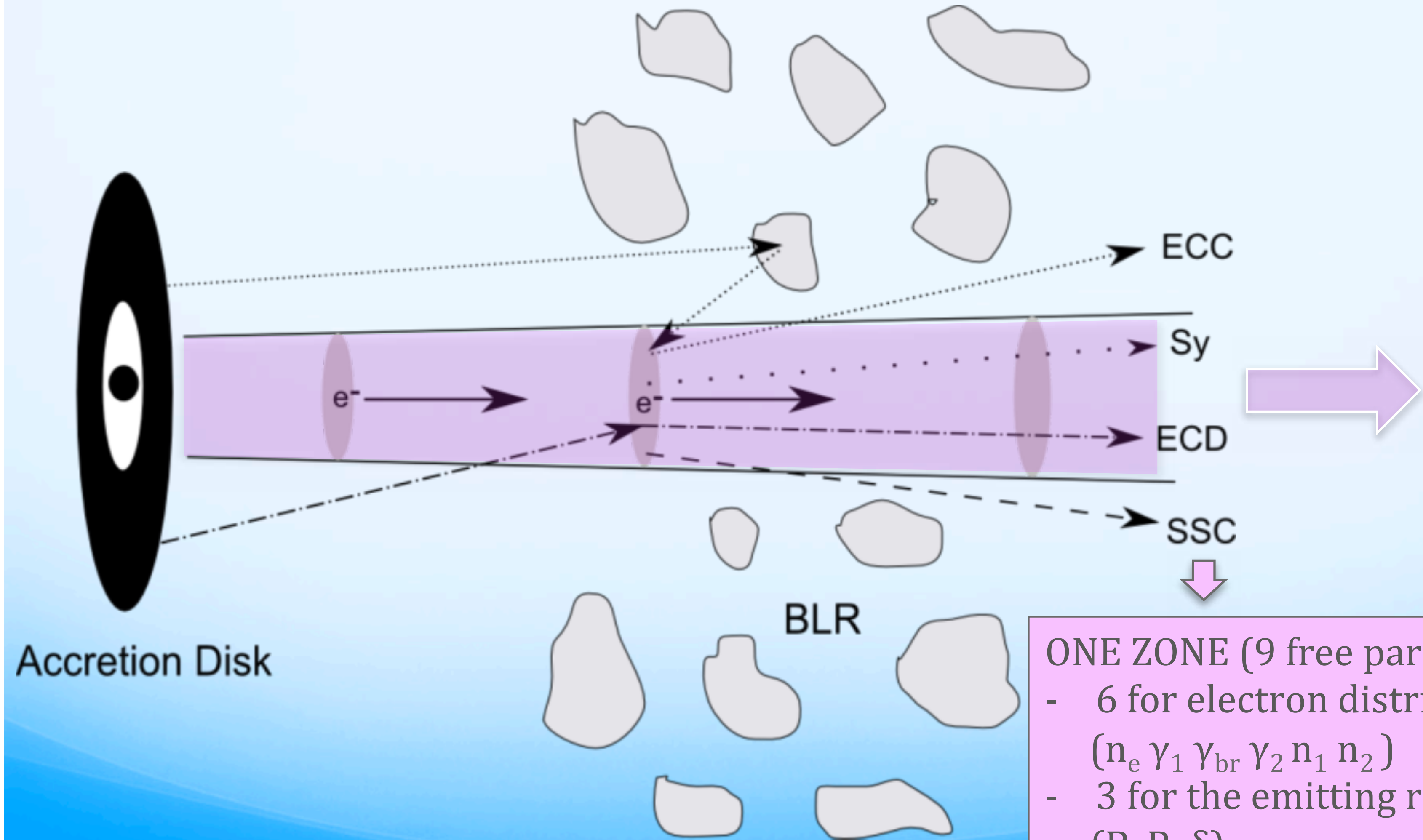
- Synchrotron Self Compton model (SSC):
  - One zone model
  - Two zone model

- External Compton model (EC):
  - External Compton from Clouds (ECC)
  - External Compton from Disk   (ECD)

EMISSION MODELS



EMISSION MODELS



ONE ZONE (9 free parameters):
- 6 for electron distribution ($n_e\ \gamma_1\ \gamma_{br}\ \gamma_2\ n_1\ n_2$)
- 3 for the emitting region (B, R, δ)

MODEL FITTING

NON LINEAR LEAST-SQAURE MINIMIZATION

$$\chi^2(\mathbf{p}) = \frac{1}{2}\sum_{i}^{1,N}\left[\frac{y_i - f(x_i;\mathbf{p})}{\sigma_i}\right]^2$$

$\mathbf{p}$ = SED model parameters
(x_i, y_i) = observational data (freq, flux)
$f(x_i, \mathbf{p})$ = SED evaluation for observed frequencies
σ_i = data uncertaintes

Mimimize χ^2 function ➔ Best values of parameters

The minimization process is performed numerically, finding a perturbation δp_j of the parameters p_j that gives a lower χ^2 value.

MODEL FITTING

LEVENBERG-MARQUARDT METHOD

1. STEEPEST DESCENT METHOD

$$\mathbf{grad}_{\mathbf{p}}\chi^2(\mathbf{p}) = -(\mathbf{y}-\mathbf{f})^T\Sigma\mathbf{J}$$

$$\mathbf{p} \rightarrow \mathbf{p} + \delta\mathbf{p}$$

$$\delta\mathbf{p} = \mu\mathbf{J}^T(\mathbf{y}-\mathbf{f})$$

2. INVERSE HEISSIAN METHOD

$$\mathbf{grad}_{\mathbf{p}}\chi^2(\mathbf{p}) = -(\mathbf{y}-\mathbf{f})^T\Sigma\mathbf{J}$$

$$\mathbf{p} \rightarrow \mathbf{p} + \delta\mathbf{p}$$

$$\delta\mathbf{p} = \mathbf{H}^{-1}\mathbf{J}^T(\mathbf{y}-\mathbf{f})$$

Combination of 2 methods:

$$\mathbf{p} \rightarrow \mathbf{p} + \delta\mathbf{p}$$

$$\delta\mathbf{p} = (\mathbf{H}+\lambda\mathrm{I})\mathbf{J}^T\Sigma(\mathbf{y}-\mathbf{f})$$

STATE OF ART

Mkn 421 data sets

- 9 data sets corresponding to different emission states.
- Simultaneous data from multi-frequency campaign (optical-Very High Energy).

STATE	DATE	ENERGY BAND	INSTRUMENTS	SOURCE ACTIVITY
1	April 2006	Otical/UV/X-rays VHE	XMM, EPIC detector Whipple	Flare decay
2	April 2006	Otical/UV/X-rays VHE	XMM, EPIC detector Whipple	Flare decay
3	Dec2002 - Jan2003	Optical X-rays VHE	Boltwood, KVA, WIYN RXTE Whipple, HEGRA-CT1	Flaring state
4	March-May 2003	Optical X-rays VHE	Whipple, Boltwood RXTE Whipple	Medium state

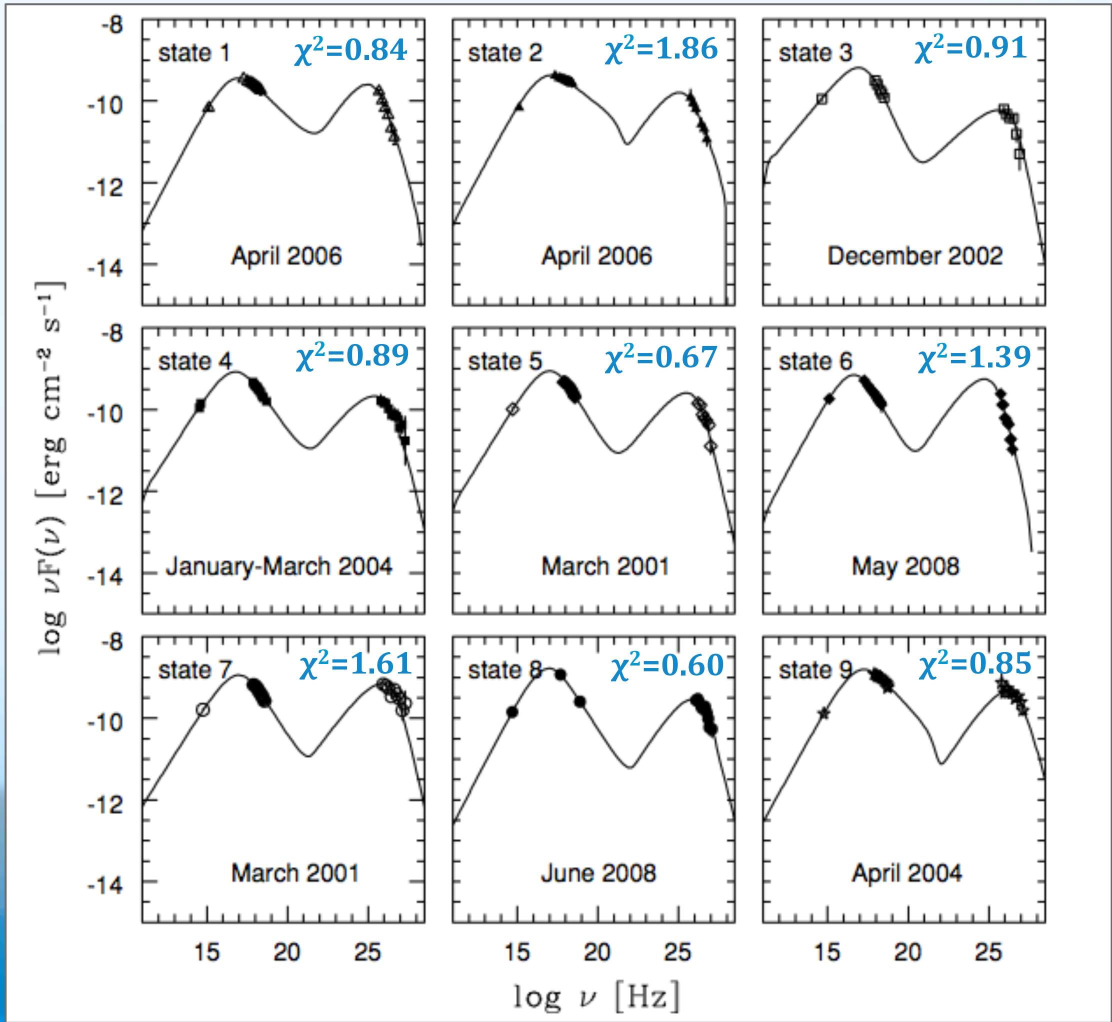
STATE OF ART (... Mkn 421)

STATE	DATE	ENERGY BAND	INSTRUMENTS	SOURCE ACTIVITY
5	March 2001	Optical X-rays VHE	Hopkins, Harvard-Smithsonian telescope RXTE Whipple	Post flare state
6	May 2008	Otical/X-rays VHE	XMM, EPIC detector VERITAS	Flare decay
7	March 2001	Optical X-rays VHE	RXTE Whipple	Flaring state
8	June 2008	Optical X-rays VHE	WEBT RXTE, Swift/BAT VERITAS	
9	April 2004	Optical X-rays VHE	Whipple, Boltwood RXTE Whipple	High state

STATE OF ART (… Mkn 421)

Mkn 421

1-zone SSC best-fit SEDs



(Mankuzhiyil et al. 2011)

STATE OF ART (… Mkn 421)

Mkn 421

1-zone SSC parameters

Source	B (G)	R (cm)	δ	χ^2_ν
State 1	$(9 \pm 3) \times 10^{-1}$	$(9 \pm 4) \times 10^{14}$	$(2 \pm 0.5) \times 10^{1}$	0.84
State 2	$(8 \pm 6) \times 10^{-1}$	$(8 \pm 4) \times 10^{14}$	$(2.7 \pm 1.1) \times 10^{1}$	1.86
State 3	$(6 \pm 6) \times 10^{-2}$	$(2.0 \pm 1.5) \times 10^{15}$	$(1.0 \pm 0.5) \times 10^{2}$	0.91
State 4	$(1.21 \pm 0.16) \times 10^{-1}$	$(1.1 \pm 1.3) \times 10^{15}$	$(8 \pm 6) \times 10^{1}$	0.89
State 5	$(1.9 \pm 1.3) \times 10^{-1}$	$(10 \pm 4) \times 10^{14}$	$(7 \pm 5) \times 10^{1}$	0.67
State 6	1.0 ± 0.7	$(6 \pm 3) \times 10^{14}$	$(2.8 \pm 1.1) \times 10^{1}$	1.39
State 7	$(4 \pm 3) \times 10^{-2}$	$(2 \pm 5) \times 10^{15}$	$(8 \pm 7) \times 10^{1}$	1.61
State 8	$(6 \pm 3) \times 10^{-2}$	$(2 \pm 1.8) \times 10^{15}$	$(1.1 \pm 0.4) \times 10^{2}$	0.60
State 9	$(4 \pm 3) \times 10^{-2}$	$(2 \pm 4) \times 10^{15}$	$(1.2 \pm 1.0) \times 10^{2}$	0.85

(Mankuzhiyil et al. 2011)

STATE OF ART (... Mkn 421)

Mkn 421

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(Mankuzhiyil et al. 2011)

STATE OF ART (… Mkn 421)

Mkn 421

1-zone SSC parameters

Source	
State 1	
State 2	
State 3	
State 4	(1.21
State 5	(1.
State 6	
State 7	
State 8	
State 9	

Source	n_e (cm^{-3})	γ_{br}
State 1	$(1.3 \pm 1.5) \times 10^3$	$(2.6 \pm 0.9) \times 10^4$
State 2	$(1 \pm 3) \times 10^3$	$(2.4 \pm 0.9) \times 10^4$
State 3	$(5 \pm 5) \times 10^3$	$(7 \pm 3) \times 10^4$
State 4	$(2 \pm 5) \times 10^3$	$(4 \pm 2) \times 10^4$
State 5	$(2 \pm 5) \times 10^3$	$(4.5 \pm 1.9) \times 10^4$
State 6	$(4 \pm 4) \times 10^3$	$(1.9 \pm 0.6) \times 10^4$
State 7	$(1 \pm 7) \times 10^3$	$(8 \pm 6) \times 10^4$
State 8	$(4 \pm 9) \times 10^1$	$(5 \pm 2) \times 10^4$
State 9	$(1 \pm 7) \times 10^2$	$(8 \pm 9) \times 10^4$

(Mankuzhiyil et al. 2011)

STATE OF ART (… Mkn 421)

Mkn 421

1-zone SSC parameters

Source	
State 1	
State 2	
State 3	
State 4	(1.21
State 5	(1.
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State 7	
State 8	
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(Mankuzhiyil et al. 2011)

STATE OF ART (… Mkn 421)

PROBLEM 1:
LARGE PARAMETERS UNCERTAINTES

→ minimum shape approximated by a quadratic function: no good approximation

PROBLEM 2:
KOLMOGOROV-SMIRNOV TEST FAILS AT 5% SIGNIFICANCE LEVEL

→ two distinct physical processes that manifest themselves at very different energies

→ The uncertaintes associated with the VHE data are much larger than those associated with optical and X-rays ones.

STATE OF ART

Mkn 501 data sets

- 8 data sets corresponding to different emission states.
- Simultaneous data from multi-frequency campaign (optical-Very High Energy).

STATE	DATE	ENERGY BAND	INSTRUMENTS	SOURCE ACTIVITY
1	Mar 2009	X-rays HE VHE	Suzaku Fermi LAT MAGIC, VERITAS	Quiescent state
2	Mar – Apr 2009	X-rays HE VHE	Swift, RXTE Fermi LAT MAGIC, VERITAS	Quiescent state
3	Jun 2006	Optical X-rays VHE	KVA Suzaku MAGIC	Quiescent state
4	Jun 1998	X-rays VHE	RXTE HEGRA	

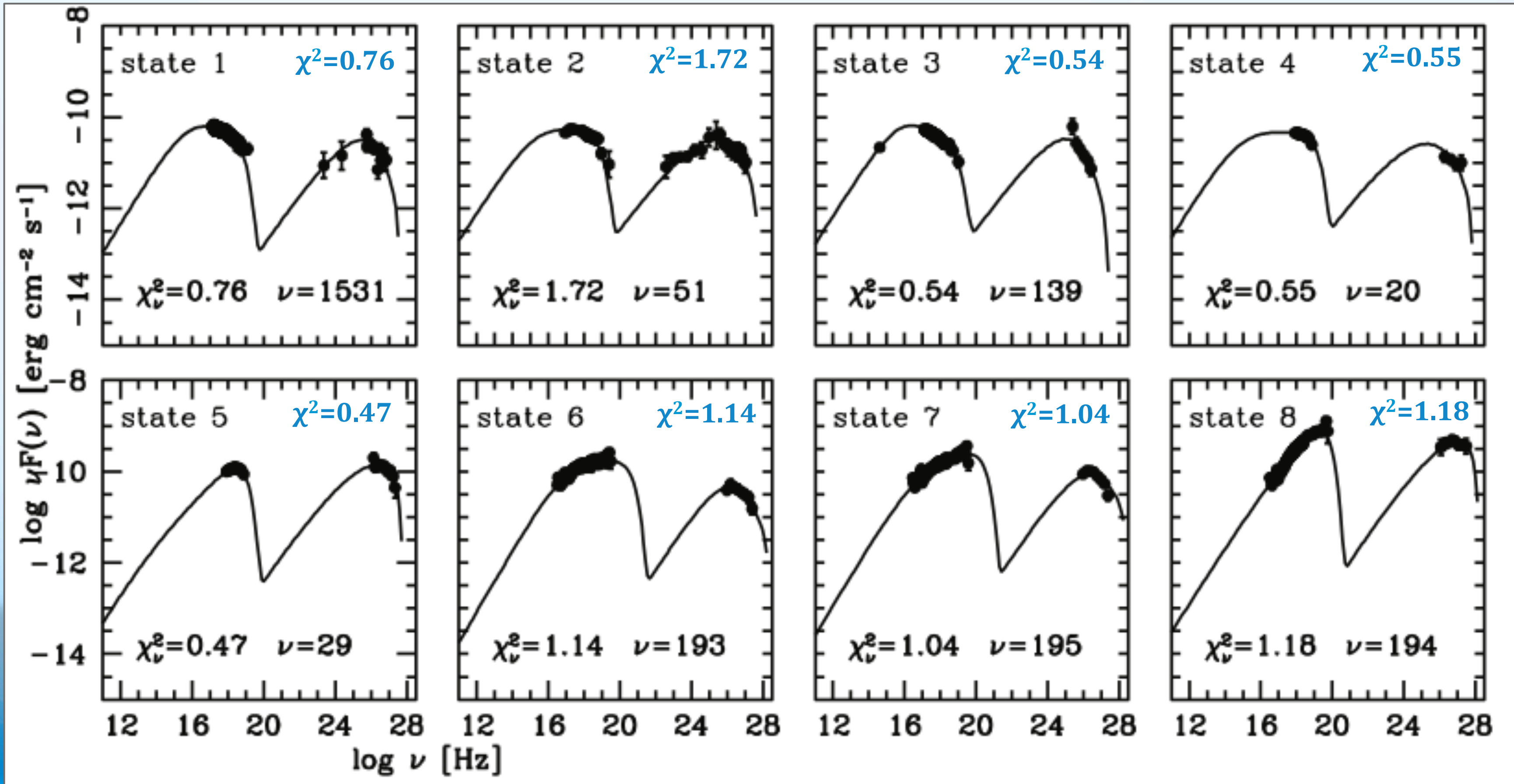
STATE OF ART (… Mkn 501)

STATE	DATE	ENERGY BAND	INSTRUMENTS	SOURCE ACTIVITY
5	Jun 1998	X-rays VHE	RXTE HEGRA	High state
6	Apr 1997	X-rays VHE	Beppo Sax CAT	Low state
7	Apr 1997	X-rays VHE	Beppo Sax CAT	Medium state
8	Apr 1997	X-rays VHE	Beppo Sax CAT	Giant flaring state

STATE OF ART (… Mkn 501)

Mkn 421

1-zone SSC best-fit SEDs



(Mankuzhiyil et al. 2012)

STATE OF ART
(…Mkn501)

	State 1	State 2	State 3	State 4
Date	2009 Mar	2009 Mar–Aug	2006 Jul	1998 Jun 15–26
Instr.		*Swift*	KVA	
	Suzaku	*RXTE*	*Suzaku*	*RXTE*
	Fermi/LAT	*Fermi*/LAT		
	MAGIC	MAGIC	MAGIC	HEGRA
	VERITAS	VERITAS		
Ref.	[1]	[2]	[3]	[4]
Param.				
K_e	$68.9^{0.2}_{0.2}$	234^{2}_{2}	$253.8^{1.7}_{1.7}$	$152.8^{1.1}_{1.1}$
γ_{min}	1	1	1	1
γ_{br}	$8.024^{0.012}_{0.012}$	$4.88^{0.03}_{0.03}$	$5.60^{0.02}_{0.02}$	$2.67^{0.02}_{0.02}$
γ_{max}	$1.86^{0.04}_{0.03}$	$2.12^{0.06}_{0.05}$	$1.61^{0.2}_{0.15}$	$2.67^{0.08}_{0.07}$
n_1	$1.727^{0.000}_{0.000}$	$1.792^{0.000}_{0.000}$	$1.779^{0.000}_{0.000}$	$1.734^{0.000}_{0.000}$
n_2	$3.376^{0.002}_{0.002}$	$3.156^{0.003}_{0.003}$	$3.610^{0.004}_{0.004}$	$3.048^{0.002}_{0.002}$
B	$2.098^{0.002}_{0.002}$	$1.530^{0.006}_{0.006}$	$5.58^{0.02}_{0.02}$	$1.513^{0.005}_{0.005}$
R	$2.332^{0.002}_{0.002}$	$1.909^{0.006}_{0.006}$	$1.620^{0.004}_{0.004}$	$1.985^{0.005}_{0.005}$
δ	$19.128^{0.011}_{0.010}$	$24.40^{0.05}_{0.05}$	$15.12^{0.02}_{0.02}$	$25.24^{0.05}_{0.05}$
$\log L$	44.57	44.58	44.59	44.54
$\log \nu_s$	16.74	16.86	16.34	17.16
$\log \nu_{IC}$	25.36	25.46	24.88	25.34

(Mankuzhiyil et al. 2012)

STATE OF ART
(...Mkn501)

	State 5	State 6	State 7	State 8
Date	1998 Jun 27–28	1997 Apr 7	1997 Apr 11	1997 Apr 16
Instr.	*RXTE*	*BeppoSAX*	*BeppoSAX*	*BeppoSAX*
	HEGRA	CAT	CAT	CAT
Ref.	[4]	[5]	[5]	[5]
Param.				
K_e	456^{4}_{4}	$93.6^{0.8}_{0.8}$	$165.4^{1.5}_{1.3}$	465^{4}_{4}
γ_{min}	1	1	1	1
γ_{br}	$1.03^{0.02}_{0.02}$	$26.7^{0.2}_{0.2}$	$30.6^{0.3}_{0.3}$	55^{2}_{2}
γ_{max}	$3.62^{0.06}_{0.06}$	13^{500}_{5}	13^{13}_{3}	$8.8^{0.4}_{0.3}$
n_1	$1.642^{0.000}_{0.000}$	$1.652^{0.000}_{0.000}$	$1.697^{0.000}_{0.000}$	$1.728^{0.000}_{0.000}$
n_2	$2.245^{0.002}_{0.002}$	$2.997^{0.011}_{0.011}$	$2.795^{0.008}_{0.009}$	$2.110^{0.010}_{0.011}$
B	$1.126^{0.006}_{0.006}$	$3.60^{0.02}_{0.02}$	$1.799^{0.009}_{0.008}$	$1.043^{0.006}_{0.005}$
R	$1.742^{0.006}_{0.006}$	$0.925^{0.003}_{0.003}$	$1.202^{0.004}_{0.003}$	$1.627^{0.004}_{0.004}$
δ	$12.85^{0.03}_{0.03}$	$16.43^{0.04}_{0.04}$	$17.53^{0.04}_{0.04}$	$13.89^{0.03}_{0.03}$
$\log L$	44.91	44.95	45.07	45.53
$\log \nu_s$	18.39	19.30	19.51	19.27
$\log \nu_{IC}$	26.35	26.17	26.53	26.93

(Mankuzhiyil et al. 2012)

STATE OF ART (… Mkn 501)

PROBLEM:
KOLMOGOROV-SMIRNOV TEST FAILS AT 5% SIGNIFICANCE LEVEL

→ two distinct physical processes that manifest themselves at very different energies
→ The uncertaintes associated with the VHE data are much larger than those associated with optical and X-rays ones.

OUTLOOK

- TO OPTIMIZE THE CODE

  in order to reduce the computational time and to better estimate the model parameters and their uncertainties

  → i.e. better estimate of the derivative

- TO IMPLEMENT DIFFERENT MODELS

  such as the 2-zone SSC and SSC+EC for different electrons' spectrum in order to efficiently rule out some models for a given source.

- TO CARRY OUT A STATISTICAL STUDY ON ALL BLAZARS

  in order to better understand the emission region properties once the bias related to different analyses is removed.

CONCLUSIONS

- Henceforth the availability of simultaneous, multi-frequency data allows to obtain, by a rigorous statistical approach, the confidence levels for the model parameters.
- Model parameters are obtained through a non-linear least-square minimization, using the Levenberg-Marquardt method and applying the Kolmogorov-Smirnov test as a goodness of fit test.
- Mkn421 and Mkn501 SED error bars are obtained, for the first time, by a standard covariance matrix approach. The results, although preliminary, show a clear statistical meaning.
- For a given source, this new approach will allow to quantify if a specific model is more suitable than others, and eventually to improve it.

THANKS !!!