Final Exam of the 10th IDPASC Sumer School

Nazaré, 6-17 September 2021

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0.1 (2 points)
The accretion rate of dark matter in a neutron star is $C_{\rm acc}$ and the annihilation rate $-C_{\rm ann}N^2$ where N is the number of dark matter particles in the star. What is the time scale required so to have an equilibrium between accretion and annihilation (i.e., accretion rate and annihilation rate become equal)? Once we have this equilibrium what is the number of dark matter particles accumulated in terms of $C_{\rm acc}$ and $C_{\rm ann}$?

0.2 (3 points)
The surface temperature of a neutron star is related to the temperature at the core as $T_{\rm surf} = \xi T_{\rm core}^{\gamma}$. In practice $\gamma = 0.55$ but for simplicity in the problem we will assume $\gamma = 0.5$. Dark matter is accumulated and annihilated into the star. The accretion rate is $C_{\rm acc}$ and the dark matter mass m.
Assume that we observe the neutron star at a time t_1 larger than 10^6 years and therefore cooling is dominated by blackbody emission by the surface of the star and not by neutrino emission (which we ignore). Furthermore let's assume that the time scale it is needed to equate the accretion rate to the annihilation rate $\tau << t_1$. This practically means that the annihilation rate is equal to the accretion rate times the energy of the particle. Calculate how the internal temperature of the neutron star involves in time after t_1. For a large time $t_2 >> t_1$ what is the temperature? Does it depend on time? Recall that the heat capacity $C_v = \lambda T$ is linearly proportional to the internal temperature (λ is just a constant) for a non-interacting degenerate fermion gas. Recall also that the cooling equation in this case will be given by

$$\frac{dT}{dt} = \frac{-L_\gamma + L_{\rm DM}}{C_v}, \tag{1}$$

where L_γ is the photon blackbody radiation from the surface of the neutron star and $L_{\rm DM}$ the luminosity produced from the annihilation of dark matter in the star. Assume that 100% of the annihilation energy converts to heat inside the star.

0.3 (2 points)
The gluon fusion cross section for the production of a 400 GeV Higgs in model A is 196 fb. The total width for this 400 GeV Higgs is 26 GeV while the partial width for the decay to tau leptons is 2 GeV. How many $\tau^+\tau^-$ pairs will be produced for a luminosity of 300 fb^{-1}?

0.4 (3 points)
Knowing that the covariant derivatives for one family of leptons is

$$D_\mu E_L = \left(\partial_\mu + ig\frac{\tau^a}{2}W^a_\mu - i\frac{g'}{2}B_\mu\right)E_L$$

$$= \left[\partial_\mu + \frac{ig}{\sqrt{2}}\left(W^+_\mu\tau^+ + W^-_\mu\tau^-\right) + ieQA_\mu + i\frac{g}{\cos\theta_W}\left(\frac{\tau^3}{2} - \sin^2\theta_W\, Q\right)Z_\mu\right]E_L$$

$$D_\mu e_R = (\partial_\mu - ig'B_\mu)e_R = (\partial_\mu + ieQA_\mu + ie\tan\theta_W\, Z_\mu)\, e_R$$

show that the the interactions of the first lepton family with the gauge bosons is given by

$$\mathcal{L}_{\text{int}} = -\frac{g}{2\sqrt{2}}\,\overline{\nu}_e\gamma^\mu(1-\gamma_s)e\, W^+_\mu - \frac{g}{2\sqrt{2}}\,\overline{e}\gamma^\mu(1-\gamma_5)\nu_e\, W^-_\mu$$

$$-\frac{g}{4\cos\theta_W}\left[\overline{\nu}_e\,\gamma^\mu(1-\gamma_5)\nu_e - \overline{e}\gamma^\mu(1+4Q_e\sin^2\theta_W\ - \gamma_5)e\right]Z_\mu$$

$$-\,(e\,Q_e)\,\overline{e}\gamma^\mu e\, A_\mu$$

where $Q_e = -1$ for the electron.

0.5 (2 points)
A theoretical model predicts a 100 GeV dark matter particle with spin-independent scattering cross section $\sigma_p = 1\times10^{-47}$ cm^2/nucleon. The local dark matter density is measured to be $\rho_0 = 0.3$ GeV/cm^3 in the solar neighbourhood, and the mean particle speed is estimated to be $\bar{v}_\chi = \sqrt{3/2}\, v_0 = 270\ km/s$, where v_0 is the circular velocity of the Sun in the galaxy. We would like to estimate the event rate predicted for an underground detector containing 10 tonnes of active liquid xenon.

1. For a spin-independent interaction, the WIMP-nucleus scattering cross section is given by:

   $$\sigma_A^{SI} \approx \frac{\mu_A^2}{\mu_p^2}\sigma_p A^2$$

   where $\mu_{A,p}$ are the WIMP-nucleus and WIMP-proton reduced masses, respectively, and A is the mass number of the target ($\langle A\rangle = 131.3$ for xenon). Calculate the scattering cross section on Xe nuclei.

2. Calculate the particle flux ϕ_χ at the detector (particles per unit time per unit area).

3. Hence estimate the scattering rate in the 10-tonne liquid xenon target for 1,000 days of operation.

4. The circular velocity of the Sun around the centre of the Milky Way is $v_0 \sim$ 220 km/s, and the *rms* velocity of dark matter particles in the galaxy is comparable, $\bar{v}_\chi \sim 270$ km/s. Is this a coincidence?

0.6 (3 points)
The coherent elastic scattering of astrophysical neutrinos with nuclei (CEνNS) will eventually pose an irreducible background in dark matter detectors searching for WIMPs. In particular, the solar boron-8 neutrino spectrum (maximum energy 14 MeV) creates a nuclear recoil spectrum very similar to that from light WIMPs.

1. What is meant by coherent nuclear scattering? Confirm that the scattering of 8B neutrinos on xenon nuclei ($M \approx 122$ GeV/c^2,$R_N \sim 5$ fm) is indeed coherent.
2. Estimate the maximum energy $E_{(r,max)}$ for xenon recoils from 8B interactions (see formulas below).
3. Now consider the most energetic WIMPs trapped in our galaxy (circular velocity $v_0 = 220$ km/s, escape velocity $v_{esc} = 544$ km/s). Calculate the WIMP mass that can generate the same maximum energy as the ^{8}B neutrino flux – and is hence affected by this background. Make sensible approximations to expedite the maths.
4. The average energy required to generate an ionisation electron or a scintillation photon in liquid xenon is 13.7 eV. Discuss briefly the detectability of 8B CEνNS events in this medium.

Recoil energy generated at lab angle α by a projectile with kinetic energy E on a target with mass M:
For a massless projectile:

$$E_r = \frac{2E^2}{Mc^2 + 2E}\cos^2\alpha$$

For a non-relativistic projectile with mass m:

$$E_r = E\frac{4mM}{(m+M)^2}\cos^2\alpha$$

0.7 (2 points)
Imagine that you plan to build an experiment to observe astrophysical gamma-rays with TeV energies. You would like/need to distinguish the photon induced from hadronically induced showers. What measurable shower caracteristic(s) (observable(s)) would you use? On average, is this quantity bigger for proton- or iron-induced showers?

0.8 (3 points)
The transparency of the Universe to photons depends on their energy. In fact, its mean free path diminishes drastically whenever it is possible to open an inelastic channel of the interaction between the travelling radiation and the Cosmic Microwave Background. For photons, the dominant process is the pair electron-positron production. What is the minimum energy that the photon must have so that the process:

$$\gamma + \gamma_{\text{CMB}} \to e^+ + e^-$$

is possible? Consider the average energy of the CMB as $\langle E_{\text{CMB}}\rangle \sim 1\,\text{meV}$.