Astroparticle Physics

Exercises + Solutions

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LIP

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Production of Cosmic Rays

1. In the Fermi acceleration mechanism, charged particles increase considerably their energies crossing back and forth many times the border of a magnetic cloud (second-order Fermi mechanism) or of a shock wave (first-order Fermi mechanism). Compute the number of crossings that a particle must do in each of the mechanisms to gain a factor 10 on its initial energy assuming:

a. $\beta = 10^{-4}$ for the magnetic cloud and $\beta = 10^{-2}$ for the shock wave;

b. $\beta = 10^{-4}$ for both acceleration mechanisms.

where β is the velocity of the astrophysical object (shock wave or cloud).

Production of Cosmic Rays

1.

$$E = E_0(1+\varepsilon)^n. \qquad n = \frac{\ln(E/E_0)}{\ln(1+\varepsilon)}.$$

a.

$$n(\varepsilon \propto \beta;\, \beta = 10^{-2}) \simeq 2.3 \times 10^2 \text{ cycles}$$
$$n(\varepsilon \propto \beta^2;\, \beta = 10^{-4}) \simeq 2.3 \times 10^8 \text{ cycles.}$$

b.

$$n(\varepsilon \propto \beta;\, \beta = 10^{-4}) \simeq 2.3 \times 10^4 \text{ cycles}$$
$$n(\varepsilon \propto \beta^2;\, \beta = 10^{-4}) \simeq 2.3 \times 10^8 \text{ cycles.}$$

The GZK effect

2. The Cosmic Microwave Background fills the Universe with photons with a peak energy of 0.37 meV and a number density of n ~ 400/cm^3.

a. Determine the minimal energy (known as the GZK threshold) that a proton should have in order that the reaction $p\gamma \rightarrow \Delta$ may occur.

b. The GZK threshold is 6×10^{19} eV. Discuss the probable origin of the discrepancy found in the previous question.

The GZK effect

2.

a. In order that the reaction $p\,\gamma \rightarrow \Delta^+$ may occur the center-of-mass energy should be greater than the mass of the Δ particle:

$$\left(p_p + p_{\gamma CMB}\right)^2 \geq m_\Delta^2,$$

$$E_p \;\geq\; \frac{m_\Delta^2 - m_{\mathrm{p}}^2}{2\,E_{\gamma CMB}\;(1-\beta\cos\theta)}.$$

Assuming head-on collisions,

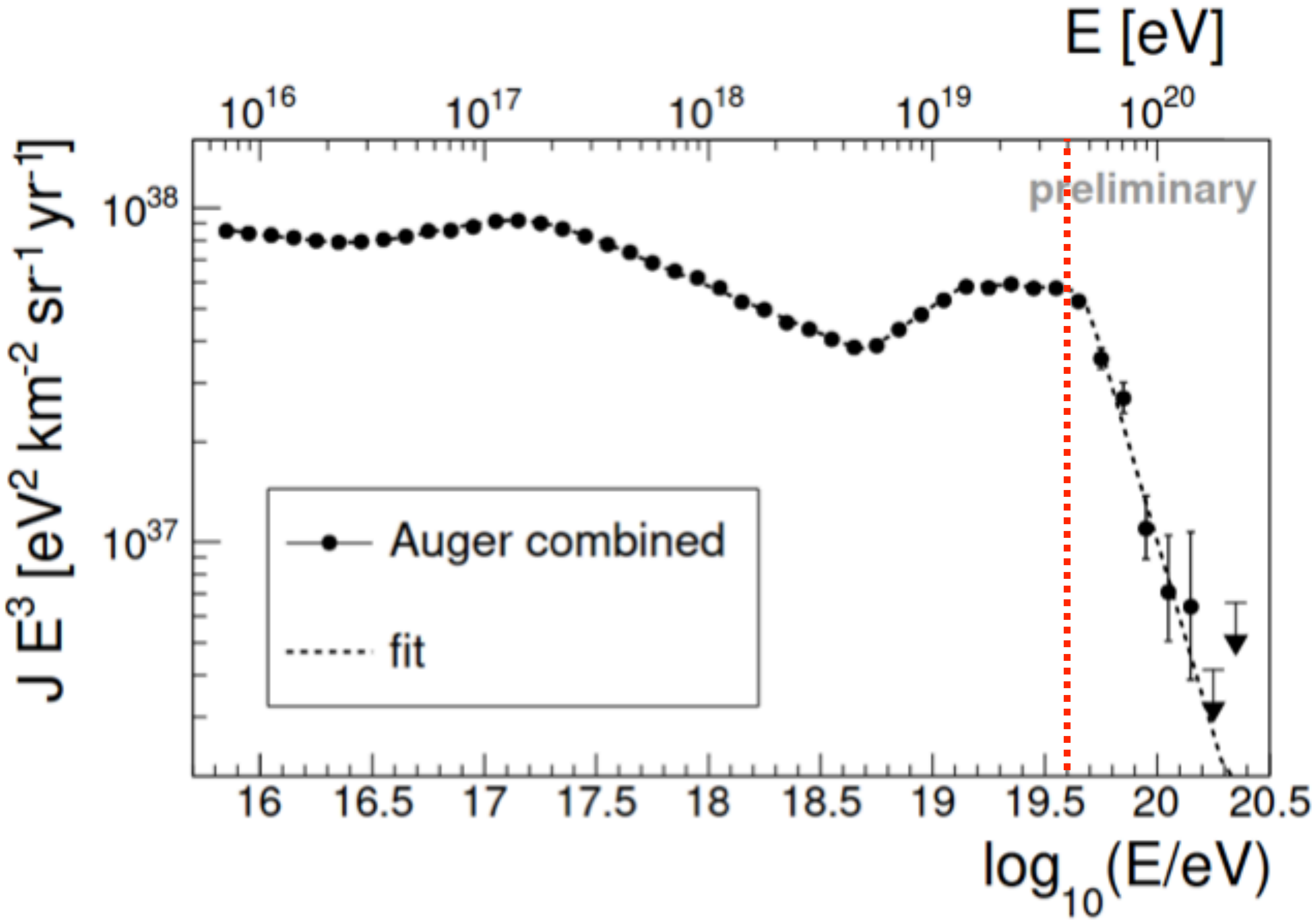
$$E_p \;\gtrsim\; \frac{m_\Delta^2 - m_{\mathrm{p}}^2}{4\,E_{\gamma CMB}} \simeq\; 3.7\times 10^{20}\mathrm{eV}.$$

The GZK effect

2.

$$E_p \gtrsim 3.7 \times 10^{20}\,\mathrm{eV} \quad \rightarrow \quad \log(E_p/\mathrm{eV}) \gtrsim 20.6$$

b.



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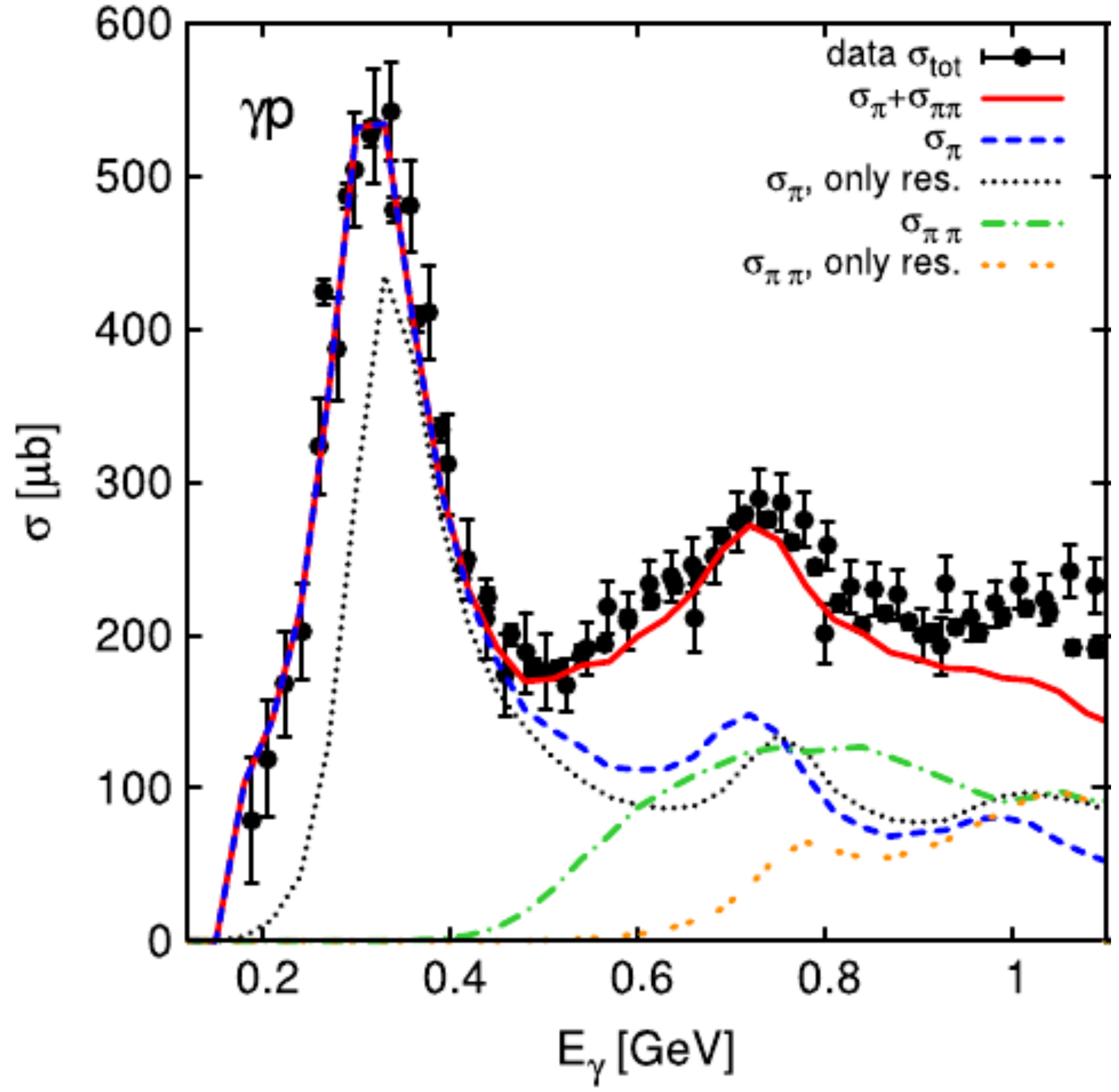
The GZK effect

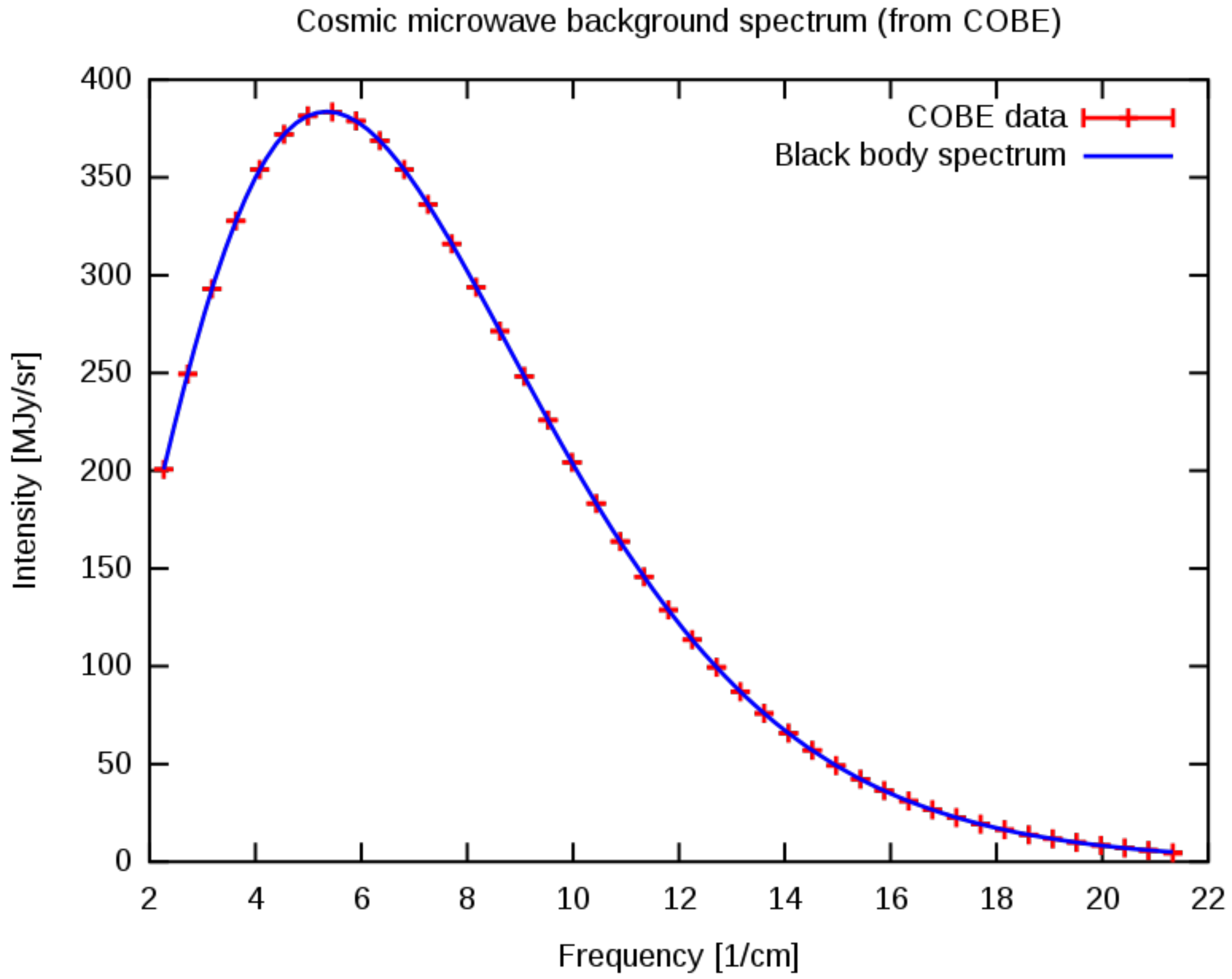
2.

b.



γp cross-section

(Photo-pion production starts before the Δ resonance)

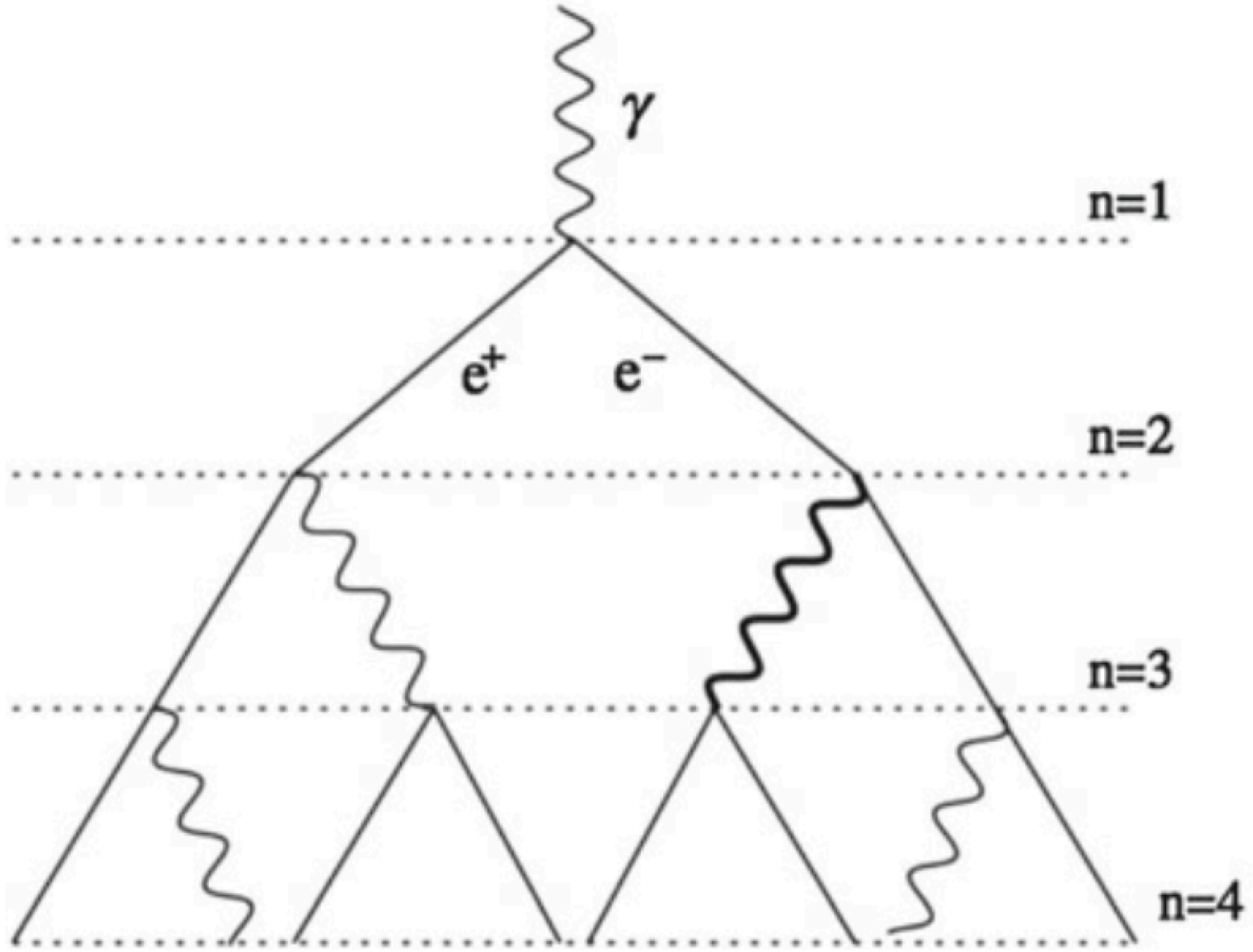


CMB spectrum

(The mean value is higher than the peak position)

The Heitler model for EAS

3. The main characteristic of an electromagnetic shower (say, initiated by a photon) can be obtained using a simple Heitler model. Let E_0 be the energy of the primary particle and consider that the electrons, positrons and photons in the cascade always interact after traveling a certain atmospheric depth $d = X_0$, and that the energy is always equally shared between the two particles. With this assumptions, we can schematically represent the cascade as in the shown in the figure.



a. Write the analytical expressions for the number of particles and for the energy of each particle at depth X as a function of d, n and E_0.

b. The multiplication of the cascade stop when the particles reach a critical energy, Ec (when the decay probability surpasses the interaction proba- bility). Using the expressions obtained in the previous question, write as a function of E_0, E_c and $\lambda = d/\ln 2$, the expressions, at the shower maximum, for:

- **i.** The average energy of the particles;
- **ii.** The number of particles, N_{max};
- **iii.** The atmospheric depth, X_{max}.

Heitler model for EAS

3.

a.

At the n-th generation,

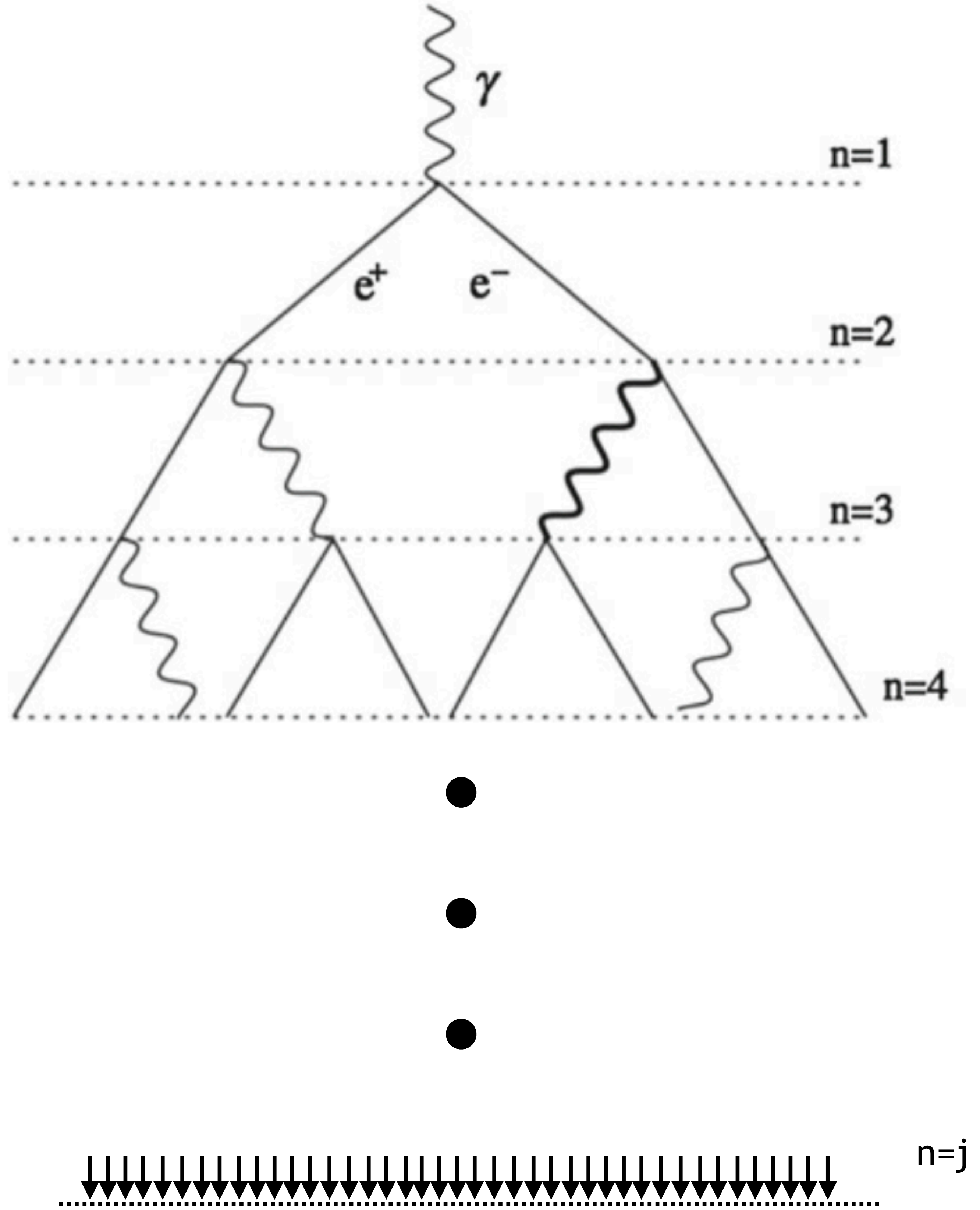
$$X = n \times d$$

and the number of produced particles is simply

$$N = 2^n \, .$$

As the energies of the particles are the same at the end of each generation, the energy of each particle is equal is the primary energy divided by the number of particles at this level, i.e.,

$$E_i = \frac{E_0}{2^n} \, .$$



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Heitler model for EAS

3.

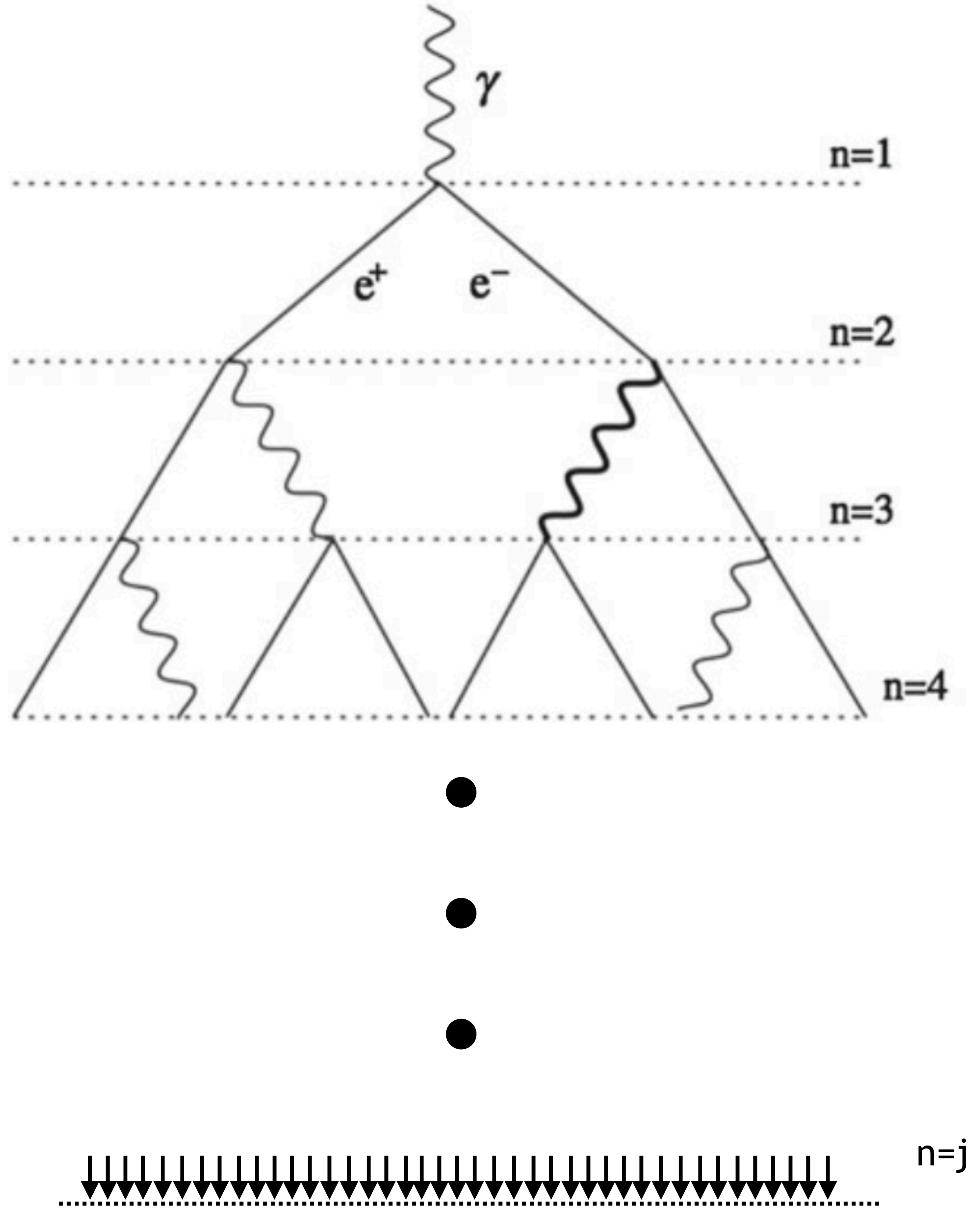
b.

i.

$$E = E_c$$

ii.

$$N_{\max} = \frac{E_0}{E_c}$$



Heitler model for EAS

3.

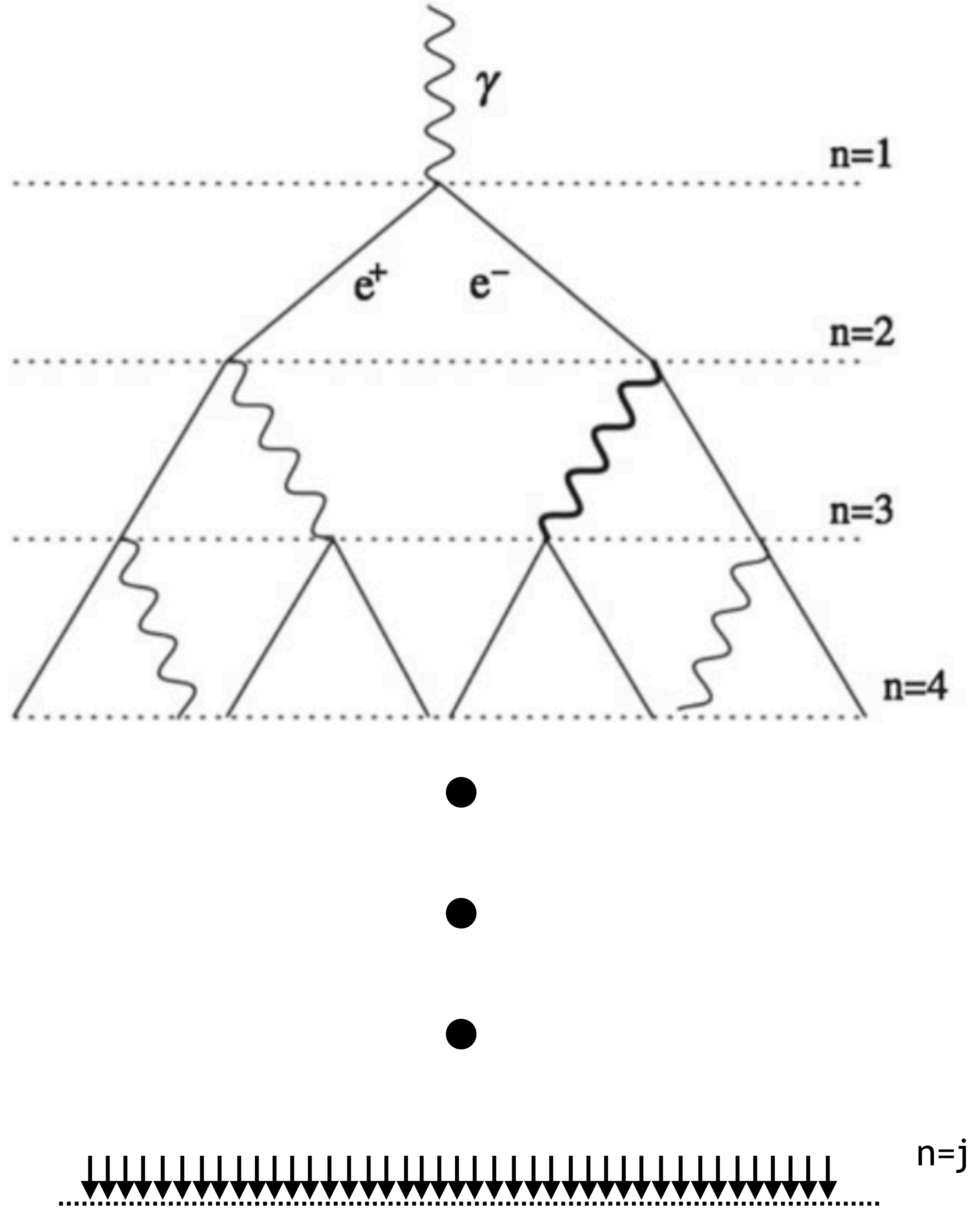
b.

iii.

$$N_{\max} = 2^{n_{\max}} \Leftrightarrow n_{\max} = \frac{\ln(N_{\max})}{\ln 2}$$

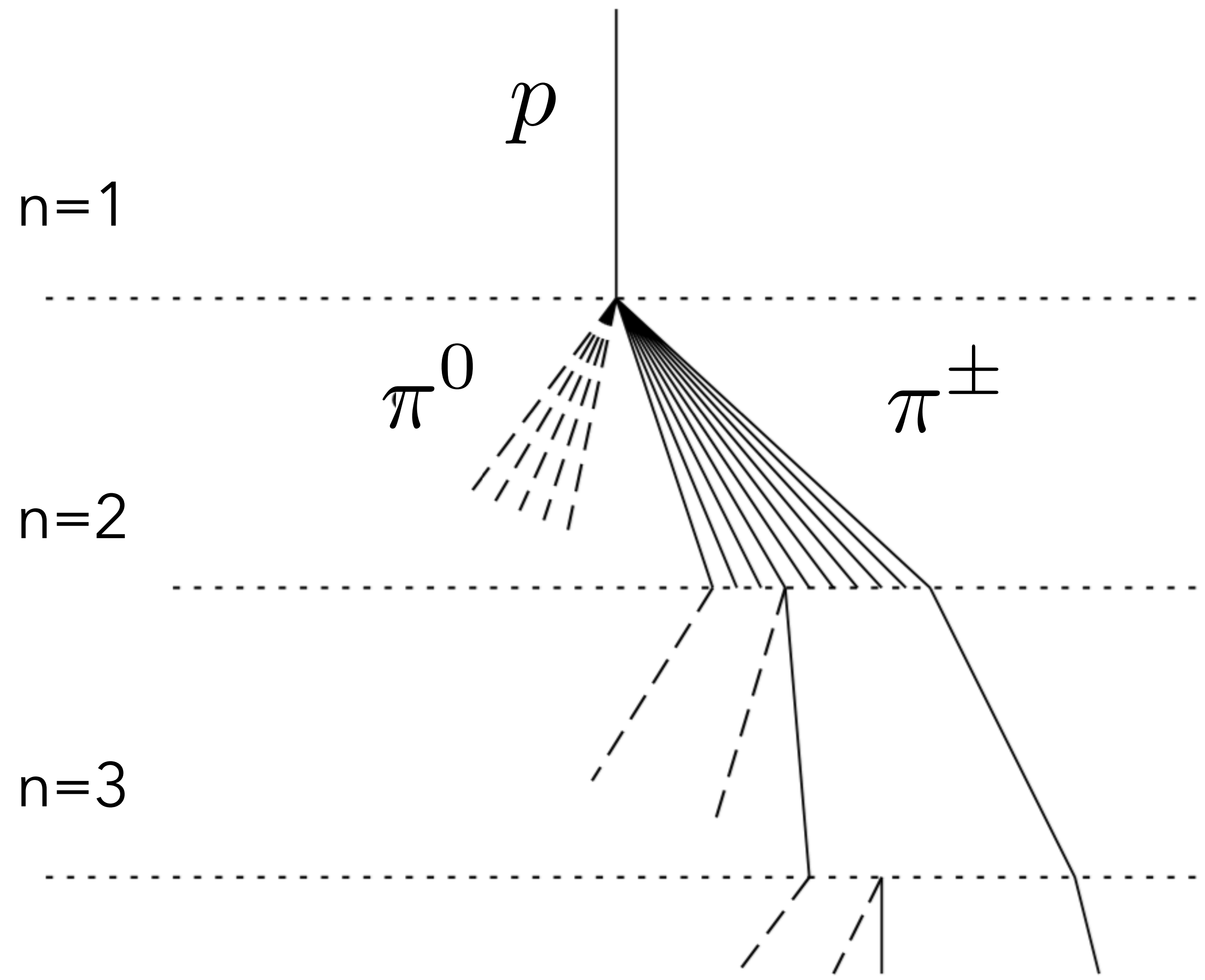
where $n_{\max}$ is the maximum number of levels.
Since $d = \lambda \ln 2$ the maximum atmospheric depth can be written as

$$X_{\max} = n_{\max} \times d = \frac{\ln(N_{\max})}{\ln 2} d = \lambda \ln\left(\frac{E_0}{E_c}\right)$$



Heitler-Matthews model for EAS

4. Consider a shower initiated by a proton of energy E0. We will describe it with a simple Heitler-like model: after each depth d an equal number of pions, n_π , and each of the 3 types is produced: π^0, π^+, π^-. Neutral pions decay through $\pi^0 \rightarrow \gamma\gamma$ and their energy is transferred to the electromagnetic cascade. Only the charged pions will feed the hadronic cascade. We consider that the cascade ends when these particles decay as they reach a given decay energy E_{dec}, after n interactions, originating a muon (plus an undetected neutrino).



Assuming the validity of the superposition principle, according to which a nucleus of mass number A and energy E_0 behaves like A nucleons of energy E_0/A, derive expressions for:

a. the depth where this maximum is reached, X_{max};

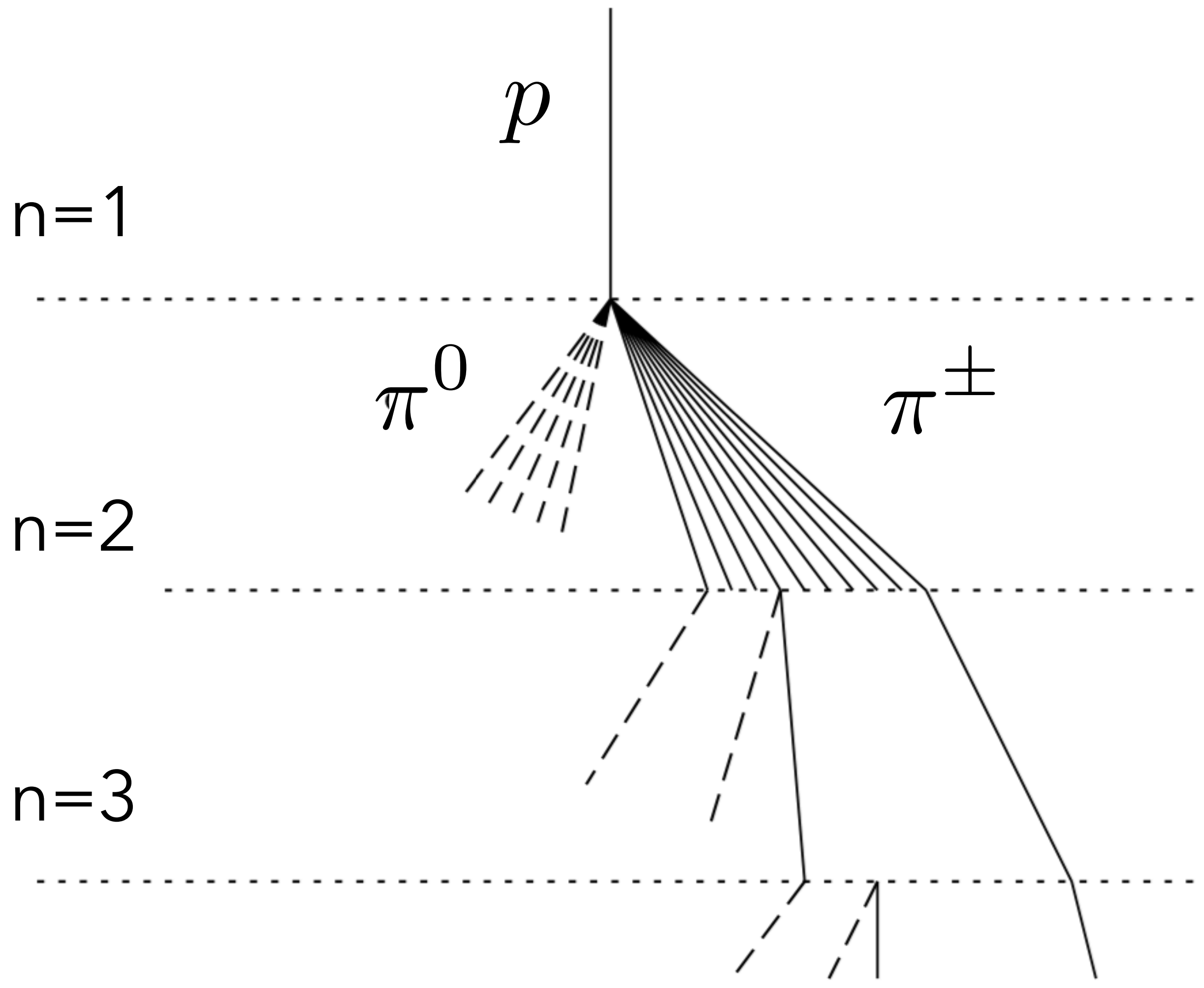
b. the number of muons produced in the shower, N_μ.

Heitler-Matthews model for EAS

4.

a.

$$N_{tot} = n_\pi^n \;;\; N_{ch} = \left(\frac{2}{3}n_\pi\right)^n \;;\; E_i = \frac{E_0}{n_\pi^n}$$



Starting with proton induced showers:

$$X_{\max} = d \times n_{dec} \qquad E_{\max} = E_0/n_\pi^{n_{dec}}$$

$$n_{dec} = \frac{\ln(E_0/E_{dec})}{\ln(n_\pi)} \qquad X_{\max} = d\,\frac{\ln(E_0/E_{dec})}{\ln(n_\pi)}$$

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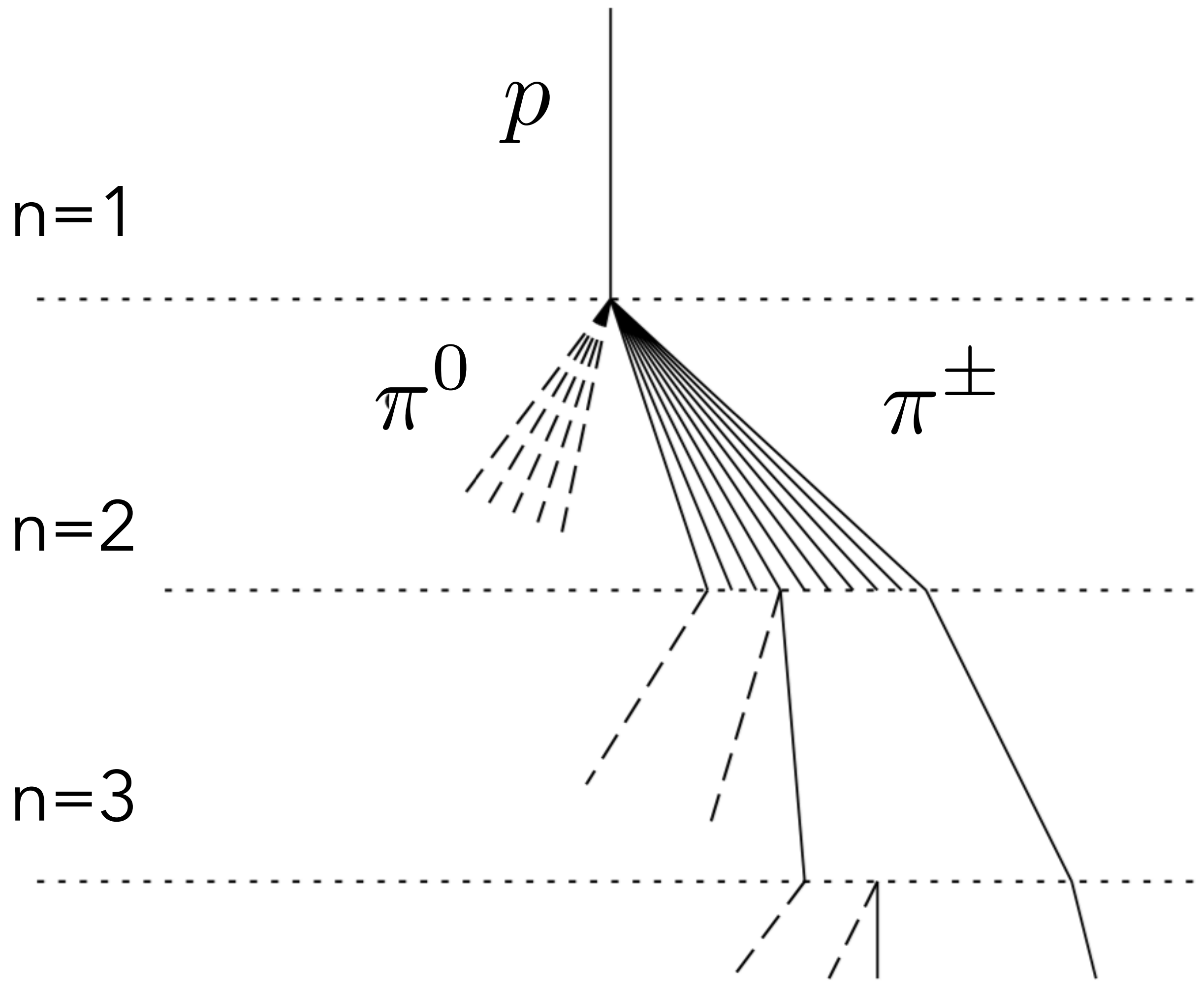
Heitler-Matthews model for EAS

4.

a.

Extrapolating for any element A protons
with energy E/A



$$X_{\max} = d\frac{\ln\left(E_0/E_{dec}\right)}{\ln(n_\pi)}$$

$$X_{\max} = d\frac{\ln\left(\frac{E_0}{AE_{dec}}\right)}{\ln(n_\pi)} = \frac{d}{\ln(n_\pi)}\Big[\,]\ln\left(\frac{E_0}{E_{dec}}\right) - \ln A\Big]$$

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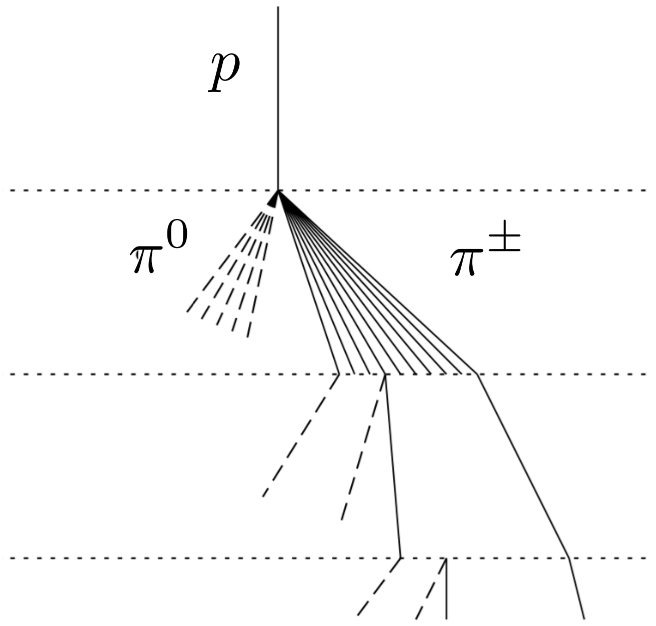
Heitler-Matthews model for EAS

4.

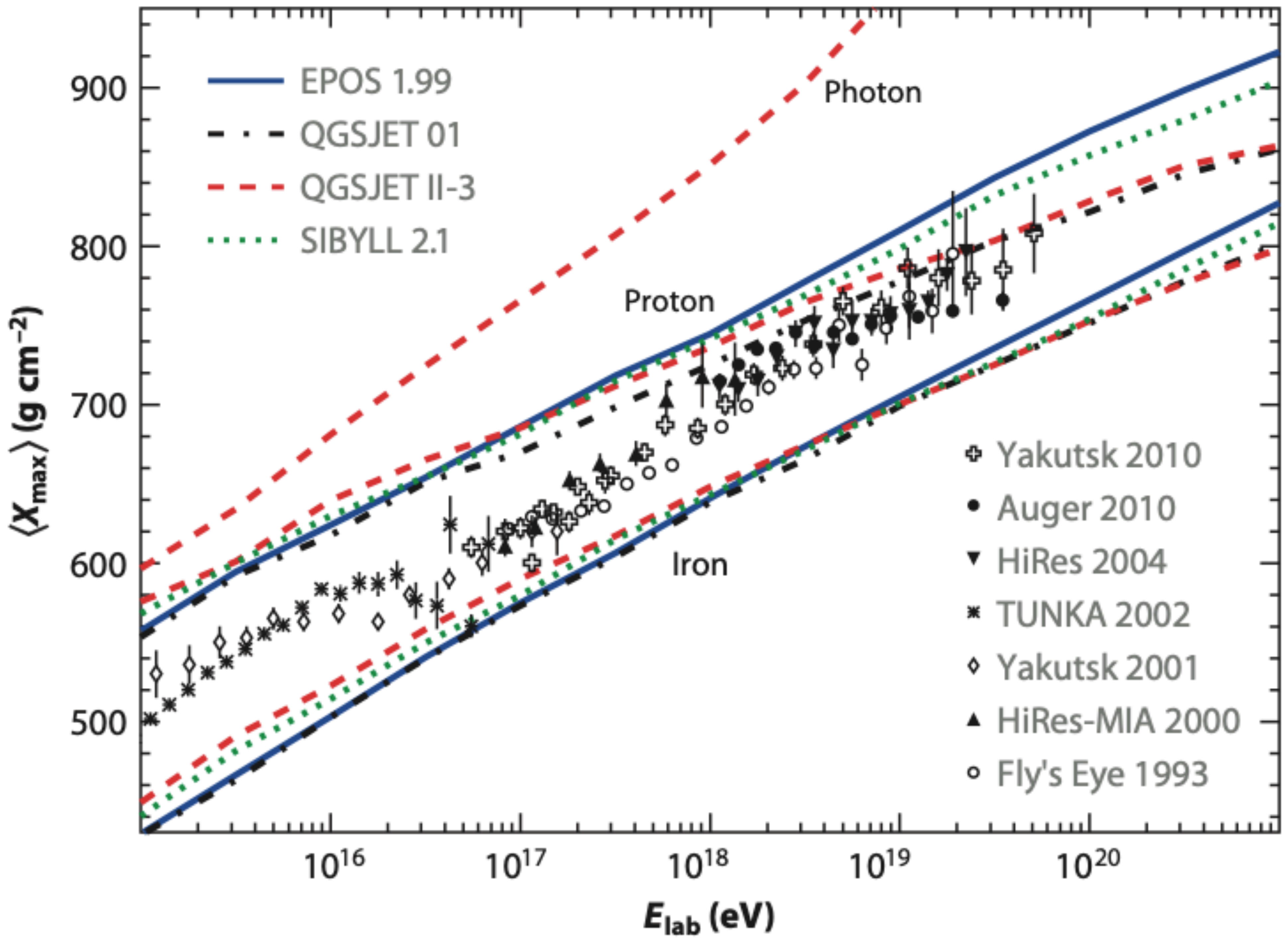
a.

Extrapolating for any element A protons with energy E/A



$$X_{\max} = d\frac{\ln\left(E_0/E_{dec}\right)}{\ln(n_\pi)}$$

$$X_{\max} = d\frac{\ln\left(\frac{E_0}{AE_{dec}}\right)}{\ln(n_\pi)} = \frac{d}{\ln(n_\pi)}\Big[\,]\ln\left(\frac{E_0}{E_{dec}}\right) - \ln A\Big]$$



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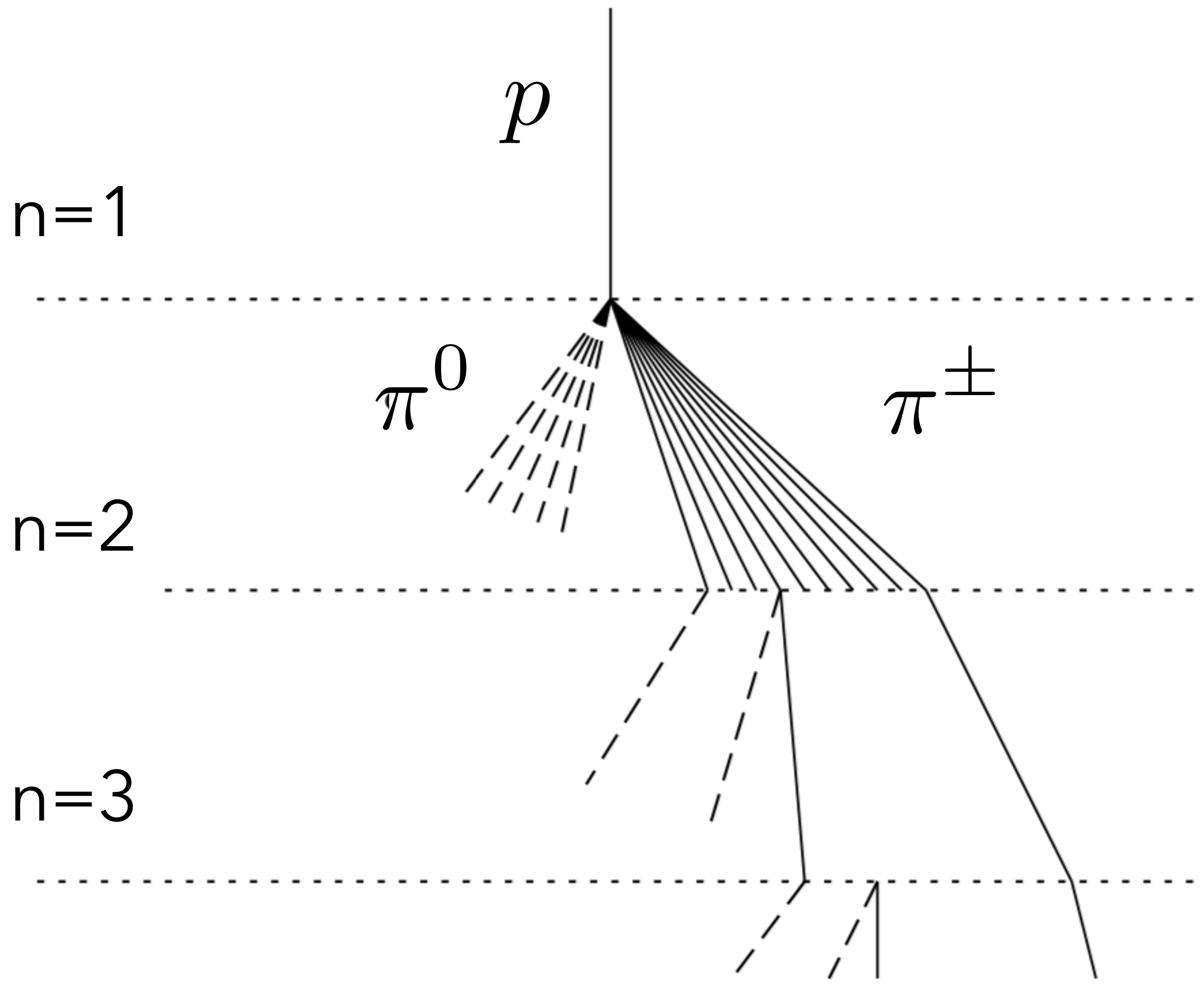
Heitler-Matthews model for EAS

4.

b.

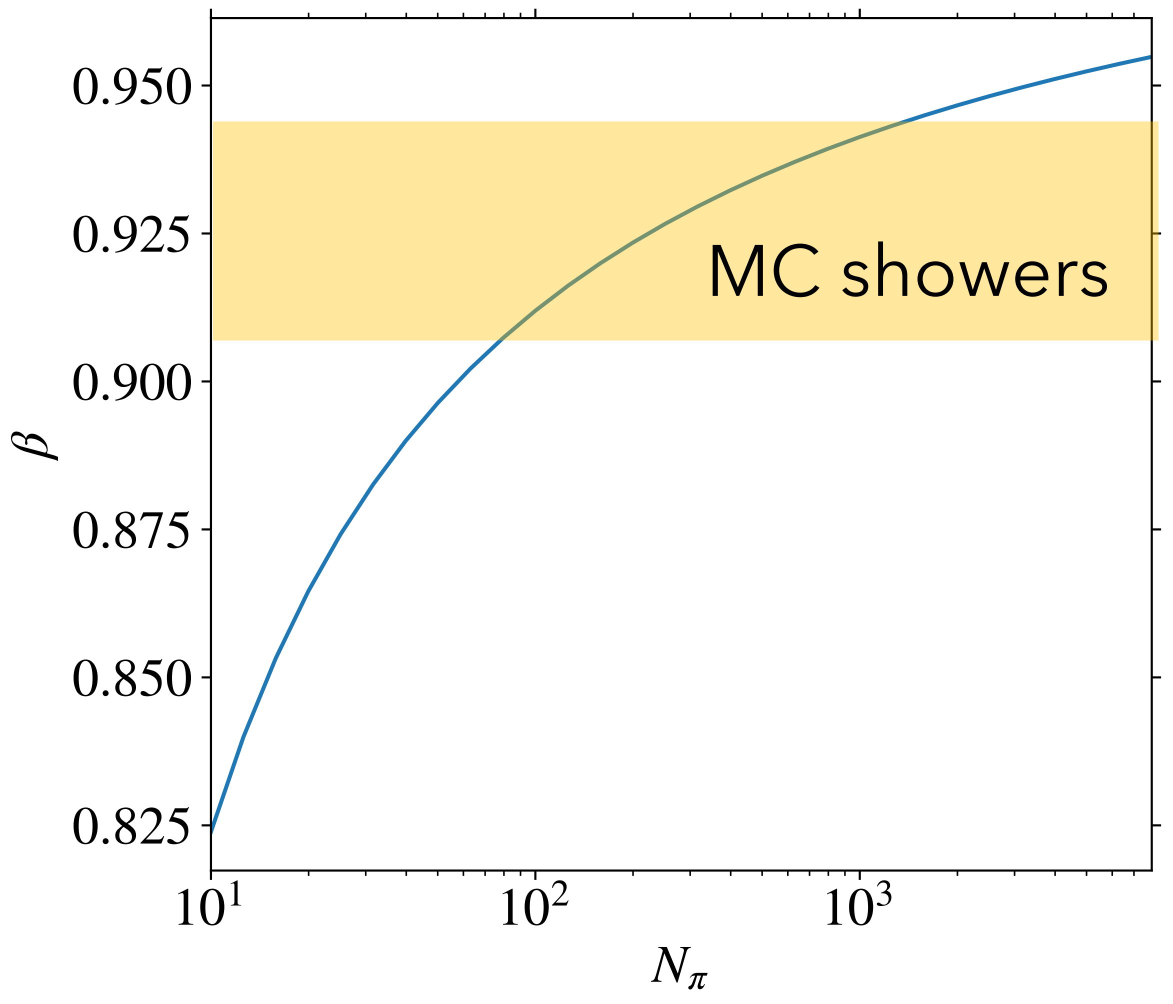
For proton induced showers

$$N_\mu = N_{ch}|_{X=X_{\max}} = \left(\frac{2}{3}n_\pi\right)^{n_{dec}}$$



$$N_\mu = \left(\frac{2}{3}n_\pi\right)^{\frac{\ln(E_0/E_{dec})}{\ln(n_\pi)}} = \left[\left(\frac{2}{3}n_\pi\right)^{\log_{\frac{2}{3}n_\pi}(E_0/E_{dec})}\right]^{\frac{\ln\left(\frac{2}{3}n_\pi\right)}{\ln(n_\pi)}}$$

$$= \left(\frac{E_0}{E_{dec}}\right)^{\frac{\ln\left(\frac{2}{3}n_\pi\right)}{\ln(n_\pi)}} = \left(\frac{E_0}{E_{dec}}\right)^{\beta}$$



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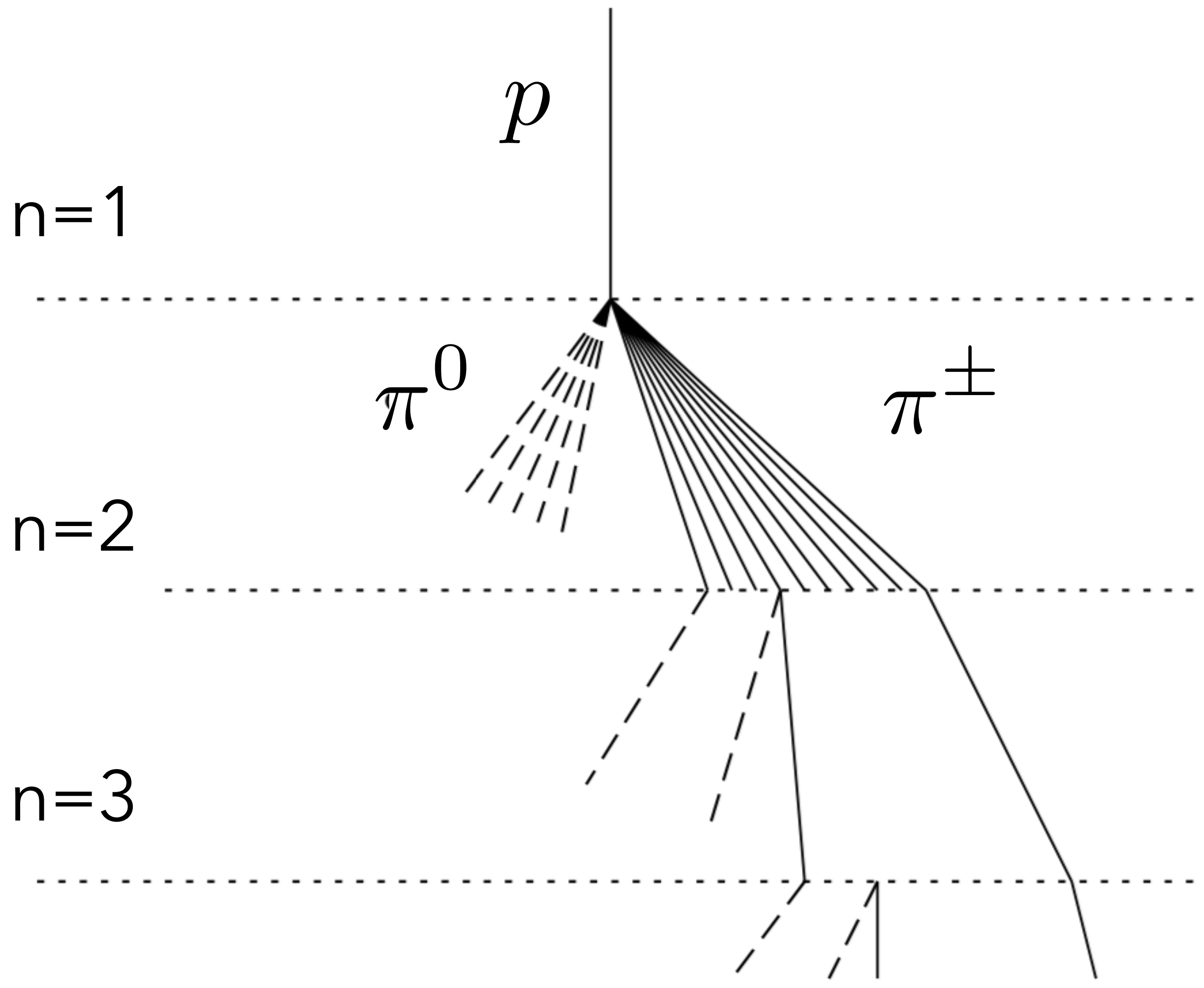
Heitler-Matthews model for EAS

4.

b.

Extrapolating for any element A protons
with energy E/A



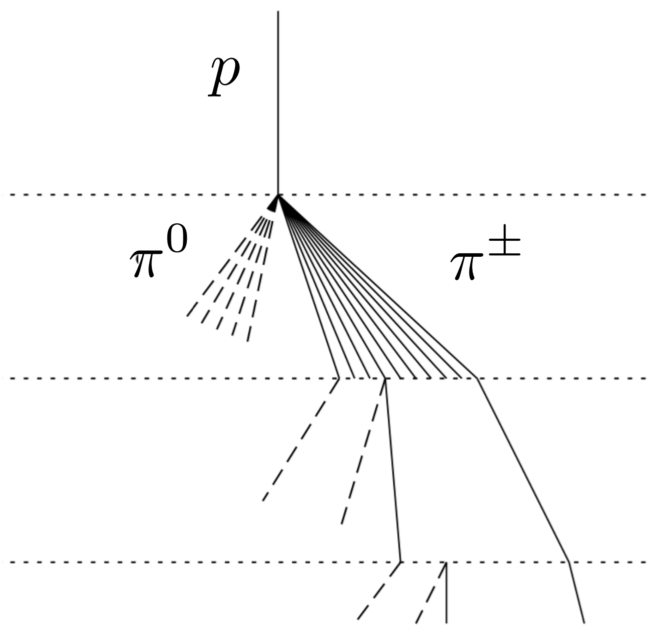
$$N_\mu = \left(\frac{E_0}{E_{dec}}\right)^\beta$$

$$N_\mu = A\left(\frac{E_0/A}{E_{dec}}\right)^\beta = A^{1-\beta}\left(\frac{E_0}{E_{dec}}\right)^\beta$$

Heitler-Matthews model for EAS

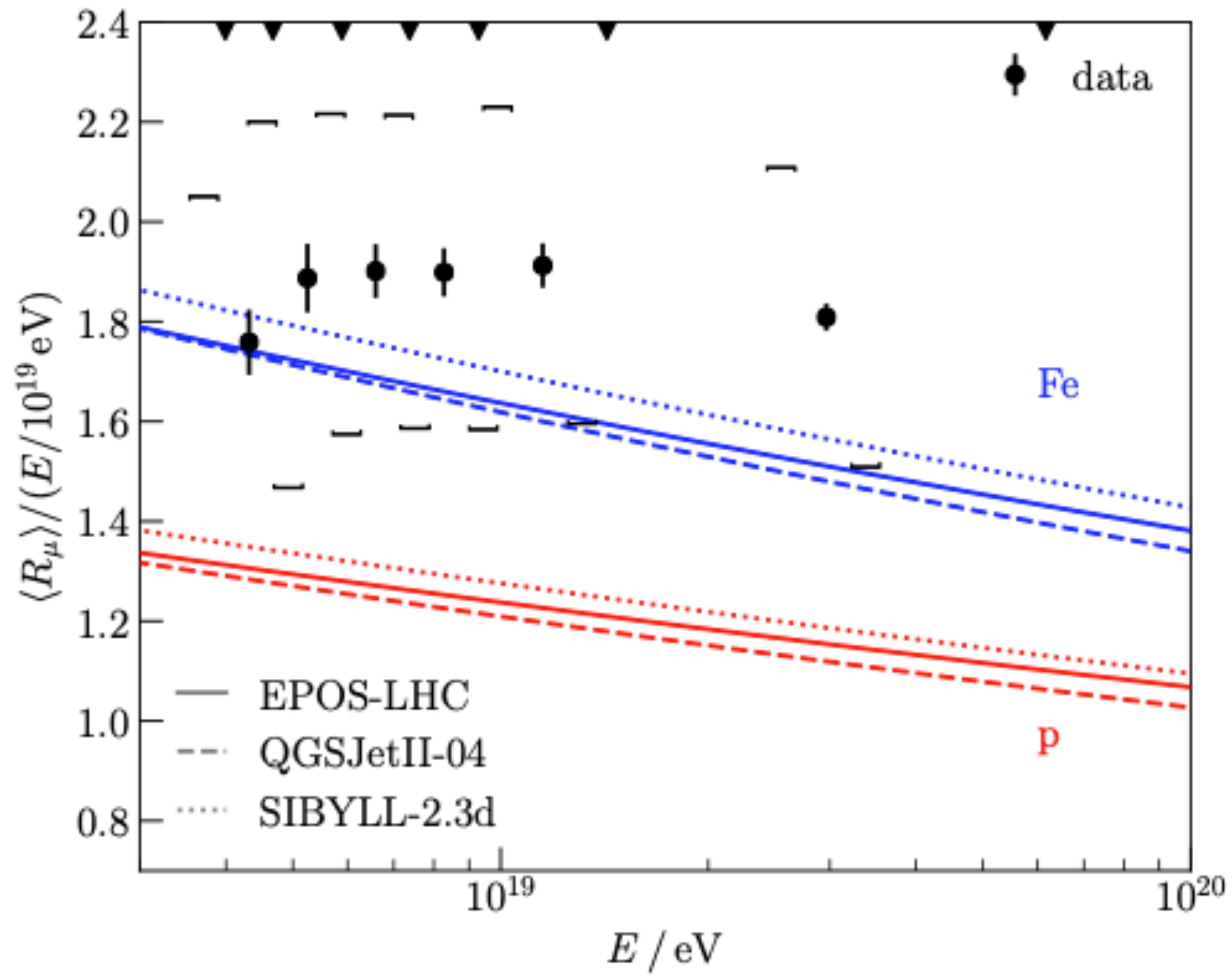
4.

b.



Extrapolating for any element A protons with energy E/A

$$N_\mu = \left(\frac{E_0}{E_{dec}}\right)^\beta$$



$$N_\mu = A\left(\frac{E_0/A}{E_{dec}}\right)^\beta = A^{1-\beta}\left(\frac{E_0}{E_{dec}}\right)^\beta$$

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Acknowledgements

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COMPETE 2020
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