Exercises for Cosmology and Particle Physics

For the following exercises consider a standard flat ΛCDM model with $\Omega_\Lambda = 0.7$, $\Omega_b = 0.05$, $\Omega_{CDM} = 0.25$, $H_0 = 70$ km/s/Mpc and $T_0 = 2.73$ K. Use natural units when necessary ($\hbar = c = k_B = 1$). Use only two significant digits.

Useful constants and conversion factors:
1 K $= 8.6 \times 10^{-5}$ eV
1 pc $= 3.1 \times 10^{16}$ m
1 y $= 3.1 \times 10^{7}$ s
$\mathrm{M}_{Pl} = 1.2 \times 10^{19}$ GeV
$(\hbar c) = 190$ MeV $\times$ f

1. Compute H_0 in units of 1/s and eV.
2. Compute the critical density today in units of eV^4 and in units of $\mathrm{GeV/m}^3$.
3. Compute the energy density of photons today in eV^4.
4. Compute Ω_γ.
5. Compute the redshift of equality between matter and radiation densities.
6. Compute the redshift when the cosmological constant starts to dominate the energy density of the Universe.
7. Make a plot showing how Ω_m, Ω_γ and Ω_Λ change with time.
8. Compute the number of photons today from the CMB per cm^3.
9. Compute the number of baryons today per cm^3.
10. Compute the baryon-to-photon ratio $\eta_b = \frac{n_b}{n_\gamma}$
11. Compute the number of neutrinos today per cm^3.
12. Compute Ω_ν for $\sum m_\nu = 0.1$ eV assuming they are nonrelativistic.
13. Compute the age of the Universe in years.
14. Compute the age of the Universe in seconds when it's temperature was 10^{12} GeV
15. Estimate the comoving sound horizon at decoupling
16. **Challenge:** For a neutrino mass that changes with time, show that the continuity equation modifies to:

$$\dot{\rho_\nu} + 3H(\rho_\nu + P_\nu) = \frac{\dot{m}}{m}(\rho_\nu - 3P_\nu) \tag{1}$$

Notice that when neutrinos are relativistic $(\rho_\nu - 3P_\nu) = 0$ and there is no effect. [Hint: start from the thermodynamical definition of energy density and pressure, change it for comoving momenta and assume the phase-space distribution does not change after decoupling.]