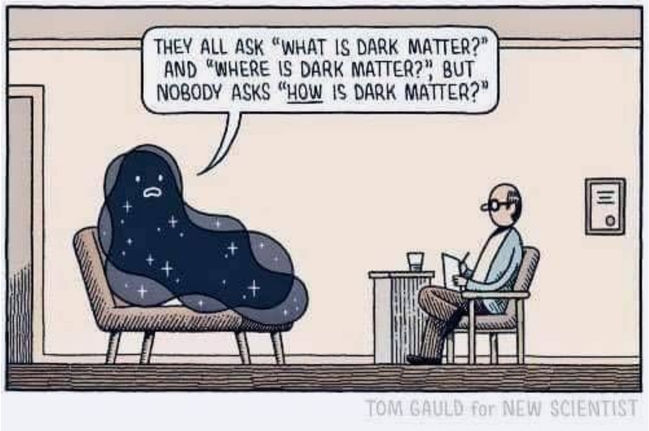


A DARK MATTER PROBLEM

14 Sep 2021

THE EXPERIMENT

Experimental parameters

- Liquid xenon target (assume natural abundance)
- Fiducial mass of 5,600 kg
- 1000-day run with 100% duty cycle
- Nuclear recoil acceptance 5 – 50 keVr
- Nuclear recoil detection efficiency of 50%
- No measurement of recoil energy, just “counts in boxes”

Keep it simple, at first

- Assume unity Form Factor
- Consider a stationary Earth in the galactic frame
- Assume an infinite galactic escape velocity
- Use ‘standard’ DM halo parameters as required

THE OBSERVATION

Nuclear recoil background

- Mean number of background counts predicted ahead of unblinding the data: **1.5±0.0**

Measurement

- Upon unblinding data in the WIMP search region, you observe **4** candidate nuclear recoil events

DISCUSSION
15 MINS

TASK (1) DISCOVERY?

Discuss these and related topics in your session

- How would you set limits using F-C statistics?
- Would you claim a discovery?
- Why/why not?

TASK (1) DISCOVERY?

TABLE IV. 90% C.L. intervals for the Poisson signal mean μ, for total events observed n_0, for known mean background b ranging from 0 to 5.

$n_0 \backslash b$	0.0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	5.0
0	0.00, 2.44	0.00, 1.94	0.00, 1.61	0.00, 1.33	0.00, 1.26	0.00, 1.18	0.00, 1.08	0.00, 1.06	0.00, 1.01	*0.00, 0.98*
1	0.11, 4.36	0.00, 3.86	0.00, 3.36	0.00, 2.91	0.00, 2.53	0.00, 2.19	0.00, 1.88	0.00, 1.59	0.00, 1.39	0.00, 1.22
2	0.53, 5.91	0.03, 5.41	0.00, 4.91	0.00, 4.41	0.00, 3.91	0.00, 3.45	0.00, 3.04	0.00, 2.67	0.00, 2.33	0.00, 1.73
3	1.10, 7.42	0.60, 6.92	0.10, 6.42	0.00, 5.92	0.00, 5.42	0.00, 4.92	0.00, 4.42	0.00, 3.95	0.00, 3.53	0.00, 2.78
4	1.47, 8.60	1.17, 8.10	0.74, 7.60	0.24, 7.10	0.00, 6.60	0.00, 6.10	0.00, 5.60	0.00, 5.10	0.00, 4.60	0.00, 3.60
5	1.84, 9.99	1.53, 9.49	1.25, 8.99	0.93, 8.49	0.43, 7.99	0.00, 7.49	0.00, 6.99	0.00, 6.49	0.00, 5.99	0.00, 4.99
6	2.21,11.47	1.90,10.97	1.61,10.47	1.33, 9.97	1.08, 9.47	0.65, 8.97	0.15, 8.47	0.00, 7.97	0.00, 7.47	0.00, 6.47
7	3.56,12.53	3.06,12.03	2.56,11.53	2.09,11.03	1.59,10.53	1.18,10.03	0.89, 9.53	0.39, 9.03	0.00, 8.53	0.00, 7.53
8	3.96,13.99	3.46,13.49	2.96,12.99	2.51,12.49	2.14,11.99	1.81,11.49	1.51,10.99	1.06,10.49	0.66, 9.99	0.00, 8.99
9	4.36,15.30	3.86,14.80	3.36,14.30	2.91,13.80	2.53,13.30	2.19,12.80	1.88,12.30	1.59,11.80	1.33,11.30	0.43,10.30
10	5.50,16.50	5.00,16.00	4.50,15.50	4.00,15.00	3.50,14.50	3.04,14.00	2.63,13.50	2.27,13.00	1.94,12.50	1.19,11.50
11	5.91,17.81	5.41,17.31	4.91,16.81	4.41,16.31	3.91,15.81	3.45,15.31	3.04,14.81	2.67,14.31	2.33,13.81	1.73,12.81
12	7.01,19.00	6.51,18.50	6.01,18.00	5.51,17.50	5.01,17.00	4.51,16.50	4.01,16.00	3.54,15.50	3.12,15.00	2.38,14.00
13	7.42,20.05	6.92,19.55	6.42,19.05	5.92,18.55	5.42,18.05	4.92,17.55	4.42,17.05	3.95,16.55	3.53,16.05	2.78,15.05
14	8.50,21.50	8.00,21.00	7.50,20.50	7.00,20.00	6.50,19.50	6.00,19.00	5.50,18.50	5.00,18.00	4.50,17.50	3.59,16.50
15	9.48,22.52	8.98,22.02	8.48,21.52	7.98,21.02	7.48,20.52	6.98,20.02	6.48,19.52	5.98,19.02	5.48,18.52	4.48,17.52
16	9.99,23.99	9.49,23.49	8.99,22.99	8.49,22.49	7.99,21.99	7.49,21.49	6.99,20.99	6.49,20.49	5.99,19.99	4.99,18.99
17	11.04,25.02	10.54,24.52	10.04,24.02	9.54,23.52	9.04,23.02	8.54,22.52	8.04,22.02	7.54,21.52	7.04,21.02	6.04,20.02
18	11.47,26.16	10.97,25.66	10.47,25.16	9.97,24.66	9.47,24.16	8.97,23.66	8.47,23.16	7.97,22.66	7.47,22.16	6.47,21.16
19	12.51,27.51	12.01,27.01	11.51,26.51	11.01,26.01	10.51,25.51	10.01,25.01	9.51,24.51	9.01,24.01	8.51,23.51	7.51,22.51
20	13.55,28.52	13.05,28.02	12.55,27.52	12.05,27.02	11.55,26.52	11.05,26.02	10.55,25.52	10.05,25.02	9.55,24.52	8.55,23.52

Feldman
& Cousins

25 MINS

TASK (2) WIMP-NUCLEUS CROSS SECTION LIMIT

Start from the differential nuclear recoil (NR) spectrum induced by WIMPs, as per derivation in Lewin & Smith:

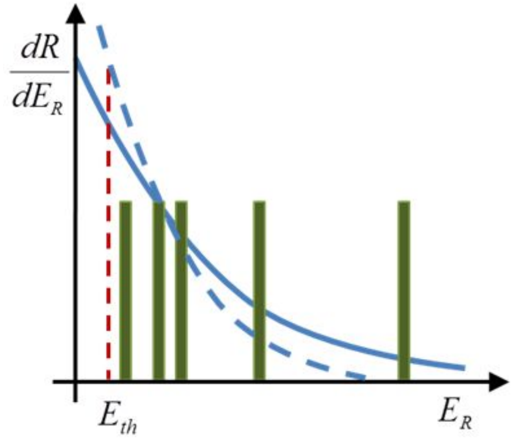
$$\frac{dR}{dE_R} = \frac{\rho_0 \sigma_A}{2 m_\chi \mu_A^2} F^2(q) \int_{v_{\min}}^{v_{\max}} \frac{f(\vec{v})}{v} d^3 v \approx \frac{R_0}{E_0 r} e^{-E_R / E_0 r}$$

(3.10)

- *R* is the NR differential rate (#/kg/day/keV)
- *R0* is a reference integral rate (normalisation)
- *E0* is the most probable WIMP kinetic energy
- *r* is a kinematic factor

This predicts an ~ exponential recoil spectrum

For our observation, derive the upper limit on the WIMP-nucleus scattering cross section.



TASK (2) WIMP-NUCLEUS CROSS SECTION LIMIT

We want to evaluate R_0, E_0 and r. Here are numbers for the quantities that we need to input:

$$\rho_0 = 0.3\ \mathrm{GeVc^{-2}cm^{-3}} \quad (28)$$
$$= 3\times 10^5\ \mathrm{GeVc^{-2}m^{-3}} \quad (29)$$
$$v_0 = 220\ \mathrm{kms^{-1}} \quad (30)$$
$$= 2.2\times 10^5\ \mathrm{ms^{-1}} \quad (31)$$
$$m_\chi = 50\ \mathrm{GeVc^{-2}} \quad (32)$$
$$m_A = 122.30\ \mathrm{GeVc^{-2}} \qquad \text{(Xenon)} \quad (33)$$

which leads to:

$$R_0 = (1.2179\times 10^7\ \mathrm{GeV^{-1}c^2m^{-2}s^{-1}})\sigma_A \quad (34)$$
$$= (6.8319\times 10^{33}\ \mathrm{kg^{-1}m^{-2}s^{-1}})\sigma_A \quad (35)$$
$$= (6.8319\times 10^{29}\ \mathrm{kg^{-1}cm^{-2}s^{-1}})\sigma_A \quad (36)$$
$$= (5.9028\times 10^{34}\ \mathrm{counts/kg/day/cm^2})\sigma_A \quad (37)$$
$$E_0 = 1.21\times 10^{12}\ \mathrm{GeVc^{-2}m^2s^{-2}} \quad (38)$$
$$= 13.463\ \mathrm{keV} \quad (39)$$
$$r = 0.82392 \quad (40)$$
$$\to \frac{R_0}{E_0 r} = 5.3215\times 10^{33}\ \mathrm{counts/kg/day/keV/cm^2})\sigma_A \quad (41)$$

$$E_0 = \frac{1}{2} M_D v_0^2$$

Most probable WIMP kinetic energy is 13.5 keV

$$r = \frac{4 m_\chi m_A}{(m_\chi + m_A)^2}$$

Kinematic factor r ~ 1:
a 122 GeV Xe nucleus
is still well matched
to a 50 GeV WIMP

$$R_0 = \frac{2\rho_0 \sigma_A v_0}{m_\chi m_A \sqrt{\pi}}$$

Integral rate (depends on XS)
(note L&S expression appears somewhat different but same…)

TASK (2) WIMP-NUCLEUS CROSS SECTION LIMIT

To find the 90% CL upper limit we will find what value of σ_A would lead to 7.10 events observed. First we integrate the differential rate:

$$R = \int_5^{50} \frac{dR}{dE_R} dE_R \tag{42}$$

$$= \frac{R_0}{E_0 r}\left[-E_0 r e^{-E_R/E_0 r}\right]_5^{50} \tag{43}$$

$$= R_0(-e^{-50/11.1} + e^{-5/11.1}) \tag{44}$$

$$= 0.62628 R_0 \tag{45}$$

i.e. 63% of recoils are in the 5-50 keV region of interest

How many events do we expect in total?

$$\text{expected count} = \text{mass} \times \text{no. days} \times \text{acceptance} \times 0.62628 R_0 \tag{46}$$

$$= 5600 \times 1000 \times 0.5 \times 0.62628 \times 5.9028 \times 10^{34} \sigma_A \text{cm}^{-2} \tag{47}$$

$$= 1.0351 \times 10^{41} \sigma_A \text{cm}^{-2} \tag{48}$$

so letting this equal 7.10, we have:

$$\sigma_A = 6.86 \times 10^{-41} \text{cm}^2 \qquad 90\% \text{ CL upper limit} \tag{49}$$

10 MINS

TASK (3) WIMP-NUCLEON CROSS SECTION LIMIT

From L&S or today's lecture, for a spin-independent interaction we have:

$$\sigma_A^{SI}(q \to 0) = \frac{4\mu_A^2}{\pi}[Zf_p + (A-Z)f_n]^2 \approx \frac{\mu_A^2}{\mu_p^2}\sigma_p A^2$$

Calculate with WIMP-nucleon cross section limit.

TASK (3) WIMP-NUCLEON CROSS SECTION LIMIT

WIMP-nucleus (A) and WIMP-proton (p) reduced masses

$$\mu_A = m_\chi m_A/(m_\chi + m_A)$$

	GeV
m_D	50
m_T	120.2
μ_A	35.3
μ_p	0.92

Then calculation of WIMP-proton XS upper limit is trivial

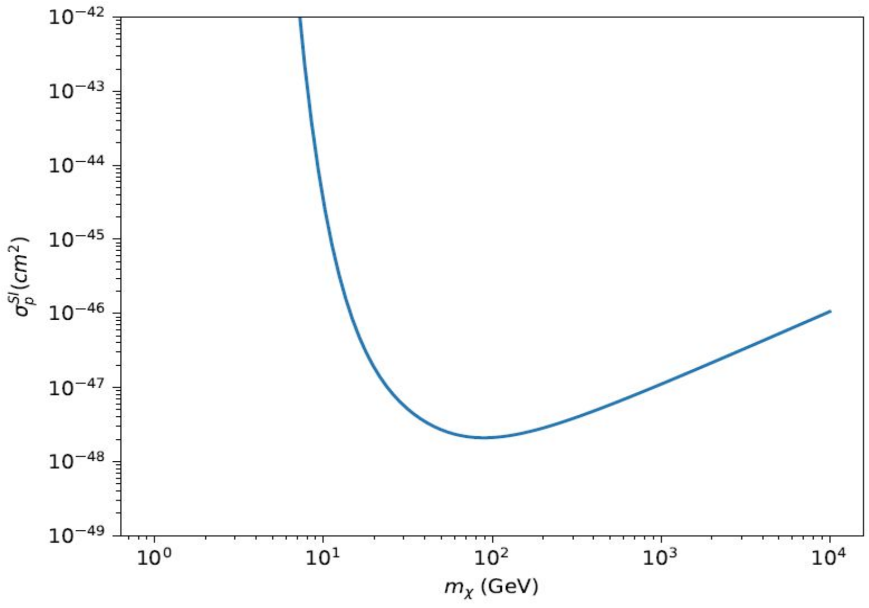
$$\sigma_p^{SI} = 2.68 \times 10^{-48}\mathrm{cm}^2 \qquad 90\% \text{ CL upper limit}$$

TASK (4) CROSS SECTION CURVE

DISCUSSION
10 MINS

This is the shape of the XS limit curve as a function of mass (you just calculated the value at 50 GeV WIMP mass), using the approximations we made.

Can you explain
the shape of the curve?

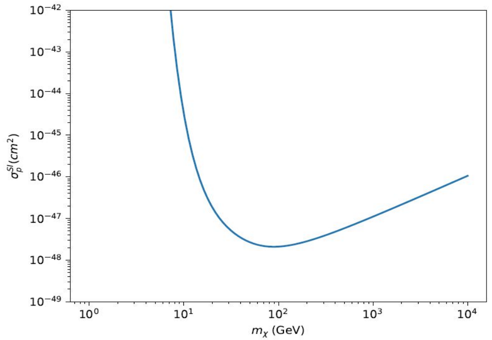


TASK (4) CROSS SECTION CURVE

Low mass part: a certain minimum WIMP velocity (v_min) is needed to create a recoil above the energy threshold of the detector. Since the velocity distribution is the same for all masses, lighter WIMPs find it harder to reach the required kinetic energy to be detected, and so the sensitivity gets worse (i.e. the curve rises).

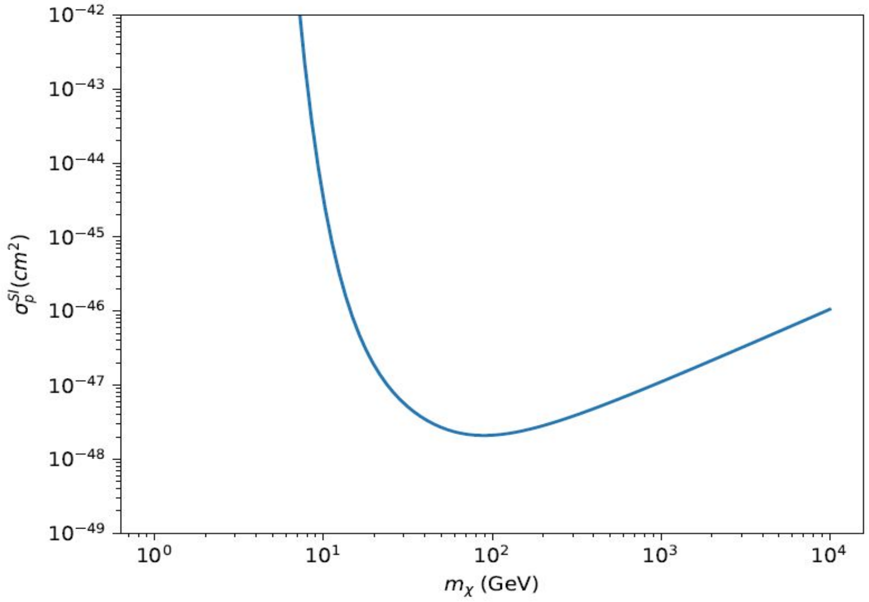
When v_esc is infinity there is always some WIMP fast enough, but when we bring v_esc down to 544 km/s this turns into a well defined mass threshold.

High mass part: there are fewer and fewer particles at higher masses since the mass density is constant (0.3 GeV/cm3), so the number density comes down. With fewer target atoms the sensitivity decreases.



TASK (4) CROSS SECTION CURVE

What would happen if the Earth is no longer stationary in the DM halo, but instead you bring v_E up to 230 km/s?

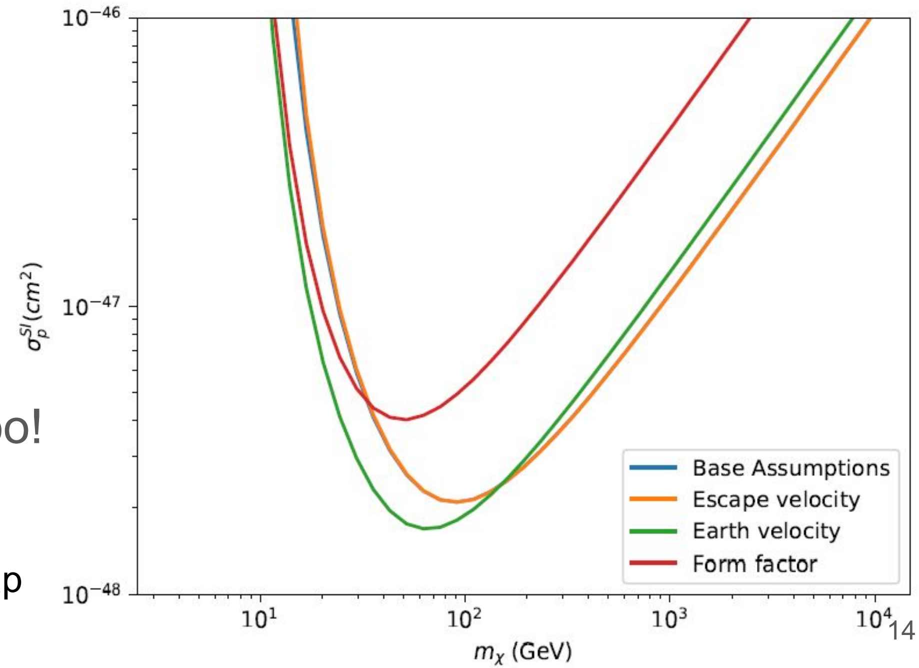


TASK (4) CROSS SECTION CURVE

This increases the relative velocity between the WIMP and the nucleus, so the recoil spectrum gets “harder” and some more WIMPs can reach v_min and cause a recoil above threshold (so the low mass sensitivity improves).

At higher masses we start losing some Events past our upper 50 keV window and so the sensitivity gets a little worse.

Note also that v_esc is boosted by v_E too!

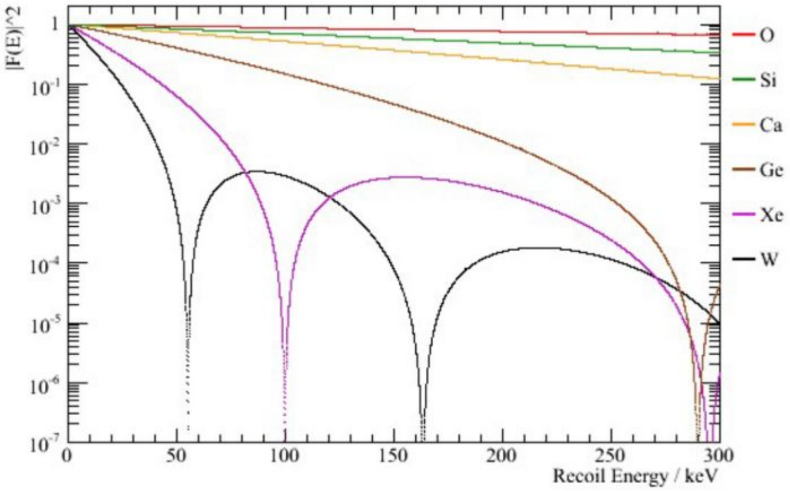


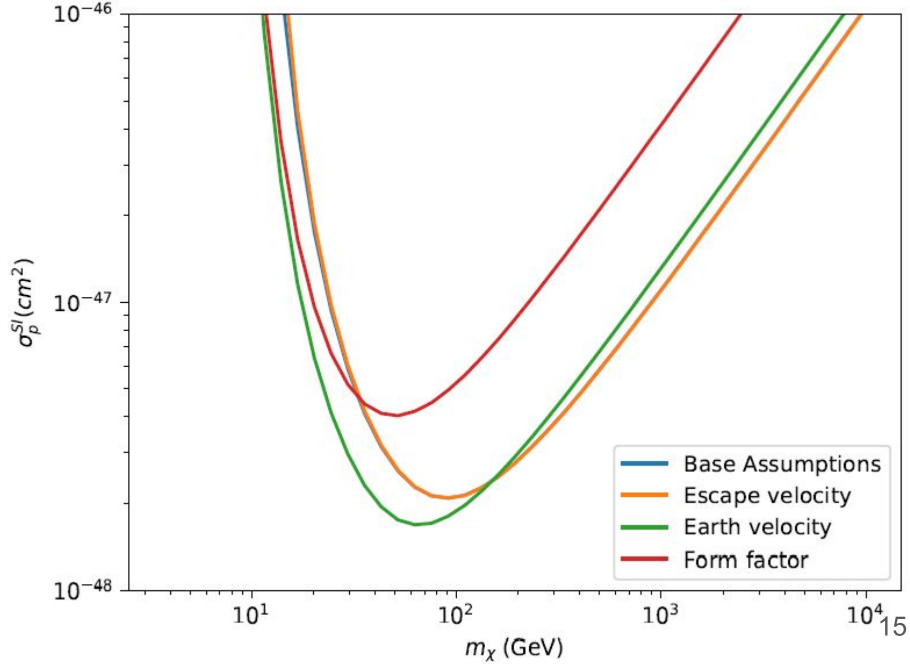
In this plot blue and orange curves overlap (difficult to see v_esc effect on this scale)

DISCUSSION
5 MINS

TASK (4) CROSS SECTION CURVE

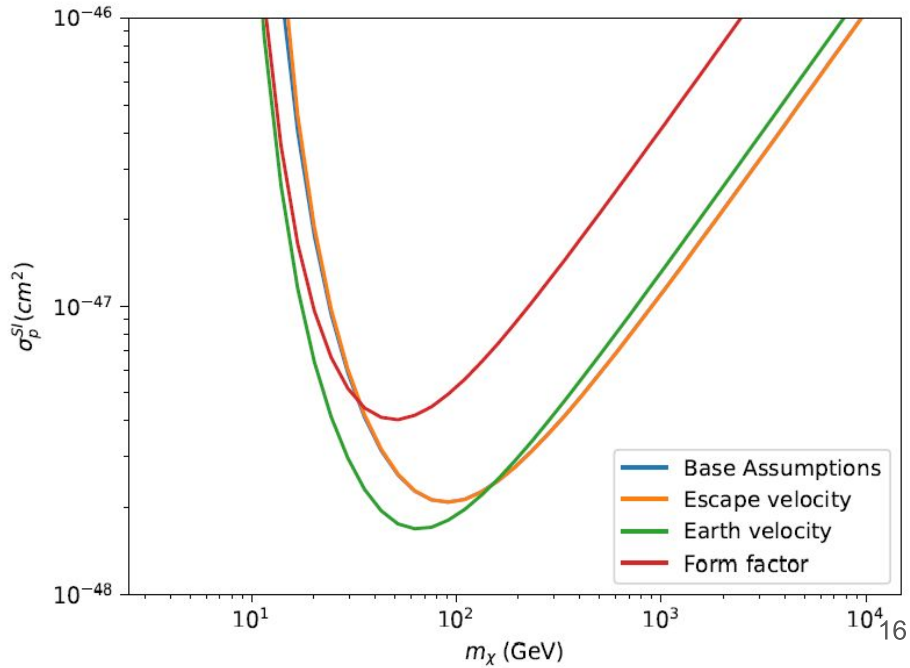
What about the form factor? What happens if we bring this in, instead of F2=1?





TASK (4) CROSS SECTION CURVE

The FF suppresses higher energy recoils and is close to unity for low energy recoils (low momentum transfer), so we do expect the sensitivity to suffer at high masses. In fact, this plot tells us that this is important already at 50 GeV...



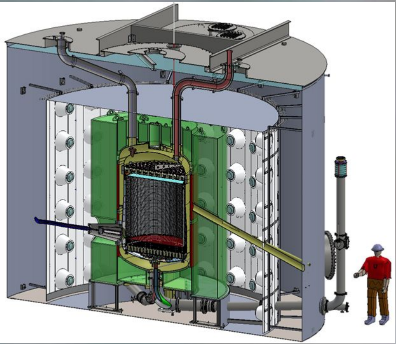
Here blue and orange overlap

LET US DROP (5) IN THE INTEREST OF TIME...

IF THIS SOUNDS FAMILIAR...

A LXe experiment with 5.6 tonnes of fiducial mass?

LUX-ZEPLIN (LZ) - coming soon!



The experiment

Consider a simplified WIMP-search experiment containing at its core a liquid xenon (LXe) target. The fiducial mass is 5,600 kg and the target operates for 1,000 days with 100% duty cycle in an underground laboratory. Natural abundance xenon is used.

The detector is instrumented as a LXe-TPC, capable of measuring scintillation (S1) and ionisation (S2) signals for each interaction. This allows 3D fiducialisation of the active volume and discrimination of interaction type: nuclear recoil (NR) events are signal candidates, electron recoils (ER) are background.

The experiment is searching for low-energy nuclear recoils in the range 5–50 keV. Let us simplify the problem by assuming that no recoil energy information is available within this energy range, i.e. treat this as a counting experiment. Given that there is some ER background in the xenon target, we must restrict our search to low values of the discrimination parameter S2/S1, thereby losing some NR signal acceptance – we assume this to be 50%. You may assume initially that the form factor $F^2(q)$ is unity.

We will work with a simplified dark matter halo model in which the Earth is stationary in the galactic frame and the galactic escape velocity is infinite. Reasonable values should be used for other parameters.

The observation

The mean number of background counts predicted in the region of interest ahead of unblinding the data is 1.5 ± 0. Upon unblinding the data, you observe 4 candidate events in this region.