

Three lectures on Cosmology and Particle Physics

Lecture I: Dynamics of the average Universe

→ Lecture II: Distances and thermal history

Lecture III: Neutrinos in cosmology

Plan for Lecture II:

- II.1 Distances in the universe
- II.2 Thermal history
- II.3 The Hubble tension (crisis)

II.1- Distances in the universe

II.1.1 – Physical and comoving distances

Universe is spatially homogeneous and isotropic on average.

On average it is described by the FLRW metric (for a spatially flat universe):

$$ds^{2} = dt^{2} - a(t)^{2} \left[dx^{2} + dy^{2} + dz^{2} \right]$$

$$r(t) = a(t) x$$
physical distance comoving distance

For a galaxy following the "Hubble flow" (expanding like the average universe):

$$\dot{r}(t) = V = \dot{a}(t)x = \frac{\dot{a}(t)}{a(t)} a(t)x = H r(t)$$

In reality galaxies will have some "peculiar velocities" due to inhomogeneities:

$$V = H r(t) + V_{nec}$$

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Comoving distance between us (z=0) and an object at redshift z:

$$ds^{2} = 0 \Rightarrow dt^{2} = a(t)^{2} d\chi^{2} \Rightarrow$$
$$\chi(z) = \int_{0}^{z} \frac{dz'}{H(z')}$$

Comoving distances can't be directly measured, but are a useful quantity in cosmology. They depend on cosmology (H(z)).

Another useful quantity is the horizon at a given redshift z: co-moving size of the causal region since the big bang ($z=\infty$) until a given redshift z.

$$r_h(z) = \int_z^\infty \frac{c \, dz'}{H(z')}$$

Sound horizon (will be important later): $C \rightarrow C_S$ Sound speed

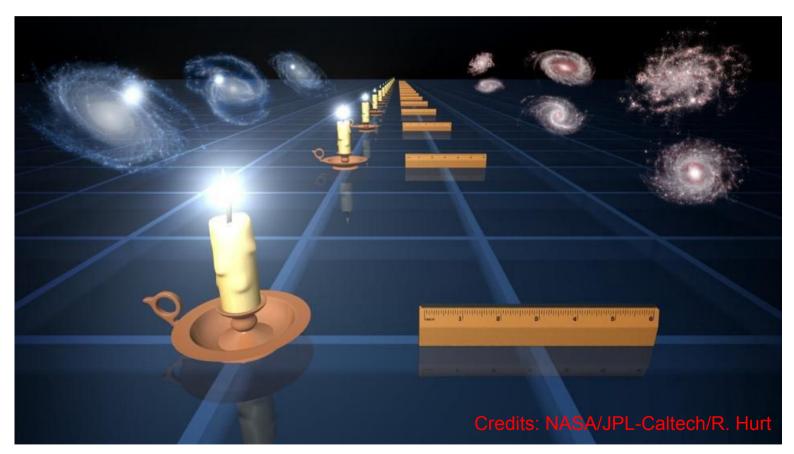
In the universe it is easy to measure the redshift of objects, using eg spectroscopy.

Measuring distances is much more difficult.

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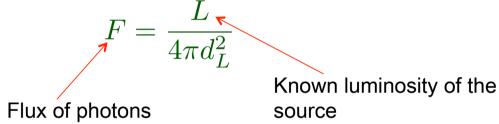
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There are 2 ways to measure large distances in the Universe: from known luminosities (standard candles – eg SNIa) – luminosity distance from known scales (standard rulers – eg BAO) - angular diameter distance



II.1.2 – Luminosity distance

Luminosity distance (d_L):



In FLRW there are 2 extra source of dilution of the flux:

- redshift of photons (1/(1+z))
- rate of arrival decrease by (1/(1+z)) time dilation

Therefore:
$$d_L = (1+z)\chi(z)$$

We can Taylor expand H(z) to first order:

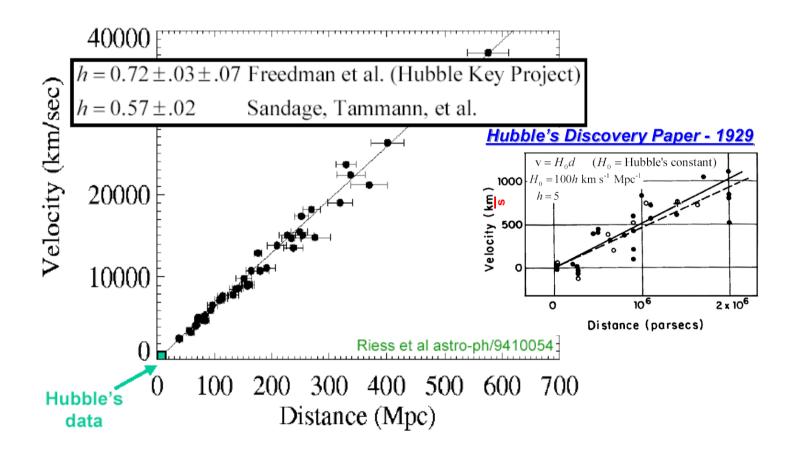
$$H(z) = H_0 + \frac{dH}{dz}z$$

to obtain the Hubble-Lemaître law:

$$d_L = \frac{z}{H_0}$$

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Hubble-Lemaître law



Hubble's diagram and cosmic expansion

Robert P. Kirshner*

Harvard-Smithsonian Center for Astrophysics, 60 Garden Street, Cambridge, MA 02138

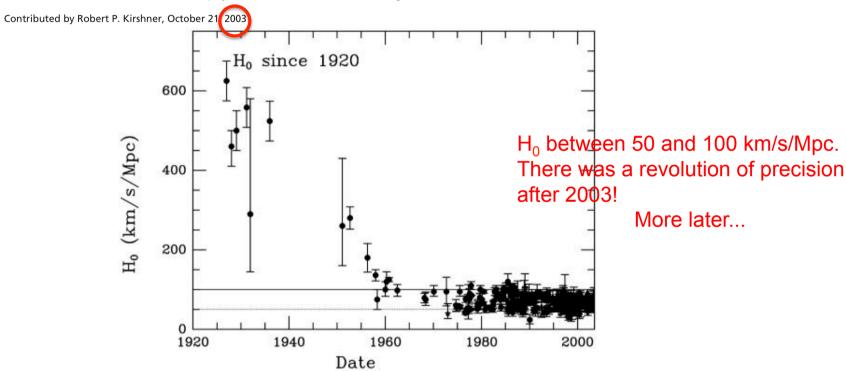


Fig. 2. Published values of the Hubble constant vs. time. Revisions in Hubble's original distance scale account for significant changes in the Hubble constant from 1920 to the present as compiled by John Huchra of the Harvard–Smithsonian Center for Astrophysics. At each epoch, the estimated error in the Hubble constant is small compared with the subsequent changes in its value. This result is a symptom of underestimated systematic errors.

Distance ladder
Riess et al 1604.01424
A 2.4% Determination of the Local
Value of the Hubble Constant $H_0 = 73.24 + /-1.74 \text{ km/sec/}$ Mpc

Riess et al 1903.07603

 $H_0 = 74.03 \pm 1.42 \text{ km/sec}$ Mpc

Tension with another measurement from CMB

 $u (z, H_0 = 73.2, q_0, j_0)$ Cepheids → Type Ia Supernovae SN Ia: m-M (mag) SN Ia: m-M (mag) Geometry → Cepheids Cepheid: m-M (mag) 14 Geometry: 5 log D [Mpc] + 25

Type Ia Supernovae \rightarrow redshift(z)

NAORO lotor

Luminosity distance at higher redshifts:

$$d_L = (1+z)\chi(z)$$

Expanding to second order in redshift:

$$H_0 d_L(z) = z + \frac{1}{2}(1 - q_0)z^2 + \cdots$$

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Cosmology: A search for two numbers

 H_0 q_0

Precision measurements of the rate of expansion and the deceleration of the universe may soon provide a major test of cosmological models

Allan R. Sandage

Mount Wilson and Palomar Observatories

$$d_L = (1+z)\chi(z)$$

We plot d₁ (z) (exact expression) for 0<z<2 for a flat Universe with

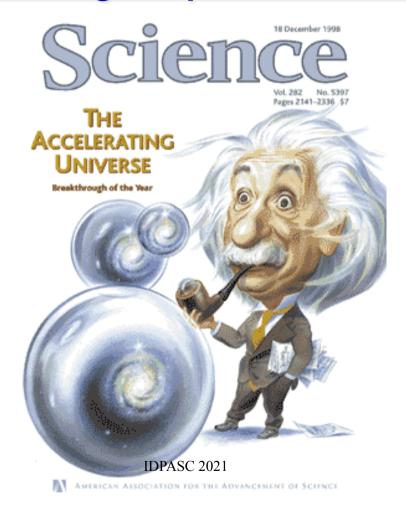
a.
$$\Omega_{\rm m}$$
 = 1 and Ω_{Λ} = 0

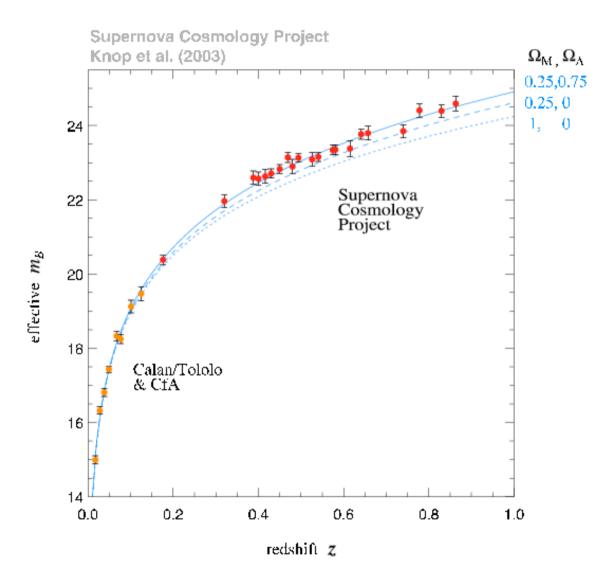
b.
$$\Omega_{\rm m}$$
 = 0.3 and Ω_{Λ} = 0.7

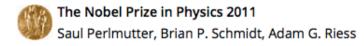
```
ln[127] = H0 = 70. (* km/s/Mpc *);
        c = 300000. (*km/s*);
 ln[130] = H[z, Om, OA] := H0 \sqrt{Om(1+z)^3 + OA}
 ln[131] = DL[z, Om, OA] := c(1+z) NIntegrate[1/H[zp, Om, OA], {zp, 0, z}]
                                         integra numéricamente
 ln[137]:= Plot[{DL[z, 1, 0], DL[z, 0.3, 0.7]}, {z, 0, 1}, PlotLegends \rightarrow "Expressions"]
        gráfico
                                                             legenda do gráfico
        6000
        5000
        4000
                                                                          DL(z, 1, 0)
Out[137]=
       3000
                                                                          DL(z, 0.3, 0.7)
        2000
        1000
                     0.2
                                           0.6
                                                      8.0
```

 D_L is larger for a Universe with Λ -> objects with same z look fainter. This is how the accelerated expansion of the Universe was discovered in 1998 using SNIa

The big surprise in 1998:















The Nobel Prize in Physics 2011



Photo: U. Montan Saul Perlmutter Prize share: 1/2



Brian P. Schmidt Prize share: 1/4

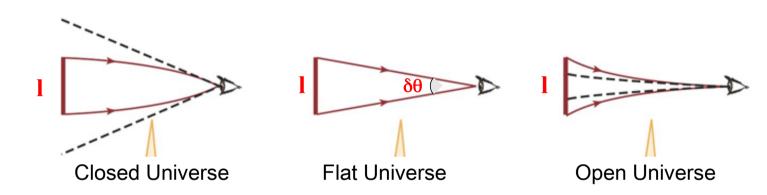


Photo: U. Montan Adam G. Riess Prize share: 1/4

The Nobel Prize in Physics 2011 was divided, one half awarded to Saul Perlmutter, the other half jointly to Brian P. Schmidt and Adam G. Riess "for the discovery of the accelerating expansion of the Universe through observations of distant supernovae".

II.1.3 – Angular diameter distance

Angular diameter distance (d_A) is related to the angle subentended by a physical scale (I)



For a flat universe:
$$d_A=\frac{l}{\delta \theta},\ l=a\chi\delta\theta\Rightarrow$$

$$d_A=\frac{1}{1+z}\chi(z)$$

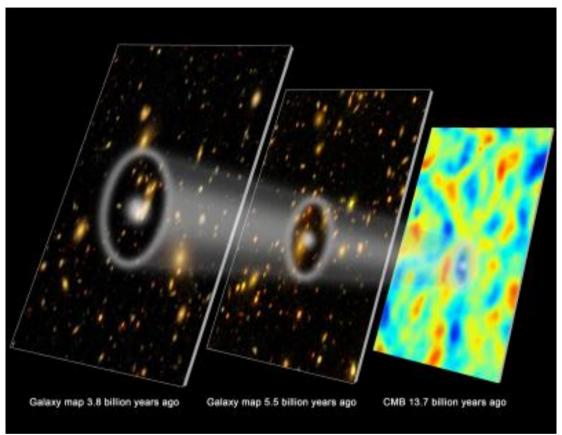
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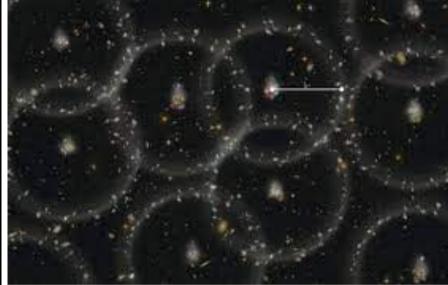
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Is there a favored physical scale in the universe?

Yes: the "acoustic horizon scale at decoupling" (r_s) – more later

This physical scale sets the angular scale for the fluctuations in the cosmic microwave background (CMB) and in the distribution of galaxies that are formed much later (baryon acoustic oscillation – BAO)!





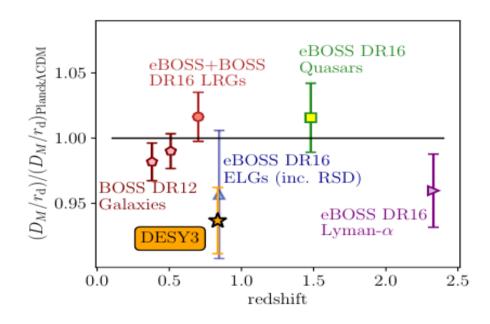
Artist impression of baryonic acoustic oscillations imprinting a mean galaxy separation of $150 \ \underline{\text{Mpc}}$ on the cosmic galaxy distribution.

Credit: Eric Huff, the SDSS-III team, and the South Pole Telescope team. Graphic by Zosia Rostomian

Astrophysics > Cosmology and Nongalactic Astrophysics

[Submitted on 9 Jul 2021]

Dark Energy Survey Year 3 Results: A 2.7% measurement of Baryon Acoustic Oscillation distance scale at redshift 0.835



DES-Y3 BAO sample: 7 million galaxies with $0.6 < z_{phot} < 1.1$ in an area of ~4100 deg²

2.3σ below Planck

II.1.4 – Hubble radius

Another useful scale: characteristic distance particles can travel in a Hubble time (c=1):

$$R_H = \frac{1}{H(t)}$$

Comoving Hubble radius:

$$r_H = \frac{1}{aH} = \frac{1}{\dot{a}}$$

Radiation dominated

$$r_H \propto a$$

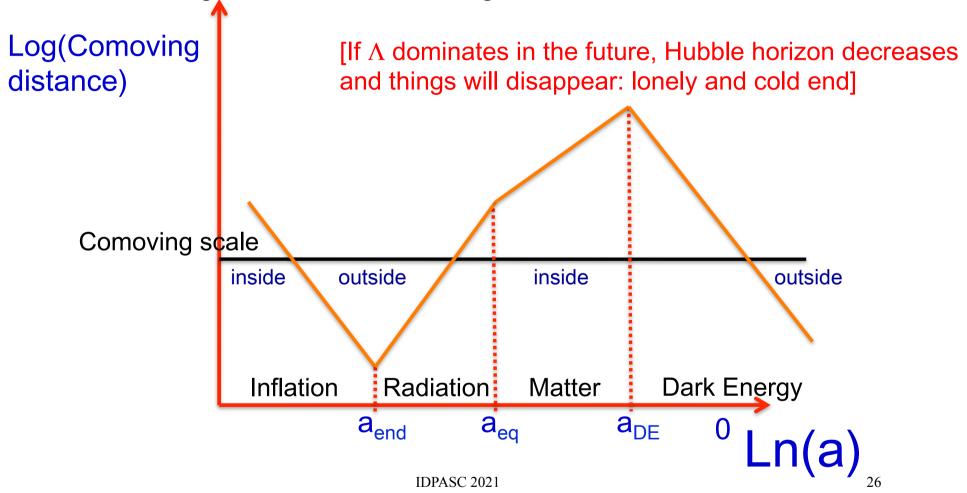
Matter dominated

$$r_H \propto a^{1/2}$$

• Λ dominated

$$r_H \propto 1/a$$

Comoving Hubble radius during the evolution of the Universe



II.2- Thermal history of the Universe

II.2.1 – Brief review of thermodynamics

Number density and energy density of a dilute, weakly interacting gas with g internal degrees of freedom:

E is the energy of a state, f(p) is its phase-space distribution and g is number of internal degrees of freedom (eg g=2 for photons, g=16 for gluons, g=12 for quarks, etc).

$$n = \frac{g}{(2\pi)^3} \int d^3p \ f(\vec{p})$$

$$\rho = \frac{g}{(2\pi)^3} \int d^3p \ E(\vec{p}) f(\vec{p})$$

$$E = \sqrt{|\vec{p}|^2 + m^2}$$

Phase-space distribution for one species in kinetic equilibrium (+ for FD, - for BE), $k_B=1$, μ chemical potential:

$$f(\vec{p}) = \frac{1}{e^{(E-\mu)/T} \pm 1}$$

Relativistic limit (T>>m) and T>> μ

$$\rho = \left(\frac{\pi^2}{30}\right) g T^4 \begin{cases} 1 \text{ (Bose - Einstein)} \\ \frac{7}{8} \text{ (Fermi - Dirac)} \end{cases}$$

$$n = \frac{\zeta(3)}{\pi^2} g T^3 \begin{cases} 1 \text{ (Bose - Einstein)} \\ \frac{3}{4} \text{ (Fermi - Dirac)} \end{cases} \zeta(3) = 1.202 \cdots$$

Exercise: compute the number of CMB photons (T=2.73 K) in 1 cm³

Non-relativistic limit (T<<m) and μ =0 [same for B-E and F-D]

$$n = g \left(\frac{mT}{2\pi}\right)^{3/2} e^{-m/T}$$

$$\rho = mn$$

Exponential Boltzmann suppression

For more than one species: density of relativistic particles in the Universe is set by the effective number of relativistic degrees of freedom g_{*}:

$$\rho_r = \frac{\pi^2}{30} g_* T^4$$

$$g_* = \sum_{\text{bosons}} g_i \left(\frac{T_i}{T}\right)^4 + \sum_{\text{fermions}} \frac{7}{8} g_i \left(\frac{T_i}{T}\right)^4$$

We assumed that in principle the particles can have a different temperature than T (the photon's temperature). If they are in chemical equilibrium then same T.

 $g_*(T)$ changes when mass thresholds are crossed as T decreases and particles become non-relativistic. At high T (>200 GeV) $g_*^{(SM)} \sim 100$.

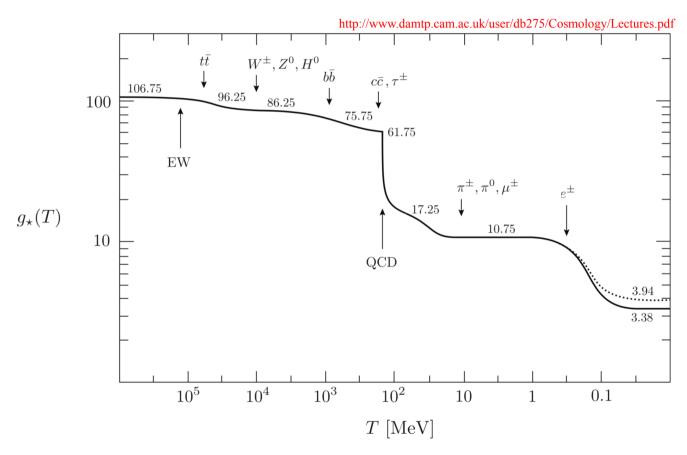


Figure 3.4: Evolution of relativistic degrees of freedom $g_{\star}(T)$ assuming the Standard Model particle content. The dotted line stands for the number of effective degrees of freedom in entropy $g_{\star S}(T)$.

II.2.2 – Relation between scale fator and temperature

Naivelly one can find the relation between the scale fator and the temperature as:

$$\rho_r \propto T^4$$
; $\rho_r \propto a^{-4} \Rightarrow a \propto T^{-1}$

But more precisely, conservation of entropy implies:

$$T \propto g_*^{-1/3} a^{-1}$$

When a mass threshold of a particle species is crossed, those particles annihilate into photons and increase the temperature.

II.2.3 – Temperature-time relationship

From Friedmann's 1st equation for a radiation-dominated era:

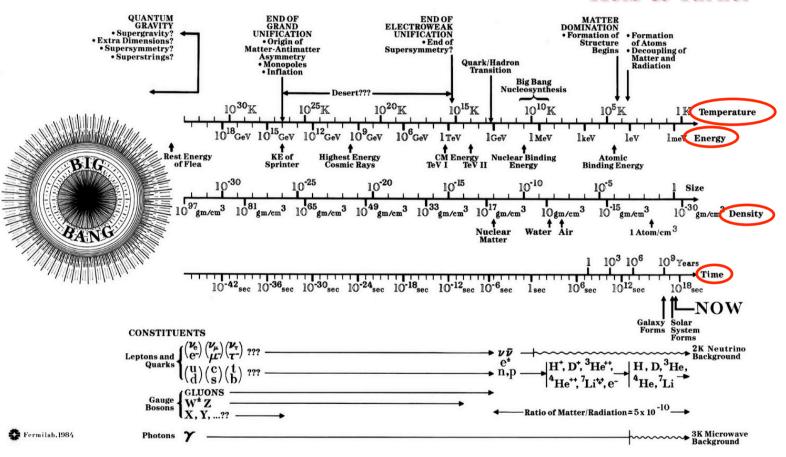
$$H = \sqrt{\frac{\rho_r}{3\tilde{M}_{\rm Pl}^2}} \sim \frac{T^2}{M_{\rm Pl}}$$

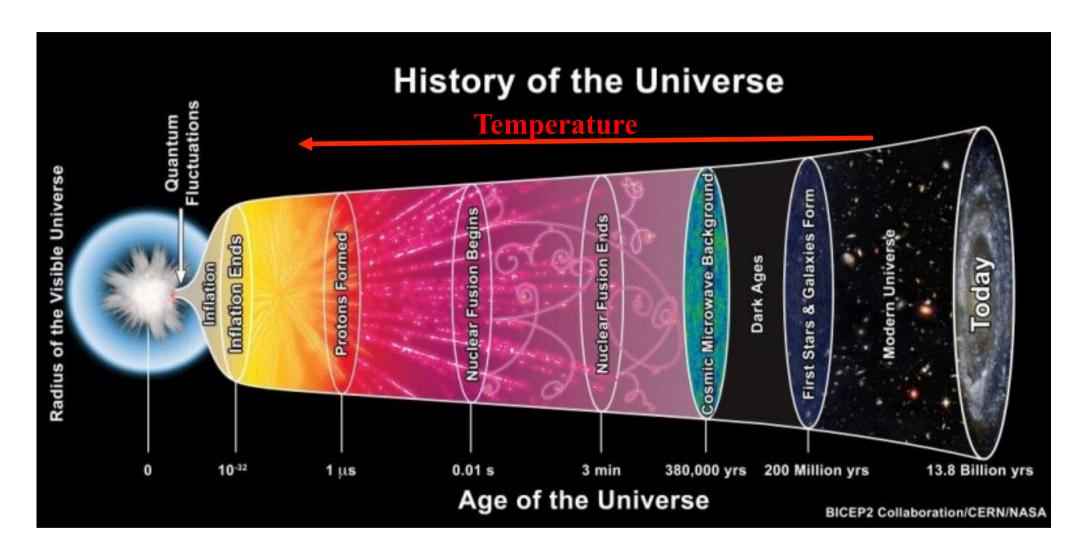
and
$$H=\frac{\dot{a}}{a}\propto t^{-1}$$
 one finds: $T\propto t^{-1/2}$

Putting numbers:
$$T(\text{MeV}) \simeq 1.5 g_*^{-1/4} t(\text{s})^{-1/2}$$

Thermal history of the Universe

Kolb & Turner





II.2.4 – The origin of the cosmic microwave background

When the universe is very hot there are no atoms! It is fully ionized. It is opaque to EM radiation. Light and matter are tightly coupled.

Decoupling of light occurs when the Universe cools down - protons and eletrons can combine to form hydrogen atoms: recombination epoch.

The cosmic microwave background (CMB) is generated after photons decouple: last scattering surface.

Naivelly one may think that this happens at a temperature T_{binding}=13.6 eV

However, one must take into account that there are many more photons than protons in the Universe!

One should study the reaction:

$$H + \gamma \longleftrightarrow e^- + p^+$$

and find at what temperature hydrogen stops being destroyed by photons. Correct treatment is to analyze a Boltzmann equation.

Here we will use a simpler, more physical estimate: find the redshift (temperature) at which the number density of photons with energy larger than the hydrogen ionization energy (E_i)equal the number density of protons:

$$n_{\gamma}^{E>E_i}(z) = n_p(z)$$

I find
$$T_{rec} = 0.47 \text{ eV}$$

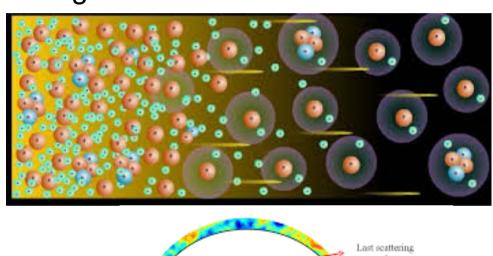
Correct result: $T_{rec} = 0.30 \text{ eV} = 0.30/8.6 \cdot 10^{-5} = 3500 \text{ K}$

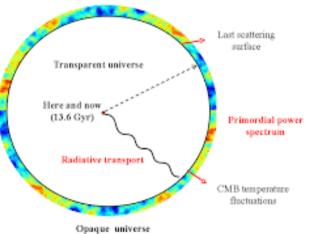
$$z_{rec} = T_{rec}/T_0 = 1300$$

The decoupling of photons occurs shortly afterwards: CMB generated at $z_{\rm dec}$ = 1100 when the universe was 380,000 years old.

Afterwards photons free stream across the universe without any significant interactions cooling as $T \propto a^{-1}$

After decoupling the Universe is neutral and becomes transparente to photons: "last scattering suface" at z=1100.





The comoving sound horizon at decoupling: r_s

$$r_s = \int_{z_{dec}}^{\infty} \frac{c_s}{H(z)}$$

This is the standard ruler measured in CMB and BAO

c_s: speed of perturbations in the coupled baryon-photon fluid – for relativistic fluids

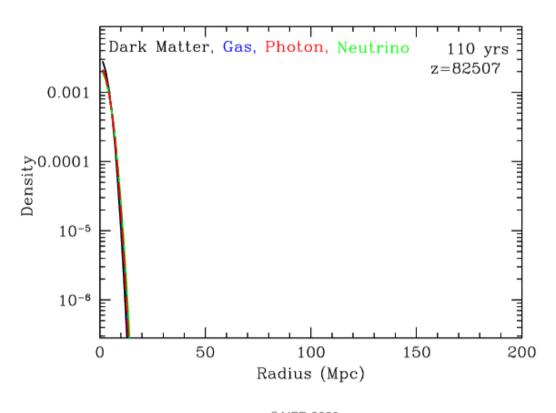
 $c_s = \frac{\delta P}{\delta \rho} = \frac{1}{\sqrt{2}}$

Exercise: show that $r_s \sim 150 \text{ Mpc}$

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Evolution of a perturbed region with dark matter, gas, photons and neutrinos

Eisenstein



SAIFR 2020 41

II.3 – The Hubble tension (crisis)

The comoving sound horizon at decoupling (z~1100) sets a physical scale in the Universe, both in the fluctuations of the CMB and the BAO.

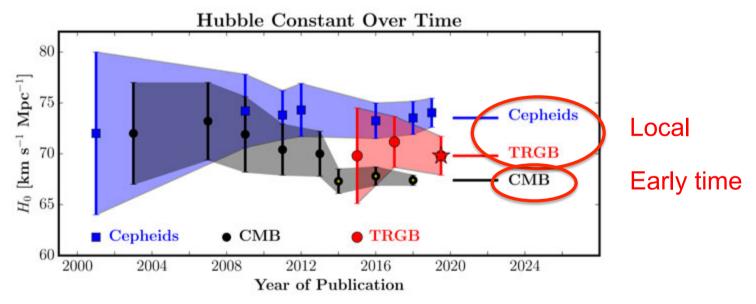
A measurement of the angular distance related to this scale can also allow for an indirect (model dependent) estimate of the Hubble constant.

This indirect measurement performed by the Planck satellite is very precise.

Greater precision brings greater possibilities for disagreement. In fact, there is a 4-6 σ tension with the local measurements.

There was a recent revolution in the measurement of H_0 with great precision (~1%)! The **Hubble tension** ~4-6 σ !

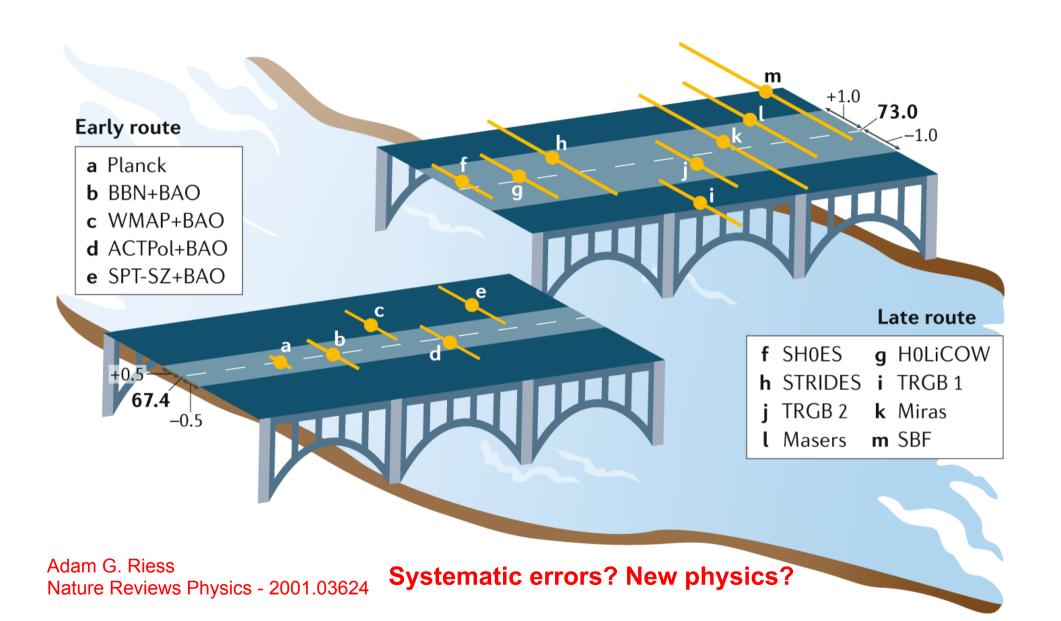
First crack in the standard Λ CDM model?



Freedman et al.,1907.05922

arXiv Search...
Help | Advance

Showing 1–50 o 750 results for all: hubble tension



II.3.1 – The CMB measurement

CMB provides an indirect (model dependent) estimate of the Hubble constant. It's a result of a complicated fit to the CMB angular power spectrum with several parameters.

However, we can have a rough idea by looking at the physical quantity measured by Planck satellite: the angular acoustic horizon scale of the CMB fluctuations, θ_* (~1°)

The angular acoustic scale is measured with high precision (0.03%) by Planck:

 $100 \theta_* = 1.0411 \pm 0.0003$

$$\theta_* = \frac{r_s}{d_A(z_{dec})}$$

$$r_{S}(z_{dec}) = \int_{z_{dec}}^{\infty} \frac{c_{S}dz}{H(z)} -$$

Early times

(expansion rate around decoupling era)

$$d_A(z_{dec}) = \int_0^{z_{dec}} \frac{c \, dz}{H(z)}$$

Late times

(expansion rate around present era)

Obs: comoving angular diameter distance

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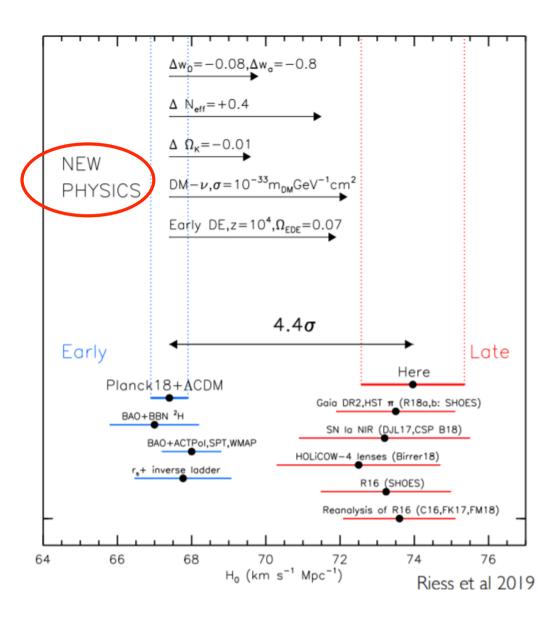
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Attempts to reconcile CMB with local measurements: New Physics!

If there is an extra contribution to the energy density (=faster expansion rate) with respect to Λ CDM around the recombination era then in order to keep θ_* fixed requires a larger value of H_0

New relativistic degrees of freedom, early dark energy, decaying dark matter,...

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Models abound:

In the Realm of the Hubble tension – a Review of Solutions 2103.01183 E. Di Valentino, O. Mena, S. Pan, L. Visinelli, W. Yang, A. Melchiorri, D. F. Mota, A. G. Riess, J. Silk

tension $\leq 1\sigma$ "Excellent models"	tension $\leq 2\sigma$ "Good models"	tension $\leq 3\sigma$ "Promising models"
Early Dark Energy [228, 235, 240, 250]	Early Dark Energy [212, 229, 236, 263]	DE in extended parameter spaces [289]
Exponential Acoustic Dark Energy [259]	Rock 'n' Roll [242]	Dynamical Dark Energy [281, 309]
Phantom Crossing [315]	New Early Dark Energy [247]	Holographic Dark Energy [350]
Late Dark Energy Transition [317]	Acoustic Dark Energy [257]	Swampland Conjectures [370]
Metastable Dark Energy [314]	Dynamical Dark Energy [309]	MEDE [399]
PEDE [394]	Running vacuum model [332]	Coupled DM - Dark radiation [534]
Vacuum Metamorphosis [402]	Bulk viscous models [340,341]	Decaying Ultralight Scalar [538]
Elaborated Vacuum Metamorphosis [401, 402]	Holographic Dark Energy [350]	BD-ΛCDM [852]
Sterile Neutrinos [433]	Phantom Braneworld DE [378]	Metastable Dark Energy [314]
Decaying Dark Matter [481]	PEDE [391,392]	Self-Interacting Neutrinos [700]
Neutrino-Majoron Interactions [509]	Elaborated Vacuum Metamorphosis [401]	Dark Neutrino Interactions [716]
IDE [637, 639, 657, 661]	IDE [659, 670]	IDE [634–636, 653, 656, 663, 669]
DM - Photon Coupling [685]	Interacting Dark Radiation [517]	Scalar-tensor gravity [855, 856]
$f(\mathcal{T})$ gravity theory [812]	Decaying Dark Matter [471, 474]	Galileon gravity [877,881]
BD-ΛCDM [851]	DM - Photon Coupling [686]	Nonlocal gravity [886]
Über-Gravity [59]	Self-interacting sterile neutrinos [711]	Modified recombination [986]
Galileon Gravity [875]	f(T) gravity theory [817]	Effective Electron Rest Mass [989]
Unimodular Gravity [890]	Über-Gravity [871]	Super ACDM [1007]
Time Varying Electron Mass [990]	VCDM [893]	Axi-Higgs [991]
∆CDM [995]	Primordial magnetic fields [992]	Self-Interacting Dark Matter [479]
Ginzburg-Landau theory [996]	Early modified gravity [859]	Primordial Black Holes [545]
Lorentzian Quintessential Inflation [979]	Bianchi type I spacetime [999]	
Holographic Dark Energy [351]	$f(\mathcal{T})$ [818]	

Table B2. Models solving the H_0 tension with R20 within 1σ , 2σ and 3σ considering *Planck* in combination with additional cosmological probes. Details of the combined datasets are discussed in the main text.

Jury is still out on the possible solutions to the Hubble tension or crisis... lots of works New physics vs Systematic errors

End of second lecture

