## COSMOLOGY AND PARTICLE PHYSICS

Rogerio Rosenfeld IFT-UNESP/ICTP-SAIFR/LIneA

Dark Energy Survey & LSST







#### Some references:

Baumann's lectures: https://cmb.wintherscoming.no/pdfs/baumann.pdf and 1807.03098 Cline's lectures: 1807.08749 Rubio's lecture notes: https://javierrubioblog.com/teaching/grcourse/

Books:

S. Weinberg: Gravitation and Cosmology (1973); Cosmology (2008)

- E. Kolb and M. Turner: The Early Universe (1994)
- J. Peebles: Principles of Physical Cosmology (1993)
- S. Dodelson and Fabian Schmidt Modern Cosmology (2020)

<u>Three lectures on Cosmology</u> <u>and Particle Physcics</u>

 Lecture I: Dynamics of the average Universe Lecture II: Distances and thermal history Lecture III: Neutrinos in cosmology

## **Plan for Lecture I:**

- I.0 Introduction and motivation
- I.1 Brief review of GR
- I.2 Dynamics of the Universe

"Our whole universe was in a hot dense state Then nearly fourteen billion years ago expansion started"



## **I.O-Introduction**

Why should a particle physicist learn cosmology?

 main evidences from BSM comes from cosmology: dark matter, dark energy, inflation;

• particle physics affect cosmology: eg origin of matter-anti-matter asymmetry, Higgs as inflaton (Higgsinflation), neutrinos and the formation of structures, phase transitions, vacum (in)stability;

 cosmology affects particle physics: eg evolution of the Universe may be responsible for solution of the strong CP problem (axion) and the breaking of electroweak symmetry breaking (relaxion idea). • early Universe is a testbed for SM and BSM: stability or metastability of SM vacuum, new physics tests from CMB, inflation, matterantimatter asymmetry, primordial non-gaussianity, primordial gravitational waves, stochatic gravitational waves from phase transitions (GUT, etc), "cosmological collider physics", modified gravity, ...

• gravity (geometry) may play an important role in particle physics: eg models with warped extra dimensions

- new particles from geometry: KK excitations, radion, etc
- models with extra dimensions can change the evolution of the Universe (and hence be tested).

Standard Model of Particle Physics works fine but it is unsatisfactory (neutrino masses, dark matter, hierarchy problem, etc). Beyond SM!

Standard Model of Cosmology ( $\Lambda$ CDM) works fine but it is unsatisfatory (value and nature of  $\Lambda$ ). Beyond  $\Lambda$ CDM!

Models abound! We have to see what Nature has chosen...

Cosmology has recently become a data driven science. Era of precision cosmology!

 $t_{U}$  = (13.799±0.021)x10<sup>9</sup> years [used to be 10<sup>9±1</sup> years]

Many experiment are taking a huge amount of data that are being analyzed in order to find out which model best describes the universe.

Cosmology became a respectable Science due in great part to Jim Peebles.

#### The Nobel Prize in Physics 2019



© Nobel Media. Photo: A. Mahmoud James Peebles Prize share: 1/2

© Nobel Media. Photo: A. Mahmoud Michel Mayor Prize share: 1/4



© Nobel Media. Photo: A. Mahmoud Didier Queloz Prize share: 1/4

The Nobel Prize in Physics 2019 was awarded "for contributions to our understanding of the evolution of the universe and Earth's place in the cosmos" with one half to James Peebles "for theoretical discoveries in physical cosmology", the other half jointly to Michel Mayor and Didier Queloz "for the discovery of an exoplanet orbiting a solar-type star."



## Several cosmological probes

- Cosmic Microwave Background (CMB)
- Big bang nucleosynthesis (BBN)
- Supernovae (type la)
- Baryon acoustic oscilation (BAO)
- Gravitational lensing
- Number count of clusters of galaxies

## **Cosmological probes**





## We know that we don't know what 95% of the Universe is made of:

#### What is dark matter?

Cold, warm, fuzzy, self-interacting...

#### What is dark energy?

New degree of freedom/MG: Quintessence, galileon, f(R), Hordensky, beyond Hordensky, massive gravity, EFTofDE... Does it interact with matter? Does it cluster?

# ky, IDPASC 2021 0.005% Neutrinos 0.0034% 15

#### **Universe today**

## Cosmology in PDG!

P.A. Zyla <i>et al.</i> (Particle Data Group), Prog. Theor. Exp. Phys. 2020, 083C01 (2020). Files can be downloaded directly by clicking on the icon: (III). Expand/Collapse All		
	Introduction, History plots, Online information Constants, Units, Atomic and Nuclear Properties	
	Standard Model and Related Topics	•
	Astrophysics and Cosmology	
<b>`</b>	Experimental tests of gravitational theory (rev.)	101
	Big-Bang cosmology (rev.)	101
	Inflation (rev.)	101
1	Big-Bang nucleosynthesis (rev.)	101
	Cosmological parameters (rev.)	101
	Neutrinos in cosmology (rev.)	101
~	Dark matter (new)	101
	Dark energy (rev.)	101
-	Cosmic microwave background (rev.)	101
	Cosmic rays (rev.)	101
	Experimental Methods and Colliders	
	Mathematical Tools	
	Kinematics, Cross-Section Formulae, and Plots	
	Particle Properties (See also Hypothetical Particles and Concepts below.)	
	Hypothetical Particles and Concepts	

## I.1- Brief Review of GR

General Relativity rules the Universe at large scales! Classical description is sufficient in most cases.

#### I.1.0 – Classical field theory in a nutshell



#### I.1.1 – Fundamental degrees of freedom

Fundamental field of gravity: the metric  $g_{\mu\nu}$ 

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$

Symmetric 4x4 matrix: 10 degrees of freedom (not all physical!)

$$g_{\mu\alpha}g^{\alpha\nu} = \delta^{\nu}_{\mu}$$
$$g_{\mu\nu}g^{\mu\nu} = 4$$

18

Flat space-time – Minkwoski metric

$$\eta_{\mu\nu} = \begin{pmatrix} 1 & & \\ & -1 & \\ & & -1 \\ & & & -1 \end{pmatrix} \qquad p_{\mu}p^{\mu} = E^2 - (\vec{p})^2$$
IDPASC 2021

#### I.1.2 – Einstein-Hilbert action

$$S_{\rm E-H}[g_{\mu\nu}] = \frac{1}{16\pi G} \int d^4x \sqrt{-g} \ R[g_{\mu\nu}]$$

For the Hilbert-Einstein dispute see: L. Corry, J. Renn, and J. Stachel, Science 278, 1270 (1997) F. Winterberg, Z. Naturforsch. **59a**, 715 – 719 (2004)

• Action is invariant under general coordinate transformations:

$$x^{\mu} \to x'^{\mu}(x^{\mu})$$

- $R[g_{\mu\nu}]$  is the Ricci scalar: second order in derivatives of the metric
- $g = \det(g_{\mu\nu})$

• G: Newton's constant

$$G = \frac{1}{M_{\rm Pl}^2} \ (\hbar = c = 1)$$
  
 $M_{\rm Pl} = 1.2 \times 10^{19} \, {\rm GeV}$ 

Obs.: sometimes the *reduced* Planck mass is used:

$$\tilde{M}_{\rm Pl} = \frac{M_{\rm Pl}}{\sqrt{8\pi}} = 2.4 \times 10^{18} \,\,{\rm GeV}$$

IDPASC 2021

20

• Dimensional analysis:  $(\hbar = c = 1)$ 

 $[g]: \text{ dimensionless; } [R]: E^{2}$  $[d^{4}x]: E^{-4}; \ [S]: \text{ dimensionless} \qquad \Rightarrow [G]: E^{-2}$  $G = \frac{1}{M_{\text{Pl}}^{2}}$ 

• Einstein equation in vacuum ( $T_{\mu\nu}$  = 0) obtained from:

$$\frac{\delta S_{\rm E-H}}{\delta g_{\mu\nu}} = 0$$

#### I.1.3 – The cosmological constant

February 1917 (~100 years ago): Einstein's "Cosmological Considerations in the General Theory of Relativity" introduces the cosmological constant in the theory without violating symmetries: a new constant of Nature!

It has an "anti-gravity" effect (repulsive force) and it was introduced to stabilize the Universe.

$$S_{\rm E-H} + S_{\Lambda} = \int d^4x \sqrt{-g} \left(\frac{R}{16\pi G} - \Lambda\right)$$

With the discovery of the expansion of the Universe (Hubble, 1929) it was no longer needed – "my biggest blunder".

## <u>George Gamow – My Worldline</u>

correct, and changing it was a mistake. Much later, when I was discussing cosmological problems with Einstein, he remarked that the introduction of the cosmological term was the biggest blunder he ever made in his life. But this "blunder," rejected by Einstein, is still sometimes used by cosmologists even today, and the cosmological constant denoted by the Greek letter  $\Lambda$  rears its ugly head again and again and again.

#### I.1.4 – Adding matter/radiation to the Universe

Matter/radiation is described by fields in a lagrangian:

$$S_{\text{matter}} = \int d^4x \sqrt{-g} \,\mathcal{L}_{\text{matter}}$$

Examples:

Electromagnetism: 
$$S_{\rm EM} = -\frac{1}{4} \int d^4 x \sqrt{-g} \ g^{\mu\alpha} g^{\nu\beta} F_{\mu\nu} F_{\alpha\beta}$$
  
Real scalar field:  $S_{\phi} = \int d^4 x \sqrt{-g} \left[ \frac{1}{2} g^{\alpha\beta} \partial_{\alpha} \phi \partial_{\beta} \phi - V(\phi) \right]$ 

#### I.1.5 – Energy-momentum tensor

Matter/radiation in GR is described by na energy-momentum tensor.

Definition:

$$\delta S_{\text{matter}} = \frac{1}{2} \int d^4 x \sqrt{-g} \ T^{\mu\nu}(x) \delta g_{\mu\nu}$$

which implies

$$T^{\mu\nu}(x) = \frac{2}{\sqrt{-g}} \frac{\delta S_{\text{matter}}}{\delta g_{\mu\nu}}$$

#### I.1.6 – Einstein's equation

Einstein's equation for GR is obtained from the requirement:

$$\delta\left(S_{\text{total}}\right) = \delta\left(S_{\text{E-H}} + S_{\Lambda} + S_{\text{matter}}\right) = 0$$



10 nonlinear differential equations. In general it must be solved numerically, eg gravitational waves from coalescence of binary black holes.

Details of Einstein's equation:

 Christoffel symbols (aka metric conection, affine connection) – first derivative of the metric :

$$\Gamma^{\mu}_{\alpha\beta} = \frac{1}{2}g^{\mu\nu} \left\{ \frac{\partial g_{\alpha\nu}}{\partial x^{\beta}} + \frac{\partial g_{\beta\nu}}{\partial x^{\alpha}} - \frac{\partial g_{\alpha\beta}}{\partial x^{\nu}} \right\}$$

• Ricci tensor – second derivative of the metric:

$$R_{\mu\nu} = \frac{\partial}{\partial x^{\alpha}} \Gamma^{\alpha}_{\mu\nu} - \frac{\partial}{\partial x^{\nu}} \Gamma^{\alpha}_{\alpha\mu} + \Gamma^{\alpha}_{\mu\nu} \Gamma^{\beta}_{\alpha\beta} - \Gamma^{\beta}_{\alpha\mu} \Gamma^{\alpha}_{\beta\nu}$$

• Ricci scalar:  $R = g^{\mu\nu} R_{\mu\nu}$ 

27

27

## **Standard Cosmological Model**



#### I.1.7 – Modified gravity

Modified gravity is anything different from E-H (+  $\Lambda$ ) action (see 1601.06133)

Example: f(R) theories (see 1002.4928)

$$S[g_{\mu\nu}] = \frac{1}{16\pi G} \int d^4x \sqrt{-g} \ f(R)$$

$$f(R) = a_0 + a_1 R + a_2 R^2 + \dots + \frac{\alpha_1}{R} + \frac{\alpha_2}{R^2} + \dots$$
cosmological GR Higher order derivatives

Issues with modified gravity:

introduces new light degrees of freedom – new forces
 Since there are stringent constraints one has to invoke "screening mechanisms" – chameleon, symmetron, Vainshtein, ...

• may have classical instabilities due to higher derivatives in equations of motion (Ostrogradski instabilities)

• may have quantum instabilities - "ghosts"

 may be brought in the form of GR with a suitable change of coordinates (Jordan frame -> Einstein frame) introducing nonstandard couplings in the matter sector

• search for MG: use simple parametrizations



#### Ezquiaga and Zumalacárregui - 1807.09241

### I.2- Dynamics of the Universe

## I.2.1 – Friedmann-Lemaître-Robertson-Walker metric

Universe is spatially homogeneous and isotropic on average.

It is described by the FLRW metric (for a spatially flat universe):

$$ds^{2} = dt^{2} - a(t)^{2} \left[ dx^{2} + dy^{2} + dz^{2} \right]$$
$$g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -a^{2} & 0 & 0 \\ 0 & 0 & -a^{2} & 0 \\ 0 & 0 & 0 & -a^{2} \end{pmatrix}$$

FLRW metric is determined by one time-dependent function: the so-called scale factor **a**(t). Distances in the universe are set by the scale factor.

Scale factor is the key function to study how the average universe evolves with time.

convention: a=1 today

OBS: conformal time (light cone has the usual 45<sup>o</sup> angle).

$$d\eta = \frac{dt}{a(t)} \qquad ds^2 = a^2(t)[d\eta^2 - d\vec{r}^2]$$

1.

#### Average evolution of the universe

- specified by <u>one function</u>: scale factor a(t)

- determines measurement of large scale distances, velocities and acceleration

a(t)  $\dot{a}(t)$   $\ddot{a}(t)$ 

- measured through standard candles (SNIa's) and standard rulers (position of CMB peak, BAO peak,...)

Redshift z:

$$a(t) = \frac{1}{1+z} \qquad z=0 \text{ today.}$$

## **Expansion of the universe**

Hubble parameter: Expansion rate of the universe  $H = \frac{\dot{a}(t)}{a(t)}$ Hubble constant: Hubble parameter today (H<sub>0</sub>)

Analogy of the expansion of the universe with a balloon:





Space itself expands and galaxies get a free "ride".

For a spatially flat FLRW metric the Ricci tensor and the Ricci scalar are given by:

$$R_{00} = -3\frac{\ddot{a}}{a}; R_{ii} = a\ddot{a} + 2\dot{a}^2$$
$$R = -6\left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2}\right)$$
# I.2.2 – The right-hand side of Einstein equation: the energy-momentum tensor simplified

It is usually assumed that one can describe the components of the Universe as "perfect fluids": at every point in the medium there is a locally inertial frame (rest frame) in which the fluid is homogeneous and isotropic (consistent with FLRW metric):



Homegeneity: density and pressure depend only on time.

Energy-momentum in the rest frame (indices are important):

$$T^{\mu}_{\nu} = \begin{pmatrix} \rho & 0 & 0 & 0 \\ 0 & -P & 0 & 0 \\ 0 & 0 & -P & 0 \\ 0 & 0 & 0 & -P \end{pmatrix}$$

In a frame with a given 4-velocity:

$$T^{\mu\nu} = -Pg^{\mu\nu} + (\rho + P)u^{\mu}u^{\nu}$$
$$u^{\mu} = \gamma (1, \vec{v})$$

[Imperfect fluids: anisotropic stress, dissipation, etc.]

# I.2.3 – Solving Einstein equation for the average Universe: Friedmann's equations

00 component:

$$R_{00} - \frac{1}{2}g_{00}R = 8\pi G T_{00} \implies \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho$$

1st Friedmann equation Expansion rate is determined by energy density.

ii component:

$$R_{ii} - \frac{1}{2}g_{ii}R = 8\pi GT_{ii} \implies$$
$$\left(\frac{\dot{a}}{a}\right)^2 + 2\frac{\ddot{a}}{a} = -8\pi GP \implies$$
$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P)$$

2nd Friedmann equation (De)acceleration is determined by energy density and pressure.

IDPASC 2021

40

## I.2.4 – Evolution of different fluids

Taking a time derivative of 1st Friedmann equation one arrives at the "continuity" equation :

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + P) = 0$$

Also follows from the conservation of the energy-momentum tensor and also from the 1st law of thermodynamics:

$$dU = -PdV, U = \rho V, V \propto a^{-3}$$

In order to study the evolution of a fluid we need to postulate a relation between density and pressure: the equation of state

Assume a simple relation:

 $P=\omega\rho$ 

 $\boldsymbol{\omega}$  is called the equation of state parameter.

Examples:

- Non-relativistic matter (dust):  $P \ll \rho \longrightarrow \omega = 0$
- Relativistic matter (radiation):  $\omega = 1/3$

• Cosmological constant:  $\omega = -1$ 

$$T^{\mu}_{\nu,\Lambda} = \begin{pmatrix} \Lambda & 0 & 0 & 0 \\ 0 & \Lambda & 0 & 0 \\ 0 & 0 & \Lambda & 0 \\ 0 & 0 & 0 & \Lambda \end{pmatrix} = \begin{pmatrix} \rho & 0 & 0 & 0 \\ 0 & -P & 0 & 0 \\ 0 & 0 & -P & 0 \\ 0 & 0 & 0 & -P \end{pmatrix}$$

From the continuity equation it is easy to show that the evolution of the energy density for a constant equation of state is:

$$\rho(t) = \rho(t_i) \left(\frac{a(t)}{a(t_i)}\right)^{-3(1+\omega)}$$

OBS: It's easy to generalize to a time-dependente equation of state

- Non-relativistic matter (dust):
- Relativistic matter (radiation):
- Cosmological constant:

 $ho \propto a^{-3}$   $ho \propto a^{-4}$   $ho \propto a^0$ 



## I.2.5 – Time evolution of the scale fator (expansion history)

Using 1st Friedmann equation and the result from last section:

$$\frac{\dot{a}}{a} \propto \sqrt{\rho}, \ \rho \propto a^{-3(1+\omega)}$$

it is easy to show that:

$$a(t) \propto t^{\frac{2}{3(1+\omega)}} = \begin{cases} t^{2/3} \text{ (matter)} \\ t^{1/2} \text{ (radiation)} \end{cases}$$

but for the case of a cosmological constant one has an exponential growth:  $\frac{\dot{a}}{a} = {\rm const.} = H o a(t) \propto e^{Ht}$ 

Exponential growth: universe is accelerating!

2nd Friedmann equation is (for w=-1):

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P) = \frac{8\pi G}{3}\rho > 0$$

## I.2.6 – Inflation

We think that the very early universe went through a phase of exponential expansion called inflation (Guth, Linde, Starobinsky, ... early 1980's).

We do not know what drove inflation. The simplest model involves a new scalar field: the inflaton.

During inflation the matter and radiation contents were rapidly diluted to ~nothing.

Any spatial curvature was erased – explains why universe is flat.

Quantum fluctuations of the inflaton field provided the initial small perturbations in homogeneity that evolved to form structures in the universe. They also produced primordial gravitational waves.

Inflation ended and the universe "reheated".



## *Encyclopædia Inflationaris* http://arxiv.org/1303.3787

## Javier Rubio

## Jérôme Martin,<sup>a</sup> Christophe Ringeval<sup>b</sup> and Vincent Vennin<sup>a</sup>

#### 3 Zero Parameter Models

3.1 Higgs Inflation (HI)

#### 4 One Parameter Models

- 4.1 Radiatively Corrected Higgs Inflation (RCHI)
- 4.2 Large Field Inflation (LFI)
- 4.3 Mixed Large Field Inflation (MLFI)
- 4.4 Radiatively Corrected Massive Inflation (RCMI)
- 4.5 Radiatively Corrected Quartic Inflation (RCQI)
- 4.6 Natural Inflation (NI)
- 4.7 Exponential SUSY Inflation (ESI)
- 4.8 Power Law Inflation (PLI)
- 4.9 Kähler Moduli Inflation I (KMII)
- 4.10 Horizon Flow Inflation at first order (HF1I)
- 4.11 Colemann-Weinberg Inflation (CWI)
- 4.12 Loop Inflation (LI)
- 4.13  $(R + R^{2p})$  Inflation (RpI)
- 4.14 Double-Well Inflation (DWI)
- 4.15 Mutated Hilltop Inflation (MHI)
- 4.16 Radion Gauge Inflation (RGI)
- 4.17 MSSM Inflation (MSSMI)
- 4.18 Renormalizable Inflection Point Inflation (RIPI)
- 4.19 Arctan Inflation (AI)
- 4.20 Constant n<sub>s</sub> A Inflation (CNAI)
- 4.21 Constant n<sub>s</sub> B Inflation (CNBI)
- 4.22 Open String Tachyonic Inflation (OSTI)
- 4.23 Witten-O'Raifeartaigh Inflation (WRI)

#### 5 Two Parameters Models

- 5.1 Small Field Inflation (SFI)
- 5.2 Intermediate Inflation (II)
- 5.3 Kähler Moduli Inflation II (KMIII)
- 5.4 Logamediate Inflation (LMI)
- 5.5 Twisted Inflation (TWI)
- 5.6 Generalized MSSM Inflation (GMSSMI)
- 5.7 Generalized Renormalizable Point Inflation (GRIPI)
- 5.8 Brane SUSY breaking Inflation (BSUSYBI)
- 5.9 Tip Inflation (TI)
- 5.10  $\beta$  exponential inflation (BEI)
- 5.11 Pseudo Natural Inflation (PSNI)
- 5.12 Non Canonical Kähler Inflation (NCKI)
- 5.13 Constant Spectrum Inflation (CSI)
- 5.14 Orientifold Inflation (OI)
- 5.15 Constant n<sub>s</sub> C Inflation (CNCI)
- 5.16 Supergravity Brane Inflation (SBI)
- 5.17 Spontaneous Symmetry Breaking Inflation (SSBI)
- 5.18 Inverse Monomial Inflation (IMI)
- 5.19 Brane Inflation (BI)

#### 6 Three parameters Models

- 6.1 Running-mass Inflation (RMI)
- 6.2 Valley Hybrid Inflation (VHI)
- 6.3 Dynamical Supersymmetric Inflation (DSI)
- 6.4 Generalized Mixed Inflation (GMLFI)
- 6.5 Logarithmic Potential Inflation (LPI)
- 6.6 Constant n<sub>s</sub> D Inflation (CNDI)

The Universe started to accelerate a couple of billion years ago. Before that there was a period of normal decelerated expansion, essential for the formation of galaxies.



## I.2.7 – Recipe of the Universe

Critical density: density at which the Universe is spatially flat.

$$\rho_c = \frac{3H_0^2}{8\pi G}$$

Hubble constant today has been measured with some precision recently and there is a strong tension between measurements from Planck (early universe) and Supernovas (late universe) – more later.

Different contributions to the energy density budget of the Universe (i=baryons, photons, neutrinos, dark matter, dark energy,...)

$$\Omega_i = \frac{\rho_i}{\rho_c}$$

Spatially flat universe:

$$\sum_{i} \Omega_i = 1$$

1st Friedmann equation:

$$\frac{H(t)^2}{H_0^2} = \sum_i \Omega_i^{(0)} a^{-3(1+\omega_i)}$$

# Exercise 1: estimate the critical density in units of GeV/m<sup>3</sup> using the time-honored convention H<sub>0</sub> = 100 x h km/s/Mpc, where h~0.7. Also estimate ( $\rho_c$ )<sup>1/4</sup> in units of eV.

Exercise 2: compute  $H_{0}^{-1}$  in years and  $H_{0}$  in units of eV.

Exercise 3: assuming that inflation happened at an energy scale of 10<sup>12</sup> GeV and approximating the whole expansion history as radiation-dominated, estimate at what time inflation took place in the universe and the scale factor at that time.

Exercise 4: given a Universe with composition

$$\Omega_{\Lambda}^{(0)} = 0.7, \ \Omega_{\text{matter}}^{(0)} = 0.3, \ \Omega_{\text{rad}}^{(0)} = 5 \times 10^{-5}$$

a. estimate the redshift  $z_{eq}$ b. estimate the redshift  $z_{\Lambda}$ c. plot these different  $\Omega$ 's as a function of log(a) (see python colab notebook)



## End of first lecture

Curiosity: what happens if w<-1 ("phantom" dark energy)?

$$\omega = -1 - \delta \rightarrow \rho_{\text{phantom}} \propto a^{-3(1+\omega)} = a^{3\delta}$$

Density increases with time. It can be shown that there is a singularity where  $a \to \infty$  at finite time: the "big rip" [see astro-ph/0302506]

Exercise 1: Show that for a real scalar field

$$T^{\mu\nu}_{\phi} = \partial^{\mu}\phi\partial^{\nu}\phi - \mathcal{L}_{\phi}g^{\mu\nu}$$

Exercise 2: Show that for a cosmological constant

$$T^{\mu\nu}_{\Lambda} = \Lambda g^{\mu\nu}$$

Hint: Use Ln(det(A)) = tr(Ln A) to show that

$$\delta\sqrt{-g} = -\frac{1}{2}\sqrt{-g}g^{\mu\nu}\delta g_{\mu\nu}$$

142 Sitsung der physikalisch-mathematischen Klasse vom 8. Februar 1917

## 43. "Cosmological Considerations in the General Theory of Relativity"

## Kosmologische Betrachtungen zur allgemeinen Relativitätstheorie.

#### Von A. EINSTEIN.

[Einstein 1917b]

Es ist wohlbekannt, daß die Posseorsche Differentialgleichung

SUBMITTED 8 February 1917 PUBLISHED 15 February 1917

in Verbindung mit der Bewegungsgleichung des materiellen Punktes die Nuwrossche Fernwirkungstheorie noch nicht vollständig ersetzt. Es muß noch die Bedingung hinzutreten, daß im räumlich Unend-

IN: Königlich Preußische Akademie der Wissenschaften (Berlin). Sitzungsberichte (1917): 142–152.

#### § 4. On an Additional Term for the Field Equations of Gravitation

Poisson's equation given by equation (2). For on the lefthand side of field equation (13) we may add the fundamental tensor  $g_{\mu\nu}$ , multiplied by a universal constant,  $-\lambda$ , at present unknown, without destroying the general covariance. In place of field equation (13) we write

$$G_{\mu\nu} - \lambda g_{\mu\nu} = -\kappa (\mathbf{T}_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\mathbf{T}) \quad . \quad (13a)$$
IDPASC 2021

59

Some modified gravity theories predict that

$$\frac{\Delta c}{c} = \frac{c_g - c}{c} = \mathcal{O}(1)$$

In 2017 GW170817 was detected by LIGO and Virgo: the first detection of a merger of 2 neutron stars at a mere 130 million light-years from Earth.

As opposed to black hole mergers, it also emitted light!!



Also seen in gamma rays by Fermi and Integral with less than 2 seconds difference!!  $\Delta c \ \ c_g - c$ 

C

 $\boldsymbol{c}$ 





# Historical interlude

# 100 years of the eclipse that confirmed GR



## First test of GR: the solar eclipse in May 29, 1919

Two expeditions organized by the Royal Society: Principe Island in Africa (Eddington) Sobral in Brazil (Crommelin)





Figure 1. The eclipse observation equipment at Sobral. The troublesome coelostats can be seen in the foreground. Copyright Science Museum/Science and Society Picture Library. Inventory no. 1922-0277.



Equipes brasileira e inglesa em Sobral, entre outras pessoas. Henrique Morize é o quarto, em pé, da esquerda para a direita. Os astrônomos ingleses estão sentados: A.C.D. Crommelin é o quarto da esquerda para a direita; C.R. Davidson é o quinto (Observatório Nacional/MCT).

### IX. A Determination of the Deflection of Light by the Sun's Gravitational Field, from Observations made at the Total Eclipse of May 29, 1919.

By Sir F. W. DYSON, F.R.S., Astronomer Royal, Prof. A. S. Eddington, F.R.S., and Mr. C. DAVIDSON.

(Communicated by the Joint Permanent Eclipse Committee.)

Received October 30,-Read November 6, 1919.

V. GENERAL CONCLUSIONS.

39. In summarising the results of the two expeditions, the greatest weight must be attached to those obtained with the 4-inch lens at Sobral. From the superiority of the images and the larger scale of the photographs it was recognised that these would prove to be much the most trustworthy. Further, the agreement of the results derived inde-

332 SIR F. W. DYSON, PROF. A. S. EDDINGTON AND MR. C. DAVIDSON ON A

Thus the results of the expeditions to Sobral and Principe can leave little doubt that a deflection of light takes place in the neighbourhood of the sun and that it is of the amount demanded by EINSTEIN'S generalised theory of relativity, as attributable to the sun's gravitational field. But the observation is of such interest that it will probably be considered desirable to repeat it at future eclipses. The unusually favourable conditions of the 1919 eclipse will not recur, and it will be necessary to photograph fainter stars, and these will probably be at a greater distance from the sun. "After a careful study of the plates I am prepared to say that there can be no doubt that they confirm Einstein's prediction. A very definite result has been obtained that light is deflected in accordance with Einstein's law of gravitation." 06/11/1919



# LIGHTS ALL ASKEW

Men of Science More or Less Agog Over Results of Eclipse Observations.

## **EINSTEIN THEORY TRIUMPHS**

Stars Not Where They Seemed or Were Calculated to be, but Nobody Need Worry.

### A BOOK FOR 12 WISE MEN

No More in All the World Could Comprehend it, Said Einstein When. His Daring Publishers Accepted It.

**IDPASC 2021** 

Special Cable to THE NEW YORK TIMES. LONDON, Nov. 9.—Efforts made to put in words intelligible to the nonWhen Einstein visited Brazil in 1925, he declared to the local newspapers: "The idea that my mind conceived was proven in the sunny sky of Brazil."



Freundlich wrote Einstein that same night, offering to help develop ways to look for light bending near the Sun or the planet Jupiter. Back in Prague, Pollak told Einstein about the young Berlin astronomer, and Einstein gave him permission to send Freundlich proofs of his article. "Prof. Einstein has given me strict orders," wrote Pollak, "to inform you that he himself very much doubts that the experiments could be done successfully with anything except the Sun." He urged Freundlich "to send further reports to me, or perhaps to Prof. Einstein, about your views on an astronomical verification."<sup>18</sup>

As luck would have it, a visitor to the Berlin Observatory in November of that year opened for Freundlich another avenue of research on the problem. Charles Dillon Perrine, who had successfully resolved the "Vulcan problem" while at Lick Observatory, had left Lick in 1909 to become director of the Southern Hemisphere observatory in Cordoba. When Freundlich told him about Einstein's light-bending prediction, Perrine suggested that he write to various astronomers who might have old eclipse plates on which star images might be measured for deflection. Naturally, he mentioned the Lick Vulcan plates. Freundlich immediately drafted a circular letter, which he sent to several observatories, including Lick, asking for "support of astronomers, who possess eclipse-plates" to test Einstein's predicted light deflection by the Sun.<sup>21</sup>

In view of the likely unsuitability of the Vulcan plates for the task at hand, Campbell offered to lend the Vulcan cameras to Perrine to try Freundlich's problem at the eclipse in Brazil on October 9–10, 1912.

# Early attempt to test GR in 1912



Perrine agreed to enlarge his eclipse program to include Freundlich's investigation and to take the photographs himself. Campbell sent the lenses down via the astronomer William Joseph Hussey. Perrine left Bue-

Material cc

59

#### EARLY INVOLVEMENT, 1911–1914

nos Aires on September 13, 1912, and the eclipse took place on October 10. A few days later Campbell received a telegram from Edward C. Pickering of Harvard, the communication center for American astronomy: "Perrine cables from Brazil, rain."<sup>31</sup>







Centenário do Eclipse Solar em Sobral/CE


### End of historical interlude

Interlude: a quick tour to extra dimensions (geometry matters!) hep-ph/0404096 hep-ph/0503177 (goes back to Kaluza and Klein in the 1920's!)

1.Flat extra dimensions (ADD - Arkani-Hamed, Dimopoulos and Dvali - 1998): n extra flat space-like dimensions compactified in a n-dimensional torus

$$S_{E-H} = \frac{1}{16\pi G_*} \int d^{4+n} x \sqrt{-g_{4+n}} R_{4+n}$$
$$[G_*] : E^{-(n+2)}$$
$$ds^2 = g_{\mu\nu} dx^{\mu} dx^{\nu} - \delta_{ab} dy^a dy^b$$



Copyright 2000 Scientific American, Inc.

In this case one can show that

$$g_{4+n} = g_4 \qquad R_{4+n} = R_4$$

$$S_{E-H} = \frac{V_n}{16\pi G_*} \int d^4x \sqrt{-g_4} \ R_4$$

Effective Newton's constant is diluted by the volume of the extra dimensions:

$$G_N = \frac{G_*}{V_n}$$

Introducing a fundamental scale

$$M_* = \left(\frac{1}{G_*}\right)^{\frac{1}{2+n}} \qquad M_P^2 = M_*^{n+2} V_n$$

Exercise: assuming that  $V_n = L^n$ 

Compute the size of the extra dimension L assuming that the fundamental scale if 1 TeV (to be relevant at colliders) for n=1 ( $\sim$ 10<sup>11</sup>m) and n=2 ( $\sim$ 0.1mm).

2. Warped extra dimension (WED) – Randall and Sundrum (1999): one extra compact dimension y (5th dimension). Each point in the 5th dimension has a 4-dimensional Minkowski background. The metric can be parametrized as:

$$ds^{2} = e^{-2A(y)}\eta_{\mu\nu}dx^{\mu}dx^{\nu} - dy^{2}$$

where A(y) is the curvature along the fifth dimension and is called the warp factor.

$$0 \le y \le L$$

Let's consider the gravity action with a bulk cosmological constant:

$$S = \int d^4x \, dy \, \sqrt{-g_5} \left(\frac{1}{2k_*^2}R_5 - \Lambda_5\right)$$

from which Einstein's equations follow:

$$G_{MN} = R_{MN} - \frac{1}{2}g_{MN}R_5 = k_*^2\Lambda_5 g_{MN}$$
  
[\Lambda\_5] = E<sup>5</sup>  
[R\_5] = E<sup>2</sup>  
[k\_\*^2] = E^{-3}  
 $k_*^2 = 8\pi G_* \equiv M_*^{-3}$ 

Solving 5-5 componente of Einstein's equations:

$$G_{55} = -k_*^2 \Lambda_5 g_{55}$$

I tried EinsteinPy and MAXIMA without success. This is GR package for Mathematica 1 warped extra dimension (Randall-Sundrum)





IDPASC 2021

Warping solves the hierarchy problem without large number (fine tuning):

$$M_{EW} = e^{-\kappa L} M_{Pl} \Rightarrow \kappa L \simeq 30$$

New particles are predict in models with exta dimensions: Kaluza-Klein excitations or resonances. More details in Chacko's lectures.

End of this interlude – back to 4 dimensions for the rest of these lectures

82

#### I.2.7 – Beyond $\Lambda$ : dynamical dark energy

For a real homogeneous scalar field the energy-momentum tensor gives:

$$T_{\phi}^{00} = \rho = \frac{1}{2}\dot{\phi}^{2} + V(\phi);$$
$$T_{\phi}^{ii} = -g^{ii}P = -\left(\frac{1}{2}\dot{\phi}^{2} - V(\phi)\right)g^{ii}$$

and therefore the time-dependent equation of state in this case is:

$$\omega(t) = \frac{P}{\rho} = \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)} \Rightarrow -1 \le \omega \le 1$$

If potential energy dominates w~-1 and scalar field resembles a cosmological constant: quintessence field. Can be ultralight (~ $H_0$ )!

### Some examples of dark energy models:

Cosmological Constant	$p_{\Lambda} = -\rho$	$\mathcal{D}_{\Lambda}$
Canonical Scalar Field: Quintessence	${\cal L}_Q = {1\over 2} \partial$	$\partial^{\mu}arphi\partial_{\mu}arphi-V\left(arphi ight)$
Perfect Fluid	$p_0 = w \rho_0$	$\delta p = c_{\rm eff}^2 \delta \rho$
Chaplygin Gas	$ \rho_{Ch} = -2 $	$A \rho_{Ch}^{-\alpha}$
K-essence	$\mathcal{L}=F\left(X,\varphi\right)$	$X = \frac{1}{2} \partial^{\mu} \varphi \partial_{\mu} \varphi$
e.g. Tachyon, Born-Infeld	$\mathcal{L}_T = V$ IDPASC 2021	$(\varphi)\sqrt{1-\partial^{\mu}\varphi\partial_{\mu}\varphi}$

84

## Klein-Gordon equation for a scalar field in an arbitrary metric

$$\mathcal{L}_{\phi} = \frac{1}{2} g^{\mu\nu} \partial_{\mu} \phi \partial_{\nu} \phi - V(\phi)$$
$$S_{\phi} = \int d^4 x \sqrt{-g} \mathcal{L}_{\phi}, \ \delta S_{\phi} = 0 \Rightarrow$$

$$\frac{1}{\sqrt{-g}}\partial_{\mu}[\sqrt{-g}\ \partial^{\mu}\phi] + \frac{dV}{d\phi} = 0$$

# Klein-Gordon equation for a homogeneous scalar field in FRWL metric

$$\sqrt{-g} = a^3(t)$$

$$\ddot{\phi} + 3H(t)\dot{\phi} + \frac{dV}{d\phi} = 0$$

### I.2.8 – Vacuum energy: the elephant in the room

Quantum mechanics – zero point energy of a harmonic oscillator:

$$E = \hbar\omega(n + 1/2)$$

In Quantum Field Theory, the energy density of the vacuum is (free scalar field of mass m):

$$\rho_{vac} = \int \frac{d^3k}{(2\pi)^3} \frac{1}{2} \sqrt{k^2 + m^2}$$

and is infinite! Integral must be cut-off at some physical energy scale - goes as (cut-off)<sup>4</sup>.

If integral is cutoff at the Planck scale, disagreement of ~  $10^{120}$  with data. This is know as the cosmological constant problem.

The Higgs field and the vacuum energy:



### Vacuum energy

 $\rho_{\Lambda} \propto \text{constant}$ 



89