TÉCNICO LISBOA

The Higgs Mechanism
The Standard Model & Multi-Higgs Models

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Topics for this Lecture

- Gauge invariance: Yang-Mills theories
- Spontaneous breaking of symmetry: simple examples
  - Complex scalar field, symmetry U(1)
- Higgs Mechanism
  - Complex scalar field, symmetry U(1)
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- The Standard Model: A quick tour
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  - The leptons and quarks. The mass of the fermions
- Multi-Higgs Models
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Yang-Mills Theories

- The same steps that lead us to QED can be followed for non-abelian groups of transformations.
- In this case we say that we have a non-abelian gauge theory, or *Yang-Mills Theory*, although strictly this name should only apply to the case of the group $\mathrm{SU}(2)$. As some of the concepts have to generalized let us go through the steps.
- We start with the Lagrangian for fermions, generalizing QED. We write,

$$\boxed{\mathcal{L} = \overline{\Psi}(i\gamma^\mu \partial_\mu - m)\Psi}$$

where Ψ is a vector column (do not forget that each ψ_i is a 4-component Dirac spinor)

$$\Psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \vdots \\ \psi_n \end{pmatrix}$$

in some internal space of dimension n

Yang-Mills Theories

- In this space acts a representation of the non-abelian group G

$$\boxed{\delta\Psi = i\varepsilon^a(x)\Omega^a\Psi\,, \quad a = 1, 2, \dots m}$$

Ω^a are m hermitian matrices $n \times n$ that obey the commutation for the group G,

$$\boxed{\left[\Omega^a, \Omega^b\right] = if^{abc}\Omega^c}$$

where f^{abc} are the structure constants of the group G. From this relation it results that m is the dimension of the adjoint representation of G, because this is also the number of generators of the algebra of G.

- The Lagrangian is not invariant under local transformations. To make it invariant we introduce the covariant derivative

$$\boxed{\partial_\mu \to D_\mu\Psi = (\partial_\mu + igA^a_\mu\Omega^a)\Psi}$$

where A^a_μ are m vector fields that will play the role that the photon had in QED. They are called *gauge fields*.

Yang-Mills Theories

- The matrices Ω^a are appropriate for the representation of Ψ with dimension n. The transformation law for A^a_μ is obtained by requiring that $D_\mu \Psi$ transforms as Ψ.
- The result is, for infinitesimal transformations,

$$\boxed{\delta A^a_\mu = -f^{bca}\ \varepsilon^b\ A^c_\mu - \frac{1}{g}\ \partial_\mu \varepsilon^a}$$

- Notice that for an abelian group this reduces to the case of the electromagnetism because $f^{abc} = 0$. In that case $m = 1$.
- Contrary to the abelian case, $F^a_{\mu\nu}$ is not invariant. In fact it transforms as a vector in the n dimensional space, that is,

$$\boxed{\delta F^a_{\mu\nu} = -f^{bca} \varepsilon^b F^c_{\mu\nu}}$$

See Problems

where

$$\boxed{F^a_{\mu\nu} \equiv \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g\ f^{bca} A^b_\mu A^c_\nu}$$

Yang-Mills Theories

- The transformation law for the gauge fields can be used to show that the generalization of the Maxwell Lagrangian,

$$\boxed{\mathcal{L}_{\rm YM} = -\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu}}$$

See Problems

is also invariant under local gauge transformations.

- Putting all the pieces together, we conclude that the generalization of the Lagrangian with local gauge invariance is

$$\boxed{\mathcal{L} = \overline{\Psi}(i\not{D} - m)\Psi - \frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu}}$$

- Let us make some comments on what we have just seen and look at applications.

Yang-Mills Theories: Some comments

- In first place the previous Lagrangian describes one physical theory for the fundamental laws of Nature, when the group G is SU(3). It is the so-called *Quantum Chromodynamics*, that describes the strong interactions
- In that theory the matter fields with spin $1/2$ are the *quarks* that belong to the fundamental representation of the group (triplets of SU(3)) and the gauge fields are the *gluons* that are in the adjoint representation, which is 8 for SU(3))
- Another observation has to do with the mass of the gauge fields. A mass term would have the form

$$\boxed{\mathcal{L}_{\text{massa}} = \frac{1}{2}\, m^2 A^a_\mu A^{a\mu}}$$

See Problems

  and it is easy to see that it is not gauge invariant. *Therefore gauge invariance requires that the photon and the gluons are massless*
- However for the weak interactions this brings a problem, because it is known that the vectors associated with the weak force should have a short range and therefore a finite mass. This problem took decades to solve and was only solved with the Higgs mechanism, as we will see later

Yang-Mills Theories: $G = \mathbf{SU}(2) \times U(1)$

- A final comment on the cases when the group is not a simple group, like

$$\boxed{G = \text{SU}(2) \times U(1)}$$

- The Lagrangian for the matter fields has the same form but now the covariant derivative is a sum over all the gauge fields with a different coupling constants. The gauge Lagrangian is the sum of the Lagrangians for each group factor.

$$\mathcal{L} = \sum_f \overline{\Psi}_f (i\not{D} - m)\Psi_f - \frac{1}{4} W^a_{\mu\nu} W^{a\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}$$

- The gauge fields are conventionally denoted by W^a_μ for SU(2) and B_μ for U(1), with the field tensors given by ($f^{bca} = \epsilon^{abc}$, $a = 1, 2, 3$)

$$W^a_{\mu\nu} \equiv \partial_\mu W^a_\nu - \partial_\nu W^a_\mu - g\ f^{bca} W^b_\mu W^c_\nu \qquad B_{\mu\nu} \equiv \partial_\mu B_\nu - \partial_\nu B_\mu$$

- The covariant derivative has a contribution of each gauge field with a different coupling constant. We have therefore

$$D_\mu \Psi = (\partial_\mu + igW^a_\mu \frac{\sigma^a}{2} + ig' \frac{Y}{2} B_\mu)\Psi$$

$\Omega^a = \frac{\sigma^a}{2}$ for SU(2)
Y the hypercharge for U(1)

Spontaneous symmetry breaking (SSB): U(1)

- Most of the symmetries observed in Nature are not exact. For instance the *Isospin* is not exact because the neutron and the proton do not have the same mass.
- One way of studying in quantum field theory the cases when symmetries are broken, is to add to the Lagrangian terms, with *small* coefficients that explicitly make the breaking of the symmetry. We call this *explicit* breaking.
- We are going to be dealing with another way of breaking the symmetry, called *spontaneous*, where the Lagrangian is symmetric under the action of a given group of transformations, but the ground state (vacuum state in QFT) it is not.
- We will start with the simplest example, a theory with a complex scalar field, which is invariant under the group U(1) (phase multiplications). The most general Lagrangian is

$$\boxed{\mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi - \mu^2 \phi^* \phi - \lambda(\phi^* \phi)^2 \equiv \partial_\mu \phi^* \partial^\mu \phi - V(\phi^* \phi)}$$

- We want to study the mass spectra of the theory. This is given by the quadratic terms in the fields. But this, has the assumption that the ground state (minimum of energy) corresponds to the configuration where the fields vanish.

Spontaneous symmetry breaking (SSB): U(1)

- For neutral scalar fields it can happen that the state with the minimum of energy will correspond to a configuration where

$$\boxed{\phi = v = \text{constant} \neq 0}$$

- In this case particles are associated with oscillations of ϕ around the minimum value, v. If we write,

$$\boxed{\phi(x) = v + \chi(x)}$$

The masses are read from the quadratic part of the Lagrangian in χ.

- Let us look for such a theory what are the states with minimum energy. The Hamiltonian is (remember that in mechanics $H = p\dot{q} - L = p\dot{q} - T + V$)

$$\boxed{\mathcal{H} = \dot{\phi}^* \dot{\phi} + (\vec{\partial}\phi^*) \cdot (\vec{\partial}\phi) + V}$$

- As the two first terms are positive, and the energy must be bounded from below, we should have that $\lambda > 0$. The sign of the term μ^2 is not determined. The minimum of the energy will correspond to a constant ϕ that minimizes the potential V.

Spontaneous symmetry breaking (SSB): U(1)

- This is given by

$$\boxed{V = \mu^2 \phi^* \phi + \lambda \left(\phi^* \phi\right)^2}$$

and the minimization equations are

$$\boxed{\begin{aligned} \frac{\partial V}{\partial \phi^*} &= \phi(\mu^2 + 2\lambda|\phi|^2) = 0 \\ \frac{\partial V}{\partial \phi} &= \phi^*(\mu^2 + 2\lambda|\phi|^2) = 0 \end{aligned}}$$

- We have therefore two possibilities:

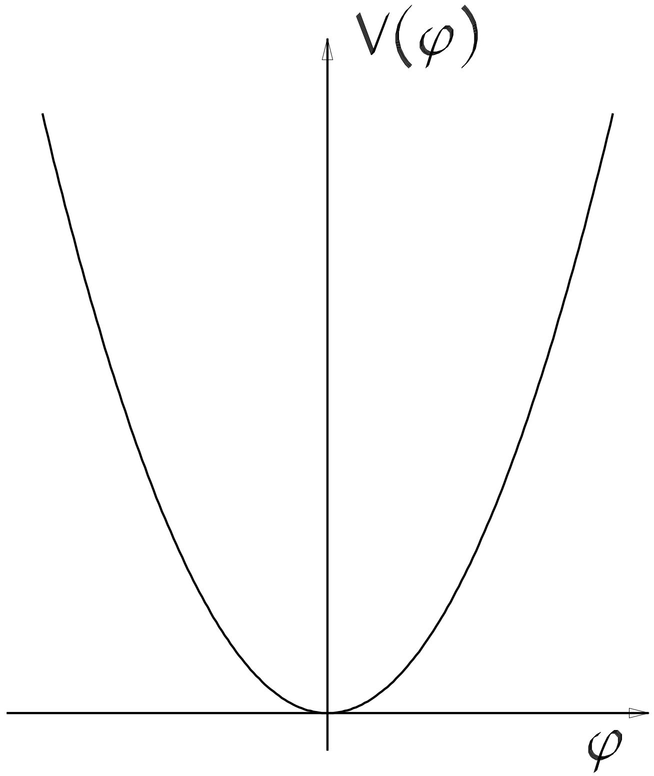
a) $\mu^2 > 0$
In this case the minimum is for $\phi = 0$. The theory describes a scalar field with mass $m = \sqrt{\mu^2}$.

b) $\mu^2 < 0$
Now the minimum corresponds to

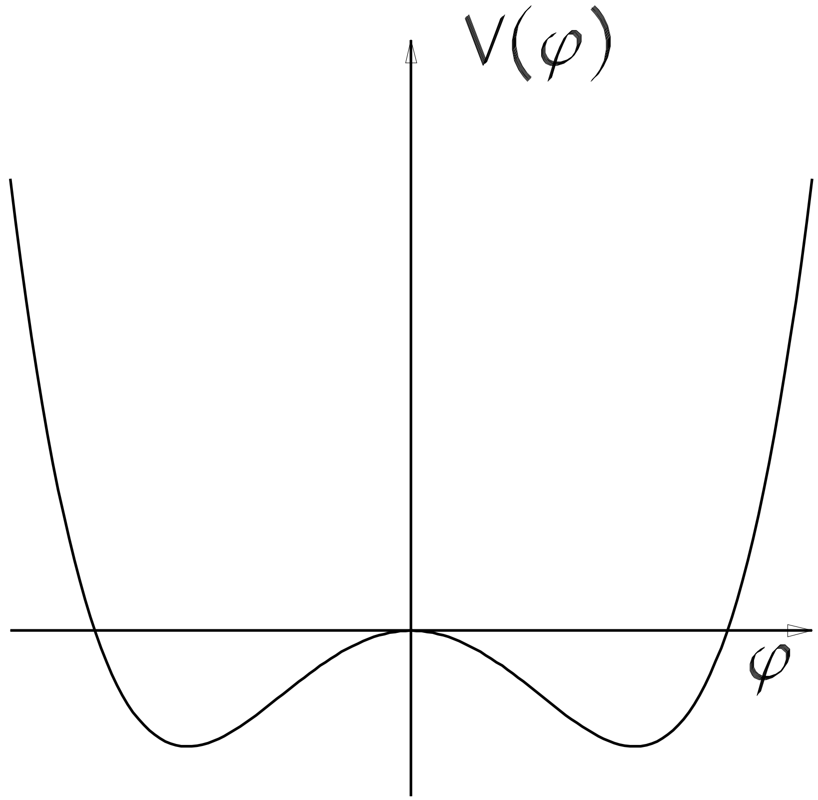
$$\boxed{\phi^* \phi = |\phi|^2 = -\frac{\mu^2}{2\lambda} \equiv v^2}$$

Spontaneous symmetry breaking (SSB): U(1)

- We have the situation shown in the figure below



$$\mu^2 > 0, \quad \langle \phi \rangle = 0$$



$$\mu^2 < 0, \quad \langle \phi \rangle = v = \sqrt{\frac{-\mu^2}{2\lambda}}$$

Spontaneous symmetry breaking (SSB): U(1)

- Let us consider the case *b)*. One possible way of analyzing the spectrum of the theory (the masses) would be to make the definition $\phi(x) = v + \chi(x)$ substitute in the Lagrangian and look for the quadratic terms.
- However this is not the easiest way to proceed. As the minimum condition corresponds to $|\phi| = v$, it is more convenient to redefine the complex field ϕ:

$$\boxed{\phi(x) = e^{\frac{i}{\sqrt{2}v}\xi(x)} \left(v + \frac{\sigma(x)}{\sqrt{2}} \right)}$$

with ξ and σ being real scalars fields. This parameterization corresponds to write the complex field in the form,

$$\boxed{\phi = e^{i\,\text{arg}(\phi)}\ |\phi|}$$

- Then

$$\boxed{\begin{aligned} \partial_\mu \phi &= \frac{i}{\sqrt{2}v}\ \partial_\mu \xi\ \phi + e^{\frac{i}{\sqrt{2}v}\xi(x)}\ \partial_\mu \sigma \\ \partial^\mu \phi^* &= \frac{-i}{\sqrt{2}v}\ \partial^\mu \xi\ \phi^* + e^{-\frac{i}{\sqrt{2}v}\xi(x)}\ \partial^\mu \sigma \end{aligned}}$$

Spontaneous symmetry breaking (SSB): U(1)

- After substituting in the Lagrangian we get, after some algebra,

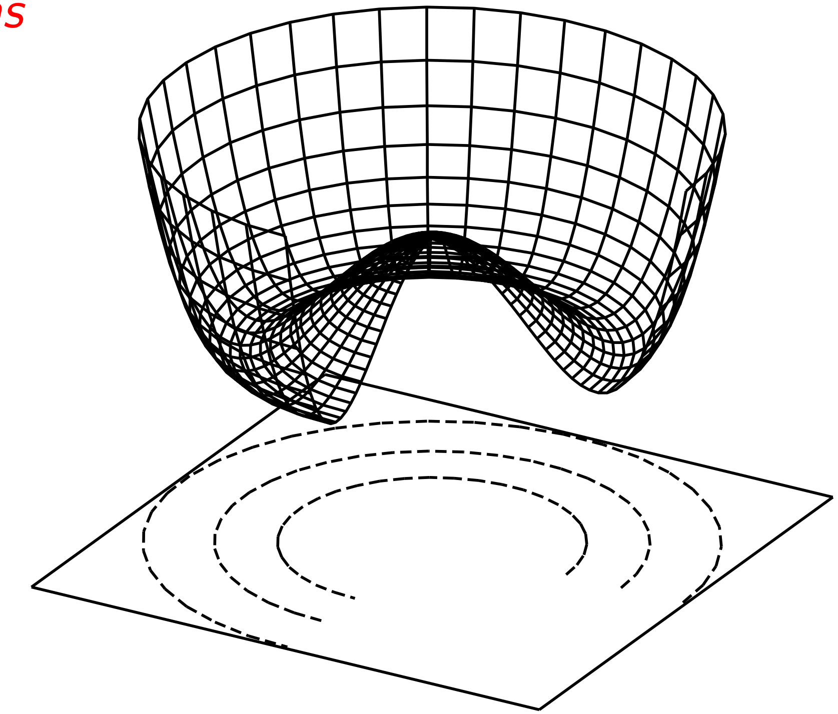
$$\begin{aligned}\mathcal{L} =& \left(\frac{-i}{\sqrt{2}v}\,\partial_\mu\xi\,\phi^* + e^{-\frac{i}{\sqrt{2}v}\xi(x)}\,\partial_\mu\sigma\right)\left(\frac{i}{\sqrt{2}v}\,\partial^\mu\xi\,\phi + e^{\frac{i}{\sqrt{2}v}\xi(x)}\,\partial^\mu\sigma\right)\\ &-\mu^2\left(v+\frac{\sigma(x)}{\sqrt{2}}\right)^2 - \lambda\left(v+\frac{\sigma(x)}{\sqrt{2}}\right)^4\\ =&\frac{1}{2}\,\partial_\mu\sigma\,\partial^\mu\sigma + \frac{1}{2}\,\partial_\mu\xi\,\partial^\mu\xi\, + \frac{1}{2}\,\partial_\mu\xi\,\partial^\mu\xi\,\left(\sqrt{2}v\sigma + \frac{\sigma^2}{2}\right)\\ &-\mu^2\left(v^2+\sqrt{2}v\sigma+\frac{\sigma^2}{2}\right) - \lambda\left(v^4 + 2\sqrt{2}v^3\sigma + 3v^2\sigma^2 + \sqrt{2}v\sigma^3 + \frac{\sigma^4}{4}\right)\end{aligned}$$

- Using the minimum equations we can simplify the expression, keeping only up the quadratic terms,

$$\begin{aligned}\mathcal{L} =&\frac{1}{2}\,\partial_\mu\sigma\,\partial^\mu\sigma + \frac{1}{2}\,\partial_\mu\xi\,\partial^\mu\xi\, - \frac{1}{2}\,\left(-2\mu^2\right)\,\sigma^2 + \text{constant}\\ &+\text{higher order terms (cubic and quartic)}\end{aligned}$$

Spontaneous symmetry breaking (SSB): U(1)

- This Lagrangian describes therefore two real scalar fields, σ and ξ, the first with mass $m_\sigma = \sqrt{-2\ \mu^2}$ and the other without mass, $m_\xi = 0$. This fact can easily be interpreted.
- In first place we note that the potential V is, in the complex plane of the field ϕ, a potential of the type *bottom of a champagne bottle*.
- With the above parameterization the field *σ corresponds to the radial oscillations and ξ to the angular oscillations*
- But while the potential has a curvature in the radial direction it is flat in the angular direction. So it does not cost energy to go along the valley at bottom of the bottle. *Then the radial excitations have mass while the angular ones do not*. The appearance of massless particles in these phenomena of spontaneous symmetry breaking is known as *Goldstone's Theorem*



Higgs Mechanism: the case of U(1)

- Having arrived here one could ask why all these details in studying theories with spontaneous breaking of symmetry.
- At first sight the fact that we need *massive spin 1 particles* to describe the weak interactions does not seem solved with these theories, as *the spontaneous breaking of symmetry gives massless particles* and as we have seen the gauge bosons of gauge theories have to be massless to preserve gauge invariance.
- The reason is that, if we have a *local gauge theory* and the phenomenon of *spontaneous symmetry braking*, then the massless Nambu- Goldstone boson disappear and *the spin one gauge bosons gets mass*.
- This phenomenon is known as the Higgs mechanism that we will now explain. We are not going to give a general proof, just two examples. Let us start by the case of the complex scalar field with local gauge invariance, U(1),

$$\boxed{\mathcal{L} = (D_\mu \phi)^* (D^\mu \phi) - \mu^2 \phi^* \phi - \lambda(\phi^* \phi)^2 - \frac{1}{4}\ F_{\mu\nu} F^{\mu\nu}}$$

where the covariant derivative is

$$\boxed{D_\mu = \partial_\mu + ieA_\mu}$$

$$\boxed{\phi(x) \to \phi'(x) = e^{i\alpha(x)} \phi(x)}$$

Higgs Mechanism: the case U(1)

- By construction the Lagrangian is invariant under the set of local gauge transformations,

$$\boxed{\begin{aligned} &\phi(x) \to \phi'(x) = e^{i\epsilon(x)}\ \phi(x) \\ &A_\mu(x) \to A'_\mu(x) = A_\mu(x) - \frac{1}{e}\ \partial_\mu \epsilon(x) \end{aligned}}$$

- If $\mu^2 > 0$ the theory is simply what is known as scalar electrodynamics. However if $\mu^2 < 0$ we should have spontaneous breaking of symmetry and the analysis of the mass spectra has to be careful.
- In first place we have to find the vacuum (lowest energy state). This is given by the values of $\langle \phi \rangle$ and $\langle A_\mu \rangle$ that minimize the energy. As the vacuum should be Lorentz invariant we must require that

$$\boxed{\langle A_\mu \rangle = 0}$$

but the scalar field ϕ can have a non-zero value, as before,

$$\boxed{\langle \phi \rangle = v = \sqrt{-\frac{\mu^2}{2\lambda}} > 0}$$

Higgs Mechanism: the case U(1)

- Instead of making the change of variables $\phi(x) \to v + \chi(x)$, we parameterize ϕ as a phase times a real term, that is,

$$\boxed{\phi(x) = e^{i\frac{\xi(x)}{\sqrt{2}v}} \left(v + \frac{\sigma(x)}{\sqrt{2}} \right)}$$

- As we have seen the field $\xi(x)$ it is associated with the spontaneous breaking of symmetry. *In the absence of the gauge field A_μ, we concluded that ξ was a massless field*

- *We now see that this is not true for a local gauge theory*. We substitute in the Lagrangian, make the derivatives and get,

$$\boxed{\begin{aligned} \mathcal{L} = &-\frac{1}{4} F_{\mu\nu}F^{\mu\nu} + \frac{1}{2} \partial_\mu\sigma \, \partial^\mu\sigma + \frac{1}{2} \partial_\mu\xi \, \partial^\mu\xi \; + e^2v^2 A_\mu A^\mu \\ &+ \sqrt{2} ve A_\mu \partial^\mu \xi + \mu^2\sigma^2 + \text{higher order terms} \end{aligned}}$$

- From this equation we see that the field σ has mass$^2 = -2\mu^2 > 0$, but that *the fields A_μ e ξ are mixed at the level of the quadratic terms*. Because of this the spectra is not immediately readable.

Higgs Mechanism: the case U(1)

- The simplest way to solve this situation is to make use of the gauge invariance of the theory. We choose the parameter of the gauge transformation as,

$$\boxed{\epsilon(x) = -\frac{\xi(x)}{\sqrt{2}v}} \quad \Rightarrow \quad \begin{cases} \boxed{\phi(x) \to \phi'(x) = e^{-i\frac{\xi(x)}{\sqrt{2}v}}\phi(x) = v + \frac{\sigma(x)}{\sqrt{2}}} \\ \boxed{A_\mu(x) \to A'_\mu(x) = A_\mu + \frac{1}{e\sqrt{2}v}\partial_\mu \xi} \end{cases}$$

- As the Lagrangian is invariant under local gauge transformations, we should have

$$\begin{aligned} \mathcal{L}(\phi, A_\mu) =& \mathcal{L}(\phi', A'_\mu) \\ =& \frac{1}{2}\left[\left(\partial_\mu - ieA'_\mu\right)\left(\sqrt{2}v + \sigma\right)\right]\left[\left(\partial^\mu + ieA'^\mu\right)\left(\sqrt{2}v + \sigma\right)\right] \\ & - \frac{1}{2}\,\mu^2\left(\sqrt{2}v + \sigma\right)^2 - \frac{1}{4}\,\lambda\left(\sqrt{2}v + \sigma\right)^4 - \frac{1}{4}\,F'_{\mu\nu}F'^{\mu\nu} \end{aligned}$$

where

$$\boxed{F'_{\mu\nu} = \partial_\mu A'_\nu - \partial_\nu A'_\mu}$$

Higgs Mechanism: the case of U(1)

- The new Lagrangian can be expanded easily,

$$\begin{aligned}\mathcal{L} = &-\frac{1}{4}\, F'_{\mu\nu}F'^{\mu\nu} + \frac{1}{2}\,\partial_\mu\sigma\,\partial^\mu\sigma + e^2v^2A'_\mu A'^\mu + \frac{1}{2}\,e^2A'_\mu A'^\mu\sigma\,(2\sqrt{2}v+\sigma)\\ &-\frac{1}{2}\,\sigma^2(6\lambda v^2+\mu^2) - \sqrt{2}\lambda v\sigma^3 - \frac{1}{4}\,\lambda\sigma^4\end{aligned}$$

- In this gauge this is no mixing for the quadratic terms, and the spectra can be read directly,

$$m_\sigma = \sqrt{6\lambda v^2+\mu^2} = \sqrt{-2\mu^2}, \quad m_A = \sqrt{2}ev$$

and the field ξ disappeared completely from the Lagrangian.

- This gauge where the spectra can be read easily is known as *unitary gauge*. *Where went the field ξ?*

- To understand this question let us do the counting of the degrees of freedom. In the original Lagrangian we have two real scalar fields and one *massless* vector field, which accounts for another two degrees of freedom. In total we have four degrees of freedom.

Higgs Mechanism: the case of U(1)

- In the redefined Lagrangian, we have just one real scalar field, but we have a *massive* vector field that corresponds to three degrees of freedom. The total is again four degrees of freedom.
- Therefore the interpretation is that the degree of freedom associated with the field ξ corresponds to the longitudinal polarization of the vector field.
- We see that, contrary to what states the Goldstone theorem, not only there are no massless scalar bosons, but also the vector field can acquire mass in this process. This process is known as the *Higgs mechanism*. It started to be known as the Brout-Englert-Higgs mechanism after the Nobel Prize for Physics in 2013, although it as been discovered independently by more people.
- The previous example is rather simple and illustrates the basics of the mechanism but it is too simple to be useful in the construction of the Standard Model of Particle Physics. *This is because the field A_μ cannot be interpreted as the photon because we know that the photon has no mass*.

Higgs Mechanism: the case of SU$(2) \times$ U(1)

- To have a more realistic model, in fact *the basis of the Standard Model of the electroweak interactions*, we consider a local gauge theory based on the group SU$(2) \times$ U(1) that we have already seen before.

- The Lagrangian with local gauge invariance is,

$$\boxed{\mathcal{L} = (D_\mu\phi)^\dagger(D^\mu\phi) - V(\phi^\dagger\phi) - \frac{1}{4}\, W^a_{\mu\nu}W^{a\mu\nu} - \frac{1}{4}\, B_{\mu\nu}B^{\mu\nu}}$$

where V is given by,

$$\boxed{V(\phi^\dagger\phi) = \mu^2\phi^\dagger\phi + \lambda\left(\phi^\dagger\phi\right)^2}$$

and where we introduced the spin one vector fields W^a_μ, $(a = 1, 2, 3)$ and B_μ corresponding to SU(2) and U(1), respectively.

- The field tensors and covariant derivatives are then

$$\boxed{\begin{aligned} &W^a_{\mu\nu} = \partial_\mu W^a_\mu - \partial_\nu W^a_\mu - g\varepsilon^{abc}W^b_\mu W^c_\nu, \quad B_{\mu\nu} = \partial_\mu B_\mu - \partial_\nu B_\mu \\ &D_\mu\phi \equiv \left(\partial_\mu + igW^a_\mu\frac{\tau^a}{2} + ig'B_\mu\frac{1}{2}\right)\phi \end{aligned}}$$

where τ^a are the Pauli matrices.

Higgs Mechanism: the case of SU$(2) \times$ U(1)

- With the insight of the previous example we can choose a gauge, known as the *unitary gauge* where,

$$\phi(x) = \begin{bmatrix} 0 \\ v + \dfrac{\sigma}{\sqrt{2}} \end{bmatrix}$$

See Problems

- Going through the same steps we get that the mass spectra is as folows:

One scalar field, σ, with mass $m_\sigma = \sqrt{-2\mu^2}$ as before. Two vector fields with mass $M_W = \sqrt{\frac{1}{2}\, g^2 v^2}$, one vector field with mass $M_Z = \sqrt{\frac{1}{2}\, v^2(g^2 + g'^2)}$ and one massless vector field.

- We see then that three of the four gauge fields acquire masses, due the spontaneous breaking of the symmetry. This is the **Higgs mechanism**

- We notice that the counting of the degrees of freedom is correct, as one massive vector field has three polarizations instead of two for the massless case. This explains the *disappearance* of the other three scalar fields of the theory.

- In pictorial language we say that they were *eaten* by the gauge fields that became *fat* with mass.

Higgs Mechanism: the case of $\mathbf{SU}(2) \times \mathbf{U}(1)$

- This mechanism made it possible to apply local gauge theories to the weak interactions because it *became possible to give mass to the intermediate vector boson*
- Notice that *one of the vector fields remained massless, becoming therefore a candidate for the photon*
- This is due to the fact that the symmetry was not completely broken, there is still a residual $\text{U}(1)$, symmetry, that is,

$$\text{SU}(2) \times \text{U}(1) \to \text{U}(1)$$

that we will see will correspond to the unbroken electromagnetism.

- Another fundamental fact about the Higgs mechanism, is that local gauge theory with spontaneous breaking of symmetry is renormalizable (G. 't Hooft, 1972), while a theory with spin one fields with mass it is not. *The model that we have been describing is in fact the Standard Model for the weak and electromagnetic interactions proposed by Glashow-Weinberg-Salam that we will describe in more detail in the remaining of the lecture*

The Gauge and Higgs Sectors: The spectra

- After the Higgs mechanism there remains a massive scalar field σ, that from now on we will denote by H, the so-called Higgs boson. Its mass is,

$$\boxed{m_H = \sqrt{-2\mu^2}}$$

See Problems

- We introduce the fields A_μ, Z_μ and $W^\pm_\mu$ by the relations,

$$A_\mu = \sin\theta_W W^3_\mu + \cos\theta_W B_\mu$$
$$Z_\mu = \cos\theta_W W^3_\mu - \sin\theta_W B_\mu$$

$$\boxed{W^\mp_\mu = \frac{1}{\sqrt{2}}\left(W^1_\mu \pm i W^2_\mu\right)}$$

- The angle θ_W is determined by the requirement that the field A_μ be the eigenstate with zero mass and we get

$$\boxed{\tan\theta_W = g'/g} \qquad \boxed{g\sin\theta_W = e}$$

- The $W^\pm_\mu$ and the Z_μ have masses (the photon is massless)

$$\boxed{M_Z = \sqrt{\tfrac{1}{2}\, v^2(g^2+g'^2)}} \quad \boxed{M_W = \sqrt{\tfrac{1}{2}\, g^2 v^2}} \quad \boxed{M_A = 0} \quad \boxed{M_W = M_Z\cos\theta_W}$$

The Weak Interactions of Leptons

- The beauty of the gauge theories is that the interactions of the gauge bosons with the matter fields are completely determined by gauge invariance. We just saw that for the Higgs fields, and the same is true for the fermions.
- In fact we already have seen that the form of the Lagrangian for any fermion should be,

$$\boxed{\mathcal{L} = \sum_f \overline{\Psi}_f (i\not{D} - m)\Psi_f}$$

where for an $SU_L(2) \times U_Y(1)$ theory we have for the covariant derivative,

$$\boxed{D_\mu \Psi = (\partial_\mu + igW_\mu^a \Omega^a + ig' \frac{Y}{2} B_\mu)\Psi}$$

- The matrices Ω^a are the appropriate for the representations in which the fermions will lie. We have therefore, before we write the interactions, to find out what are the representations of the group $SU_L(2) \times U_Y(1)$ in which are the different fermions. We start with the leptons.

The Representations and Quantum Numbers: Leptons

- The known fermions are distributed by three families with identical properties, only differing in the mass. This repetition it is not explained by the theory, but it is incorporated to be in agreement with the known phenomenology.
- In what follows we will talk just about the family of the electron (the electron and its neutrino), but all that we will say it is also true for the muon and tau.
- As we saw before, the charged currents that mediate the weak interaction ($W_\mu^\pm$ exchange) are exactly $V - A$, that is only the left-handed component of the fermions participates in those interactions.
- To obtain this it is necessary to treat differently the two chiralities of the charged particles. Therefore, taking in account that the group that emerges from the phenomenology is $SU_L(2) \times U_Y(1)$, we assign the electron and its neutrino to the following representations of $SU_L(2)$, known as *weak isospin*

$$\boxed{E_L \equiv \frac{1-\gamma_5}{2}\begin{bmatrix}\nu_e \\ e\end{bmatrix} \equiv \begin{bmatrix}\nu_{eL} \\ e_L\end{bmatrix} \quad ; \quad e_R \equiv \frac{1+\gamma_5}{2}\, e}$$

The Representations and Quantum Numbers: Leptons

- How to fix the value of Y? It is not arbitrary, as the photon should couple to the electromagnetic current proportional to the electric charge

$$\begin{aligned}\mathcal{L}_{\rm int} &= -\bar{\ell}\gamma^\mu \left[g W^3_\mu T^3 + \frac{1}{2} g' B_\mu Y \right] \ell \\ &= -\bar{\ell}\gamma_\mu \left\{ A^\mu e \left(T^3 + \frac{1}{2} Y \right) + Z^\mu \frac{g}{\cos\theta_W} \left[T^3 - \sin^2\theta_W \left(T^3 + \frac{1}{2} Y \right) \right] \right\} \ell\end{aligned}$$

where T^3 is the numeric value of the weak isospin of the lepton ℓ.

- Comparing with what one should get for the electromagnetic current,

$$\mathcal{L}^{\rm QED}_{\rm int} = -eQ\,\bar{\ell}\gamma^\mu \ell\, A_\mu \qquad \boxed{Q = T^3 + \tfrac{1}{2} Y}$$

See Problems

where $e = |e|$ and therefore Q is the charge of the particle in units e

- We show in the table on the right the quantum numbers for the leptons

	e_L	e_R	ν_L
T^3	$-1/2$	0	$+1/2$
Y	-1	-2	-1
Q	-1	-1	0

The Representations and Quantum Numbers: Quarks

- The weak interactions of the hadrons can be obtained from the weak interactions of the quarks. We will make the following assumptions:

i) Quarks appear in different flavours. Experimentally six were observed: $u, d, s, c, b,$ *e* t.

ii) For each flavour the quarks appear in three different colours, but hadrons are colour singlets.

iii) The electromagnetic and weak interactions are also colour singlets and act only in the space of the quarks flavours.

- With these assumptions, that incorporate what is known experimentally, we need to specify the transformation properties of the left and right chiralities under the action of the group $SU(2) \times U(1)$. We give in the table below the values of T^3, Y and Q for each flavour and chirality.

$$\boxed{Q = T^3 + Y/2}$$

	u_L	d_L	c_L	s_L	t_L	b_L	u_R	d_R	c_R	s_R	t_R	b_R
T^3	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	0	0	0	0
Y	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{4}{3}$	$-\frac{2}{3}$	$\frac{4}{3}$	$-\frac{2}{3}$	$\frac{4}{3}$	$-\frac{2}{3}$
Q	$\frac{2}{3}$	$-\frac{1}{3}$	$\frac{2}{3}$	$-\frac{1}{3}$	$\frac{2}{3}$	$-\frac{1}{3}$	$\frac{2}{3}$	$-\frac{1}{3}$	$\frac{2}{3}$	$-\frac{1}{3}$	$\frac{2}{3}$	$-\frac{1}{3}$

The Mass of the Leptons and Quarks

- As we said already, because the transformations of $SU_L(2) \times U_Y(1)$ treat in a different form the two chiralities of the fermions, a mass term for these is not invariant under the action of the group.
- The way to solve this problem is to require that before the spontaneous breaking of the symmetry the leptons are massless, and its mass has origin in the same Higgs mechanism that gives mass to the gauge bosons.
- This is possible if we add to the Lagrangian new interactions between the leptons and the scalar fields, known as Higgs fields. To achieve this we have therefore to construct new terms that are invariant under $SU_L(2) \times U_Y(1)$.
- As we have

$$Y(E_L) = -1, \quad Y(e_R) = -2, \quad Y(\phi) \quad = +1$$

we conclude that the Lorentz invariant Lagrangian that is also invariant for the group $SU_L(2) \times U_Y(1)$ is

$$\boxed{\mathcal{L}_{\rm Yukawa} = -f_e \overline{E}_L \phi\, e_R + \text{h.c.}}$$

where f_e is a coupling constant without dimensions.

The Mass of the Leptons and Quarks

- Now we show that this Lagrangian gives indeed mass to the electron when we have the spontaneous breaking of symmetry. In this case we have

$$\phi = \begin{bmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{bmatrix} + \cdots$$

- We then get (we take f_e real)

$$\boxed{\mathcal{L}_{\text{Yukawa}} = -f_e v(\overline{e}_R e_L + \overline{e}_L e_R) + \cdots}$$

from where we conclude that ($v \simeq 246$ GeV)

$$f_e = \frac{\sqrt{2} m_e}{v} = 2.9 \times 10^{-6}$$

- The introduction of the other leptons and quarks is now similar. There are some important details, specially for quarks leading to the understanding of the Cabibbo-Kobayashi-Maskawa (CKM) matrix, but we will not cover these details here (however, see problems for these lectures).

Multi-Higgs Models

Why only 1 Higgs Boson?

Summary
Yang-Mills Theories
SSB
Higgs Mechanism
Standard Model
Multi-Higgs
- ρ parameter
- 2HDM
- Mass Matrices
- C2HDM
- Higgs Basis
- Parameters
- Couplings
- New Physics
- Problems

Number of particles with Spin 1

Fixed by the choice of Symmetry Group $\mathbf{SU(3)} \times \mathbf{SU(2)} \times \mathbf{U(1)}$

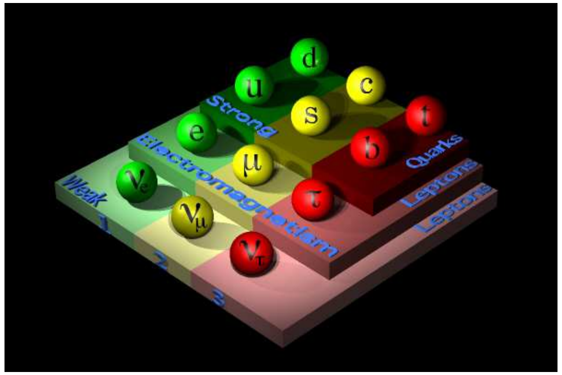
Properties of the Interactions

The strengths of the interactions (forces) are shown relative to the strength of the electromagnetic force for two u quarks separated by the specified distances.

Property	Gravitational Interaction	Weak Interaction (Electroweak)	Electromagnetic Interaction	Strong Interaction
Acts on:	Mass – Energy	Flavor	Electric Charge	Color Charge
Particles experiencing:	All	Quarks, Leptons	Electrically Charged	Quarks, Gluons
Particles mediating:	Graviton (not yet observed)	W^+ W^- Z^0	γ	Gluons
Strength at 10^{-18} m	10^{-41}	0.8	1	25
Strength at 3×10^{-17} m	10^{-41}	10^{-4}	1	60

Number of Particles with Spin $\frac{1}{2}$

There is no principle. Fixed by experiment



Number of particles with Spin 0

There is no principle. $\boxed{\text{Therefore should be fixed by experiment!}}$

The ρ parameter

- While the number of spin one gauge bosons is determined by the gauge group, the same is not true for spin 1/2 and 0 particles. The number of fermion light families was experimentally determined at LEP and we expect that the same situation holds for the spin 0 scalars.
- So we expect that the simple structure of the standard model (SM) doublet

$$\phi = \begin{bmatrix} G^+ \\ \frac{1}{\sqrt{2}}\left(v + h + iG^0\right) \end{bmatrix}$$

should be generalized.

- Of course the generalization has to respect the experimental results. One of the important relations from the SM is the relation between the W and Z masses. One defines the so-called ρ-parameter by,

$$\boxed{\rho = \frac{M_W^2}{M_Z^2 \cos^2\theta_W} = 1.0004^{+0.0003}_{-0.0004}}$$

so experimentally it is very close to 1

The ρ parameter: N Higgs Doublets

- This experimental result tell us that we better keep it close to 1. It turns out that for a scalar in a given complex representation i of the $SU(2)_L$, with weak isospin T_i, vev v_i, and weak hypercharge Y_i the ρ parameter is given by

$$\boxed{\rho = \frac{\sum_i \left[4T_i(T_i+1) - Y_i^2\right] |v_i|^2}{\sum_i 2Y_i^2 |v_i|^2}}$$

See Problems

satisfying the relation $Q = T_3 + \frac{1}{2}Y$.

- From here we immediatetly see that a theory with any number of complex Higgs doublets ($T_i = \frac{1}{2}, Y = \pm 1$) and $SU(2)_L$ singlets automatically give the desired $\rho = 1$.

- So we are naturally lead to consider models with N doublets

$$\boxed{\phi_i = \begin{bmatrix} \varphi_i^+ \\ \frac{1}{\sqrt{2}}\left(v_i + \eta_i + i\,\chi_i\right) \end{bmatrix}, \quad i = 1, \ldots, N}$$

The two Higgs doublet model (2HDM)

- To keep things simple we do not consider the most general 2HDM, but choose all the parameters real (caution here) and impose a Z_2 symmetry $\phi_1 \to \phi_1, \phi_2 \to -\phi_2$ violated softly to allow decoupling.
- The Higgs potential is

$$\boxed{\begin{aligned} V_H =& m_{11}^2|\phi_1|^2 + m_{22}^2|\phi_2|^2 - m_{12}^2 \left(\phi_1^\dagger\phi_2 + \phi_2^\dagger\phi_1\right) \\ &+ \frac{\lambda_1}{2}|\phi_1|^4 + \frac{\lambda_2}{2}|\phi_2|^4 + \lambda_3|\phi_1|^2|\phi_2|^2 + \lambda_4\left(\phi_1^\dagger\phi_2\right)\left(\phi_2^\dagger\phi_1\right) \\ &+ \frac{\lambda_5}{2}\left[\left(\phi_1^\dagger\phi_2\right)^2 + \left(\phi_2^\dagger\phi_1\right)^2\right]. \end{aligned}}$$

- The minimization conditions can be solved for the quadratic parameters

$$\begin{aligned} 2m_{11}^2 =& -\lambda_1 v_1^2 - \lambda_{345} v_2^2 + 2m_{12}^2\frac{v_2}{v_1} \\ 2m_{22}^2 =& -\lambda_2 v_2^2 - \lambda_{345} v_1^2 + 2m_{12}^2\frac{v_1}{v_2} \\ \lambda_{345} \equiv& \lambda_3 + \lambda_4 + \lambda_5 \end{aligned}$$

CP Odd Mass Matrix

- The mass matrix for the CP odd Higgs is

$$\boxed{\mathcal{M}^P = (m_{12}^2 - \lambda_5 v^2 s_\beta c_\beta) \begin{bmatrix} t_\beta & -1 \\ -1 & \frac{1}{t_\beta} \end{bmatrix}}$$

where, as usual, we have defined $t_\beta = \tan\beta \equiv \frac{v_2}{v_1}$, $s_\beta = \sin\beta$ and $c_\beta = \cos\beta$.

- It is clear that this mass matrix has zero determinant, corresponding to the neutral Goldstone boson, G^0. We define the mass eigenstates through the rotation

$$\mathcal{O}_\beta = \begin{bmatrix} c_\beta & s_\beta \\ -s_\beta & c_\beta \end{bmatrix}, \quad \mathcal{O}_\beta \mathcal{M}^P \mathcal{O}_\beta^T = \begin{bmatrix} 0 & 0 \\ 0 & m_A^2 \end{bmatrix}, \quad \begin{bmatrix} G^0 \\ A \end{bmatrix} = \mathcal{O}_\beta \begin{bmatrix} \chi_1 \\ \chi_2 \end{bmatrix} .$$

- From here one finds that we can write the parameter λ_5 in terms of the other parameters and the physical mass of the CP odd scalar A, obtaining,

$$\boxed{\lambda_5 = -\frac{m_A^2}{v^2} + \frac{m_{12}^2}{v^2 s_\beta c_\beta}}$$

The charged Higgs mass matrix

- In a similar way, the charged Higgs mass matrix

$$\mathcal{M}^C = \begin{bmatrix} m_{12}^2 t_\beta - \frac{1}{2}(\lambda_4+\lambda_5)v^2 s_\beta^2 & -m_{12}^2 + \frac{1}{2}(\lambda_4+\lambda_5)v^2 s_\beta c_\beta \\ -m_{12}^2 + \frac{1}{2}(\lambda_4+\lambda_5)v^2 s_\beta c_\beta & -\frac{1}{2}(\lambda_4+\lambda_5)v^2 c_\beta^2 + \frac{m_{12}^2}{t_\beta} \end{bmatrix}$$

or

$$\mathcal{M}^C = \left(m_{12}^2 - \frac{1}{2}(\lambda_4+\lambda_5)v^2 s_\beta c_\beta\right) \begin{bmatrix} t_\beta & -1 \\ -1 & \frac{1}{t_\beta} \end{bmatrix}$$

is diagonalized by the same matrix

$$\mathcal{O}_\beta \mathcal{M}^C \mathcal{O}_\beta^T = \begin{bmatrix} 0 & 0 \\ 0 & m_{H^+}^2 \end{bmatrix}, \qquad \begin{bmatrix} G^+ \\ H^+ \end{bmatrix} = \mathcal{O}_\beta \begin{bmatrix} \varphi_1^+ \\ \varphi_2^+ \end{bmatrix} .$$

- From here one can exchange the parameter λ_4 for the mass of the charged Higgs boson, the parameter m_{12}^2 and λ_5 that is already known

$$\lambda_4 = -\lambda_5 - \frac{2m_{H^+}^2}{v^2} + \frac{2m_{12}^2}{v^2 s_\beta c_\beta}$$

The CP even mass matrix

- Finally we look at the CP even mass matrix.

$$\boxed{\mathcal{M}^S = \begin{bmatrix} \lambda_2 v^2 c_\beta^2 + m_{12}^2 t_\beta & -m_{12}^2 + \lambda_{345} v^2 s_\beta c_\beta \\ -m_{12}^2 + \lambda_{345} v^2 s_\beta c_\beta & \frac{m_{12}^2}{t_\beta} + \lambda_2 v^2 s_\beta^2 \end{bmatrix}}$$

- The mass eigenstates are obtained through the following rotation

$$\mathcal{O}_\alpha = \begin{bmatrix} c_\alpha & s_\alpha \\ -s_\alpha & c_\alpha \end{bmatrix}, \quad \mathcal{O}_\alpha \mathcal{M}^S \mathcal{O}_\alpha^T = \begin{bmatrix} m_h^2 & 0 \\ 0 & m_H^2 \end{bmatrix}, \quad \begin{bmatrix} h \\ H \end{bmatrix} = \mathcal{O}_\alpha \begin{bmatrix} \eta_1 \\ \eta_2 \end{bmatrix} .$$

- We obtain the remaining λ's

$$\boxed{\begin{aligned} &\lambda_1 = \frac{m_h^2 c_\alpha^2 + m_H^2 s_\alpha^2 - m_{12}^2 t_\beta}{v^2 c_\beta^2}, \quad \lambda_2 = \frac{m_h^2 s_\alpha^2 + m_H^2 c_\alpha^2 - \frac{m_{12}^2}{t_\beta}}{v^2 s_\beta^2} \\ &\lambda_3 = -\lambda_4 - \lambda_5 + \frac{2 m_{12}^2 + (m_h^2 - m_H^2) s_{2\alpha}}{v^2 s_{2\beta}} \end{aligned}}$$

- Independent parameters: $\alpha, \beta, m_{12}^2, m_h, m_H, m_A, m_{H^+}$

The C2HDM Potential

- We consider a model with two Higgs doublets, ϕ_1 and ϕ_2, with the Z_2 symmetry $\phi_1 \to \phi_1, \phi_2 \to -\phi_2$ violated softly

$$\boxed{\begin{aligned} V_H =& m_{11}^2|\phi_1|^2 + m_{22}^2|\phi_2|^2 - m_{12}^2\, \phi_1^\dagger\phi_2 - (m_{12}^2)^*\, \phi_2^\dagger\phi_1 \\ & + \frac{\lambda_1}{2}|\phi_1|^4 + \frac{\lambda_2}{2}|\phi_2|^4 + \lambda_3|\phi_1|^2|\phi_2|^2 + \lambda_4\left(\phi_1^\dagger\phi_2\right)\left(\phi_2^\dagger\phi_1\right) \\ & + \frac{\lambda_5}{2}(\phi_1^\dagger\phi_2)^2 + \frac{\lambda_5^*}{2}(\phi_2^\dagger\phi_1)^2. \end{aligned}}$$

- Hermiticity implies that all couplings are real, except m_{12}^2 and λ_5. If $\arg(\lambda_5) \neq 2\arg(m_{12}^2)$, then the phases cannot be removed.
- If $\arg(\lambda_5) = 2\arg(m_{12}^2)$, then we can choose a basis where m_{12}^2 and λ_5 become real, and, we talk about the real 2HDM.
- The stationarity conditions become

$$\boxed{\begin{aligned} -2\, m_{11}^2 &= -\,\text{Re}\left(m_{12}^2\right)\frac{v_2}{v_1} + \lambda_1 v_1^2 + \lambda_{345}\, v_2^2, \quad & 2\,\text{Im}\left(m_{12}^2\right) &= v_1 v_2\,\text{Im}\left(\lambda_5\right) \\ -2\, m_{22}^2 &= -\,\text{Re}\left(m_{12}^2\right)\frac{v_1}{v_2} + \lambda_2 v_2^2 + \lambda_{345}\, v_1^2, \quad & \lambda_{345} &= \lambda_3 + \lambda_4 + \text{Re}\left(\lambda_5\right) \end{aligned}}$$

Higgs Basis and Mass Eigenstates

- We can now transform the fields into the Higgs basis by

$$\boxed{\begin{bmatrix} H_1 \\ H_2 \end{bmatrix} = \begin{bmatrix} c_\beta & s_\beta \\ -s_\beta & c_\beta \end{bmatrix} \begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix}} \tag{1}$$

where $\tan\beta = v_2/v_1$, $c_\beta = \cos\beta$, and $s_\beta = \sin\beta$.

- The Higgs basis was introduced such that the second Higgs does not get a vev:

$$\boxed{H_1 = \begin{bmatrix} G^+ \\ \frac{1}{\sqrt{2}}(v + H^0 + iG^0) \end{bmatrix}, \qquad H_2 = \begin{bmatrix} H^+ \\ \frac{1}{\sqrt{2}}(R_2 + iI_2) \end{bmatrix}} \tag{2}$$

- In this basis, G^+ and G^0 are massless and, in the unitary gauge, will become the longitudinal components of W^+ and Z^0, respectively.
- There remains a charged pair $H^\pm$ with mass $m_{H^\pm}$.

The neutral eigenstates

- In the usual notation for the C2HDM, $\eta_3 = I_2$, and the three neutral components mix into the neutral mass eigenstates through

$$\boxed{\begin{bmatrix} h_1 \\ h_2 \\ h_3 \end{bmatrix} = R \begin{bmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \end{bmatrix} = \begin{bmatrix} c_1 c_2 & s_1 c_2 & s_2 \\ -(c_1 s_2 s_3 + s_1 c_3) & c_1 c_3 - s_1 s_2 s_3 & c_2 s_3 \\ -c_1 s_2 c_3 + s_1 s_3 & -(c_1 s_3 + s_1 s_2 c_3) & c_2 c_3 \end{bmatrix} \begin{bmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \end{bmatrix}}$$

- The orthogonal matrix R diagonalizes the neutral mass matrix through

$$R\,\mathcal{M}^2\,R^T = \text{diag}\left(m_1^2, m_2^2, m_3^2\right),$$

- We take as nine independent parameters v and the 8 input parameters β, $m_{H^\pm}$, α_1, α_2, α_3, m_1, m_2, and $\text{Re}(m_{12}^2)$.
- With this choice, m_3 is given by

$$\boxed{m_3^2 = \frac{m_1^2\,R_{13}(R_{12}\tan\beta - R_{11}) + m_2^2\,R_{23}(R_{22}\tan\beta - R_{21})}{R_{33}(R_{31} - R_{32}\tan\beta)}}$$

The parameters of the potential

- The Higgs potential can be reconstructed through the following expressions

$$
\begin{aligned}
v^2\,\lambda_1 =& -\frac{1}{\cos^2\beta}\left[-m_1^2\,c_1^2c_2^2-m_2^2(c_3s_1+c_1s_2s_3)^2-m_3^2\,(c_1c_3s_2-s_1s_3)^2+\mu^2\,\sin^2\beta\right]\\
v^2\,\lambda_2 =& -\frac{1}{\sin^2\beta}\left[-m_1^2\,s_1^2c_2^2-m_2^2\,(c_1c_3-s_1s_2s_3)^2-m_3^2\,(c_3s_1s_2+c_1s_3)^2+\mu^2\,\cos^2\beta\right]\\
v^2\,\lambda_3 =& \frac{1}{\sin\beta\cos\beta}\Big[\big(m_1^2\,c_2^2+m_2^2\,(s_2^2s_3^2-c_3^2)+m_3^2\,(s_2^2c_3^2-s_3^2)\big)\,c_1s_1\\
&+(m_3^2-m_2^2)(c_1^2-s_1^2)s_2c_3s_3\Big]-\mu^2+2m_{H^\pm}^2\\
v^2\,\lambda_4 =& m_1^2\,s_2^2+(m_2^2\,s_3^2+m_3^2\,c_3^2)c_2^2+\mu^2-2m_{H^\pm}^2\\
v^2\,\mathrm{Re}(\lambda_5) =& -m_1^2\,s_2^2-(m_2^2\,s_3^2+m_3^2\,c_3^2)c_2^2+\mu^2\\
v^2\,\mathrm{Im}(\lambda_5) =& \frac{2}{\sin\beta}c_2\left[(-m_1^2+m_2^2\,s_3^2+m_3^2\,c_3^2)c_1s_2+(m_2^2-m_3^2)s_1s_3c_3\right]\\
\mu^2 =& \frac{v^2}{v_1\,v_2}\,\mathrm{Re}(m_{12}^2)
\end{aligned}
$$

Couplings in the real 2HDM and C2HDM

- The relevant couplings are defined in the Lagrangian

$$\mathcal{L}_Y = -\left(\sqrt{2}G_\mu\right)^{1/2} m_f\, \bar{\psi}\,(a+ib\gamma_5)\,\psi\, h_i, \qquad [\textbf{SM}: a=1, b=0]$$

$$\mathcal{L}_{h_i H^+ H^-} = \lambda_i\, v\, h_i H^+ H^-, \qquad [\textbf{SM}: \lambda_i = 0]$$

$$\mathcal{L}_{h_i VV} = C_i \left[g\, m_W W_\mu^+ W^{\mu -} + \frac{g}{2c_W} m_Z Z_\mu Z^\mu \right] h \quad [\textbf{SM}: C_1=1, C_{i\neq 1}=0]$$

where a, b, and C_i are real, $c_W = \cos\theta_W$, and θ_W is the Weinberg angle. In the limit, $a = C_1 = 1$, and $b = \lambda = 0$, we obtain the SM

- We can write both C_i and λ_i in compact form

$$\begin{aligned} C_i =& c_\beta R_{i1} + s_\beta R_{i2} \\ -\lambda_i =& c_\beta \left[s_\beta^2 \lambda_{145} + c_\beta^2 \lambda_3 \right] R_{i1} + s_\beta \left[c_\beta^2 \lambda_{245} + s_\beta^2 \lambda_3 \right] R_{i2} \\ & + s_\beta c_\beta\, \mathrm{Im}(\lambda_5)\, R_{i3} \\ \lambda_{145} =& \lambda_1 - \lambda_4 - \mathrm{Re}(\lambda_5), \quad \lambda_{245} = \lambda_2 - \lambda_4 - \mathrm{Re}(\lambda_5) \end{aligned}$$

See Problems

Couplings in the real 2HDM and C2HDM

- The couplings C_i obey the sum rule (orthogonality of R)

$$\boxed{\sum_i C_i^2 = 1}$$

- One can check that, with our conventions, in the appropriate limit, ($\alpha_2 = \alpha_3 = 0$ and $\alpha_1 = \alpha$), these expressions reduce to those of the real 2HDM. In fact in this limit we have

$$R = \begin{bmatrix} c_1 & s_1 & 0 \\ -s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

leading to the decoupling of the CP odd scalar from the CP even.

- For instance

$$\boxed{C_1 = \cos(\beta - \alpha), \quad C_2 = \sin(\beta - \alpha), \quad C_1^2 + C_2^2 = 1} \tag{3}$$

Couplings to Fermions

- Finally the fermion couplings depend on the type of couplings chosen
- To avoid FCNC one needs that each fermion type will couple to only one Higgs doublet.
- In the 2HDM there are four possiblities, depending which Higgs doublet couples to which fermion. We have

	Up Quarks	Down Quarks	Leptons
Type I	ϕ_2	ϕ_2	ϕ_2
Type II	ϕ_2	ϕ_1	ϕ_1
Lepton Specific (Type X)	ϕ_2	ϕ_2	ϕ_1
Flipped (Type Y)	ϕ_2	ϕ_1	ϕ_2

Table 1: Different type of fermion couplings in the 2HDM.

Couplings to Fermions

- For fermions in the C2HDM in the form

$$(a_i + ib_i\gamma_5)$$

	Type I	Type II	Lepton Specific	Flipped
Up	$\frac{R_{i2}}{s_\beta} - ic_\beta\frac{R_{i3}}{s_\beta}\gamma_5$	$\frac{R_{i2}}{s_\beta} - ic_\beta\frac{R_{i3}}{s_\beta}\gamma_5$	$\frac{R_{i2}}{s_\beta} - ic_\beta\frac{R_{i3}}{s_\beta}\gamma_5$	$\frac{R_{i2}}{s_\beta} - ic_\beta\frac{R_{i3}}{s_\beta}\gamma_5$
Down	$\frac{R_{i2}}{s_\beta} + ic_\beta\frac{R_{i3}}{s_\beta}\gamma_5$	$\frac{R_{i1}}{c_\beta} - is_\beta\frac{R_{i3}}{c_\beta}\gamma_5$	$\frac{R_{i2}}{s_\beta} + ic_\beta\frac{R_{i3}}{s_\beta}\gamma_5$	$\frac{R_{i1}}{c_\beta} - is_\beta\frac{R_{i3}}{c_\beta}\gamma_5$
Leptons	$\frac{R_{i2}}{s_\beta} + ic_\beta\frac{R_{i3}}{s_\beta}\gamma_5$	$\frac{R_{i1}}{c_\beta} - is_\beta\frac{R_{i3}}{c_\beta}\gamma_5$	$\frac{R_{i1}}{c_\beta} - is_\beta\frac{R_{i3}}{c_\beta}\gamma_5$	$\frac{R_{i2}}{s_\beta} + ic_\beta\frac{R_{i3}}{s_\beta}\gamma_5$

- In the 2HDM: $R_{11} \to \cos\alpha, \quad R_{12} \to \sin\alpha, \quad R_{13} \to 0, R_{33} \to 1$
- So we get, as an example, for down quarks and Type II

$$a_1^d = \frac{c_\alpha}{c_\beta}, \quad b_1^d = 0$$

$$a_2^d = -\frac{s_\alpha}{c_\beta}, \quad b_2^d = 0$$

$$b_1^d = b_2 = 0, \quad b_3^d = t_\beta$$

Minimal Extension: 2HDM. Many Novelties

- Several particles with Spin 0
  - Neutral:
    - Scalars: h e H
    - Pseudoscalar: A
    - Or mixed: h_1, h_2, h_3 like in the C2HDM
  - Charged: $H^{\pm}$
- Properties of the minimum of the potential changed
  - Minima with charge breaking (to avoid!)
  - More than one minimum. What is the absolute minimum?

Problem Session

- A collection of problems is already available at the School Web Page
- This is a large collection and some of them are difficult
- We will try to discuss the following
  - Prob. 2.1
  - Prob. 3.1
  - Prob. 4.1
  - Prob. 4.4
  - Prob. 5.1
  - Prob. 5.3
- Try them before class