# Lecture 3

## Cooling of Neutron Stars

$n \to p + e^- + \bar{\nu}_e \sim k_B T$ Direct

$p + e^- \to n + \nu_e \sim k_B T$ Urca

$\mu_n = \mu_p + \mu_e$ $\quad \varepsilon \sim T^6$

$n_p = n_e$

Momentum Conservation

$\vec{k}_{Fn} = \vec{k}_{Fp} + \vec{k}_{Fe} + \vec{k}_\nu$ $\sim k_B T$

Modified Urca

$$\left.\begin{aligned} n + n &\to n + p + e^- + \bar{\nu}_e \\ n + p + e &\to n + n + \nu_e \end{aligned}\right] \quad \varepsilon \sim T^8$$

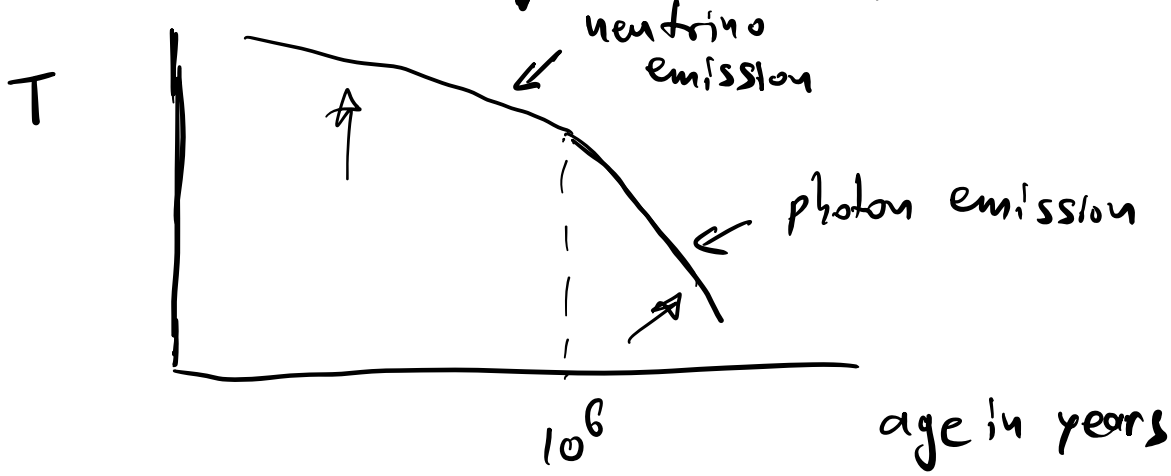
NS also loses energy by photon emission from blackbody radiation

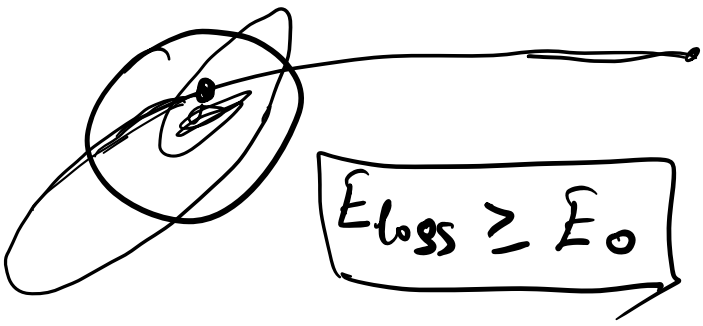
$$\varepsilon_\gamma \sim T_{surf}^4 \qquad T_{surf} \sim T^{0.55}$$

$$\varepsilon_\gamma \sim T^{2.2}$$

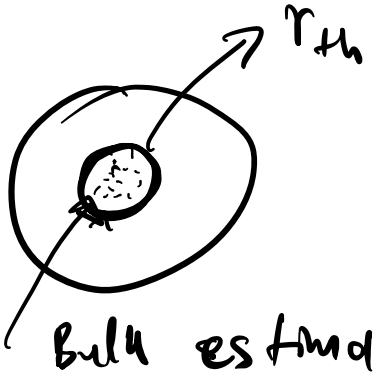
$$C_v \frac{dT}{dt} = -(L_v + L_\gamma)$$

$$\frac{dT}{dt} = \frac{-L_v - L_\gamma}{C_v} \sim \frac{-\#T^8 - \#T^{2.2}}{T}$$






$t = 10^{-5}$ years for 1 GeV




virial theorem

$$KE = \frac{1}{2}|PE|$$

Bulk estimate

$$\frac{3}{2} k_B T = \frac{1}{3} G \rho_m M r_{th}^2$$

$$r_{th} \simeq \left( \frac{9 k_B T}{2 G \rho_m M} \right)^{1/2}$$

$M$: mass of DM

# Annihilations of DM

$$\Gamma = n^2 \langle \sigma v \rangle \quad \left( \text{units } \frac{\#}{cm^3 \, sec} \right)$$

Total annihilation rate

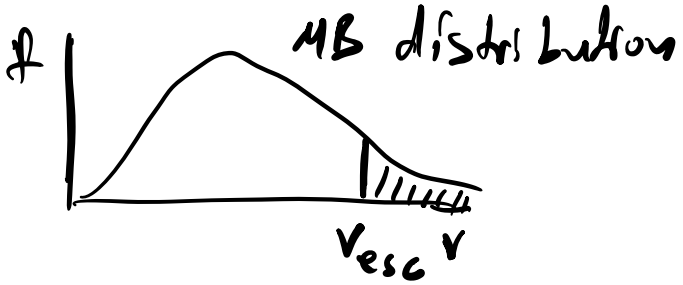
$$\Gamma_t = \underbrace{n^2}_{\uparrow} \langle \sigma v \rangle V_{th} = \frac{N^2}{\cancel{V_{th}^2}} \cancel{V_{th}} \langle \sigma v \rangle =$$

$$= \frac{\langle \sigma v \rangle}{V_{th}} N^2 = C_{ann} N^2$$

$$V_{th} = \frac{4}{3} \pi r_{th}^3$$

How the population of DM changes inside the NS

$$\frac{dN}{dt} = \overset{\text{accretion}}{\uparrow} C_{acc} + \overset{\text{self-capture}}{\uparrow} C_{sc} N - C_{ann} N^2 - \underset{\text{evaporation}}{\uparrow} C_{ev} N$$



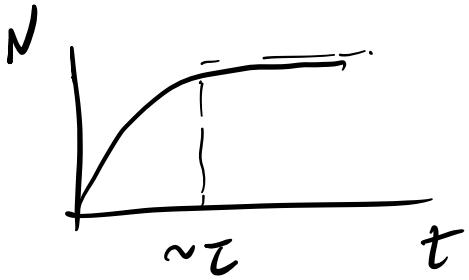

For the Sun
$m < 1\text{-}2$ GeV

$$\frac{dN}{dt} = C_{ace} - C_{ann} N^2$$

$$\int \frac{dN}{C_{ace} - C_{ann} N^2} = \int dt$$

$$N = \sqrt{\frac{C_{ace}}{C_{ann}}} \tanh\left(\sqrt{C_{ace} C_{ann}}\, t\right)$$

$$= \sqrt{\frac{C_{ace}}{C_{ann}}} \tanh \frac{t}{\tau} \qquad \tau = \frac{1}{\sqrt{C_{ace} C_{ann}}}$$



Once $t \gg \tau$ Accretion rate = accretion rate

$$\frac{dT}{dt} = \frac{-L_\nu - L_\gamma + L_{DM}}{C_V}$$