Modified Gravity: f(Q) and cosmological constraints.

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General Relativity

- The current theory of gravity is General Relativity. It was proposed by Albert Einstein in 1915.

- Gravity is considered as a consequence of the deformation of the space-time by the matter.

- Its representative action is:

$$S = \frac{1}{\kappa^2}\int d^4x\sqrt{-g}R + \int d^4x\sqrt{-g}\mathcal{L}_{mat}$$

where, in order to obtain the equations of motion, one should apply variations with respect the degrees of freedom.

- The most usual procedure considers the metric as the only degree of freedom, and the equations of motions are:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G T_{\mu\nu}$$

Cosmological Model

GR as gravitational theory + The Universe is considered homogeneous and isotropic → FLRW- metric + Observations

↓ ΛCDM model

- Big Bang
- Cold Dark Matter (CDM) to explain the velocity curves of galaxies and the structure formation
- Dark Energy (DE) related with the Cosmological Constant (CC) to explain the current acceleration of the Universe → Introduction of Λ
- The Universe is (nearly) flat
- There are three epochs dominated by radiation, matter and CC respectively
- Early acceleration to solve the horizon, flatness and monopole problem

Results

Achievements

- GR has passed all precision tests
  - Anomalous perihelion advance of Mercury
  - Gravitational lensing
  - Gravitational time dilation
  - …
- GR predicts gravitational waves
- ΛCDM is able to explain almost all observations until now

Problems

- Singularities in GR
- GR can not be quantized
- CDM has not been detected (directly)
- About the Cosmological Constant:
  - The CC Problem, related with theoretical predictions of its value
  - The Coincidence Problem
  - The Fine-Tuning Problem

P. Bull et al. Phys. Dark Univ. **12** (2016) [arXiv:1512.05356 [astro- ph.CO]]

Attempts to explain the current acceleration of the Universe without a cosmological constant

Introducing a new theory of gravity

Modified gravity

Emanuele Berti et al, Class.Quant.Grav. **32** (2015) 243001 [arXiv:0706.2151 [astro-ph]]

Maintaining GR as gravitational theory:

Changing the structure of the space-time

Introducing inhomogeneities and anisotropies

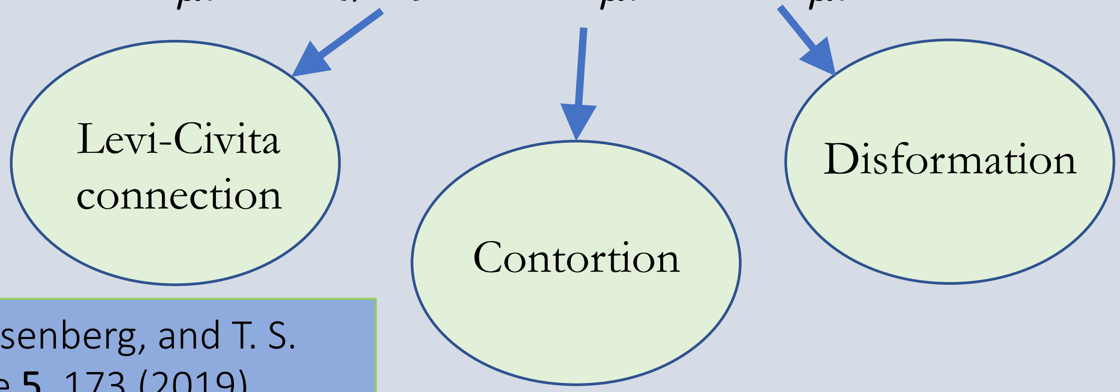
Backreaction models

Modifying the matter content

Modify the gravity through the connection

- Are we sure that the metric is the only fundamental degree of freedom?
- The connection tells us how tensors are transported on the manifold through the covariant derivative. In principle, one could consider it as a degree of freedom, looking for the more general assumptions.
- In General Relativity the connection is defined as the Levi-Civita connection expressed as the Christoffel symbols. However, the most general connection reads:

$$\Gamma^{\alpha}_{\mu\nu} = \left\{{}^{\,\alpha}_{\mu\nu}\right\} + K^{\alpha}{}_{\mu\nu} + L^{\alpha}{}_{\mu\nu}$$



J. B. Jiménez, L. Heisenberg, and T. S. Koivisto, Universe **5**, 173 (2019), [arXiv:1903.06830 [hep-th]]

Modify the gravity through the connection

- Let me assume the connection as a degree of freedom → metric – affine formalism.

- Thus, in this formalism, one should apply variations with respect to the metric and the connection. However, **the peculiarity of the Einstein-Hilbert action, i.e. the GR action, implies that the connection becomes the Levi-Civita one.**

- Therefore, for this action of gravity, both formalisms converges and reproduces the same gravity equations.

- However, **it will not be true for modified theories of gravity any more**.

Modify the gravity through the connection

- Are we sure that the metric is the only fundamental degree of freedom?
- The connection tells us how tensors are transported on the manifold. In principle, one could consider it as a degree of freedom, looking for the more general assumptions.
- A general connection can be decomposed as follows:

$$\Gamma^{\alpha}_{\mu\nu} = \left\{ {}^{\alpha}_{\mu\nu} \right\} + K^{\alpha}{}_{\mu\nu} + L^{\alpha}{}_{\mu\nu}$$

and:

$$K^{\alpha}{}_{\mu\nu} = \frac{1}{2}T^{\alpha}{}_{\mu\nu} + T_{(\mu}{}^{\alpha}{}_{\nu)} \qquad L^{\alpha}{}_{\mu\nu} = \frac{1}{2}Q^{\alpha}{}_{\mu\nu} - Q_{(\mu}{}^{\alpha}{}_{\nu)}$$

where

$T^{\alpha}_{\mu\nu} \equiv \Gamma^{\alpha}_{\mu\nu} - \Gamma^{\alpha}_{\nu\mu}$ Torsion → antisymmetric part of the connection

$Q_{\alpha\mu\nu} \equiv \nabla_{\alpha} g_{\mu\nu}$ Non-metricity tensor

Can we build GR from the non-metricity scalar?

- So in GR:

$$S_{GR} = \frac{1}{16\pi G}\int d^x \sqrt{-g} R(\{\})$$

- And therefore the scalar of curvature:

$$R(\hat{\Gamma}) = R(\{\}) + \nabla_\alpha \left(Q^\alpha - \tilde{Q}^\alpha\right) + \frac{1}{4}Q^{\alpha\beta\gamma}Q_{\alpha\beta\gamma} - \frac{1}{2}Q^{\gamma\alpha\beta}Q_{\alpha\gamma\beta} - \frac{1}{4}Q^\alpha Q_\alpha + \frac{1}{2}\tilde{Q}^\alpha Q_\alpha$$

We are going to work with the constraint of a flat space-time. This framework of modified gravity theories is called teleparallel.

Non-metricity scalar Q

$$Q_\alpha = Q_\alpha{}^\mu{}_\mu$$

$$\tilde{Q}_\alpha = Q^\mu{}_{\alpha\mu}$$

$$R(\{\}) = -Q - \nabla_\alpha^{\{\}}(Q^\alpha - \tilde{Q}^\alpha)$$

And what about a modified gravity theory from Q?

$$S_{STEGR} = -\frac{1}{16\pi G}\int d^4x\sqrt{-g}Q \quad \Rightarrow \quad S = \int d^4x\sqrt{-g}\left[-\frac{1}{2}f(Q) + \mathcal{L}_M\right]$$

- Working in the coincident gauge, allows to use a null connection:

$$_{cg}\Gamma^{\alpha}_{\mu\nu} = 0 \quad \rightarrow \quad \nabla_\alpha g_{\mu\nu} = \partial_\alpha g_{\mu\nu}$$

and for a FLRW space-time:

$$ds^2 = -N^2(t)dt^2 + a^2(t)\left[dx^2 + dy^2 + dz^2\right]$$

the non-metricity scalar becomes:

$$Q = 6\frac{H^2}{N^2}$$

Q retains a residual time-reparameterization invariance, so we can choose N(t)=1.

J. B. Jiménez, L. Heisenberg, and T. S. Koivisto, JCAP **1808**, 039 (2018), [arXiv:1803.10185 [gr-qc]]

Ismael Ayuso

And what about a modified gravity theory from Q?

$$S_{STEGR} = -\frac{1}{16\pi G}\int d^4x\sqrt{-g}Q \quad \Rightarrow \quad S = \int d^4x\sqrt{-g}\left[-\frac{1}{2}f(Q) + \mathcal{L}_M\right]$$

- The equations of motion for Coincident gauge, FLRW space-time and N(t)=1:

$$6f_Q(Q)H^2 - \frac{1}{2}f(Q) = \rho$$

$$\left[12H^2 f_{QQ}(Q) + f_Q(Q)\right]\dot{H} = -\frac{1}{2}(\rho + p)$$

An example of a f(Q) model

$$f(Q) = \frac{1}{8\pi G}\left[Q - 6\lambda M^2\left(\frac{Q}{6M^2}\right)^{\alpha}\right]$$

J. B. Jiménez, L. Heisenberg, T. S. Koivisto, and S. Pekar, Phys. Rev. D **101,** 103507 (2020), [arXiv:1906.10027 [gr-qc]]

- We are going to focus on the case α=-1, which leads to:

$$H_{\pm}^2 = \frac{4\pi G}{3}\rho\left(1 \pm \sqrt{1 - \frac{27\lambda M^4}{(4\pi G\rho)^2}}\right)$$

- We consider the three usual kinds of matter-energy: cosmic dust, radiation and cosmological constant.

$$\dot{\rho} = -3H(\rho + p) \Longrightarrow \rho = \frac{3H_0^2}{8\pi G}\left(\Omega_\Lambda + \Omega_m(1+z)^3 + \Omega_r(1+z)^4\right)$$

- We will do the normalization of the Hubble function: $E(z=0) = \dfrac{H_{\pm}(z=0)}{H_0} = 1$

$$-\frac{M^4\lambda}{H_0^4} = \frac{1}{3}(1 - \Omega_\Lambda - \Omega_m - \Omega_r) \equiv \frac{\Omega_Q}{3}$$

It is necessary to impose the condition:

$$0 < \Omega_m + \Omega_\Lambda + \Omega_r < 2$$

An example of a f(Q) model

- At the end of the day, the model becomes:

$$H_{\pm}^2 = \frac{H_0}{2}\left[\Omega_\Lambda + \Omega_m(1+z)^3 + \Omega_r(1+z)^4\right]\left(1 \pm \sqrt{1 + \frac{4\Omega_Q}{\left[\Omega_\Lambda + \Omega_m(1+z)^3 + \Omega_r(1+z)^4\right]^2}}\right)$$

  We will focus on the positive branch which could have a similar behaviour to **Λ**CDM.

- The case, where the term associated with Q vanishes, converges to General Relativity with a Cosmological Constant.

- How can we fit these parameters?

Fitting the free parameters through a MCMC

- The tests, to fit the free parameters, will be implemented using a Markov Chain Monte Carlo (MCMC) code, upon minimization of a total χ^2 (which measures how far the parameters are from fitting data)

- Our results are drawn under the assumption of some priors, which give some room from modified gravity features, enforce the choice of the right branch and preclude nonphysical behaviour and pronounced departures from the well established standard evolution:

$$0 < \Omega_m$$

$$0 < \Omega_r < \Omega_b < \Omega_m$$

$$0 < h < 1$$

$$0 < \Omega_m + \Omega_\Lambda + \Omega_r < 2$$

Set of data

Pantheon

- Sample of 1048 Type Ia Supernovae ($0.01<z<2.26$) used as standard candles, though the use of the distance modulus.

Hubble

- Sample of values of H(z) ($0.007<z<1.965$), obtained from Early-type galaxies with a passive evolution (from the age difference of these galaxies at somewhat different redshifts)

CMB

- This set of data is condensed into the three so called shift parameters, which inform us of the position of the first peak in the temperature angular power spectrum.

BAO

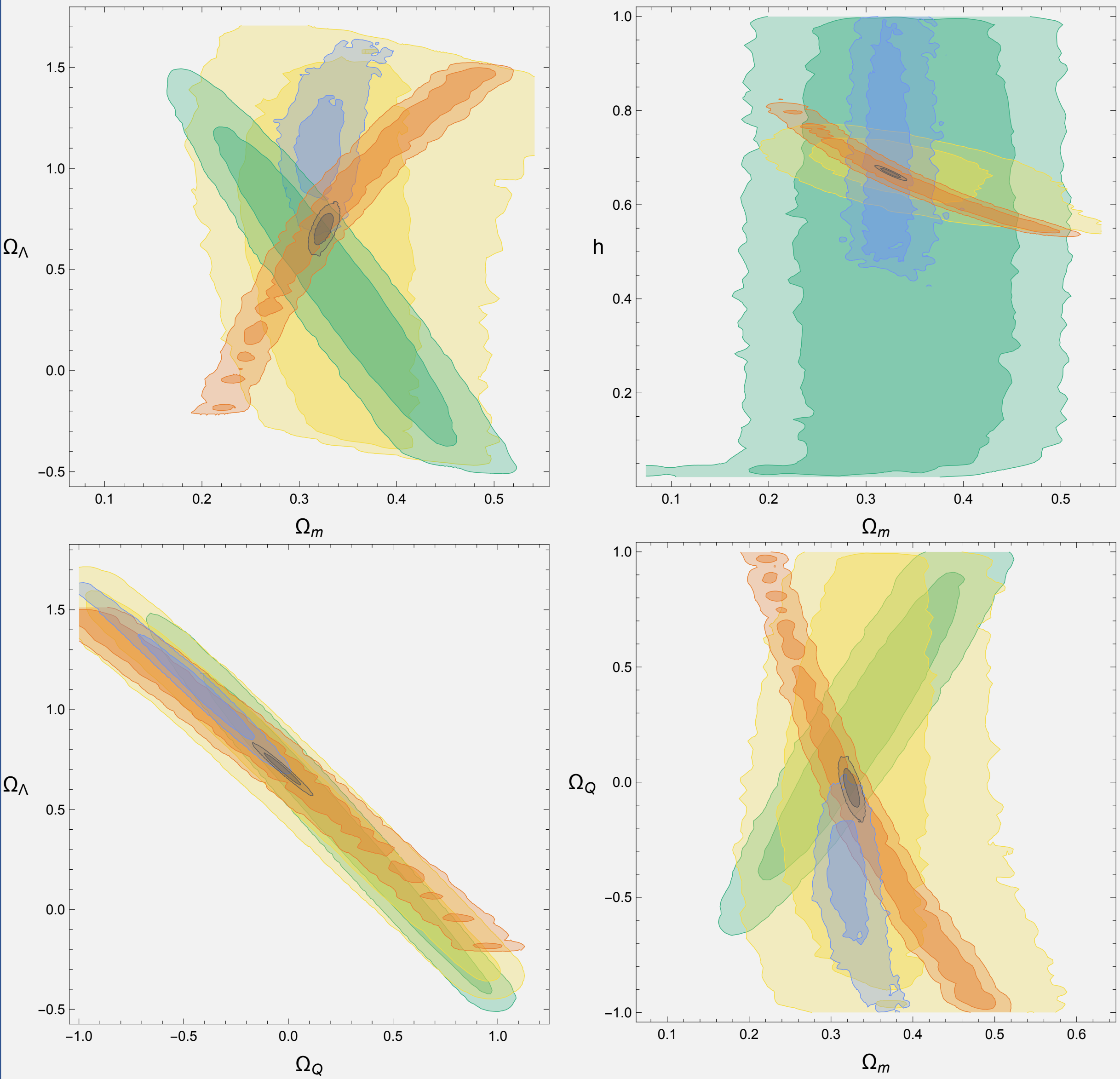
- Sets of data about the Baryon Acoustic Oscillations. We use 5 sets:
  - ✓ WiggleZ
  - ✓ BOSS
  - ✓ eBOSS
  - ✓ Boos-Lyman α
  - ✓ Voids-galaxy cross-correlation

Results: best fits

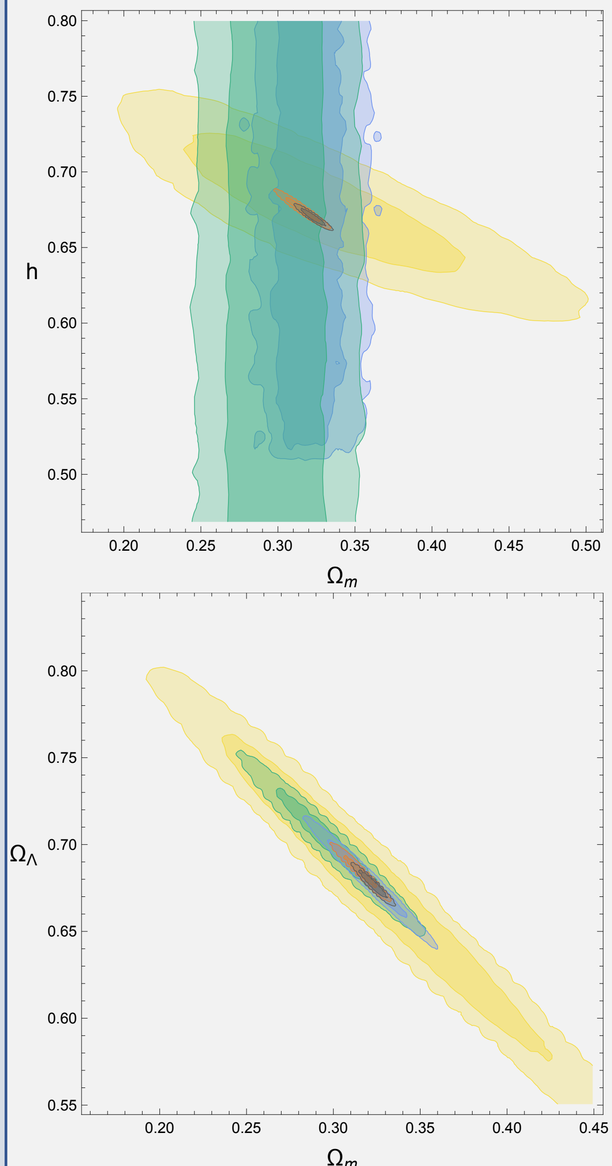
		Pantheon	Hubble	CMB	BAO	Total
Ω_m	ΛCDM	$0.297^{+0.023}_{-0.023}$	$0.327^{+0.067}_{-0.056}$	$0.3156^{+0.0074}_{-0.0073}$	$0.320^{+0.016}_{-0.015}$	$0.3229^{+0.0048}_{-0.0048}$
	$f(Q)_{\Omega_\Lambda\neq 0}$	$0.337^{+0.075}_{-0.073}$	$0.341^{+0.070}_{-0.060}$	$0.346^{+0.092}_{-0.080}$	$0.323^{+0.020}_{-0.017}$	$0.32503^{+0.0067}_{-0.0065}$
	$f(Q)_{\Omega_\Lambda=0}$	$0.400^{+0.024}_{-0.024}$	$0.350^{+0.057}_{-0.049}$	$0.2384^{+0.0058}_{-0.0057}$	$0.348^{+0.016}_{-0.016}$	$0.2848^{+0.0044}_{-0.0042}$
h	ΛCDM	$0.50^{+0.32}_{-0.32}$	$0.677^{+0.031}_{-0.031}$	$0.675^{+0.0054}_{-0.0053}$	$0.73^{+0.16}_{-0.14}$	$0.6700^{+0.0034}_{-0.0034}$
	$f(Q)_{\Omega_\Lambda\neq 0}$	$0.51^{+0.31}_{-0.30}$	$0.674^{+0.039}_{-0.054}$	$0.645^{+0.090}_{-0.071}$	$0.70^{+0.18}_{-0.14}$	$0.6674^{+0.0065}_{-0.0067}$
	$f(Q)_{\Omega_\Lambda=0}$	$0.51^{+0.31}_{-0.31}$	$0.703^{+0.029}_{-0.030}$	$0.7768^{+0.0065}_{-0.0065}$	$0.76^{+0.14}_{-0.12}$	$0.7301^{+0.0040}_{-0.0039}$
Ω_Λ	ΛCDM	$0.703^{+0.022}_{-0.023}$	$0.673^{+0.056}_{-0.067}$	$0.6843^{+0.0073}_{-0.0074}$	$0.680^{+0.015}_{-0.016}$	$0.6770^{+0.0048}_{-0.0048}$
	$f(Q)_{\Omega_\Lambda\neq 0}$	$0.43^{+0.47}_{-0.49}$	$0.64^{+0.59}_{-0.60}$	$0.87^{+0.43}_{-0.57}$	$1.11^{+0.21}_{-0.18}$	$0.701^{+0.054}_{-0.053}$
	$f(Q)_{\Omega_\Lambda=0}$	-	-	-	-	-
Ω_Q	ΛCDM	-	-	-	-	-
	$f(Q)_{\Omega_\Lambda\neq 0}$	$0.23^{+0.42}_{-0.40}$	$0.03^{+0.58}_{-0.61}$	$-0.22^{+0.65}_{-0.52}$	$-0.43^{+0.18}_{-0.22}$	$-0.027^{+0.057}_{-0.058}$
	$f(Q)_{\Omega_\Lambda=0}$	$0.599^{+0.023}_{-0.024}$	$0.650^{+0.049}_{-0.057}$	$0.7615^{+0.0057}_{-0.0058}$	$0.651^{+0.016}_{-0.016}$	$0.7151^{+0.0042}_{-0.0044}$
Ω_b	ΛCDM	-	-	$0.04905^{+0.00060}_{-0.00060}$	$0.063^{+0.012}_{-0.031}$	$0.04962^{+0.00041}_{-0.00040}$
	$f(Q)_{\Omega_\Lambda\neq 0}$	-	-	$0.057^{+0.014}_{-0.012}$	$0.081^{+0.019}_{-0.036}$	$0.0501^{+0.0010}_{-0.0010}$
	$f(Q)_{\Omega_\Lambda=0}$	-	-	$0.03706^{+0.00050}_{-0.00048}$	$0.042^{+0.011}_{-0.021}$	$0.04073^{+0.00037}_{-0.00036}$

Results: contour plots

f(Q)



ΛCDM



Results: how good best fits are

		Pantheon	Hubble	CMB	BAO Total	Total
χ^2	ΛCDM	1035.77	14.4942	0.00104677	16.5496	1072.19
	$f(Q)_{\Omega_\Lambda \neq 0}$	1035.72	14.3987	0.00534963	11.3411	1072.01
	$f(Q)_{\Omega_\Lambda = 0}$	1036.48	14.5340	0.00287402	51.3390	1207.96

Results: effective matter component $p_{\text{eff}} = w_{\text{eff}}\,\rho$

$$H^2 = H_0 \Omega_m (1+z)^3 + \frac{8\pi G}{3}\rho_{\text{eff}}$$

$$w_{\text{eff}}(z) = \frac{\frac{2}{3}(1+z)\frac{d\ln E(z)}{dz} - 1}{1 - E^{-2}(z)\Omega_m(1+z)^3}$$

$$w_{\text{eff}}\big|_{z=0} = -0.987^{+0.032}_{-0.027}$$

Summarize and conclusions

- We are able to build modified gravity models from the non-metricity.
- We have focused on the case given by:

$$f(Q) = \frac{1}{8\pi G}\left[Q - 6\lambda M^2 \left(\frac{Q}{6M^2}\right)^{-1}\right]$$

$$\Downarrow$$

$$H^2 = \frac{H_0}{2}\left[\Omega_\Lambda + \Omega_m(1+z)^3 + \Omega_r(1+z)^4\right]\left(1 + \sqrt{1 + \frac{4\Omega_Q}{\left[\Omega_\Lambda + \Omega_m(1+z)^3 + \Omega_r(1+z)^4\right]^2}}\right)$$

- and fitted the free parameters through an MCMC to compare them with the values of the parameters of the **Λ**CDM model.
- Ω_Q and Ω_{Λ} are super-correlated because both parameters play a similar role. But the introduction of Ω_Q breaks the correlation between other parameters as Ω_m and Ω_{Λ}

Summarize and conclusions

- The f(Q) model seems as good as the **Λ**CDM model. However, this is because the best fit is when the f(Q) model converges to **Λ**CDM, i.e. $\Omega_Q=0$.

- From the Bayesian evidence, according to Jeffreys' scale no model is preferred over the other. However, this is an obvious fact since the new phenomenology of the model vanishes when we obtain the best fit.

- Concerning to the cosmographic parameters, its values reflect the striking similarity between both models. However, for the modified gravity model, these parameters are more poorly constrained as their complexity penalizes error propagation.

- This model is not able to reproduce the phenomenology of **Λ**CDM without the problems associated with the Cosmological Constant.

Thanks for your attention!