Neutral meson oscillations and matter-antimatter asymmetry

Miguel Nebot

CFTP - IST Lisbon

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Introduction

P, T, C Transformations

Discrete transformations

- P, space inversion: $(t,\vec{r}) \mapsto (t,-\vec{r})$
- T, time reversal: $(t,\vec{r}) \mapsto (-t,\vec{r})$
- C, charge conjugation: $Q \mapsto -Q$ (in general $\mathfrak{r} \mapsto \mathfrak{r}^*$)

Key points

- Electromagnetic and strong interactions are C, P and T invariant
- Weak interactions badly violate C and P,

  . . . and they also violate CP
- Sakharov conditions in order to obtain a net baryon asymmetry in the Universe
  - baryon number violation
  - departure from thermal equilibrium
  - *C and CP violation*

Discrete symmetries C, P, T

The CPT Theorem

Any quantum field theory which is local, Lorentz invariant, and which respects the spin-statistics connection,

is invariant under the product CPT

Some consequences

- Particle/Antiparticle have equal mass and width
- $\not{C}\not{P} \Rightarrow \not{T}$
- ...

but, it is a property of <u>QFT</u>

- one can e.g. build an observable sensitive to CP violation but unrelated to T violation

Neutral Meson Systems

Neutral meson systems

- SM content

  3 generations of quarks: $\begin{pmatrix} u \\ d \end{pmatrix} \quad \begin{pmatrix} c \\ s \end{pmatrix} \quad \begin{pmatrix} t \\ b \end{pmatrix}$

- Bound states quark-antiquark: $M^0 = q_1\bar{q}_2$, $\bar{M}^0 = \bar{q}_1 q_2$

  N.B. $Q(q_1) = Q(q_2)$, $q_1 \neq q_2$ i.e. $M^0 \neq \bar{M}^0$

- Best known cases, lightest pseudoscalar mesons

  $K^0 = \bar{s}d$, $D^0 = \bar{c}u$, $B_d^0 = \bar{b}d$, $B_s^0 = \bar{b}s$

- M^0 and $\bar{M}^0$ have the same strong and electromagnetic properties

- Strong and electromagnetic interactions conserve flavour

  (i.e. no $M^0 \leftrightarrows \bar{M}^0$ transitions)

  but this is not the case for weak interactions

  - they violate C, P, CP, T
  - they induce flavour transitions

Neutral meson mixing

A simple description (QM)

- System controlled by a two level effective Hamiltonian $\mathbf{H}$

$$\mathbf{H} = \mathbf{M} - i\mathbf{\Gamma}/2 \quad \text{with} \quad \mathbf{M}^\dagger = \mathbf{M},\ \mathbf{\Gamma}^\dagger = \mathbf{\Gamma}$$

- Dispersive $\mathbf{M}$ and absorptive $\mathbf{\Gamma}$ parts
- $\mathbf{H}_{ij}$ from 2^{nd} order perturbation theory (in the weak interaction)

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$$[\mathbf{M}]_{ij} = M_0\delta_{ij} + \langle i|\mathcal{H}_{\text{w}}|j\rangle + \sum_n \mathrm{P}\frac{\langle i|\mathcal{H}_{\text{w}}|n\rangle\langle n|\mathcal{H}_{\text{w}}|j\rangle}{M_0 - E_n}$$

$$[\mathbf{\Gamma}]_{ij} = 2\pi\sum_n \delta(M_0 - E_n)\,\langle i|\mathcal{H}_{\text{w}}|n\rangle\langle n|\mathcal{H}_{\text{w}}|j\rangle$$

$$\{|M^0\rangle, |\bar{M}^0\rangle\} = \{|1\rangle, |2\rangle\}$$

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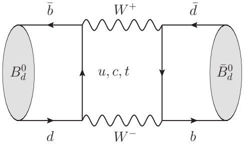
- $\mathbf{H}_{ij}$ from 2$^{\text{nd}}$ order perturbation theory (in the weak interaction)

$[\mathbf{M}]_{21} \rightarrow$



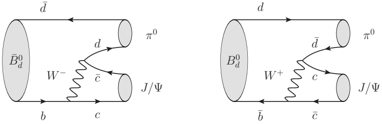
Neutral meson mixing

A simple description (QM)

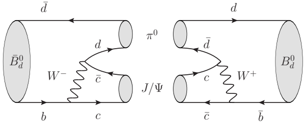
- System controlled by a two level effective Hamiltonian **H**

$$\mathbf{H} = \mathbf{M} - i\mathbf{\Gamma}/2 \quad \text{with} \quad \mathbf{M}^\dagger = \mathbf{M},\ \mathbf{\Gamma}^\dagger = \mathbf{\Gamma}$$

- Dispersive **M** and absorptive **Γ** parts
- $\mathbf{H}_{ij}$ from 2[nd] order perturbation theory (in the weak interaction)



$[\mathbf{\Gamma}]_{12} \quad \rightarrow$



Effective Hamiltonian for Neutral Meson Mixing

Eigenvectors[1]:

$$\mathbf{H}|M_H\rangle = \mu_H|M_H\rangle, \qquad |M_H\rangle = p_H|M^0\rangle + q_H|\bar{M}^0\rangle,$$
$$\mathbf{H}|M_L\rangle = \mu_L|M_L\rangle, \qquad |M_L\rangle = p_L|M^0\rangle - q_L|\bar{M}^0\rangle.$$

Eigenvalues: $\mu_{H,L} = M_{H,L} - \frac{i}{2}\Gamma_{H,L}$

$$\mu = \mu_H + \mu_L \equiv M - \frac{i}{2}\Gamma, \quad \Delta\mu = \mu_H - \mu_L \equiv \Delta M - \frac{i}{2}\Delta\Gamma,$$

Evolution

$$i\hbar\frac{d}{dt}|\Psi(t)\rangle = \mathbf{H}|\Psi(t)\rangle \;\Rightarrow\; |\Psi(0)\rangle \mapsto |\Psi(t)\rangle = e^{-i\mathbf{H}t/\hbar}|\Psi(0)\rangle$$

[1] N.B. "H" and "L" correspond to the "heavy" and "light" states respectively, $\Delta M > 0$ and the sign of $\Delta\Gamma$ is not a matter of convention; fine for B_d, B_s but bad notation for K

Effective Hamiltonian for Neutral Meson Mixing

Hamiltonian:

$$\mathbf{H} = \begin{pmatrix} \mu - \frac{\Delta\mu}{2}\theta & \frac{p}{q}\frac{\Delta\mu}{2}\sqrt{1-\theta^2} \\ \frac{q}{p}\frac{\Delta\mu}{2}\sqrt{1-\theta^2} & \mu + \frac{\Delta\mu}{2}\theta \end{pmatrix} .$$

Mixing parameters $\theta, q/p \in \mathbb{C}$:

$$\frac{q_H}{p_H} = \frac{q}{p}\sqrt{\frac{1+\theta}{1-\theta}}\,, \qquad \frac{q_L}{p_L} = \frac{q}{p}\sqrt{\frac{1-\theta}{1+\theta}}\,, \qquad \delta = \frac{1-|q/p|^2}{1+|q/p|^2}\,.$$

$$\theta = \frac{\mathbf{H}_{22}-\mathbf{H}_{11}}{\Delta\mu}\,, \quad \left(\frac{q}{p}\right)^2 = \frac{\mathbf{H}_{21}}{\mathbf{H}_{12}}\,.$$

- θ is CP and CPT violating,
- δ is CP and T violating.

(N.B. p is $\mathbf{X}$ violating $\Leftrightarrow$ for $p \neq 0$, $\mathbf{X}$ is not a symmetry)

Effective Hamiltonian for Neutral Meson Mixing

Hamiltonian:

$$\mathbf{H} = \begin{pmatrix} \mu - \frac{\Delta\mu}{2}\theta & \frac{p}{q}\frac{\Delta\mu}{2}\sqrt{1-\theta^2} \\ \frac{q}{p}\frac{\Delta\mu}{2}\sqrt{1-\theta^2} & \mu + \frac{\Delta\mu}{2}\theta \end{pmatrix} .$$

- θ is CP and CPT violating,
- δ is CP and T violating.

$$\mathbf{H}_{\mathrm{O}} = (O)^{\dagger}\,\mathbf{H}\,(O)$$

$$\mathrm{CP}|M^0\rangle = e^{+i\alpha}|\bar{M}^0\rangle,\ \mathrm{TCP}|M^0\rangle = e^{i\beta}|\bar{M}^0\rangle,$$
$$\mathrm{CP}|\bar{M}^0\rangle = e^{-i\alpha}|M^0\rangle,\ \mathrm{TCP}|\bar{M}^0\rangle = e^{i\beta}|M^0\rangle.$$

Effective Hamiltonian for Neutral Meson Mixing

Hamiltonian:

$$\mathbf{H} = \begin{pmatrix} \mu - \frac{\Delta\mu}{2}\theta & \frac{p}{q}\frac{\Delta\mu}{2}\sqrt{1-\theta^2} \\ \frac{q}{p}\frac{\Delta\mu}{2}\sqrt{1-\theta^2} & \mu + \frac{\Delta\mu}{2}\theta \end{pmatrix}.$$

- θ is CP and CPT violating,
- δ is CP and T violating.

$$\langle M^0|(\mathbf{H}-\mathbf{H}_{\text{CP}})|M^0\rangle = \langle M^0|\mathbf{H}|M^0\rangle - \langle \bar{M}^0|\mathbf{H}|\bar{M}^0\rangle = -\Delta\mu\,\theta$$

$$\langle M^0|(\mathbf{H}-\mathbf{H}_{\text{TCP}})|M^0\rangle = \langle M^0|\mathbf{H}|M^0\rangle - \langle \bar{M}^0|(T)^\dagger\,\mathbf{H}\,T|\bar{M}^0\rangle = \langle M^0|\mathbf{H}|M^0\rangle - \langle \bar{M}^0|\,\mathbf{H}\,|\bar{M}^0\rangle^* = -\text{Re}(\Delta\mu\,\theta)$$

Time evolution of $|M^0\rangle$, $|\bar{M}^0\rangle$

- Consider an initial state $|M^0\rangle$, then (with no CPT violation)

$$
\begin{aligned}
e^{-i\mathbf{H}t}|M^0\rangle = e^{-i\mathbf{H}t}\frac{1}{2p}\left\{|M_H\rangle + |M_L\rangle\right\} \\
&= \frac{1}{2p}\left\{e^{-i\mu_H t}|M_H\rangle + e^{-i\mu_L t}|M_L\rangle\right\} \\
&= \frac{1}{2}\left\{(e^{-i\mu_H t} + e^{-i\mu_L t})|M^0\rangle + \frac{q}{p}(e^{-i\mu_H t} - e^{-i\mu_L t})|\bar{M}^0\rangle\right\} \\
&= \frac{e^{-i\mu t}}{2}\left\{g_+(t)|M^0\rangle + g_-(t)\frac{q}{p}|\bar{M}^0\rangle\right\}
\end{aligned}
$$

$$
e^{-i\mathbf{H}t}|M^0\rangle = |M^0(t)\rangle = \frac{e^{-i\mu t}}{2}\left\{g_+(t)|M^0\rangle + g_-(t)\frac{q}{p}|\bar{M}^0\rangle\right\}
$$

and

$$
e^{-i\mathbf{H}t}|\bar{M}^0\rangle = |\bar{M}^0(t)\rangle = \frac{e^{-i\mu t}}{2}\left\{g_+(t)\frac{p}{q}|M^0\rangle - g_-(t)|\bar{M}^0\rangle\right\}
$$

Time evolution of $|M^0\rangle$, $|\bar{M}^0\rangle$

- Including CPT violation

$$|M^0(t)\rangle = \frac{e^{-i\mu t}}{2}\left\{\left[g_+(t) - \theta\, g_-(t)\right]|M^0\rangle + \left[g_-(t)\frac{q}{p}\sqrt{1-\theta^2}\right]|\bar{M}^0\rangle\right\}$$

and

$$|\bar{M}^0(t)\rangle = \frac{e^{-i\mu t}}{2}\left\{\left[g_-(t)\frac{p}{q}\sqrt{1-\theta^2}\right]|M^0\rangle + \left[g_+(t) + \theta\, g_-(t)\right]|\bar{M}^0\rangle\right\}$$

Observables (i.e. include decays!)

- Consider a decay at time t into a state $|f\rangle$ with amplitudes

$$M^0 \to f:\ A_f \quad \text{and} \quad \bar{M}^0 \to f:\ \bar{A}_f$$

Compute the probabilities $\Pr(M^0(t) \to f)$ and $\Pr(\bar{M}^0(t) \to f)$!
First

$$\mathcal{A}(M^0(t) \to f) \to e^{-i\mu t}\frac{A_f}{2}\left\{g_+(t) + g_-(t)\frac{q}{p}\frac{\bar{A}_f}{A_f}\right\}$$

$$\mathcal{A}(\bar{M}^0(t) \to f) \to e^{-i\mu t}\frac{\bar{A}_f}{2}\left\{g_+(t)\frac{p}{q}\frac{A_f}{\bar{A}_f} - g_-(t)\right\}$$

- λ_f collects information of mixing $\times$ decay:

$$\lambda_f \equiv \frac{q}{p}\frac{\bar{A}_f}{A_f}, \quad C_f \equiv \frac{1-|\lambda_f|^2}{1+|\lambda_f|^2}, \quad S_f \equiv \frac{2\mathrm{Im}(\lambda_f)}{1+|\lambda_f|^2}, \quad R_f \equiv \frac{2\mathrm{Re}(\lambda_f)}{1+|\lambda_f|^2}$$

- Time dependence

$$|g_\pm(t)|^2 = 2\left\{\cosh\left(\frac{\Delta\Gamma}{2}t\right) \pm \cos(\Delta M t)\right\}$$
$$g_+(t)^* g_-(t) = -2\left\{\sinh\left(\frac{\Delta\Gamma}{2}t\right) + i\sin(\Delta M t)\right\}$$

- Rearrange terms

$$\Gamma(M^0(t)\to f) = e^{-\Gamma\, t}\frac{|\bar{A}_f|^2}{1-C_f}$$
$$\left\{\cosh\left(\frac{\Delta\Gamma}{2}t\right) + C_f\cos(\Delta M t) + S_f\sin(\Delta M t) - R_f\sinh\left(\frac{\Delta\Gamma}{2}t\right)\right\}$$
$$\Gamma(\bar{M}^0(t)\to f) = e^{-\Gamma\, t}\frac{|A_f|^2}{1+C_f}$$
$$\left\{\cosh\left(\frac{\Delta\Gamma}{2}t\right) - C_f\cos(\Delta M t) - S_f\sin(\Delta M t) - R_f\sinh\left(\frac{\Delta\Gamma}{2}t\right)\right\}$$

General time dependence

- Functional form

$$\Gamma\left(\begin{smallmatrix} M^0 \\ \bar{M}^0 \end{smallmatrix}(t) \to f\right) = e^{-\Gamma\, t} \times$$

$$\left\{\mathscr{C}_h \cosh\left(\frac{\Delta\Gamma}{2} t\right) + \mathscr{C}_c \cos(\Delta M t) + \mathscr{S}_c \sin(\Delta M t) + \mathscr{S}_h \sinh\left(\frac{\Delta\Gamma}{2} t\right)\right\}$$

- where [N.B. $\Gamma_f = \frac{1}{2}(|A_f|^2 + |\bar{A}_f|^2)$]

$$\mathscr{C}_h = \frac{2\Gamma_f(1 \mp \delta)}{(1-\delta C_f)} \left\{\begin{matrix} [1 \pm C_f](1+|\theta|^2) + |1-\theta^2|[1 \mp C_f] \\ \mp 2\mathrm{Re}\left(\theta^* \sqrt{1-\theta^2}\right) R_f + 2\mathrm{Im}\left(\theta^* \sqrt{1-\theta^2}\right) S_f \end{matrix}\right\}$$

$$\mathscr{C}_c = \frac{2\Gamma_f(1 \mp \delta)}{(1-\delta C_f)} \left\{\begin{matrix} [1 \pm C_f](1-|\theta|^2) - |1-\theta^2|[1 \mp C_f] \\ \pm 2\mathrm{Re}\left(\theta^* \sqrt{1-\theta^2}\right) R_f - 2\mathrm{Im}\left(\theta^* \sqrt{1-\theta^2}\right) S_f \end{matrix}\right\}$$

$$\mathscr{S}_h = \frac{4\Gamma_f(1 \mp \delta)}{(1-\delta C_f)} \left\{\begin{matrix} -\mathrm{Re}\left(\sqrt{1-\theta^2}\right) R_f \pm \mathrm{Im}\left(\sqrt{1-\theta^2}\right) S_f \\ +\mathrm{Re}(\theta)\,[C_f \pm 1] \end{matrix}\right\}$$

$$\mathscr{S}_c = \frac{4\Gamma_f(1 \mp \delta)}{(1-\delta C_f)} \left\{\begin{matrix} \pm\mathrm{Re}\left(\sqrt{1-\theta^2}\right) S_f + \mathrm{Im}\left(\sqrt{1-\theta^2}\right) R_f \\ +\mathrm{Im}(\theta)\,[-C_f \mp 1] \end{matrix}\right\}$$

Some facts

	Quarks	M	Γ	ΔM	$\Delta\Gamma$
K^0–$\bar{K}^0$	sd	0.498 GeV	5.56 ns^{-1}	5.30 ns^{-1}	5.54 ns^{-1}
D^0–$\bar{D}^0$	cu	1.864 GeV	2.439 ps^{-1}	9.5 ns^{-1}	3.15 ns^{-1}
B_d^0–$\bar{B}_d^0$	bd	5.280 GeV	0.658 ps^{-1}	0.506 ps^{-1}	$\simeq 0$
B_s^0–$\bar{B}_s^0$	bs	5.367 GeV	0.664 ps^{-1}	17.76 ps^{-1}	0.08 ps^{-1}

	$\Delta M/\Gamma$	$\Delta\Gamma/\Gamma$
K^0–$\bar{K}^0$	0.953	0.996
D^0–$\bar{D}^0$	$3.9 \cdot 10^{-3}$	$1.3 \cdot 10^{-3}$
B_d^0–$\bar{B}_d^0$	0.769	0
B_s^0–$\bar{B}_s^0$	26.75	0.12

[N.B. $\hbar = 6.58 \times 10^{-22}$ MeV s $\Rightarrow$ 1 $\text{ps}^{-1} \simeq 0.658$ meV]

Why do you bother us with *this*???

Why do you bother us with *this*???

. . . because it allows us to play some simple "games" to show:

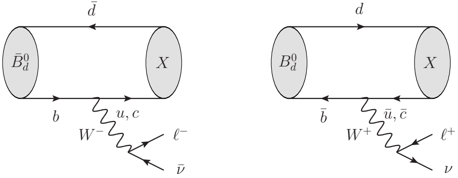
- that these mesons oscillate,
- that we can observe matter-antimatter asymmetry,
  that is, CP violation!
- One can even study time reversal asymmetries!

Meson oscillations

We need one (simple) extra ingredient

- **Flavour Tag** (for B_d^0 here, $\ell = e, \mu$):

  $\bar{B}_d^0$ decays into ℓ^- and B_d^0 decays into ℓ^+



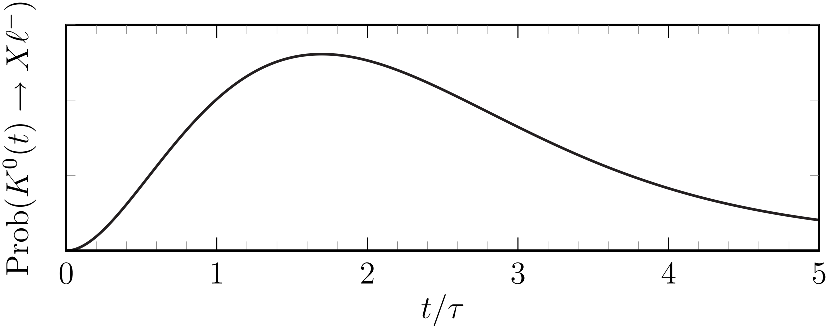
  but $\bar{B}_d^0$ does not decay into ℓ^+ and B_d^0 does not decay into ℓ^-

- That is

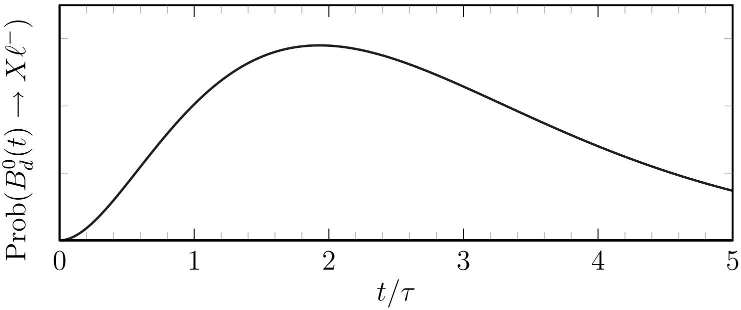
$$\lambda_f = \frac{q}{p}\frac{\bar{A}_f}{A_f} \;\rightarrow\; \left\{ \begin{array}{c} \lambda_{\ell^+} \rightarrow 0 \\ \lambda_{\ell^-} \rightarrow \infty \end{array} \right\} \;\rightarrow\; C_{\ell^\pm} = \pm 1,\; S_{\ell^\pm} = R_{\ell^\pm} = 0$$

- In general
  - An initial M^0 can only decay into ℓ^- if it mixes with $\bar{M}^0$ in the time evolution
  - An initial $\bar{M}^0$ can only decay into ℓ^+ if it mixes with M^0 in the time evolution

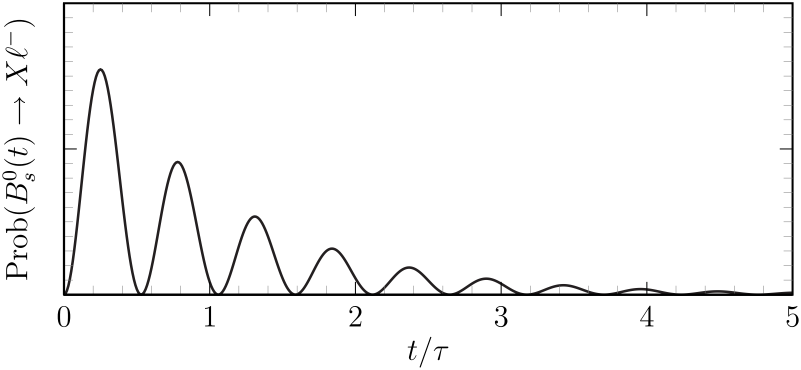
$\Pr(K^0(t) \to X\ell^-)$ i.e. $\Pr(K^0(t) \to \bar{K}^0)$



$\Pr(B_d^0(t) \to X\ell^-)$ i.e. $\Pr(B_d^0(t) \to \bar{B}_d^0)$



$\Pr(B_s^0(t) \to X\ell^-)$ i.e. $\Pr(B_s^0(t) \to \bar{B}_s^0)$



Time-dependent $B_s^0, \bar{B}_s^0 \to B_s^0, \bar{B}_s^0$ (LHCb)

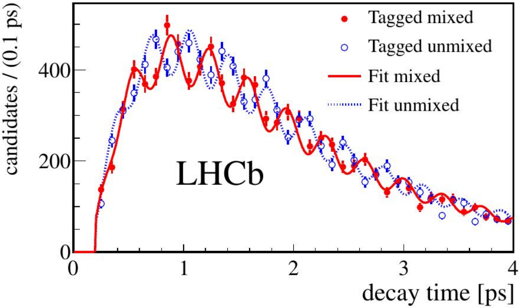


Figure 2: Decay time distribution for the sum of the five decay modes for candidates tagged as mixed (different flavour at decay and production; red, continuous line) or unmixed (same flavour at decay and production; blue, dotted line). The data and the fit projections are plotted in a signal window around the reconstructed B_s^0 mass of $5.32 - 5.55$ GeV/c^2.

CP Violation
Matter-Antimatter asymmetry

The discovery of CP Violation [Historical interlude]

If CP is a good symmetry in the neutral kaon system

- Eigenstates of the Hamiltonian are also CP eigenstates:

$$CP|K^0\rangle = |\bar{K}^0\rangle, \qquad (CP)^2 = \mathbf{1},$$

$$|K^0_\pm\rangle = \frac{1}{\sqrt{2}}\left[|K^0\rangle \pm |\bar{K}^0\rangle\right], \qquad CP|K^0_\pm\rangle = \pm|K^0_\pm\rangle$$

- Decays

$$K^0_+ \to \pi\pi, \quad K^0_- \nrightarrow \pi\pi, \qquad K^0_- \to \pi\pi\pi$$

but $m_K \sim 500$ MeV and $m_\pi \sim 140$ MeV

$\Rightarrow$ much more phase space for $\pi\pi$ than $\pi\pi\pi$

- K^0_+ short-lived, K^0_- long-lived

The discovery of CP Violation [Historical interlude]

VOLUME 13, NUMBER 4 PHYSICAL REVIEW LETTERS 27 JULY 1964

EVIDENCE FOR THE 2π DECAY OF THE K_2^0 MESON*†

J. H. Christenson, J. W. Cronin,‡ V. L. Fitch,‡ and R. Turlay§
Princeton University, Princeton, New Jersey
(Received 10 July 1964)

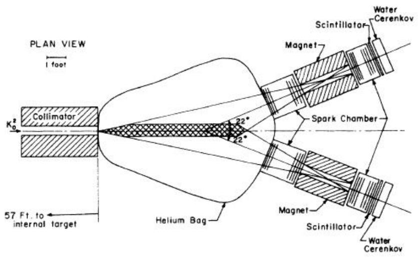


FIG. 1. Plan view of the detector arrangement.

The discovery of CP Violation [Historical interlude]

The Christenson, Cronin, Fitch & Turlay experiment

- Prepare a beam of kaons
- Propagate: the short-lived component disappears
- If CP is a good symmetry, no decays $\to \pi\pi$ should be observed
- ... *they were observed!*

> three-body decays of the K_2^0. The presence of a two-pion decay mode implies that the K_2^0 meson is not a pure eigenstate of CP. Expressed as

CP Violation / Matter-antimatter asymmetry

- Consider a decay channel which is a CP *eigenstate*

$$CP|f_{CP}\rangle = \eta_{f_{CP}}|f_{CP}\rangle$$

- Then, if there is no matter-antimatter asymmetry, i.e. CP invariance,

$$\Pr(M^0(t) \to f_{CP}) = \Pr(\bar{M}^0(t) \to f_{CP})$$

- and thus, if

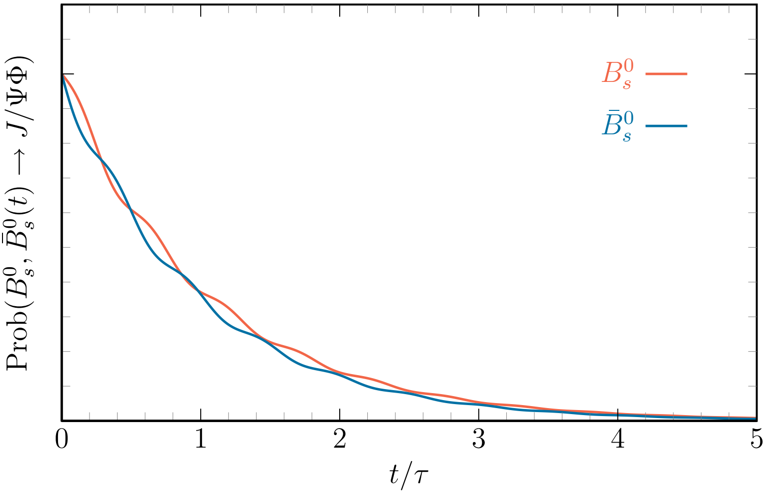
$$\Pr(M^0(t) \to f_{CP}) - \Pr(\bar{M}^0(t) \to f_{CP}) \neq 0$$

we have **matter-antimatter asymmetry!**

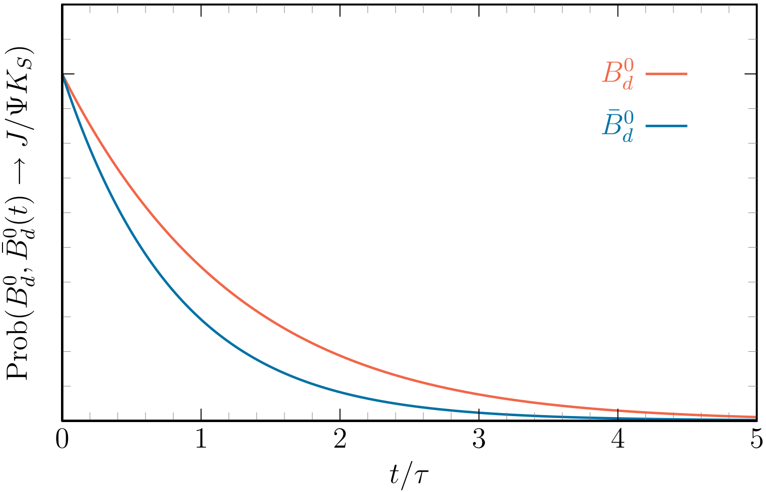
Example ($\delta = 0$)

$$\Pr(M^0(t) \to f_{CP}) - \Pr(\bar{M}^0(t) \to f_{CP}) \propto$$
$$e^{-\Gamma\, t}\{C_{f_{CP}} \cos(\Delta M t) + S_{f_{CP}} \sin(\Delta M t)\}$$

$\Pr(B_s^0, \bar{B}_s^0(t) \to f_{CP})$



$\text{Pr}(B_d^0, \bar{B}_d^0(t) \to f_{CP})$



Time-dependent $B_d^0, \bar{B}_d^0 \to f_{CP}$ (BaBar)

MEASUREMENT OF TIME-DEPENDENT *CP* ASYMMETRY ... PHYSICAL REVIEW D **79**, 072009 (2009)

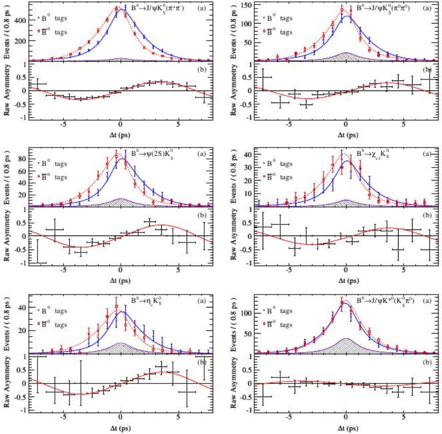


FIG. 4 (color online). (a) Number of B^0 candidates in the signal region with a B^0 tag (N_{B^0}) and with a $\bar{B}^0$ tag ($N_{\bar{B}^0}$), and (b) the corresponding raw asymmetry, $(N_{B^0} - N_{\bar{B}^0})/(N_{B^0} + N_{\bar{B}^0})$, as functions of Δt for each B^0 decay mode. The solid (dashed) curves represent the fit projections in Δt for $B^0(\bar{B}^0)$ tags. The right-hatched (left-hatched) shaded regions represent the estimated background contributions in $\bar{B}^0(B^0)$ tagged events.

This is the end

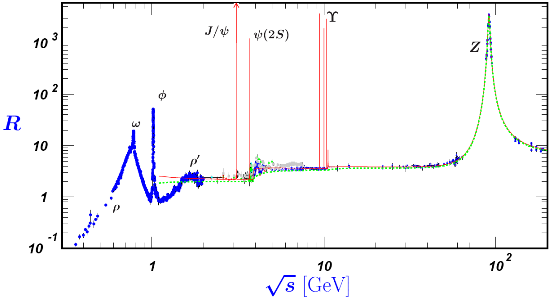
Not covered

- How do you “prepare” initial states for meson evolution?
  - Strong production (e.g. LHCb) vs. Entangled states (Factories)
- How do you relate parameters in meson evolution & decay with fundamental parameters of your lagrangian?
- How do you probe time reversal?

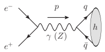
Preparing Mesons. . .

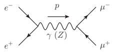
Factories & Entangled States

Resonances in e^+e^-

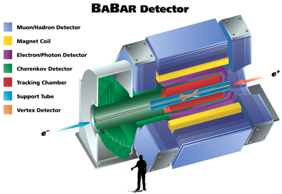


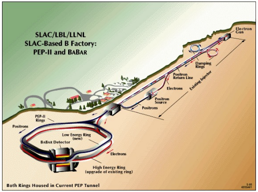
$$R(s=p^2) = \frac{\sigma(e^+e^- \to \text{ hadrons })}{\sigma(e^+e^- \to \mu^+\mu^-)} = \frac{\left|\ \cdots\ \right|^2}{\left|\ \cdots\ \right|^2}$$

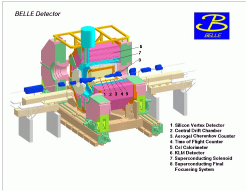


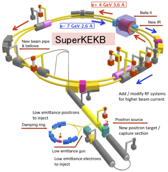


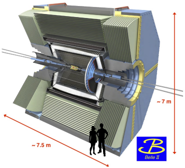
Υ Resonances and B mesons: B-factories











$\Upsilon(4S) \to B\bar{B}\,(B_d^0\bar{B}_d^0, B^+B^-)$

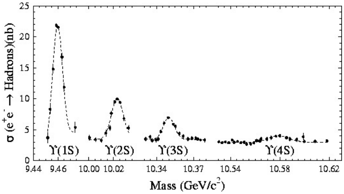
Decay $\Upsilon(4S) \to B\bar{B}$

- $J^{PC}[\Upsilon(4S)] = 1^{--}$, $J^P[B] = J^P[\bar{B}] = 0^-$
- $B\bar{B}$ state with $C = -$

$$|\Psi(0)\rangle \sim \Big(|B_d^0(\vec{p})\rangle|\bar{B}_d^0(-\vec{p})\rangle - |\bar{B}_d^0(\vec{p})\rangle|B_d^0(-\vec{p})\rangle\Big)/\sqrt{2}$$
$$= \Big(|B_L\rangle|B_H\rangle - |B_H\rangle|B_L\rangle\Big)/\left[\sqrt{2}(p_L q_H + p_H q_L)\right]$$

- Antisymmetric entangled state, extremely important

Υ Resonances and B mesons



$\Upsilon(3S)$ $I^G(J^{PC}) = 0^-(1^{--})$

$\Upsilon(3S)$ MASS

VALUE (MeV)	*DOCUMENT ID*	*TECN*	*COMMENT*
10355.2±0.5	[1] ARTAMONOV 00	MD1	$e^+e^- \to$ hadrons

$\Upsilon(4S)$ or $\Upsilon(10580)$ $I^G(J^{PC}) = 0^-(1^{--})$

$\Upsilon(4S)$ MASS

VALUE (MeV)	*DOCUMENT ID*	*TECN*	*COMMENT*
10579.4±1.2 OUR AVERAGE			

BOTTOM MESONS
$(B = \pm 1)$

$B^+ = u\overline{b}$, $B^0 = d\overline{b}$, $\overline{B}^0 = \overline{d}\,b$, $B^- = \overline{u}\,b$, similarly for B^*'s

$B^\pm$ $I(J^P) = \frac{1}{2}(0^-)$

I, J, P need confirmation. Quantum numbers shown are quark-model predictions.

Mass $m_{B^\pm} = 5279.29 \pm 0.15$ MeV (S = 1.1)
Mean life $\tau_{B^\pm} = (1.638 \pm 0.004) \times 10^{-12}$ s
$c\tau = 491.1$ μm

B^0 $I(J^P) = \frac{1}{2}(0^-)$

I, J, P need confirmation. Quantum numbers shown are quark-model predictions.

Mass $m_{B^0} = 5279.61 \pm 0.16$ MeV (S = 1.1)
$m_{B^0} - m_{B^\pm} = 0.32 \pm 0.06$ MeV
Mean life $\tau_{B^0} = (1.520 \pm 0.004) \times 10^{-12}$ s
$c\tau = 455.7$ μm
$\tau_{B^+}/\tau_{B^0} = 1.076 \pm 0.004$ (direct measurements)

B^0-$\overline{B}^0$ mixing parameters

Υ Resonances and B mesons

$\Upsilon(3S)$ DECAY MODES

	Mode	Fraction (Γ_i/Γ)	Scale factor/ Confidence level
Γ_1	$\Upsilon(2S)$ anything	(10.6 ± 0.8) %	
Γ_2	$\Upsilon(2S)\pi^+\pi^-$	(2.82 ± 0.18) %	S=1.6
Γ_3	$\Upsilon(2S)\pi^0\pi^0$	(1.85 ± 0.14) %	
Γ_4	$\Upsilon(2S)\gamma\gamma$	(5.0 ± 0.7) %	
Γ_5	$\Upsilon(2S)\pi^0$	$< 5.1 \times 10^{-4}$	CL=90%
Γ_6	$\Upsilon(1S)\pi^+\pi^-$	(4.37 ± 0.08) %	
Γ_7	$\Upsilon(1S)\pi^0\pi^0$	(2.20 ± 0.13) %	

$\Upsilon(4S)$ DECAY MODES

	Mode	Fraction (Γ_i/Γ)	Confidence level
Γ_1	$B\overline{B}$	> 96 %	95%
Γ_2	B^+B^-	(51.4 ± 0.6) %	
Γ_3	D_s^+ anything + c.c.	(17.8 ± 2.6) %	
Γ_4	$B^0\overline{B}^0$	(48.6 ± 0.6) %	
Γ_5	$J/\psi K_S^0 + (J/\psi,\ \eta_c)K_S^0$	$< 4 \times 10^{-7}$	90%
Γ_6	non-$B\overline{B}$	< 4 %	95%
Γ_7	e^+e^-	$(1.57 \pm 0.08) \times 10^{-5}$	

Antisymmetric entangled state

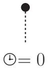
- **Time evolution**

$$|\Psi(t_0)\rangle = e^{-i(\mu_H+\mu_L)t_0}|\Psi(0)\rangle = e^{-\Gamma t_0}e^{-i2Mt_0}|\Psi(0)\rangle$$

- + **Flavour Tag**:
(Reminder: $\bar{B}_d^0 \to \ell^-$, $B_d^0 \to \ell^+$ $\bar{B}_d^0 \nrightarrow \ell^+$, $B_d^0 \nrightarrow \ell^-$)

$\Rightarrow$ controlled state for the evolution of one (anti-)meson

$$\Upsilon(4S) \to B\bar{B}$$



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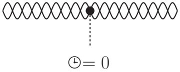

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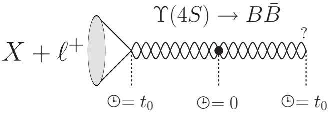

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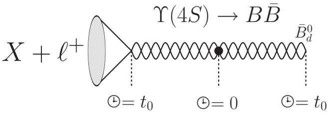

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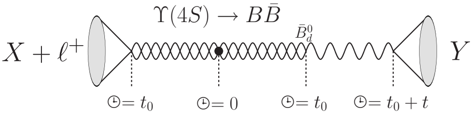

Measuring Time Reversal Violation

(independent of CPT)

Motion reversal and time reversal asymmetries

Short summary

1. Time reversal in the B_d^0–$\bar{B}_d^0$ Hilbert space
   - Reference transition $B_1 \to B_2(t)$ *among meson states* compared to $B_2 \to B_1(t)$
   - Probability $\mathrm{P}_{12}(t) = |\langle B_2|U(t,0)|B_1\rangle|^2$
   - Proposed T violating asymmetry $\mathrm{P}_{12}(t) - \mathrm{P}_{21}(t)$
2. Going beyond $B_{1,2} = B_d^0, \bar{B}_d^0$ ($\Rightarrow$ independent of CP)
   - Reference $B_d^0 \to B_+$ with a defined $CP = +$ decay channel $f_{CP=+}$
   - How to measure the reverse transition?
3. Importance of entangled nature of the initial state
   - To connect meson transitions to double decay rates
   - To identify the reverse transition

Starting with the initial antisymmetric entangled state,

- if at time t_0 we observe a decay product f in one side, the still living meson state in the opposite side is

$$|B_{\nrightarrow f}\rangle = \frac{1}{\sqrt{|A_f|^2 + |\bar{A}_f|^2}} \left(\bar{A}_f|B_d^0\rangle - A_f|\bar{B}_d^0\rangle\right)$$

- The orthogonal state $\langle B^{\perp}_{\nrightarrow f}|B_{\nrightarrow f}\rangle = 0$ is

$$|B^{\perp}_{\nrightarrow f}\rangle = \frac{1}{\sqrt{|A_f|^2 + |\bar{A}_f|^2}} \left(A_f^*|B_d^0\rangle + \bar{A}_f^*|\bar{B}_d^0\rangle\right)$$

This is the state filtered by a decay f

- The *filtering identity*

$$\left|\langle f|T_{\mathrm{w}}|B_1\rangle\right|^2 = \left|\langle f|T_{\mathrm{w}}\left(|B_{\nrightarrow f}\rangle\langle B_{\nrightarrow f}| + |B^{\perp}_{\nrightarrow f}\rangle\langle B^{\perp}_{\nrightarrow f}|\right)|B_1\rangle\right|^2 =$$
$$\left|\langle f|T_{\mathrm{w}}|B^{\perp}_{\nrightarrow f}\rangle\right|^2 \left|\langle B^{\perp}_{\nrightarrow f}|B_1\rangle\right|^2 = \left(|A_f|^2 + |\bar{A}_f|^2\right)\left|\langle B^{\perp}_{\nrightarrow f}|B_1\rangle\right|^2$$

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- if at time t_0 we observe a decay product f in one side, the still living meson state in the opposite side is

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This is the state filtered by a decay f

- The *filtering identity*

$$\left|\langle B_{\nrightarrow f}^{\perp} | B_1\rangle\right|^2 = \frac{\left|\langle f | T_{\text{w}} | B_1\rangle\right|^2}{|A_f|^2 + |\bar{A}_f|^2}$$

relates "probabilities to decay rates"

- For $B_1 = B_{\not\to g}(t)$, this is exactly the reduced intensity $\hat{\mathrm{I}}(g, f; t)$:

$$\hat{\mathrm{I}}(g, f; t) = \frac{\left|\langle f|T|B_{\not\to g}(t)\rangle\right|^2}{|A_f|^2 + |\bar{A}_f|^2} = \left|\langle B^{\perp}_{\not\to f}|B_{\not\to g}(t)\rangle\right|^2 .$$

This is the precise connection between
meson transition probabilities and double decay rates

- Measuring $\hat{\mathrm{I}}(f_1, f_2; t)$, "decays (f_1, f_2)", we access the transition probability $\mathrm{P}_{12}(t)$ between meson states $(B_1, B_2) = (B_{\not\to f_1}, B^{\perp}_{\not\to f_2})$
- To compare with the reverse $\mathrm{P}_{21}(t)$ we would need $(B^{\perp}_{\not\to f_2}, B_{\not\to f_1})$:
this is not what we access experimentally
- How do we bypass this?

- Reference: (f_1, f_2) gives transition probability for $(B_{\nrightarrow f_1}, B^{\perp}_{\nrightarrow f_2})$;
  we now need $(B^{\perp}_{\nrightarrow f_2}, B_{\nrightarrow f_1})$
- Two new decay channels (f'_2, f'_1) give $(B_{\nrightarrow f'_2}, B^{\perp}_{\nrightarrow f'_1})$;
  provided they fulfill

$$|B_{\nrightarrow f'_i}\rangle = |B^{\perp}_{\nrightarrow f_i}\rangle,$$

  this new transition (f'_2, f'_1) gives the reversed meson transition
- For flavour specific decay channels

$$|B^0_d\rangle = |B_{\nrightarrow \ell^-}\rangle \quad \text{and} \quad |\bar{B}^0_d\rangle = |B_{\nrightarrow \ell^+}\rangle$$

- The identity is obviously $|\bar{B}^0_d\rangle = |(B^0_d)^{\perp}\rangle$:
  if $f_1 = X\ell^+\nu_\ell$, then $f'_1 = X'\ell^-\bar{\nu}_\ell$

- For the CP decay channel the condition is

$$\lambda_{f_2}\lambda^*_{f'_2} = -\left|\frac{q}{p}\right|^2$$

the original proposal used $f_2 = J/\psi K_+$ and $f'_2 = J/\psi K_-$.
From now on, K_S for K_+ and K_L for K_-, which is accurate up to CP violation in the kaon system.

- Considering that

$$\lambda_{J/\psi K_S} \equiv \lambda_{K_S} \sim \left|\frac{q}{p}\right| e^{-i2\beta} \text{ and } \lambda_{J/\psi K_L} \equiv \lambda_{K_L} \sim -\left|\frac{q}{p}\right| e^{-i2\beta}$$

we parameterise

$$\lambda_{K_S} = \left|\frac{q}{p}\right| \rho\,(1+\epsilon_\rho)\, e^{-i(2\beta+\epsilon_\beta)}\,, \quad \lambda_{K_L} = -\left|\frac{q}{p}\right| \frac{1}{\rho}\,(1+\epsilon_\rho)\, e^{-i(2\beta-\epsilon_\beta)}$$

with $\{\rho, \beta, \epsilon_\rho, \epsilon_\beta\}$ real *to control deviations from the requirement*

- Recapitulating: if $\epsilon_\rho = 0$ and $\epsilon_\beta = 0$, the considered channels allow to compare $B_1 \to B_2(t)$ with the reversed transition $B_2 \to B_1(t)$ (even if $\rho \neq 1$)
- At last, for that motion reversal asymmetry to be truly a time reversal asymmetry, one needs decay channels f such that, in the limit of T invariance, $S_f = 0$
- For CP eigenstates this amounts to no CP violation in the decay in the limit of T invariance, the additional condition is

$$\rho = 1$$

- Overall, deviations from

$$\epsilon_\rho = \epsilon_\beta = 0,\ \rho = 1$$

$$C_{K_S} = C_{K_L} = \delta\,,\ S_{K_S} + S_{K_L} = 0\,,\ R_{K_S} + R_{K_L} = 0$$

will be a source of fake T violation

N.B. In the absence of wrong flavour decays in $B_d^0 \to J/\psi K^0$ and $\bar{B}_d^0 \to J/\psi \bar{K}^0$

$$\lambda_{K_S} + \lambda_{K_L} = 0 \quad \Leftrightarrow \quad \rho = 1 \ \&\ \epsilon_\beta = 0$$

- Out of experimental convenience, BaBar fixed the normalization of the constant term and used the normalized decay intensity

$$\mathbf{g}_{f,g}(t) \propto e^{-\Gamma t}\{1 + \mathrm{C}[f,g]\cos(\Delta M\, t) + \mathrm{S}[f,g]\sin(\Delta M\, t)\}$$

- Two quantities,

$$\mathrm{C}[f,g] = \frac{\mathscr{C}_c[f,g]}{\mathscr{C}_h[f,g]}, \quad \mathrm{S}[f,g] = \frac{\mathscr{S}_c[f,g]}{\mathscr{C}_h[f,g]}$$

are measured for each pair (f,g)

- Notice that

$$\mathrm{C}[f,g] = \mathrm{C}[g,f], \quad \mathrm{S}[f,g] = -\mathrm{S}[g,f]$$

	Transition	$\mathbf{g}_{f,g}(t)$	$\mathbf{g}_{g,f}(t)$	Transition	
Ref.	$\bar{B}_d^0 \to B_-$	(ℓ^+, K_S)	(K_S, ℓ^+)	$B_+ \to B_d^0$	Ref.
T(Ref.)	$B_- \to \bar{B}_d^0$	(K_L, ℓ^-)	(ℓ^-, K_L)	$B_d^0 \to B_-$	T(Ref.)
CP(Ref.)	$B_d^0 \to B_-$	(ℓ^-, K_S)	(K_S, ℓ^-)	$B_+ \to \bar{B}_d^0$	CP(Ref.)
CPT(Ref.)	$B_- \to B_d^0$	(K_L, ℓ^+)	(ℓ^+, K_L)	$\bar{B}_d^0 \to B_+$	CPT(Ref.)

Decay channels, corresponding *filtered* meson *states* and transformed transitions

- 16 experimentally independent measurements
- theoretically only 8 are independent

- BaBar asymmetries

$$A_{\mathrm{T}}(t) = \mathbf{g}_{K_L,\ell^-}(t) - \mathbf{g}_{\ell^+,K_S}(t)$$
$$A_{\mathrm{CP}}(t) = \mathbf{g}_{\ell^-,K_S}(t) - \mathbf{g}_{\ell^+,K_S}(t)$$
$$A_{\mathrm{CPT}}(t) = \mathbf{g}_{K_L,\ell^+}(t) - \mathbf{g}_{\ell^+,K_S}(t)$$

$$A_{\mathrm{S}}(t) = e^{-\Gamma t}\left\{\Delta C_{\mathrm{S}}[\ell^+,K_S]\,\cos(\Delta M\,t) + \Delta S_{\mathrm{S}}[\ell^+,K_S]\,\sin(\Delta M\,t)\right\}$$
$$\mathrm{S} = \mathrm{T}, \mathrm{CP}, \mathrm{CPT}$$

where

$$\Delta C^+_{\mathrm{T}} \equiv \Delta C_{\mathrm{T}}[\ell^+,K_S] = \mathrm{C}[K_L,\ell^-] - \mathrm{C}[\ell^+,K_S]$$
$$\Delta C^+_{\mathrm{CP}} \equiv \Delta C_{\mathrm{CP}}[\ell^+,K_S] = \mathrm{C}[\ell^-,K_S] - \mathrm{C}[\ell^+,K_S]$$
$$\Delta C^+_{\mathrm{CPT}} \equiv \Delta C_{\mathrm{CPT}}[\ell^+,K_S] = \mathrm{C}[K_L,\ell^+] - \mathrm{C}[\ell^+,K_S]$$

$$\Delta S^+_{\mathrm{T}} \equiv \Delta S_{\mathrm{T}}[\ell^+,K_S] = \mathrm{S}[K_L,\ell^-] - \mathrm{S}[\ell^+,K_S]$$
$$\Delta S^+_{\mathrm{CP}} \equiv \Delta S_{\mathrm{CP}}[\ell^+,K_S] = \mathrm{S}[\ell^-,K_S] - \mathrm{S}[\ell^+,K_S]$$
$$\Delta S^+_{\mathrm{CPT}} \equiv \Delta S_{\mathrm{CPT}}[\ell^+,K_S] = \mathrm{S}[K_L,\ell^+] - \mathrm{S}[\ell^+,K_S]$$

One can of course compute them; e.g. to linear order in θ

$$
\begin{aligned}
\Delta S^{+}_{\mathrm{T}} &\simeq S_{K_S} - S_{K_L} - \mathrm{Re}(\theta)\,(S_{K_S}R_{K_S} + S_{K_L}R_{K_L}) \\
&\qquad + \mathrm{Im}(\theta)\,(S^2_{K_S} - S^2_{K_L} + C_{K_S} + C_{K_L}) \\
\Delta S^{+}_{\mathrm{CP}} &\simeq 2S_{K_S} + 2\mathrm{Im}(\theta)\,(S^2_{K_S} - 1) \\
\Delta S^{+}_{\mathrm{CPT}} &\simeq S_{K_L} + S_{K_S} - \mathrm{Re}(\theta)\,(S_{K_L}R_{K_L} + S_{K_S}R_{K_S}) \\
&\qquad + \mathrm{Im}(\theta)\,(-2 + S^2_{K_S} + S^2_{K_L} + C_{K_S} + C_{K_L}) \\
\Delta C^{+}_{\mathrm{T}} &\simeq C_{K_S} + C_{K_L} + \mathrm{Re}(\theta)\,(R_{K_S}(1 - C_{K_S}) + R_{K_L}(1 + C_{K_L})) \\
&\qquad + \mathrm{Im}(\theta)\,(S_{K_L}(1 + C_{K_L}) - S_{K_S}(1 - C_{K_S})) \\
\Delta C^{+}_{\mathrm{CP}} &\simeq 2C_{K_S} + 2\mathrm{Re}(\theta)\,R_{K_S} + 2\mathrm{Im}(\theta)\,S_{K_S}C_{K_S} \\
\Delta C^{+}_{\mathrm{CPT}} &\simeq C_{K_S} - C_{K_L} + \mathrm{Re}(\theta)\,(R_{K_S}(1 - C_{K_S}) - R_{K_L}(1 - C_{K_L})) \\
&\qquad + \mathrm{Im}(\theta)\,(S_{K_L}(1 - C_{K_L}) - S_{K_S}(1 - C_{K_S}))
\end{aligned}
$$

Important: $\Delta S^{+}_{\mathrm{T}} \neq \Delta S^{+}_{\mathrm{CP}}$ & $\Delta C^{+}_{\mathrm{T}} \neq \Delta C^{+}_{\mathrm{CP}}$

Genuine T-reverse and fake asymmetries

- As discussed, candidate T-asymmetries can be "contaminated", they can receive contributions not truly T-violating
- This occurs when $\lambda_{K_S}\lambda^*_{K_L} = -|q/p|^2$ is not fulfilled
- One can disentangle *fake* effects from *true* violations in T and CPT asymmetries!

Input from BaBar, PRL 109 (2012) 211801 [arXiv:1207.5832]

TABLE II: Measured values of the $(S^{\pm}_{\alpha,\beta}, C^{\pm}_{\alpha,\beta})$ coefficients. The first uncertainty is statistical and the second systematic. The indices $\alpha = \ell^-, \ell^+$ and $\beta = K^0_S, K^0_L$ stand for reconstructed final states that identify the B meson as $\overline{B}^0$, B^0 and B_-, B_+, respectively.

Transition		Parameter	Result
$B_- \to \overline{B}^0$	$(J/\psi K^0_L, \ell^- X)$	$S^-_{\ell^-,K^0_L}$	$-0.83 \pm 0.11 \pm 0.06$
$B^0 \to B_-$	$(\ell^- X, c\bar{c}K^0_S)$	$S^+_{\ell^-,K^0_S}$	$-0.76 \pm 0.06 \pm 0.04$
$B_- \to B^0$	$(J/\psi K^0_L, \ell^+ X)$	$S^-_{\ell^+,K^0_L}$	$0.70 \pm 0.19 \pm 0.12$
$\overline{B}^0 \to B_-$	$(\ell^+ X, c\bar{c}K^0_S)$	$S^+_{\ell^+,K^0_S}$	$0.55 \pm 0.09 \pm 0.06$
$B^0 \to B_+$	$(\ell^- X, J/\psi K^0_L)$	$S^+_{\ell^-,K^0_L}$	$0.51 \pm 0.17 \pm 0.11$
$B_+ \to \overline{B}^0$	$(c\bar{c}K^0_S, \ell^- X)$	$S^-_{\ell^-,K^0_S}$	$0.67 \pm 0.10 \pm 0.08$
$\overline{B}^0 \to B_+$	$(\ell^+ X, J/\psi K^0_L)$	$S^+_{\ell^+,K^0_L}$	$-0.69 \pm 0.11 \pm 0.04$
$B_+ \to B^0$	$(c\bar{c}K^0_S, \ell^+ X)$	$S^-_{\ell^+,K^0_S}$	$-0.66 \pm 0.06 \pm 0.04$
$B_- \to \overline{B}^0$	$(J/\psi K^0_L, \ell^- X)$	$C^-_{\ell^-,K^0_L}$	$0.11 \pm 0.12 \pm 0.08$
$B^0 \to B_-$	$(\ell^- X, c\bar{c}K^0_S)$	$C^+_{\ell^-,K^0_S}$	$0.08 \pm 0.06 \pm 0.06$
$B_- \to B^0$	$(J/\psi K^0_L, \ell^+ X)$	$C^-_{\ell^+,K^0_L}$	$0.16 \pm 0.13 \pm 0.06$
$\overline{B}^0 \to B_-$	$(\ell^+ X, c\bar{c}K^0_S)$	$C^+_{\ell^+,K^0_S}$	$0.01 \pm 0.07 \pm 0.05$
$B^0 \to B_+$	$(\ell^- X, J/\psi K^0_L)$	$C^+_{\ell^-,K^0_L}$	$-0.01 \pm 0.13 \pm 0.08$
$B_+ \to \overline{B}^0$	$(c\bar{c}K^0_S, \ell^- X)$	$C^-_{\ell^-,K^0_S}$	$0.03 \pm 0.07 \pm 0.04$
$\overline{B}^0 \to B_+$	$(\ell^+ X, J/\psi K^0_L)$	$C^+_{\ell^+,K^0_L}$	$-0.02 \pm 0.11 \pm 0.08$
$B_+ \to B^0$	$(c\bar{c}K^0_S, \ell^+ X)$	$C^-_{\ell^+,K^0_S}$	$-0.05 \pm 0.06 \pm 0.03$

Input from BaBar, PRL 109 (2012) 211801 [arXiv:1207.5832]

TABLE III: Statistical correlation coefficients for the vector of $(S^{\pm}_{\alpha,\beta}, C^{\pm}_{\alpha,\beta})$ measurements given in the same order as in Table II. Only lower off-diagonal terms are written, in %.

100															
0	100														
-14	0	100													
2	-6	0	100												
8	0	41	0	100											
0	18	0	38	0	100										
6	0	19	0	-7	0	100									
0	10	0	16	0	-9	1	100								
-45	0	38	-1	31	0	9	0	100							
0	-33	0	31	0	28	0	6	0	100						
27	0	-9	0	23	0	18	0	-14	0	100					
0	28	0	-14	0	23	0	18	1	-15	0	100				
15	0	21	0	-21	0	27	0	-16	0	22	0	100			
0	18	0	21	0	-18	0	29	0	-16	0	21	0	100		
1	0	25	0	31	0	-37	0	22	0	-15	0	-20	0	100	
0	7	0	23	0	31	0	-41	0	20	0	-17	0	-20	0	100

Input from BaBar, PRL 109 (2012) 211801 [arXiv:1207.5832]

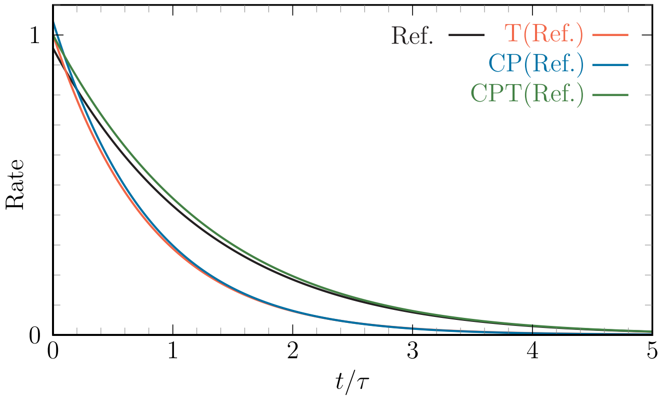
TABLE IV: Systematic correlation coefficients for the vector of $(S^{\pm}_{\alpha,\beta}, C^{\pm}_{\alpha,\beta})$ measurements given in the same order as in Table II. Only lower off-diagonal terms are written, in %.

100															
6	100														
18	-14	100													
44	3	66	100												
16	-4	57	58	100											
37	-19	67	66	44	100										
-5	-5	10	8	-4	-3	100									
30	-19	57	59	10	58	6	100								
-28	-10	39	13	43	21	-8	-1	100							
42	-20	60	68	57	72	-6	47	30	100						
-31	0	23	17	20	8	11	6	58	18	100					
41	-27	70	66	46	64	0	71	32	81	20	100				
31	-16	63	63	39	67	-23	59	39	63	24	73	100			
1	-1	15	7	2	2	-31	5	23	7	5	18	49	100		
28	-23	73	72	52	61	-1	64	43	69	28	84	83	39	100	
-14	-13	12	-6	-34	11	2	34	23	0	31	17	26	15	15	100

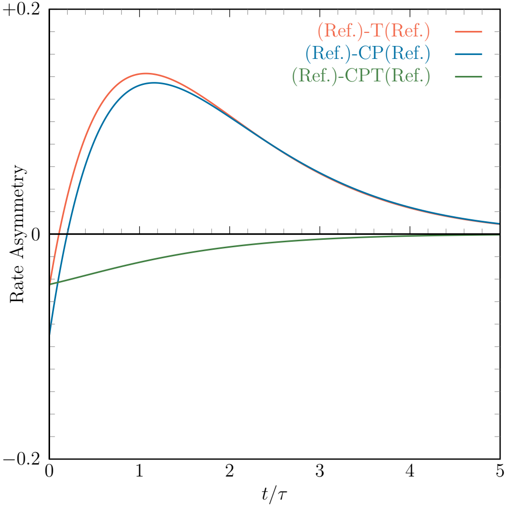
Results – Fit summary (I)

BaBar Asymmetries			
$\Delta S_{\mathrm{T}}^{+}$	-1.317 ± 0.050	$\Delta S_{\mathrm{CP}}^{+}$	-1.360 ± 0.038
$\Delta S_{\mathrm{CPT}}^{+}$	$(7.6 \pm 4.8)10^{-2}$		
$\Delta C_{\mathrm{T}}^{+}$	$(4.7 \pm 3.7)10^{-2}$	$\Delta C_{\mathrm{CP}}^{+}$	$(8.9 \pm 3.2)10^{-2}$
$\Delta C_{\mathrm{CPT}}^{+}$	$(4.4 \pm 3.6)10^{-2}$		
Genuine T-reverse		Fake	
$\Delta S_{\mathrm{T}}^{+}$ g.	-1.318 ± 0.050	$\Delta S_{\mathrm{T}}^{+}$ f.	$(0.9 \pm 2.0)10^{-3}$
$\Delta S_{\mathrm{CPT}}^{+}$ g.	$(5.6 \pm 4.3)10^{-2}$	$\Delta S_{\mathrm{CPT}}^{+}$ f.	$(1.9 \pm 4.7)10^{-2}$
$\Delta C_{\mathrm{T}}^{+}$ g.	$(0.2 \pm 2.5)10^{-2}$	$\Delta C_{\mathrm{T}}^{+}$ f.	$(4.5 \pm 2.6)10^{-2}$
$\Delta C_{\mathrm{CPT}}^{+}$ g.	$(8.9 \pm 5.2)10^{-2}$	$\Delta C_{\mathrm{CPT}}^{+}$ f.	$(-4.5 \pm 6.2)10^{-2}$

Rates of reference and transformed transitions



Rate asymmetries



Best existing limits on CPT violating Re(θ) [Propaganda]

$$\left\{\begin{matrix} \mathrm{Re}(\theta) = \pm(5.92 \pm 3.03) \times 10^{-2} \\ \mathrm{Im}(\theta) = (0.22 \pm 1.90) \times 10^{-2} \end{matrix}\right\}$$

$$\text{and } \left\{\begin{matrix} \mathrm{Re}(\theta) = \pm(3.92 \pm 1.43) \times 10^{-2} \\ \mathrm{Im}(\theta) = (-0.22 \pm 1.64) \times 10^{-2} \end{matrix}\right\} \text{ with } \lambda_{K_S} + \lambda_{K_L} = 0,$$

Significant improvement on the uncertainty of Re(θ) quoted by the Particle Data Group:

$$\mathrm{Re}(\theta)_{\mathrm{PDG}} = \pm(1.9 \pm 3.7 \pm 3.3) \times 10^{-2}, \quad \mathrm{Im}(\theta)_{\mathrm{PDG}} = (-0.8 \pm 0.4) \times 10^{-2}.$$

With $\Delta\Gamma = 0$

$$\left\{\begin{matrix} \mathbf{M}_{22} - \mathbf{M}_{11} = \pm(2.0 \pm 1.0) \\ \mathbf{\Gamma}_{22} - \mathbf{\Gamma}_{11} = -0.1 \pm 1.3 \end{matrix}\right\} 10^{-5}\mathrm{eV}$$

$$\text{and } \left\{\begin{matrix} \mathbf{M}_{22} - \mathbf{M}_{11} = \pm(1.3 \pm 0.5) \\ \mathbf{\Gamma}_{22} - \mathbf{\Gamma}_{11} = 0.1 \pm 1.1 \end{matrix}\right\} 10^{-5}\mathrm{eV} \text{ with } \lambda_{K_S} + \lambda_{K_L} = 0.$$