Diverse Phenomenological studies for the LHC

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LIP
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LIP

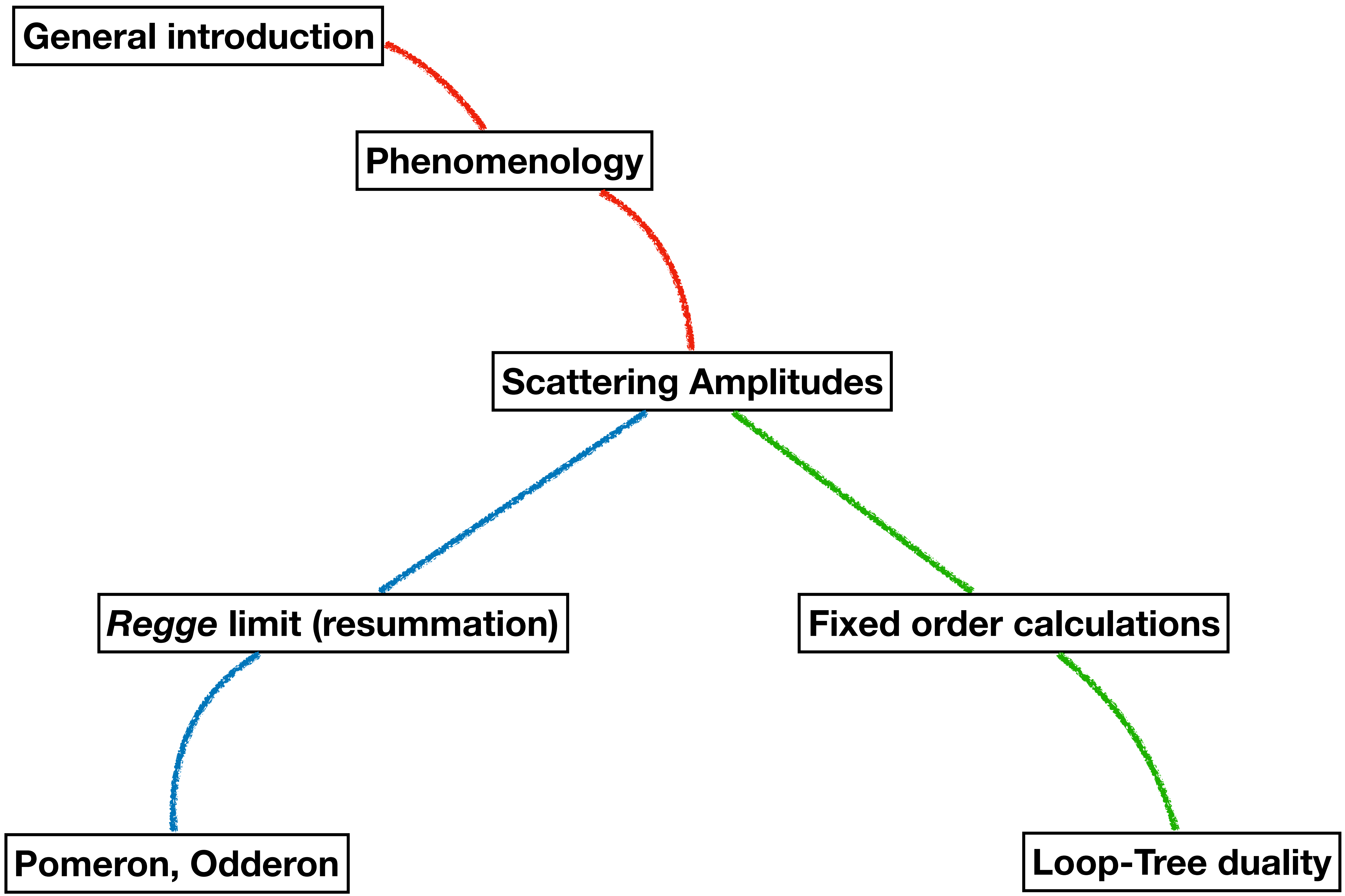
Research line

Career path so far: mainly phenomenology

- Monte Carlo (and generally software) development
- High energy scattering in QCD (and N=4 SYM)
- NNLO calculations
- Techniques on computing Feynman diagrams
- Asymptotic expansions
- Loop-tree duality method, automatization of NLO calculations

Present and future wish list:

- Machine learning
- Heavy Ion physics
- Physics of Cosmic rays


General introduction
Phenomenology
Scattering Amplitudes
Regge limit (resummation)
Fixed order calculations
Pomeron, Odderon
Loop-Tree duality

Phenomenology

- Philosophy

Phenomenology (from Greek: phenomenon = "that which appears" and logos = "study") is the philosophical study of the structures of subjective experience and consciousness.

- Science in general

Observe "that which appears", a collection of phenomena that share a unifying principle, and try to find patterns to describe it. The patterns might or might not be of fundamental nature or they might be up to a certain degree.

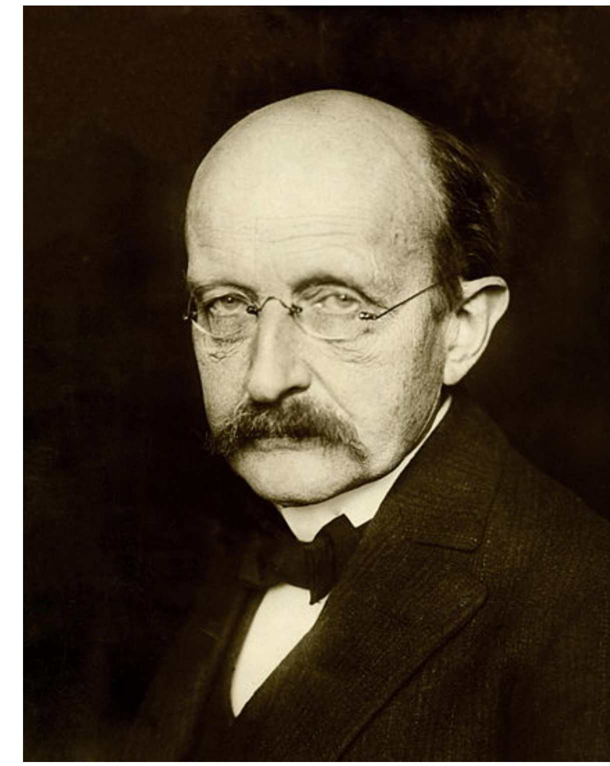
- Particle Physics (our familiar SM phenomenology, for example)

Use assumed fundamental laws to produce theoretical predictions for physical observables and then compare against experimental data to validate or falsify the assumed laws.

Extremely important the close collaboration between theorists and experimentalists.

A great example: Planck's Law

$$B_\nu(\nu, T) = \frac{2h\nu^3}{c^2} \frac{1}{e^{\frac{h\nu}{k_\mathrm{B}T}} - 1}$$



Describes the spectral density of electromagnetic radiation emitted by a black body in thermal equilibrium at a given temperature T

h is the Planck constant… fundamental importance for quantum mechanics

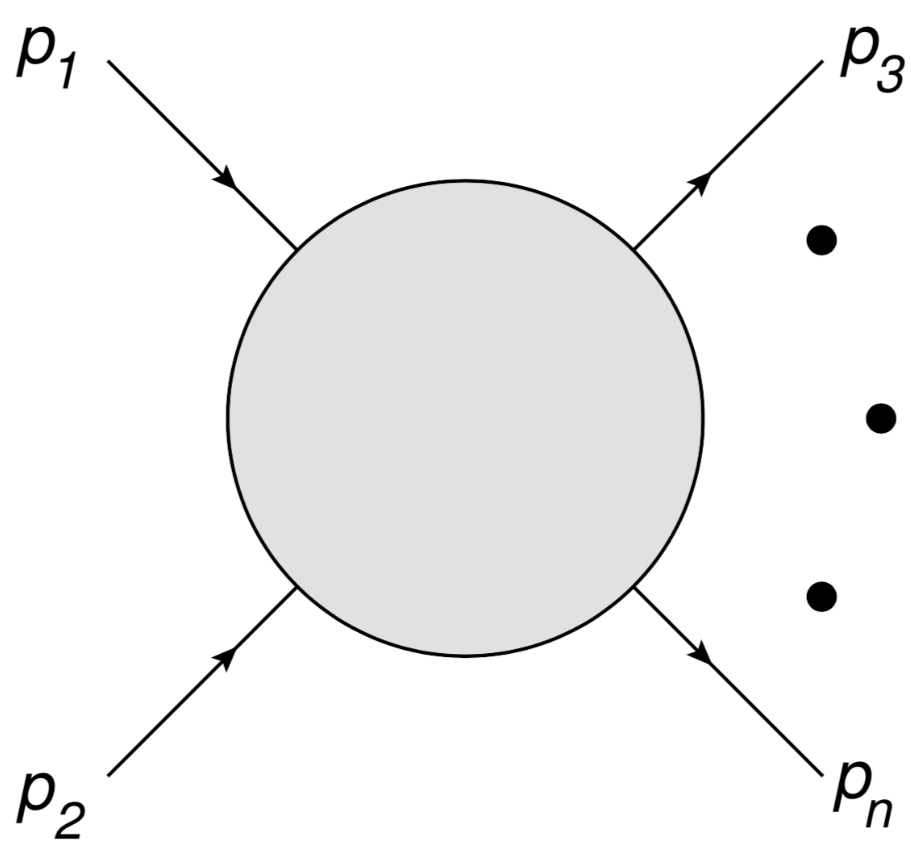
The central role of scattering amplitudes in modern phenomenology

From the excellent recent article by C. White "Aspects of High Energy Scattering" (1909.05177), we quote a few reasons below of the **importance of studying scattering amplitudes**:

- "What physical behaviour occurs in a given theory?"
- "What mathematical structures can amplitudes contain?"
- "Can we find common languages, that make e.g. QCD and gravity look the same?"

Perturbation theory

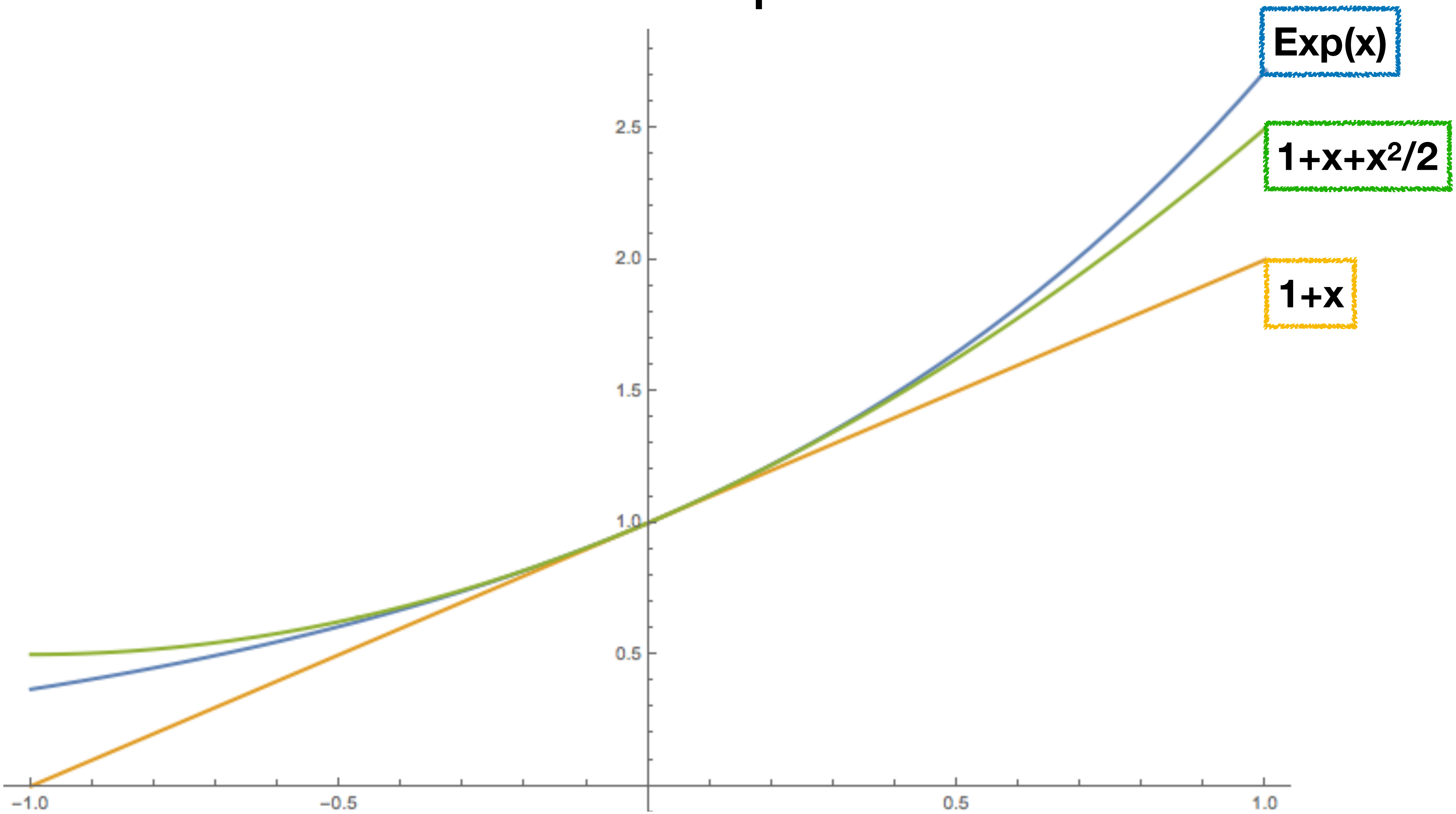
Feynman Calculus



Go at a certain limit of the theory

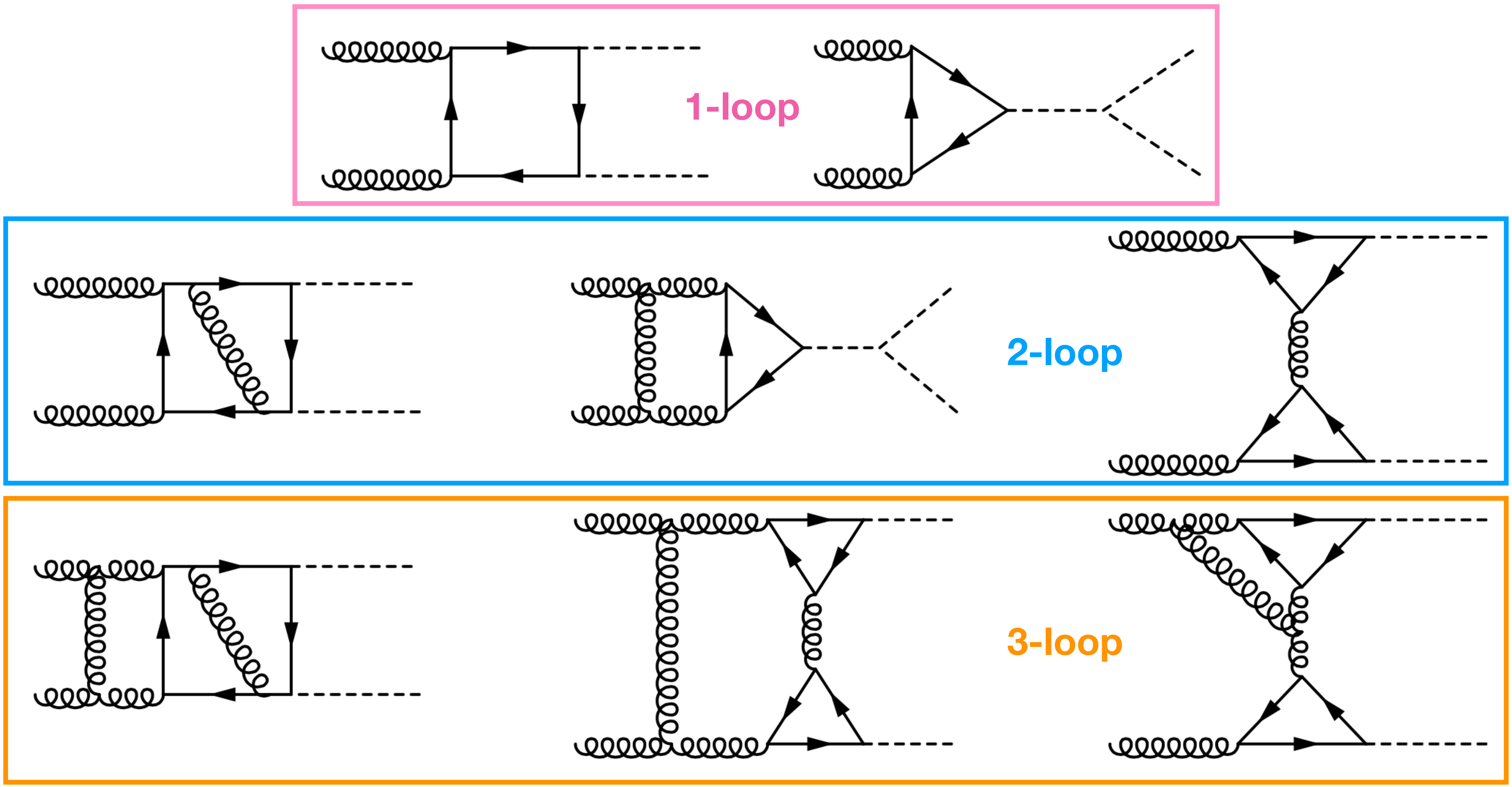
What does this diagram tell us?

Fixed order expansion



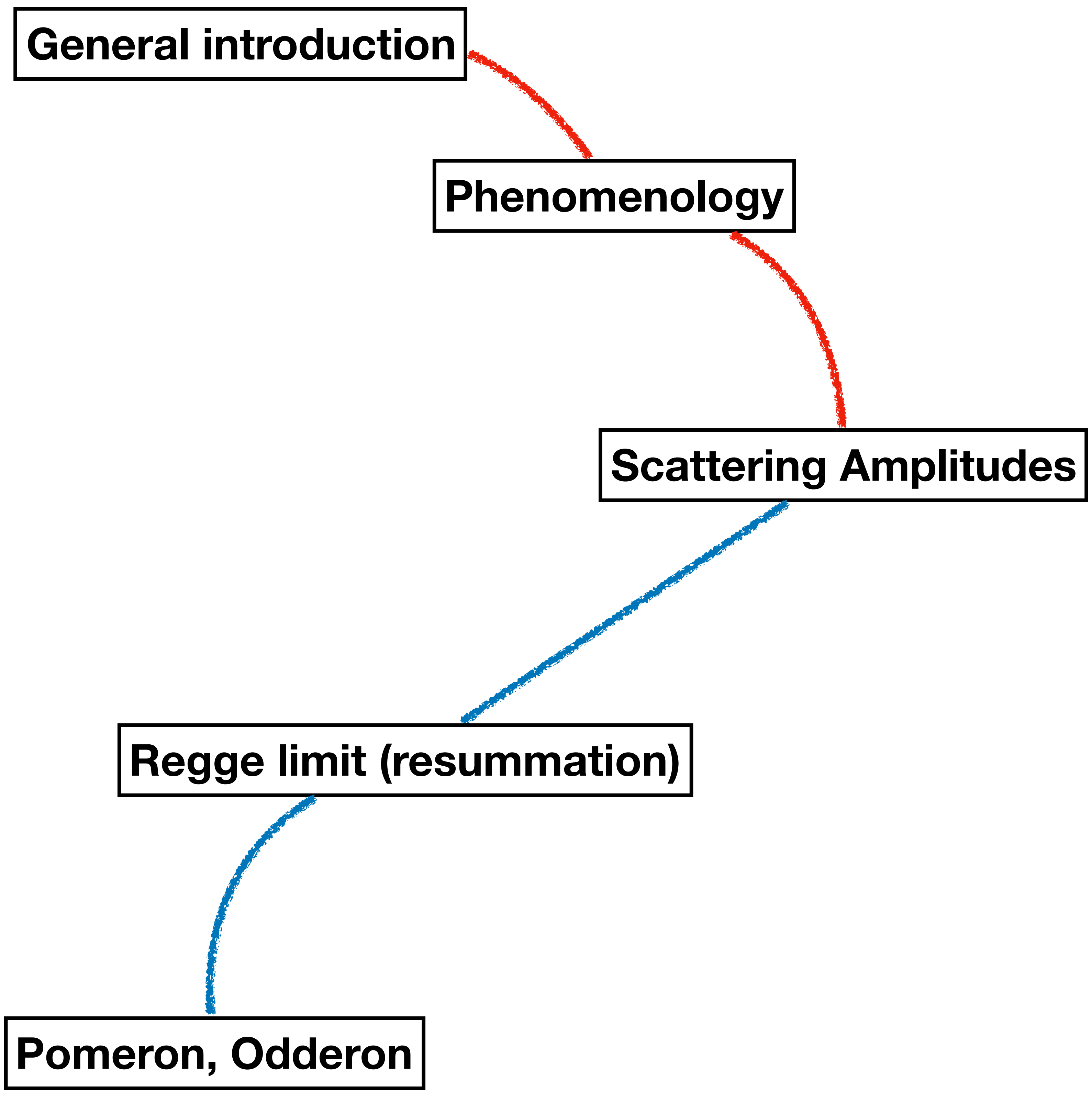
$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = \boxed{\boxed{1 + x} + \frac{x^2}{2!}} + \frac{x^3}{3!} + \cdots$$

Perturbative expansion (fixed order) in α_s



$gg \to HH$ Grigo, Hoff, Steinhauser, 2015

NOTE: at the end, we study the properties of the expanded amplitudes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Regge limit (resummation)
Pomeron, Odderon

The high energy or *Regge* limit

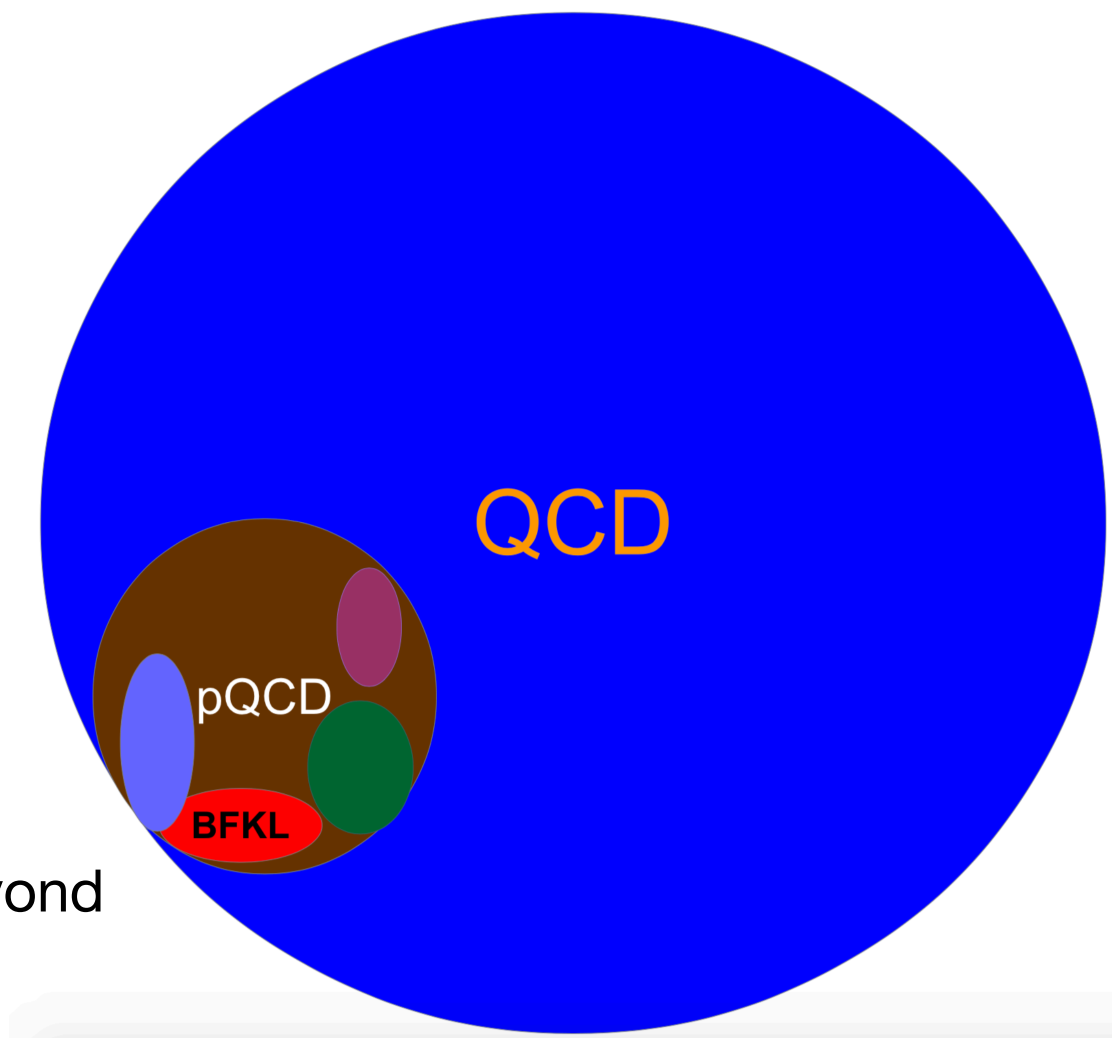
$$\boxed{s \gg -t \gg m^2}$$

There is a plethora or things we access from studying that limit:

- Integrability
- Gravity, black holes
- AdS/CFT
- Bern-Dixon-Smirnov amplitudes
- Factorization
- Separation between transverse and longitudinal d.o.f
- Transition from hard to soft scale physics
- Glueballs
- *Phenomenology*

Furthermore, in Mathematics:
number theory, abstract algebra, special functions, …

A crucial tool to study the *Regge* limit is
Balitsky-Fadin-Kuraev-Lipatov (BFKL)
dynamics. In its essence, BFKL resums to all orders
diagrams that carry large logarithms in energy. It goes beyond
fixed order.



Relevant considerations for the *Regge* limit

- Q: Is a fixed order calculation enough?
- A: It depends on the energy and the order, for asymptotic energies, no
- Q: What is the most relevant scale in high energy scattering?
- A: The center-of-mass energy squared s
- Q: In which functional form does s appear in the Feynman diagrams?
- A: $\alpha_s^m \ln(s)^n$
- Q: Can one isolate those Feynman diagrams that come with a numerically important [$\alpha_s^m \ln(s)^n \sim 1$] contribution?
- A: It depends (for this talk the answer is yes)
- Q: Can one resum all these diagrams with important $\alpha_s^m \ln(s)^n$ contributions to all orders in α_s?
- A: It depends (for this talk the answer is yes)

Key question: What is the applicability energy window for BFKL? Is it at LHC energies?

CMS-FSQ-12-002

CERN-PH-EP/2015-309
2016/01/26

Azimuthal decorrelation of jets widely separated in rapidity in pp collisions at $\sqrt{s} = 7\,\mathrm{TeV}$

The CMS Collaboration*

Key question: What is the applicability energy window for BFKL? Is it at LHC energies?

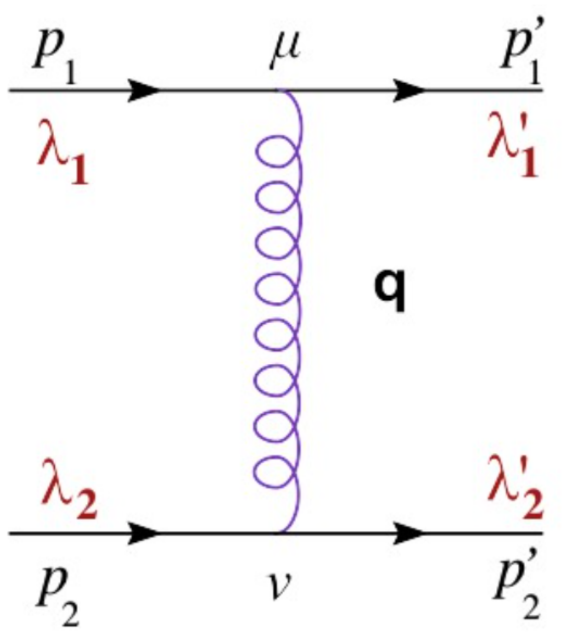
In the Conclusions of that paper, it reads:

The observed sensitivity to the implementation of the colour-coherence effects in the DGLAP MC generators and the reasonable data-theory agreement shown by the NLL BFKL analytical calculations at large Δy, may be considered as indications that the kinematical domain of the present study lies in between the regions described by the DGLAP and BFKL approaches. Possible manifestations of BFKL signatures are expected to be more pronounced at increasing collision energies.

Large logs from virtual corrections

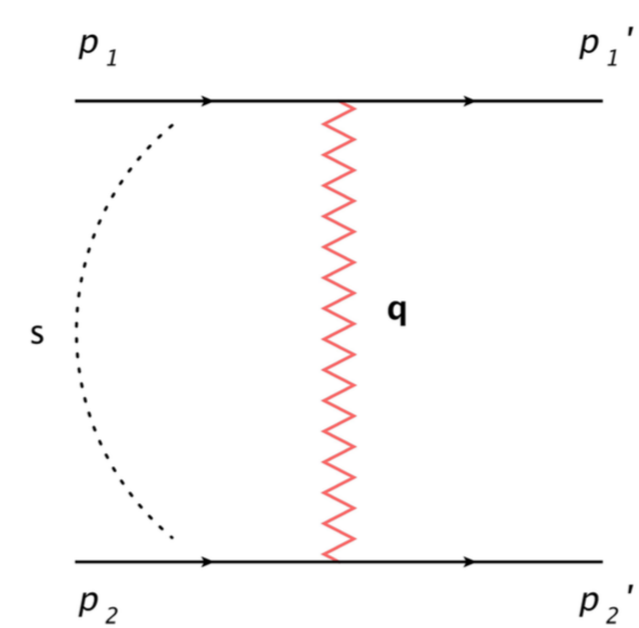
At leading logarithmic accuracy we resum terms of the form $(\alpha_s \log(s))^n$

A normal gluon propagator: $$D_{\mu\nu}(s,q^2) = -i\frac{g_{\mu\nu}}{q^2}$$



The reggeized gluon is a gluon with modified propagator:

$$D_{\mu\nu}(s,q^2) = -i\frac{g_{\mu\nu}}{q^2}\boxed{\left(\frac{s}{\mathbf{k}^2}\right)^{\omega(q^2)}}$$



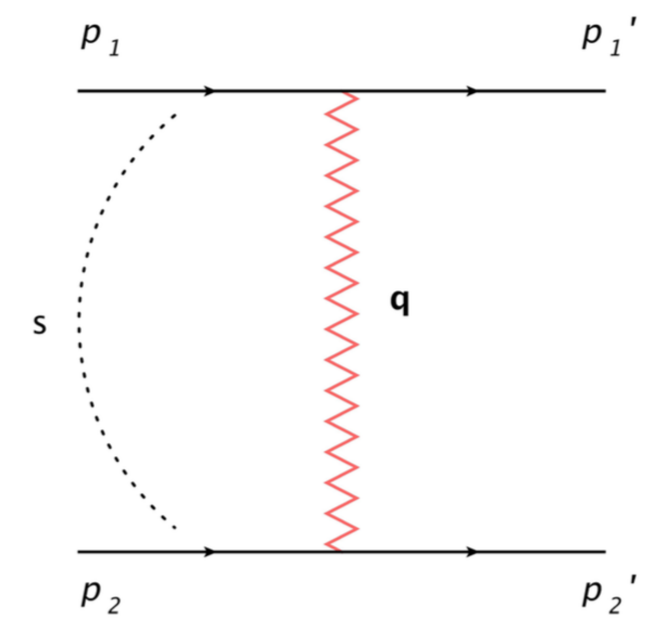
The Reggeon

All the <u>virtual corrections</u> that carry leading-logs in s are accounted for

The reggeized gluon is a gluon with modified propagator:

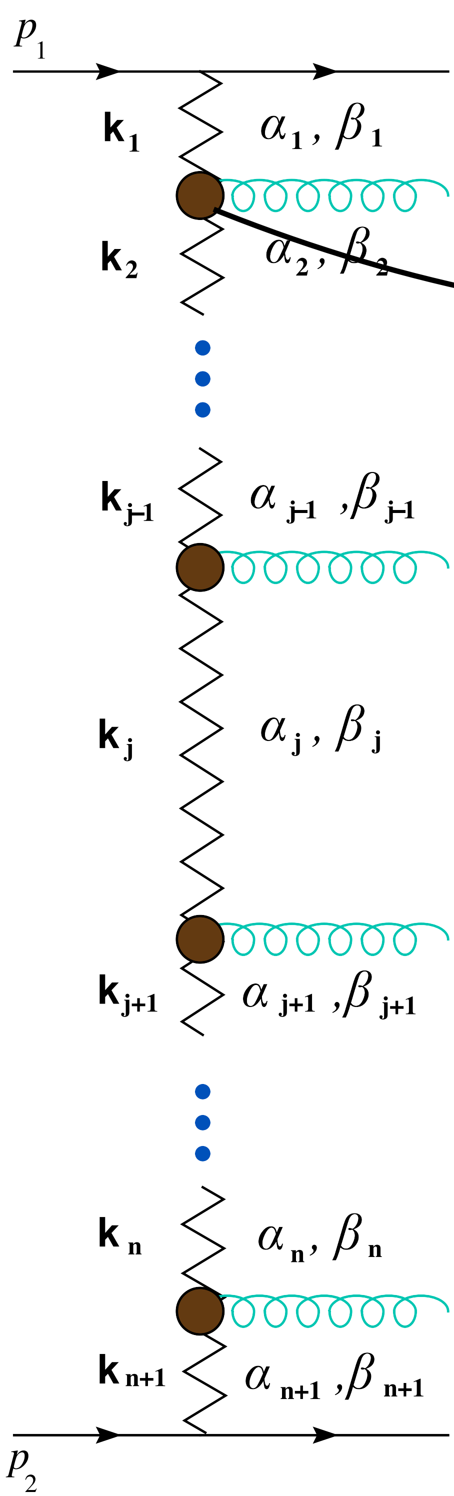
$$D_{\mu\nu}(s, q^2) = -i\frac{g_{\mu\nu}}{q^2}\boxed{\left(\frac{s}{\mathbf{k}^2}\right)^{\omega(q^2)}}$$

(where $\mathbf{k}^2$ is a hard scale in the process at hand)



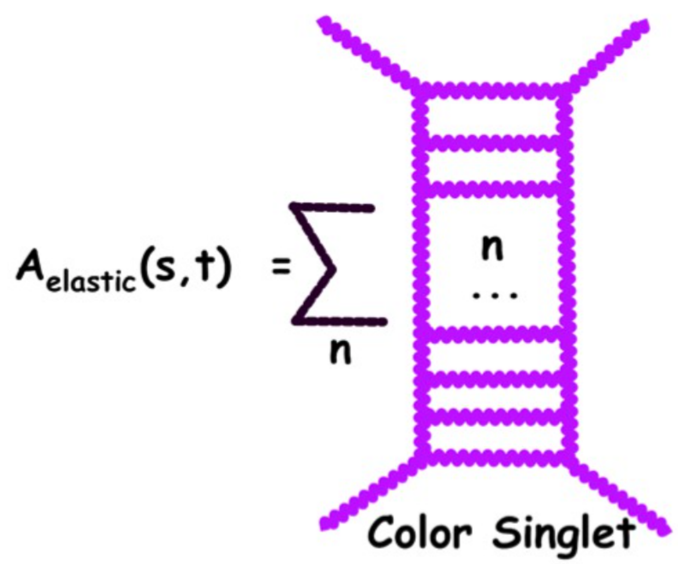
<u>From now on, vertical propagators represent Reggeons</u>

Large logs from real emission corrections



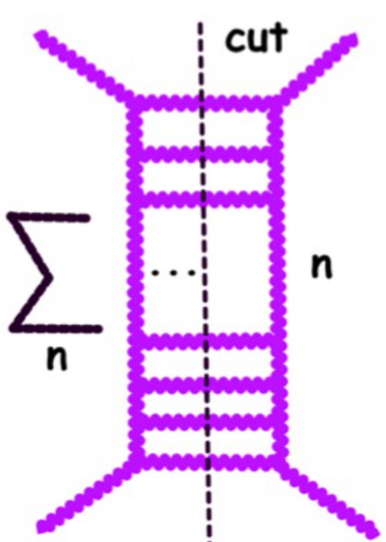
Lipatov's effective vertex

BFKL equation



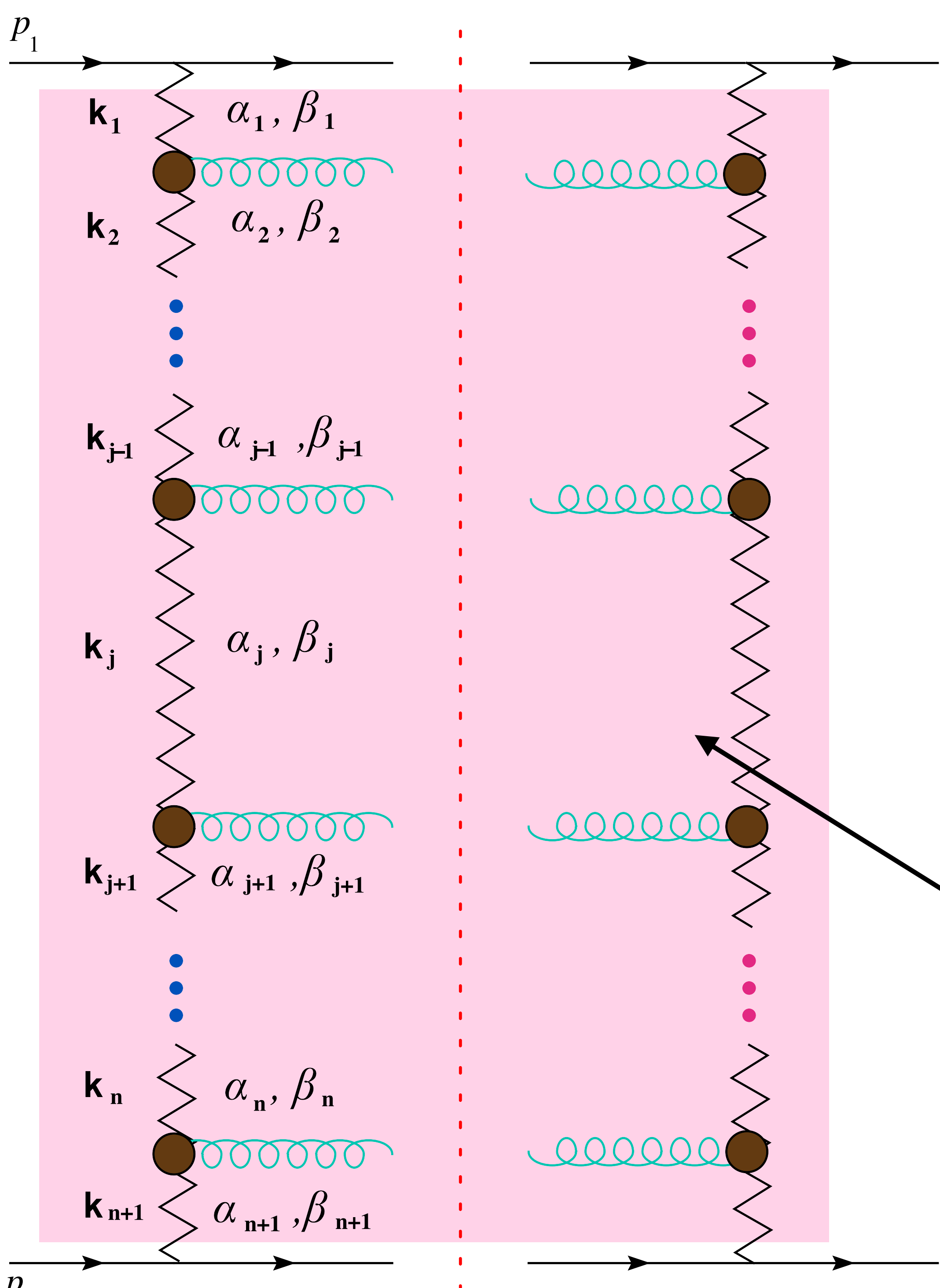
Optical Theorem :

$$\sigma_{\mathrm{TOT}} \simeq \frac{1}{s}\mathrm{Im}\mathcal{A}_{\mathrm{elastic}}(s, t=0) = \frac{1}{s}\sum_n$$



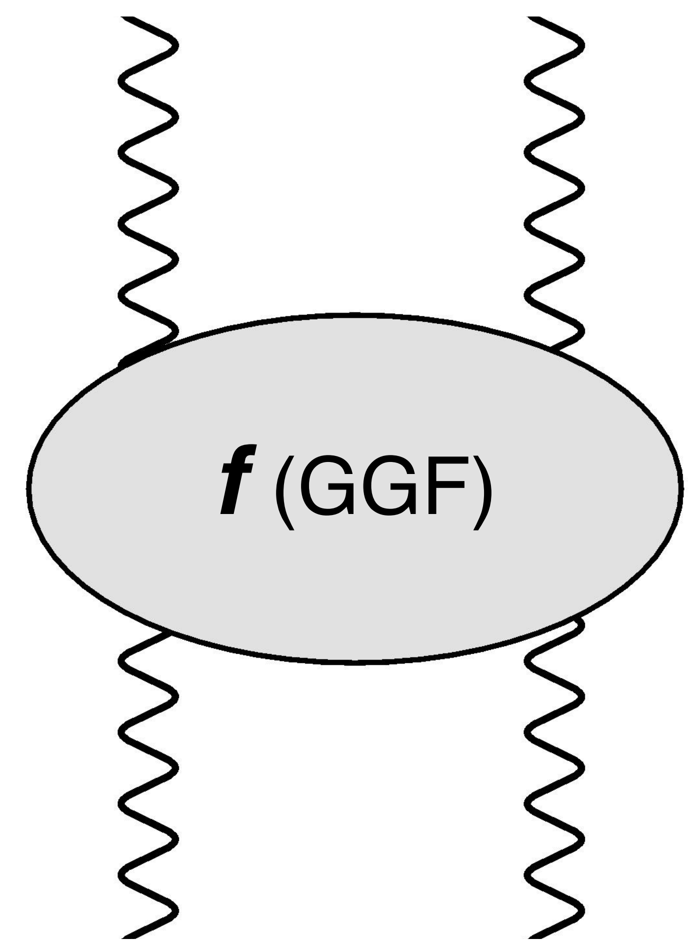
$$= \frac{1}{s}\sum_n |A_n(s,t)|^2$$

The Pomeron



From now on, ‘rungs’ instead of real emissions

$$\xrightarrow[\text{equation}]{\text{BFKL}}$$

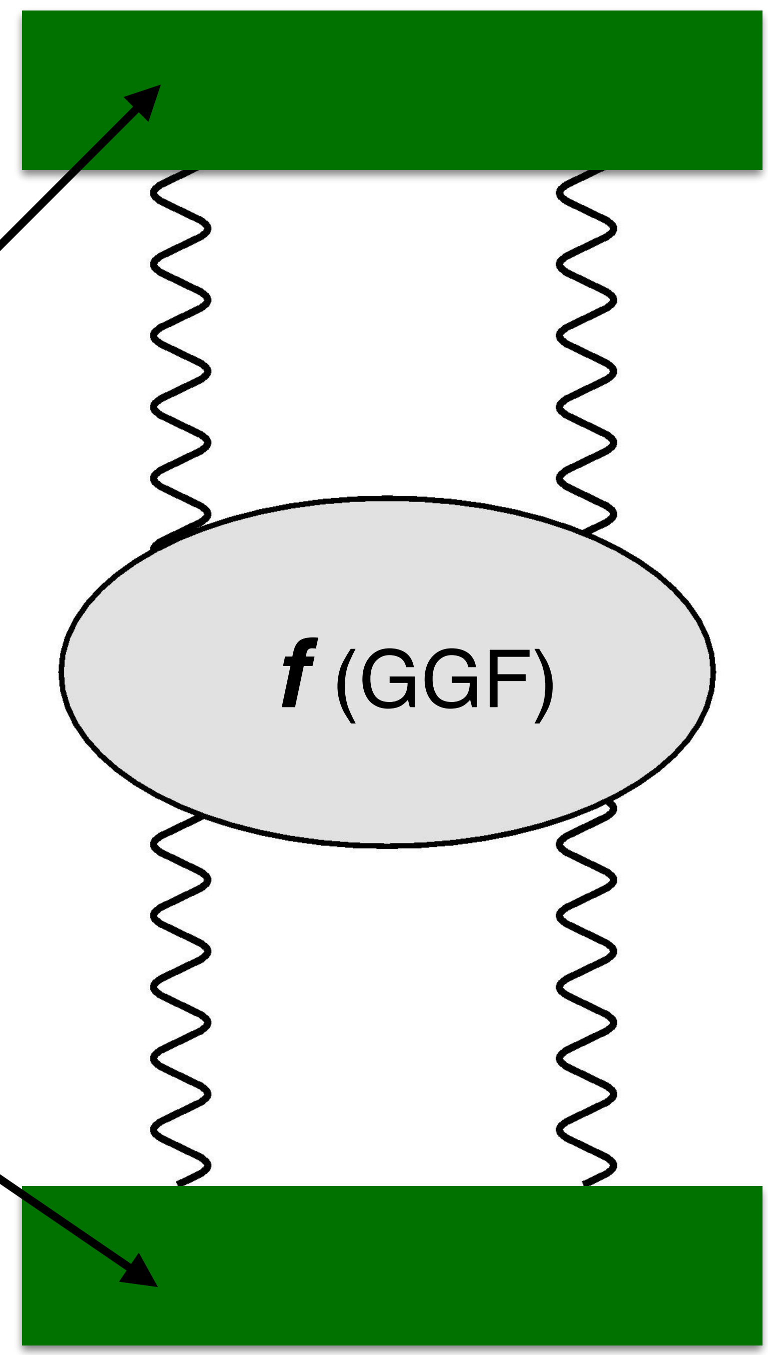


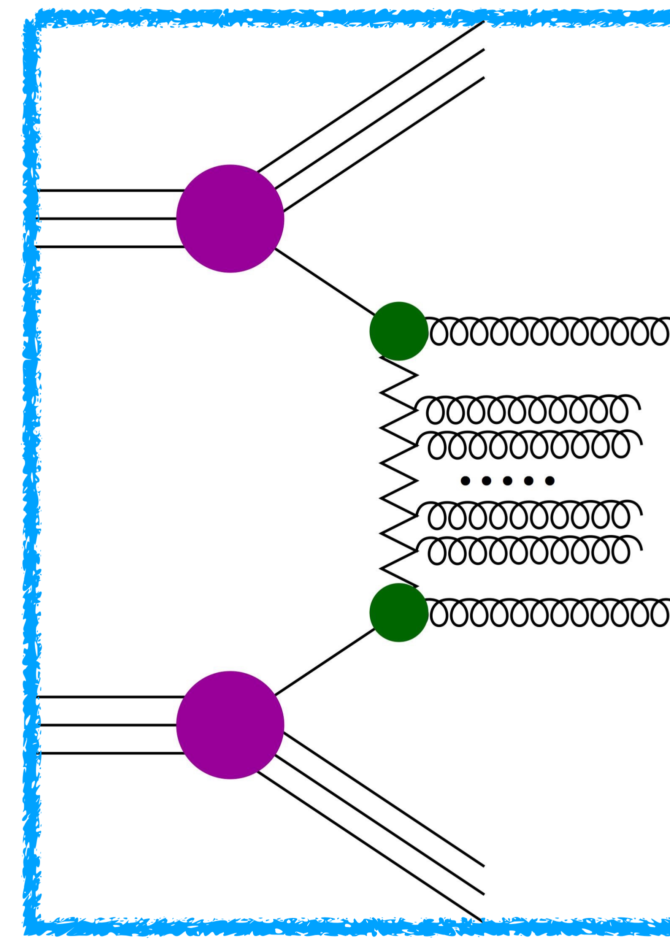
Many many
phase-space diagrams

Impact factors

The gluon Green's function
is process independent.

The effective couplings
to the colliding projectiles though
which are called
Impact Factors
are process dependent and
need to be calculated for each
different process.





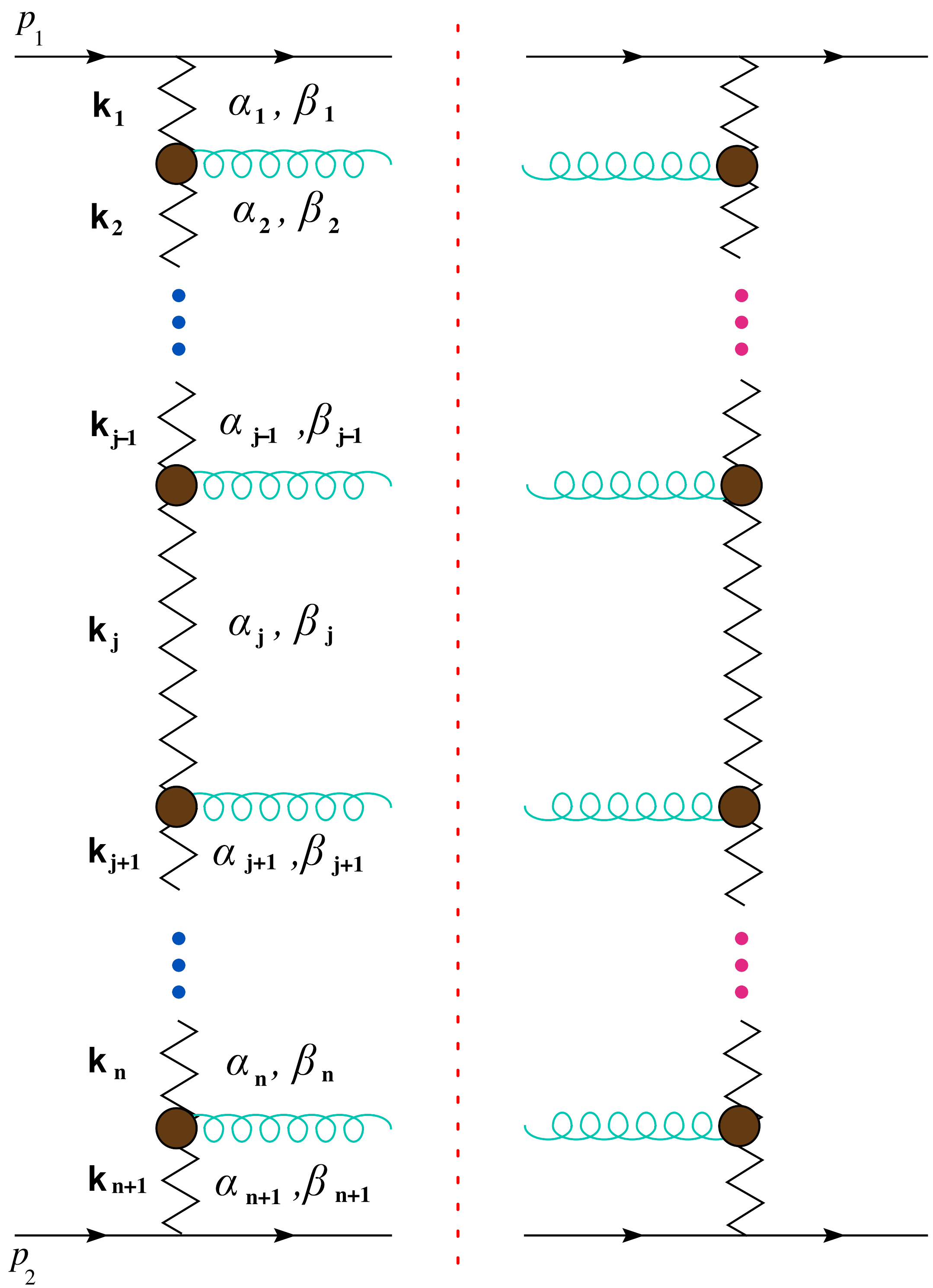
Why a Monte Carlo approach?

- We don't always know the analytic solution
- Even if we know it, we still want to store and analyze information about “differential” quantities (e.g. rapidities, transverse momenta, angles) that will be lost once we perform the integrations analytically.

  We want this for two reasons:

  1. Because then we can compare theoretical predictions to a greater set of observables
  2. Because there are lots of things we can still learn about concepts we use every day and maybe we don't fully understand
- We want to have a common language with people that work and are familiar with fixed order calculations and who are the majority in the “pheno” community – the interaction will help both sides
- We want to work in momentum space
- Connect to Heavy Ion physics
- Connect to physics of Cosmic Rays

Large logs from real emission corrections in a Monte Carlo setup



- Assume Reggeons in the t-channel
- Assume you have only one real emission
- Do the phase-space integration —> res1
- Now assume you have two real emissions
- Do the phase-space integration —> res2
- Add the results: RES = res1+res2
- Now assume you have three real emissions
- Do the phase-space integration —> res3
- Add the results: RES = RES + res3
- Repeat until you have N real emissions with resN so tiny compared to RES such that you are allowed to claim convergence

NOTE: The phase-space integration is over rapidity and transverse momenta.

BFKLex, a BFKL Monte Carlo

- The main goal was to have a tool that calculates the gluon Green's function (GGF) and other differential observables.

- The GGF is the solution to the BFKL equation. Use the iterative form:

$$f = e^{\omega\left(\vec{k}_A\right)Y}\Bigg\{\delta^{(2)}\left(\vec{k}_A-\vec{k}_B\right)+\sum_{n=1}^{\infty}\prod_{i=1}^{n}\frac{\alpha_s N_c}{\pi}\int d^2\vec{k}_i\frac{\theta\left(k_i^2-\lambda^2\right)}{\pi k_i^2}$$

$$\int_0^{y_{i-1}} dy_i e^{\left(\omega\left(\vec{k}_A+\sum_{l=1}^{i}\vec{k}_l\right)-\omega\left(\vec{k}_A+\sum_{l=1}^{i-1}\vec{k}_l\right)\right)y_i}\delta^{(2)}\left(\vec{k}_A+\sum_{l=1}^{n}\vec{k}_l-\vec{k}_B\right)\Bigg\}$$

$\omega\left(\vec{q}\right)=-\frac{\alpha_s N_c}{\pi}\log\frac{q^2}{\lambda^2}$ is the gluon Regge trajectory

The implementation of the BFKLex is in C++

Some results with BFKLex

A Comparative study of small x Monte Carlos with and without QCD coherence effects
G. C, M. Deak, A.Sabio Vera, P. Stephens
Nucl.Phys. B849 (2011) 28-44

The Colour Octet Representation of the Non-Forward BFKL Green Function
G. C, A. Sabio Vera.
Phys.Lett. B709 (2012) 301-308

The NLO N =4 SUSY BFKL Green function in the adjoint representation
G. C, A.Sabio Vera
Phys.Lett. B717 (2012) 458-461

Bootstrap and momentum transfer dependence in small x evolution equations
G. C, A. Sabio Vera, C. Salas
Phys.Rev. D87 (2013) no.1, 016007

A study of the diffusion pattern in N = 4 SYM at high energies
F. Caporale, G. C, J.D. Madrigal, B. Murdaca, A. Sabio Vera
Phys.Lett. B724 (2013) 127-132

Monte Carlo study of double logarithms in the small x region
G. C, A. Sabio Vera
Phys.Rev. D93 (2016) no.7, 074004

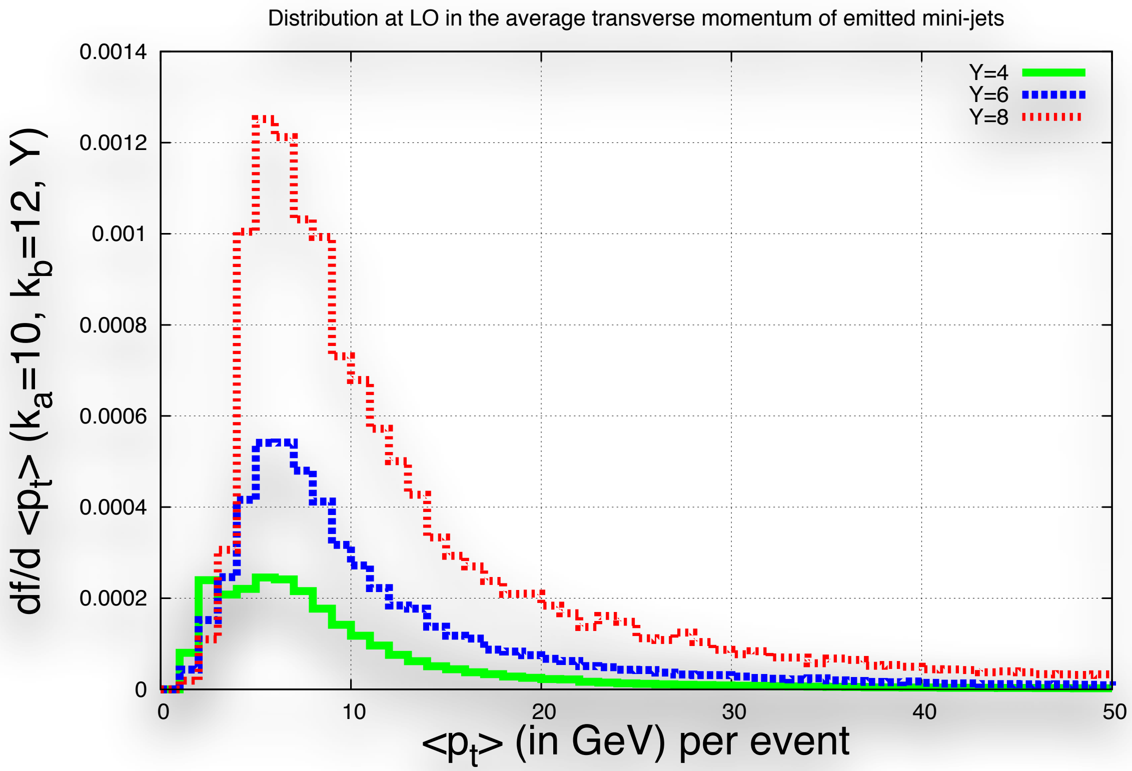
The high-energy radiation pattern from BFKLex with double-log collinear contributions
G. C, A. Sabio Vera
JHEP 1602 (2016) 064

The high-energy radiation pattern from BFKLex with double-log collinear contributions

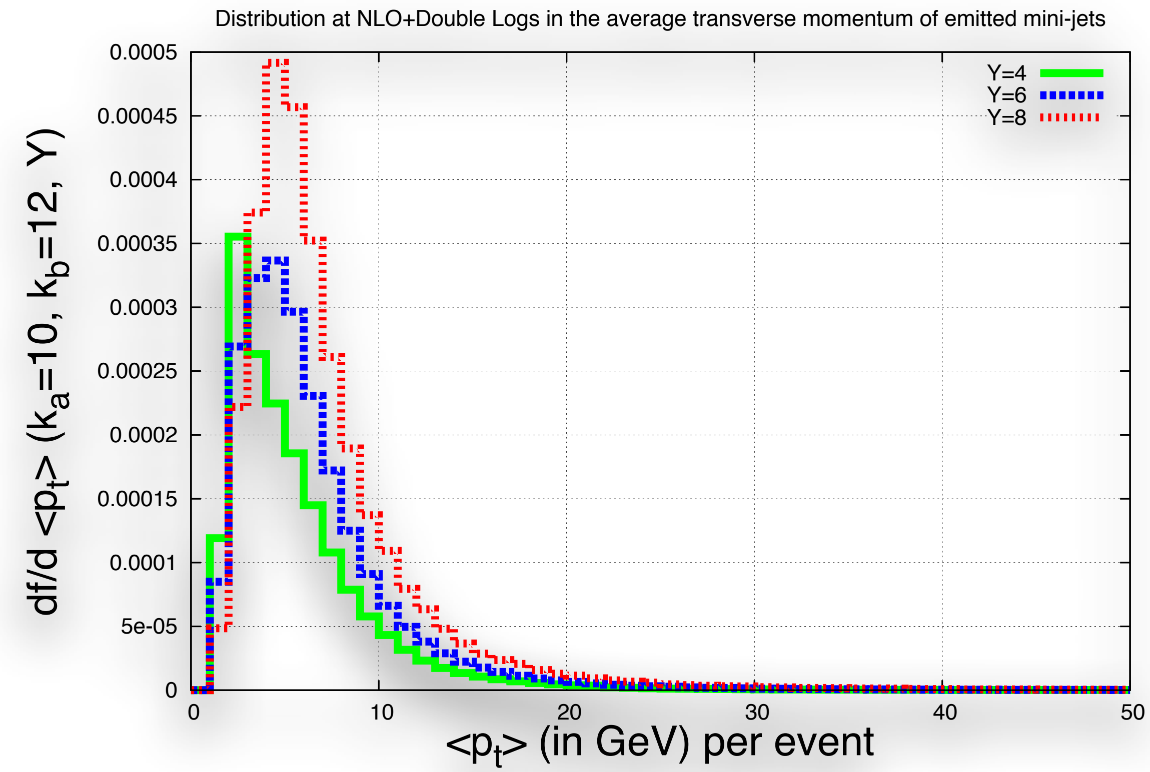
- Introduce three quantities related to the jet activity along the ladder. These characterize uniquely the event (but not fully).

- average p_t $\quad \langle p_t \rangle = \frac{1}{N}\sum_{i=1}^{N} |k_i|$
- average azimuthal angle $\quad \langle \phi \rangle = \frac{1}{N}\sum_{i=1}^{N} \phi_i$
- rapidity ratio between subsequent jets $\quad \langle \mathcal{R}_y \rangle = \frac{1}{N+1}\sum_{i=1}^{N+1} \frac{y_i}{y_{i-1}}$

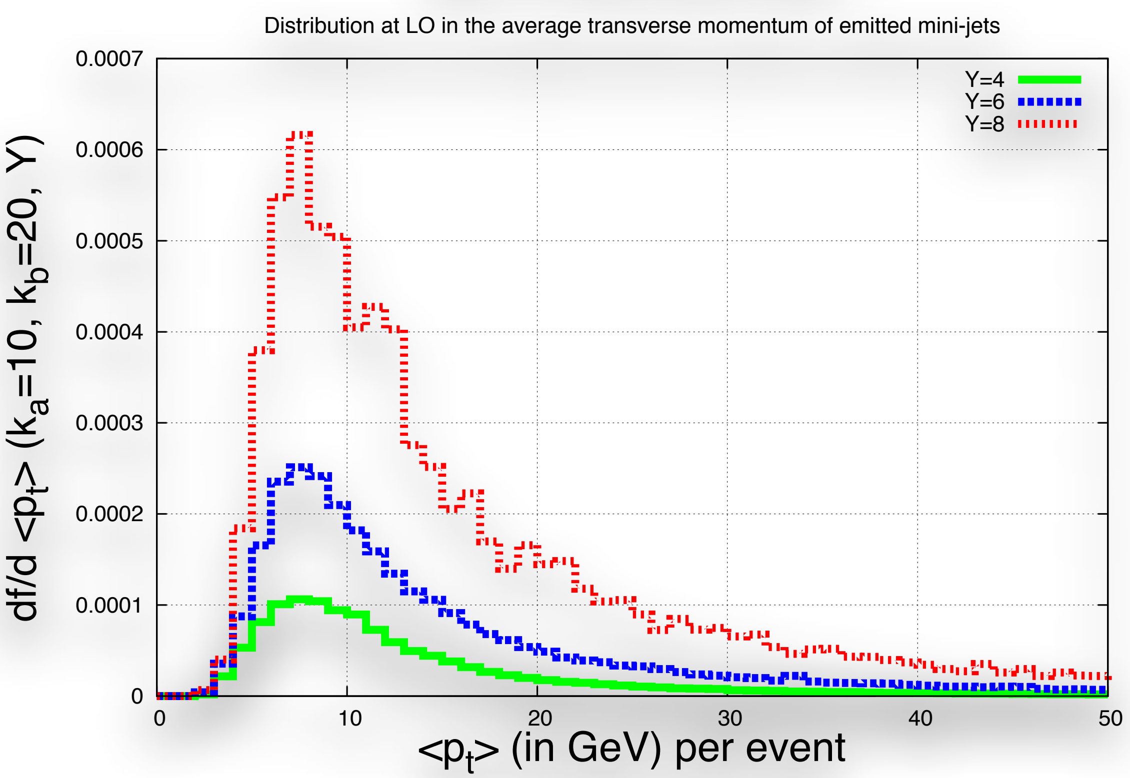
Average p_t



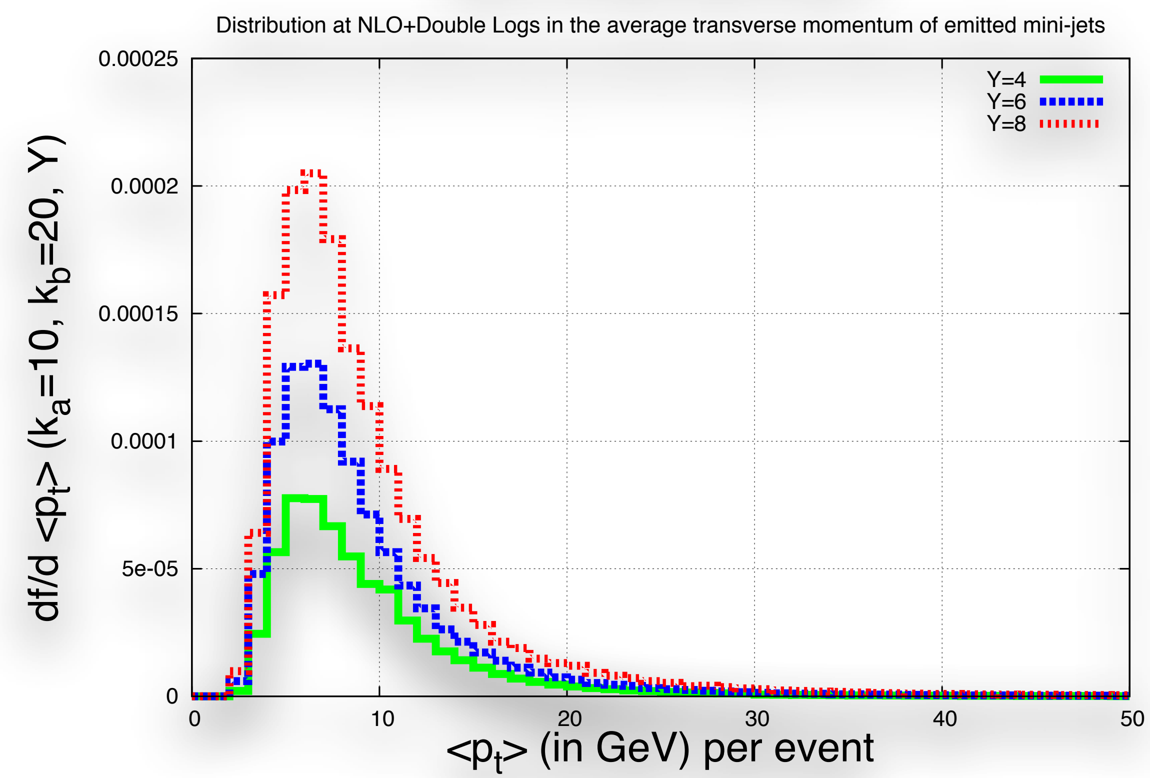
Distribution at LO in the average transverse momentum of emitted mini-jets
Y=4
Y=6
Y=8
df/d <pt> (ka=10, kb=12, Y)
<pt> (in GeV) per event



Distribution at NLO+Double Logs in the average transverse momentum of emitted mini-jets
Y=4
Y=6
Y=8
df/d <pt> (ka=10, kb=12, Y)
<pt> (in GeV) per event

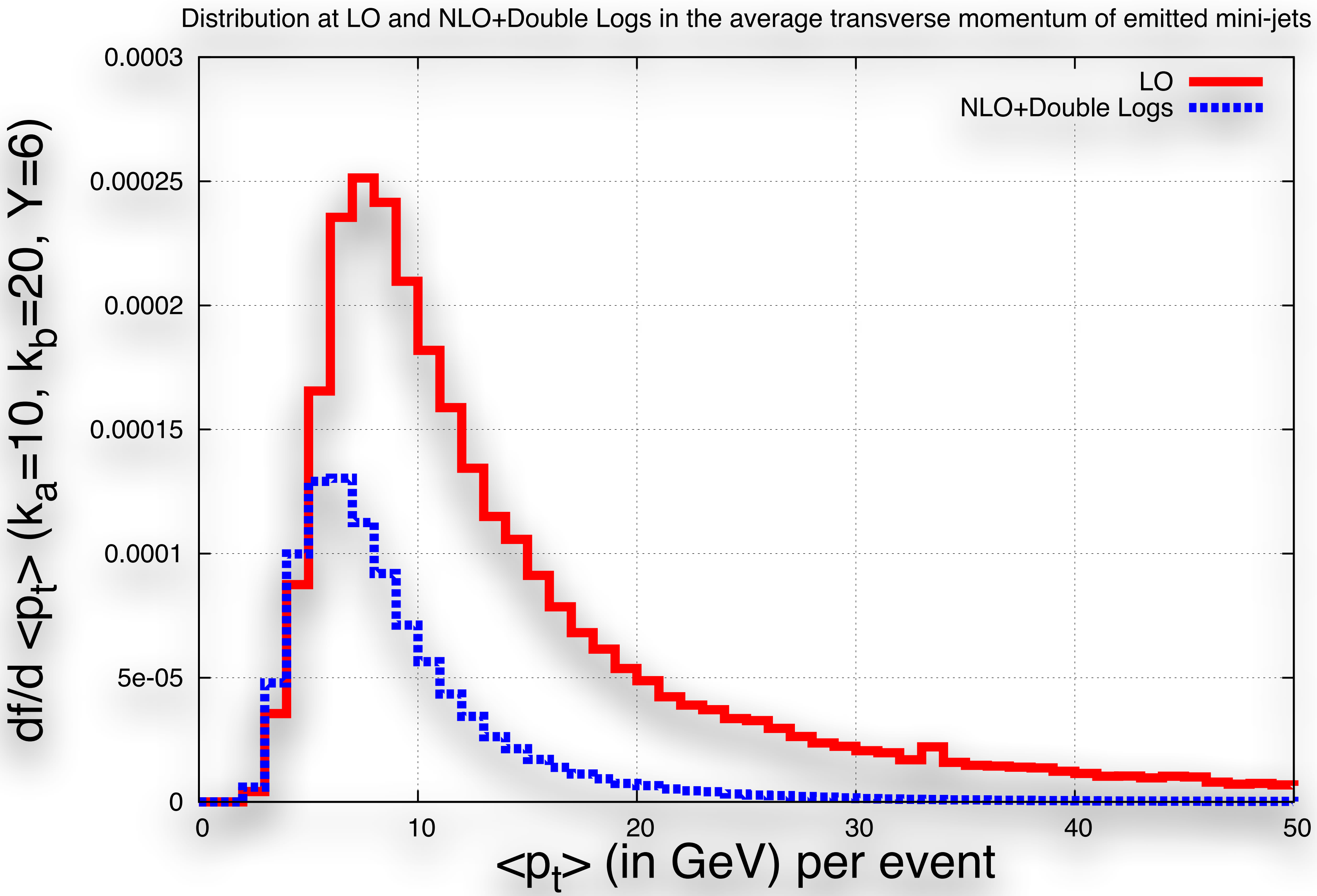


Distribution at LO in the average transverse momentum of emitted mini-jets
Y=4
Y=6
Y=8
df/d <pt> (ka=10, kb=20, Y)
<pt> (in GeV) per event



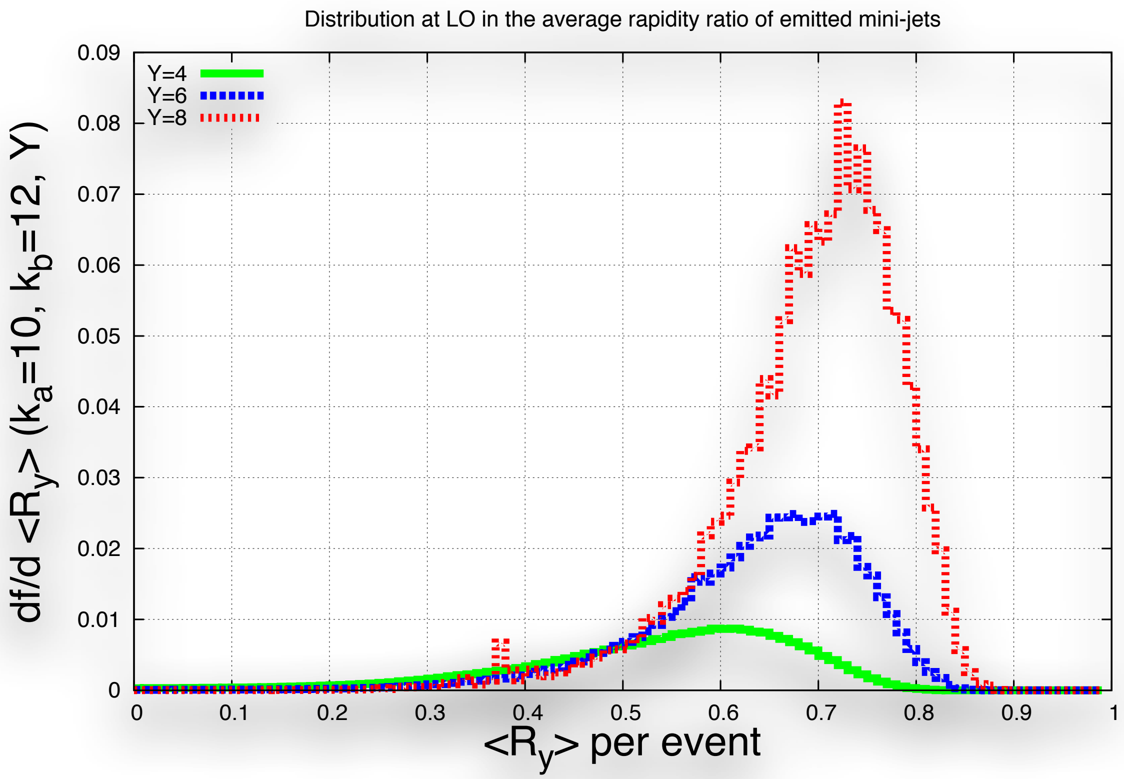
Distribution at NLO+Double Logs in the average transverse momentum of emitted mini-jets
Y=4
Y=6
Y=8
df/d <pt> (ka=10, kb=20, Y)
<pt> (in GeV) per event

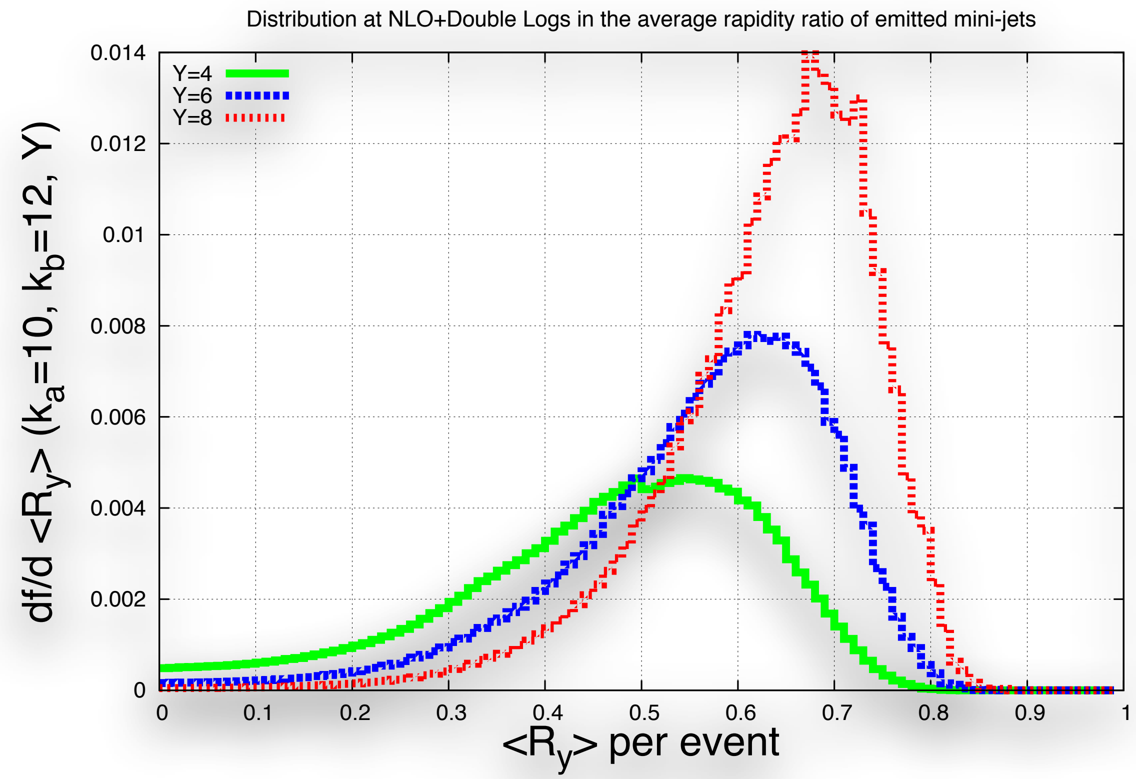
Average p_t

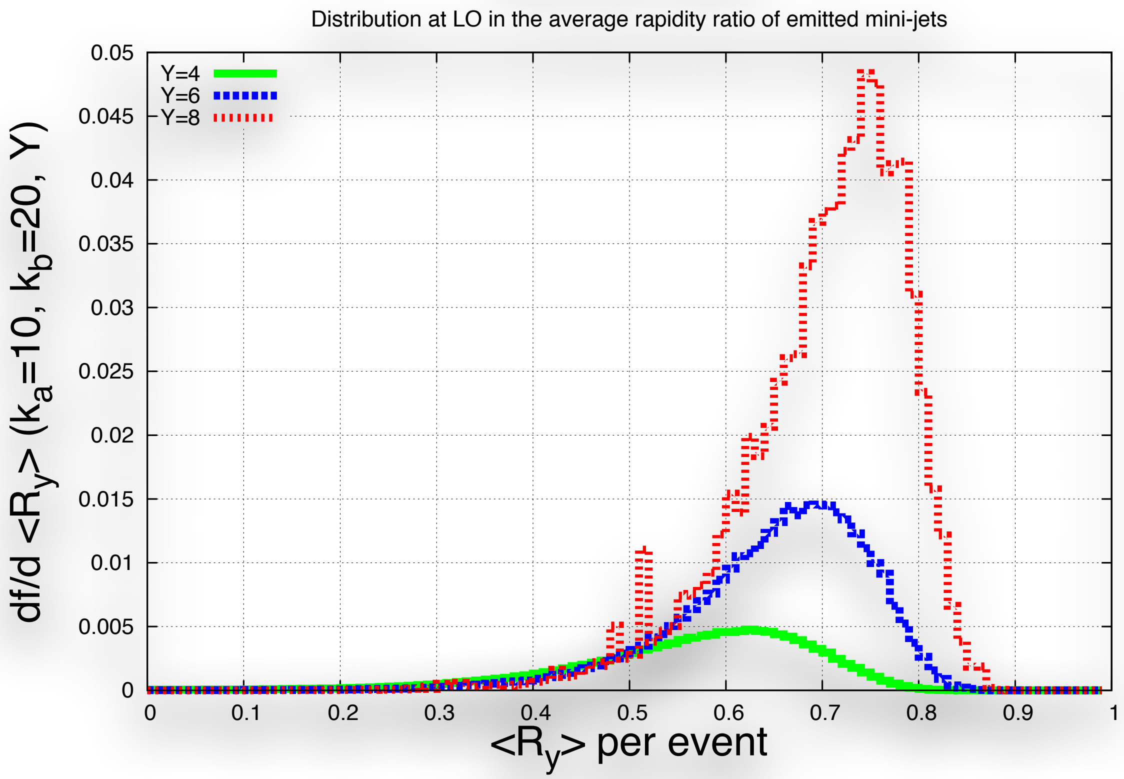


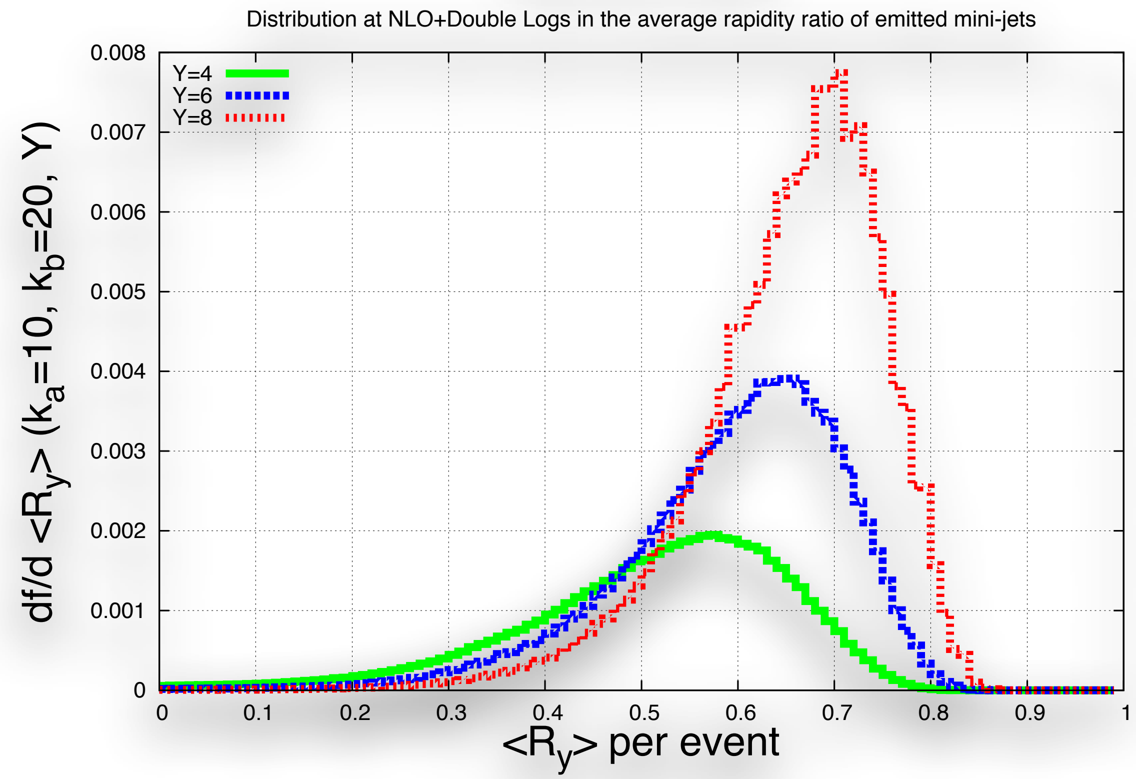
Distribution at LO and NLO+Double Logs in the average transverse momentum of emitted mini-jets
LO
NLO+Double Logs
df/d <p_t> (k_a=10, k_b=20, Y=6)
<p_t> (in GeV) per event
0.0003
0.00025
0.0002
0.00015
0.0001
5e-05
0
0
10
20
30
40
50

Average rapidity ratio

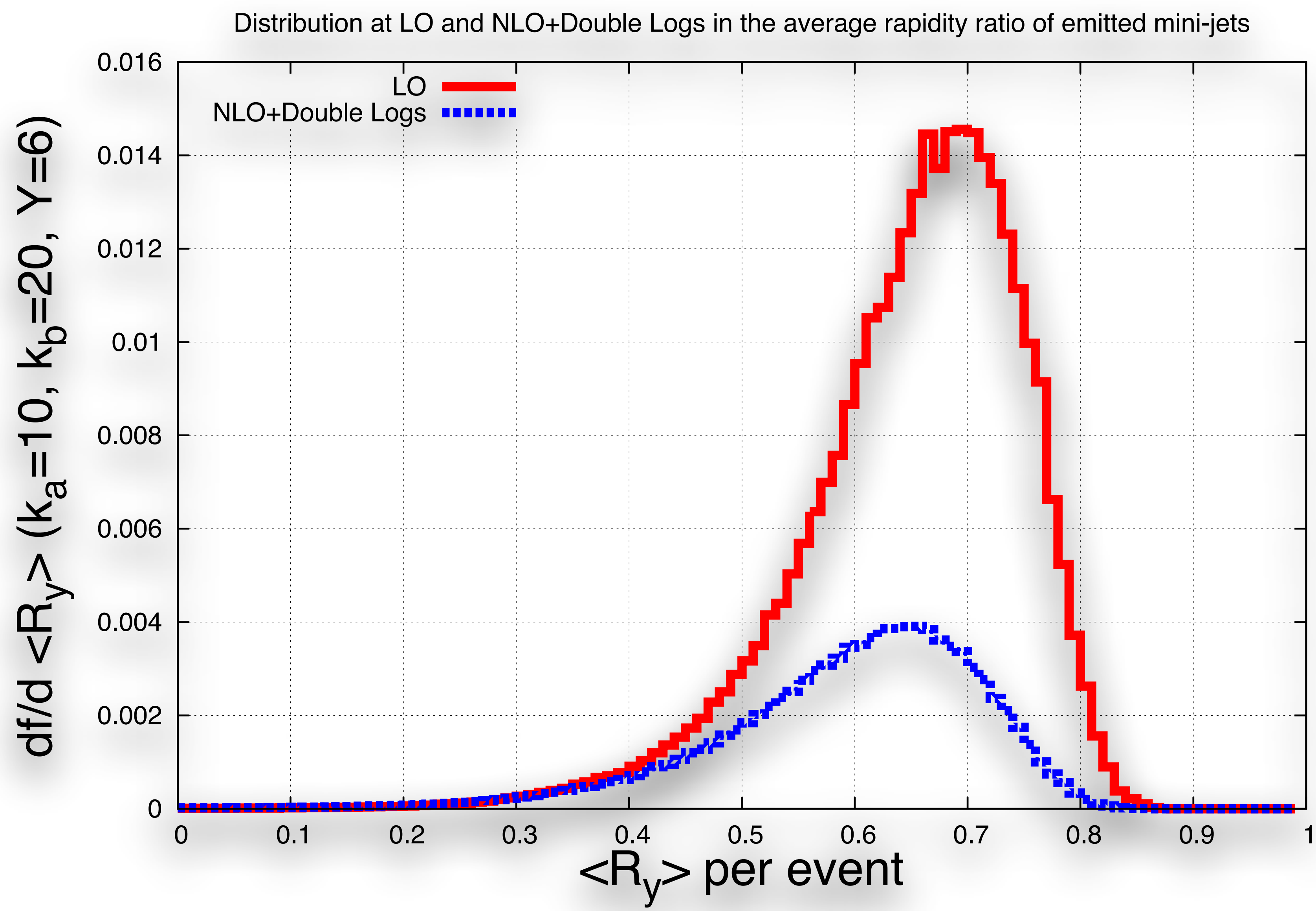




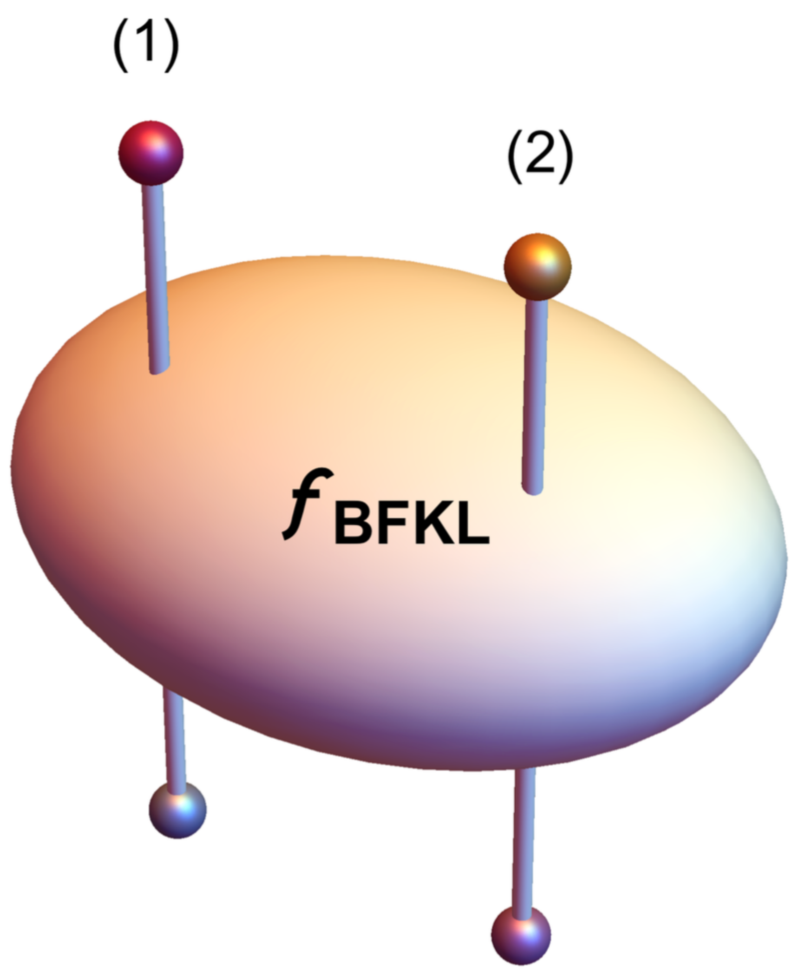


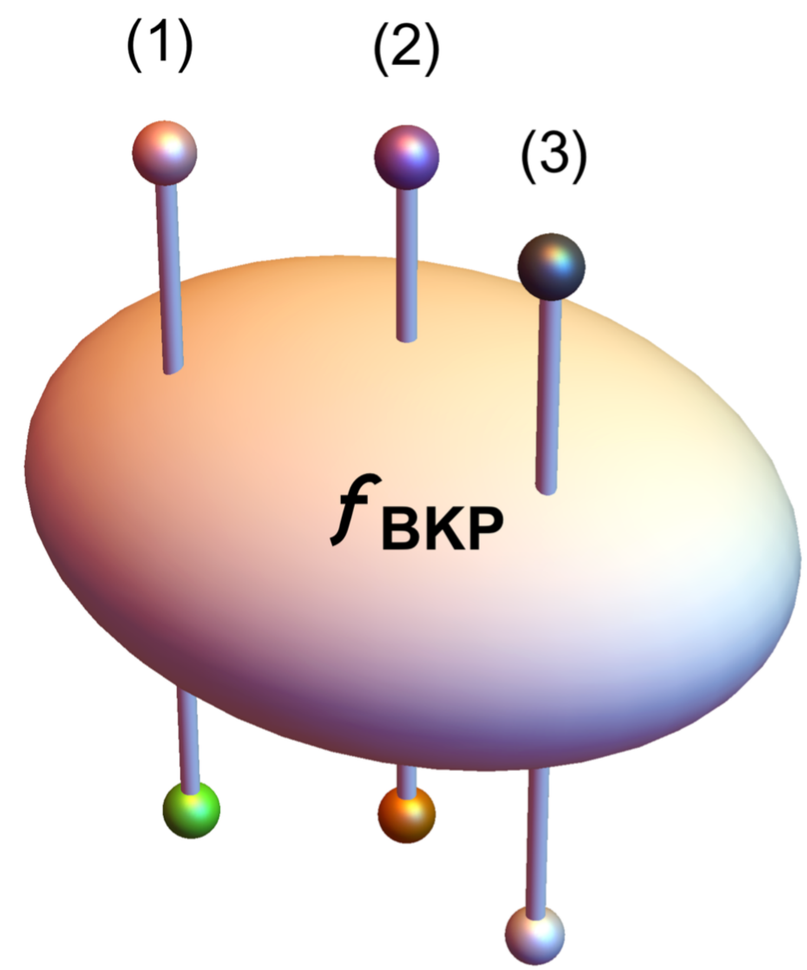


Average rapidity ratio



From two to three Reggeons: the Odderon





BKP equation (*Bartels-Kwiecinski-Praszalowicz*)

TOTEM-2017-002
30 November 2018

CERN-EP-2017-335-v3
30 November 2018

First determination of the ρ parameter at $\sqrt{s} = 13$ TeV – probing the existence of a colourless three-gluon bound state

Physics Letters B 778 (2018) 414–418

Contents lists available at ScienceDirect

Physics Letters B

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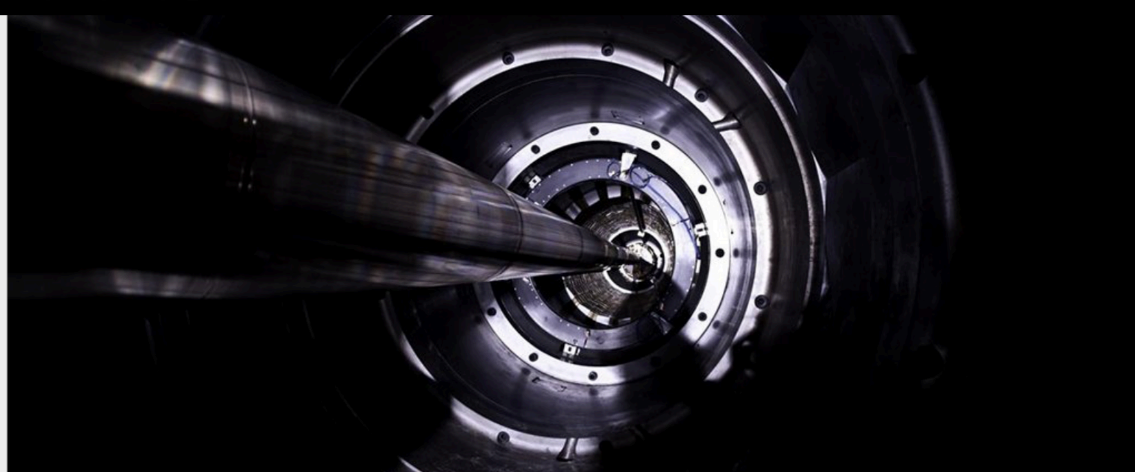
Did TOTEM experiment discover the Odderon?

Evgenij Martynov [a,*], Basarab Nicolescu [b]

[a] Bogolyubov Institute for Theoretical Physics, Metrologichna 14b, Kiev, 03680, Ukraine
[b] Faculty of European Studies, Babes-Bolyai University, Emmanuel de Martonne Street 1, 400090 Cluj-Napoca, Romania

The Odderon makes headlines

sciencealert Trending



(CERN)

PHYSICS

CERN May Have Evidence of a Quasiparticle We've Been Hunting For Decades

DAVID NIELD 6 FEB 2018

GIZMODO

LATEST REVIEWS SCIENCE IO9 FIELD GUIDE EARTHER DESIGN PALEOFUTURE

PHYSICS

Physicists Spot Evidence of 'Odderon' First Predicted in the 1970s

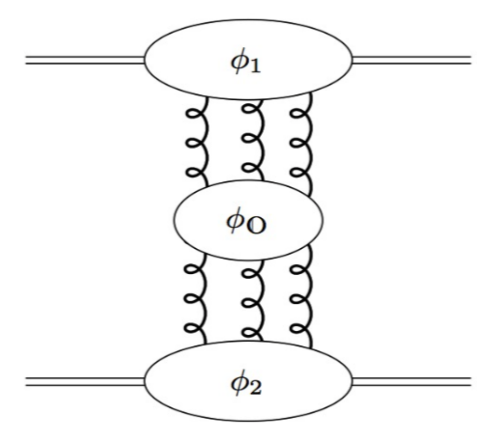
Ryan F. Mandelbaum
2/02/18 3:50pm
• Filed to: THIS TOOK A VERY LONG TIME TO DISTILL

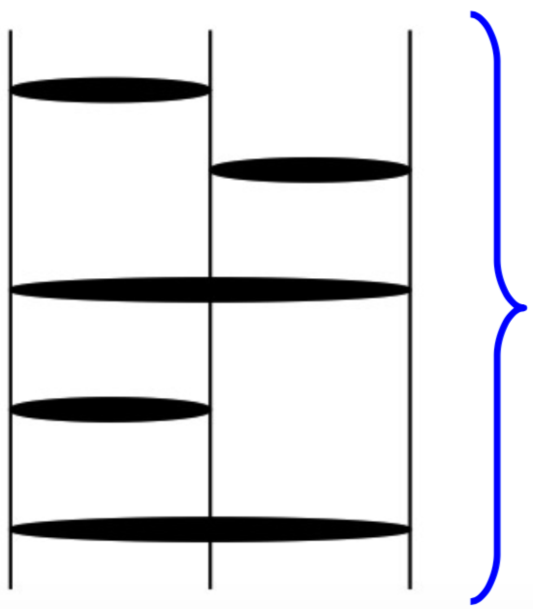
105.4K 25 12

…Physicists at the TOTEM experiment on the Large Hadron Collider in Geneva, Switzerland have found evidence for a physical effect called an "Odderon." It is not direct evidence, but rather some results that would make more sense if this particle existed. […]
"… the Odderon is one of the possible ways by which protons can interact without breaking, whose manifestations have never been observed," **Simone Giani, spokesperson at the TOTEM experiment**, told me. "This could be the first evidence of that."

The Pomeron vs the Odderon

- Pomeron is the state of two interacting reggeized gluons in the t-channel in the color singlet. It has the quantum numbers of the vacuum
- Odderon is the state of three interacting gluons exchanged in the t-channel in the color singlet but with C =-1 and P=-1
- Any pair of two gluons in the Odderon forms symmetric color octet subsystems

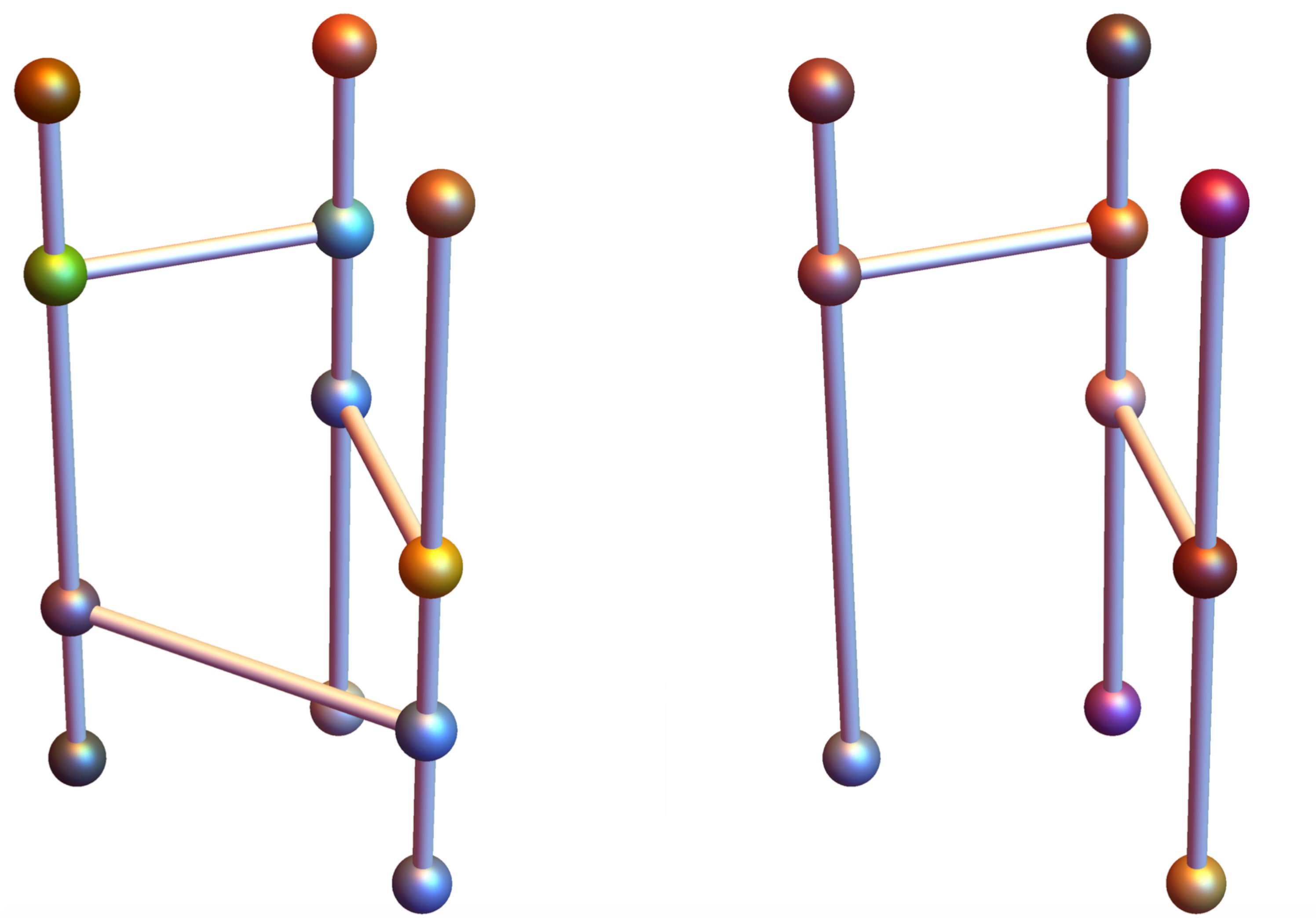




Ladder structure of the Odderon. BKP resums term of the form $\alpha_s(\alpha_s \log s)^n$

NLO corrections recently available
Bartels, Fadin, Lipatov, Vacca (2012)

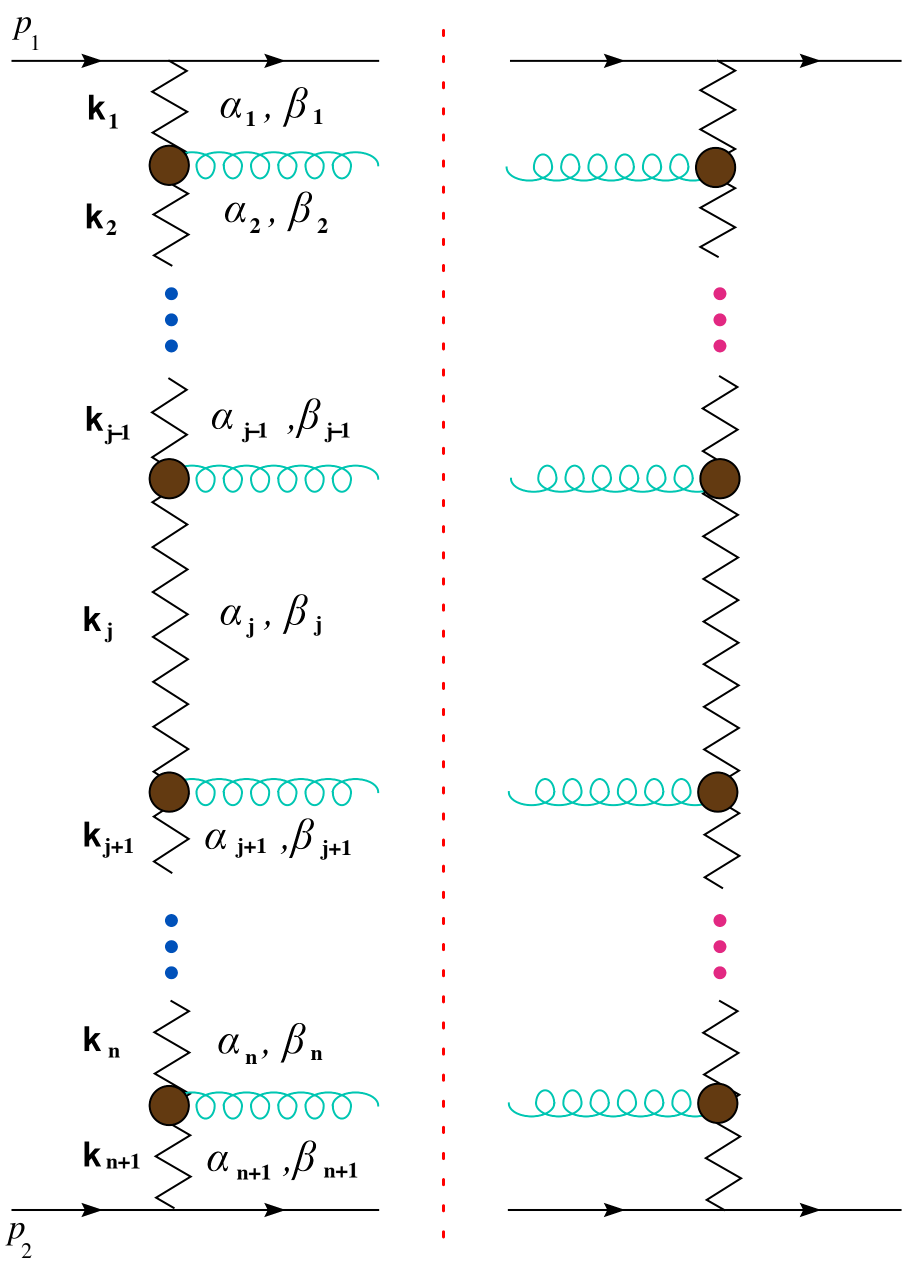
Closed vs Open

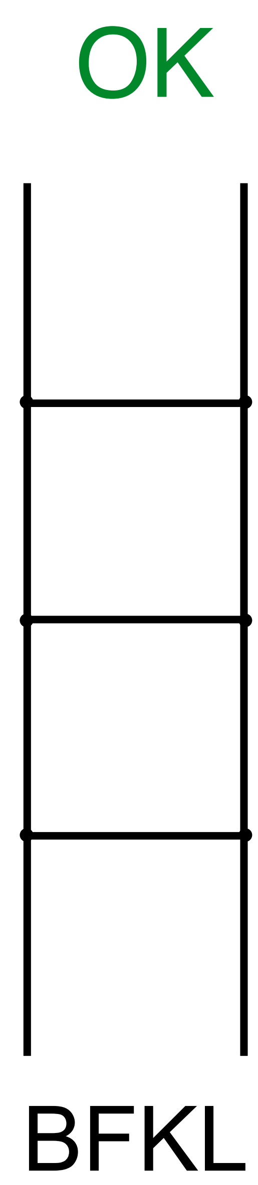


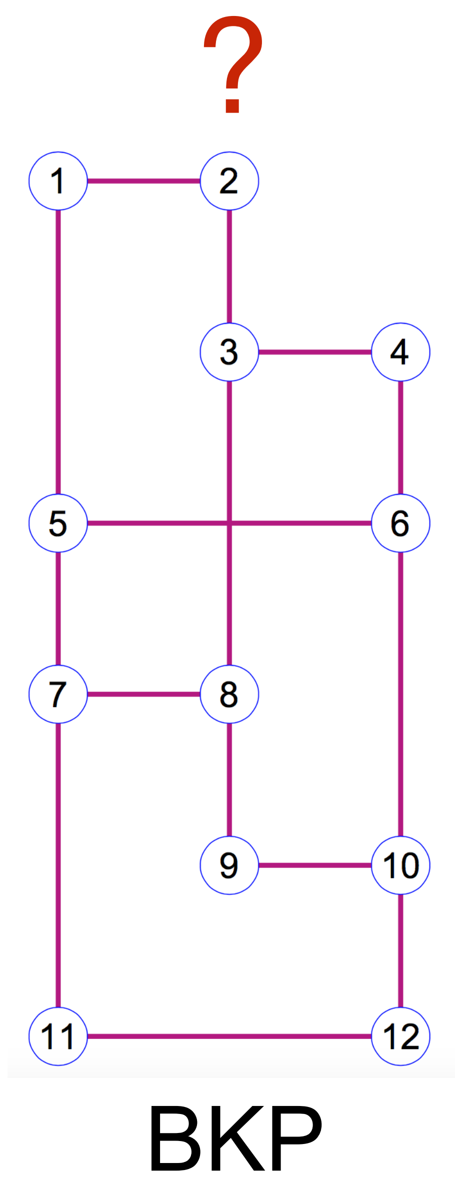
Side note: BKP was found to have a hidden integrability being equivalent to a periodic spin chain of a XXX Heisenberg ferromagnet. This was the first example of the existence of integrable systems in QCD

Lipatov (1986, 1990, 1993)

Are Monte Carlo techniques adequate to compute the Odderon GGFs?

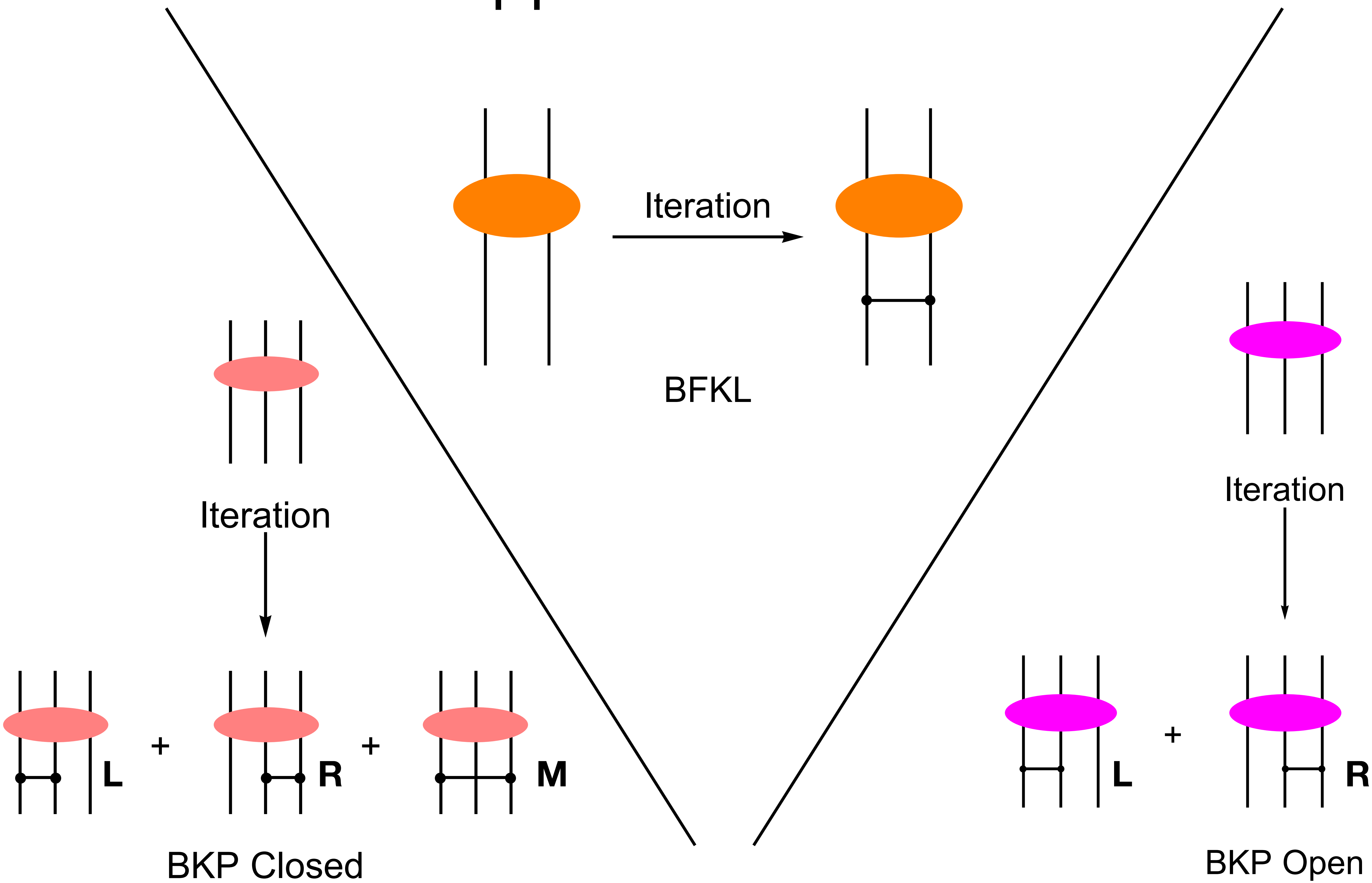






Vertical lines are Reggeons horizontal ones are gluons

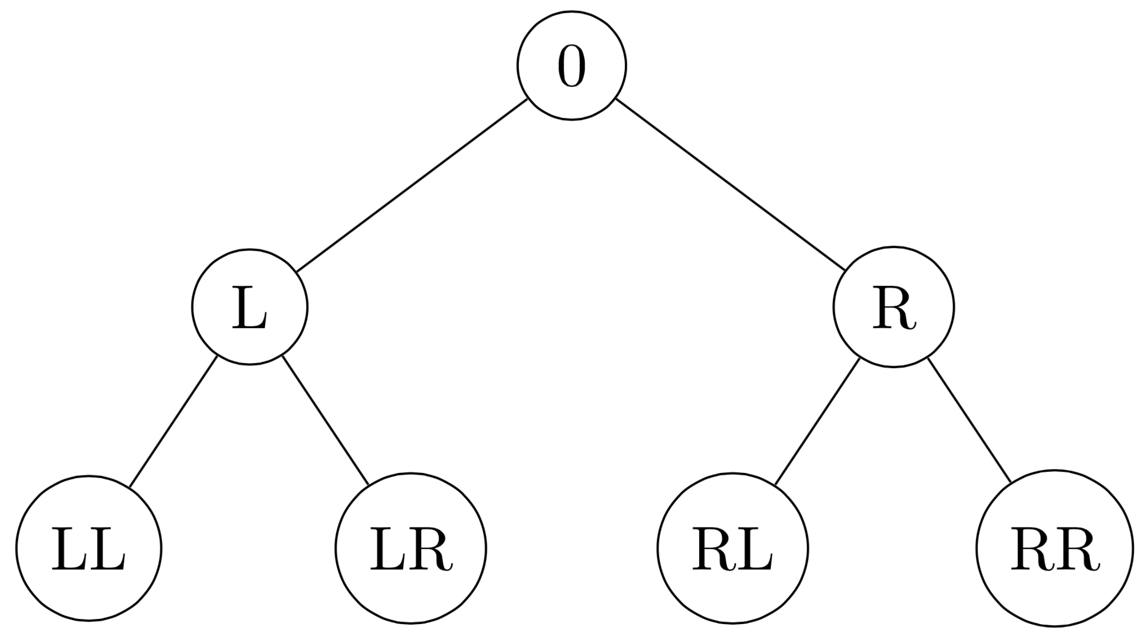
Monte Carlo approach is based on iteration

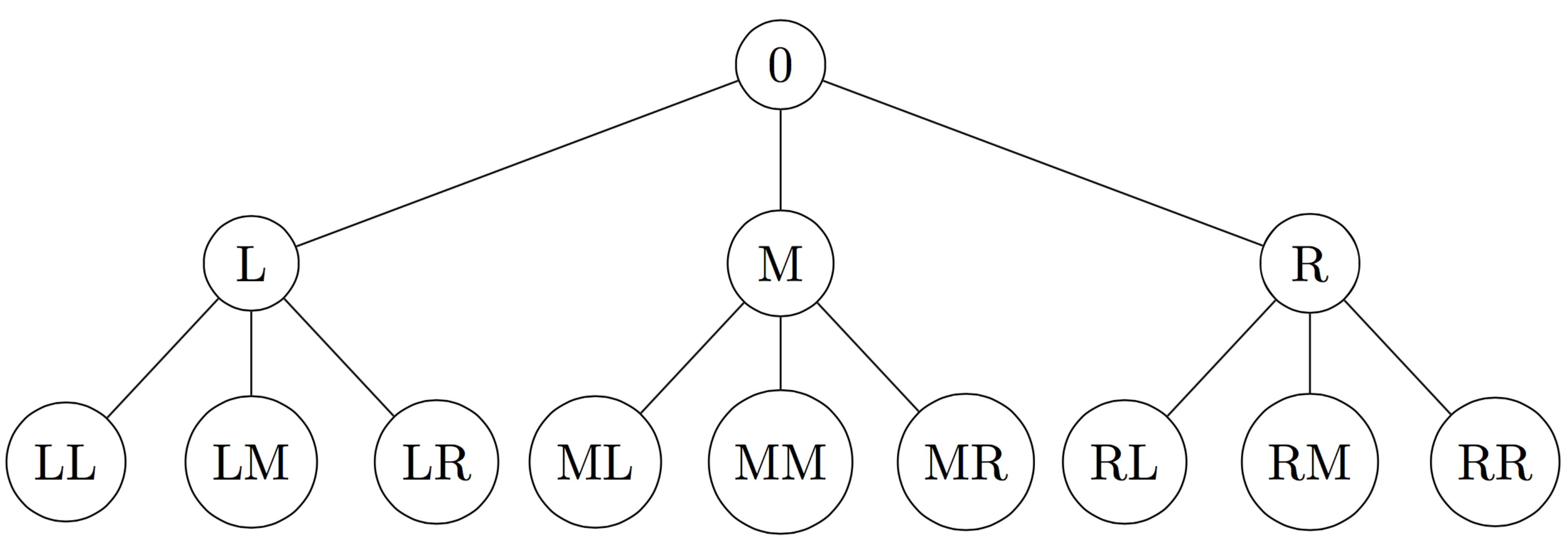


Binary/ternary tree structure

number of diagrams:
2^n (open) and 3^n (closed)

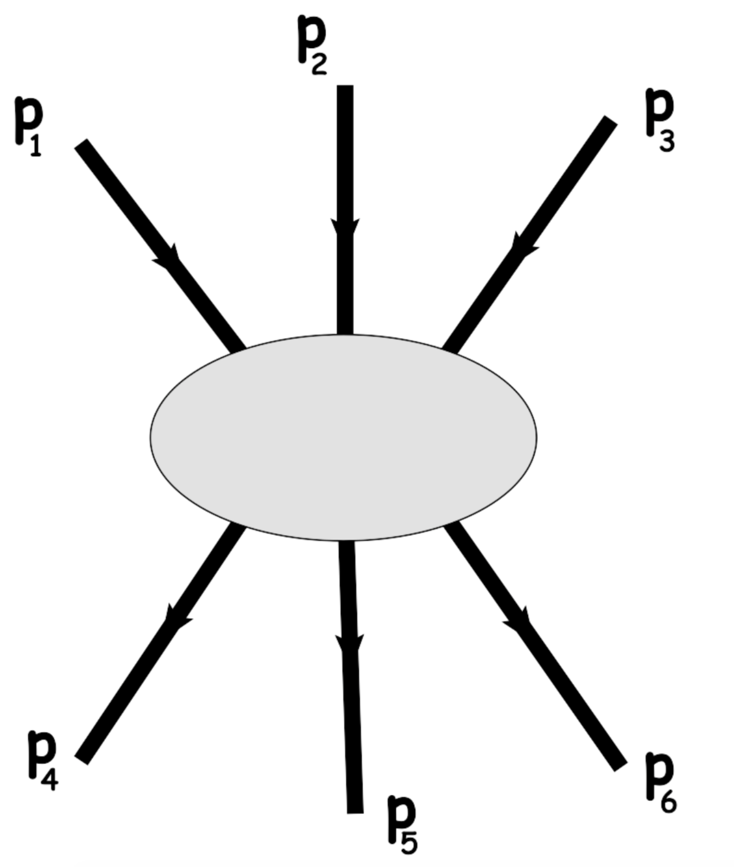
n rungs	Number of diagrams
2	9
3	27
4	81
5	243
6	729
7	2187
8	6561
9	19683
10	59049
11	177147
12	531441
13	1594323
14	4782969.





Results:
kinematical configuration

All vectors live in the transverse momentum space



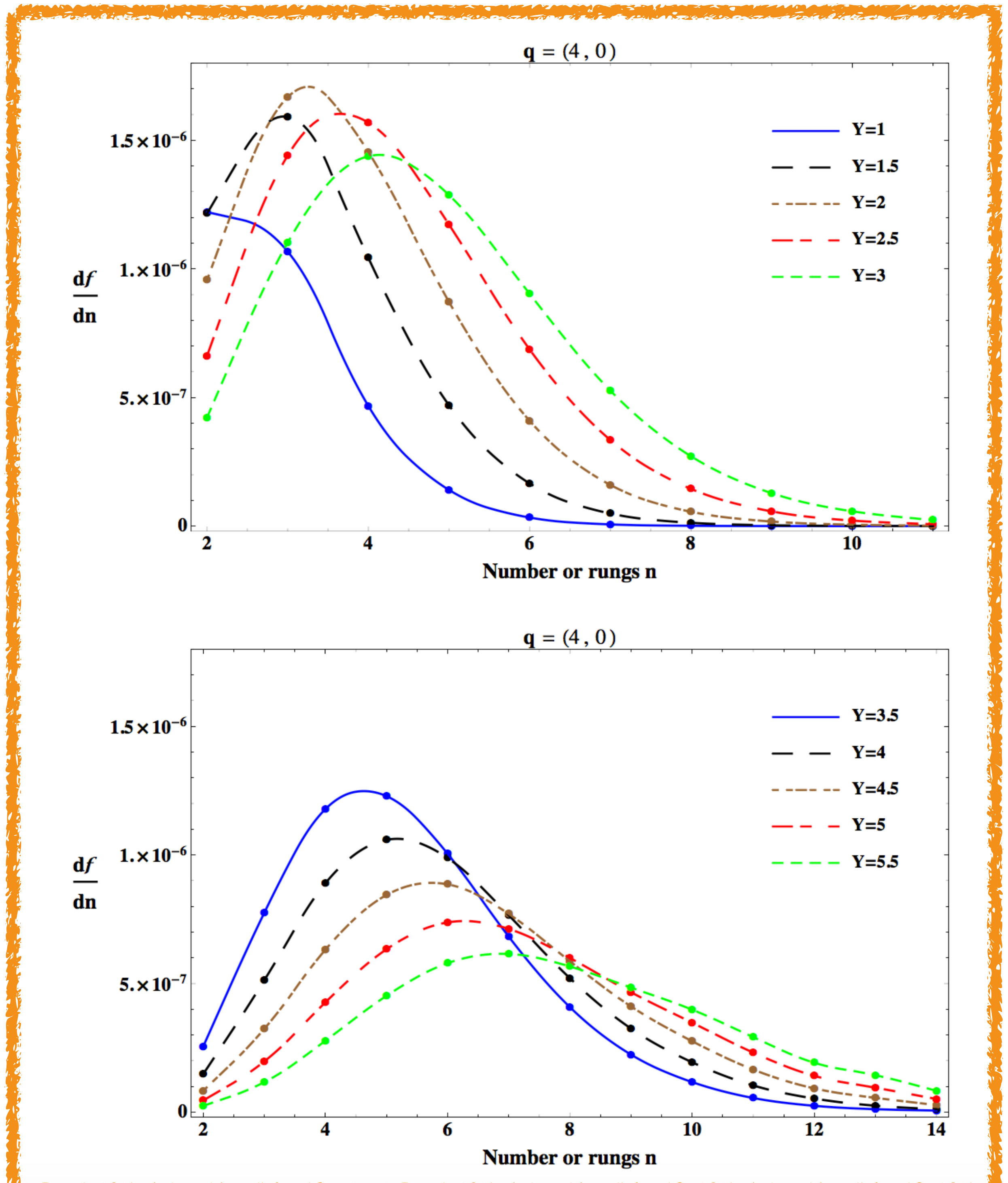
$$\mathbf{q} = (4,0)$$
$$\mathbf{p}_1 = (10,0)$$
$$\mathbf{p}_2 = (20,\pi)$$
$$\mathbf{p}_3 = (\mathbf{q}-\mathbf{p}_1) - \mathbf{p}_2 = (14,0)$$
$$\mathbf{p}_4 = (20,0)$$
$$\mathbf{p}_5 = (25,\pi)$$
$$\mathbf{p}_6 = (\mathbf{q}-\mathbf{p}_4) - \mathbf{p}_5 = (9,0)$$

$$\mathbf{q} = (31,0)$$
$$\mathbf{p}_1 = (10,0)$$
$$\mathbf{p}_2 = (20,\pi)$$
$$\mathbf{p}_3 = (\mathbf{q}-\mathbf{p}_1) - \mathbf{p}_2 = (41,0)$$
$$\mathbf{p}_4 = (20,0)$$
$$\mathbf{p}_5 = (25,\pi)$$
$$\mathbf{p}_6 = (\mathbf{q}-\mathbf{p}_4) - \mathbf{p}_5 = (36,0)$$

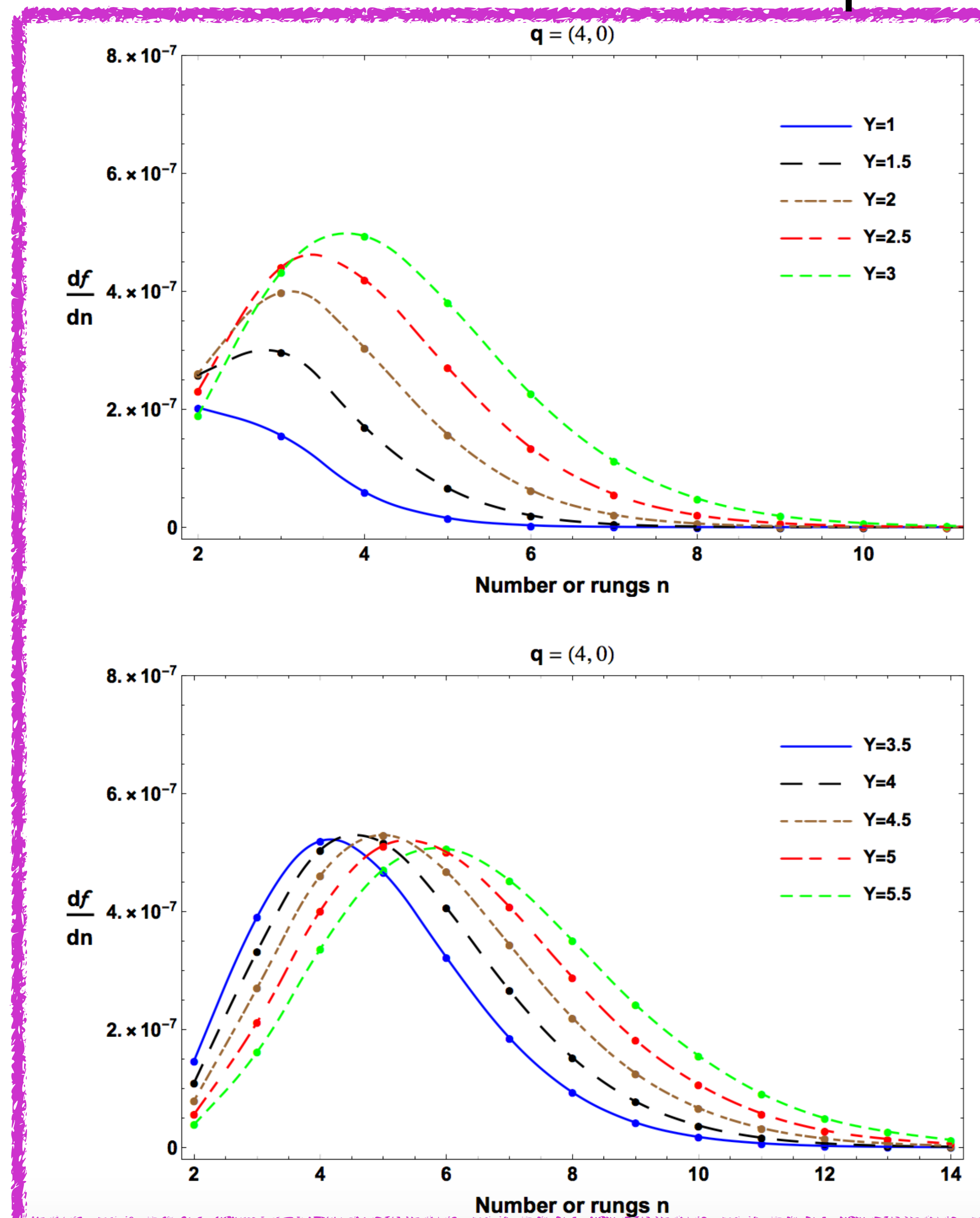
(r,θ): first component is in GeV, the second component in radians

Results:
“multiplicity”

Closed

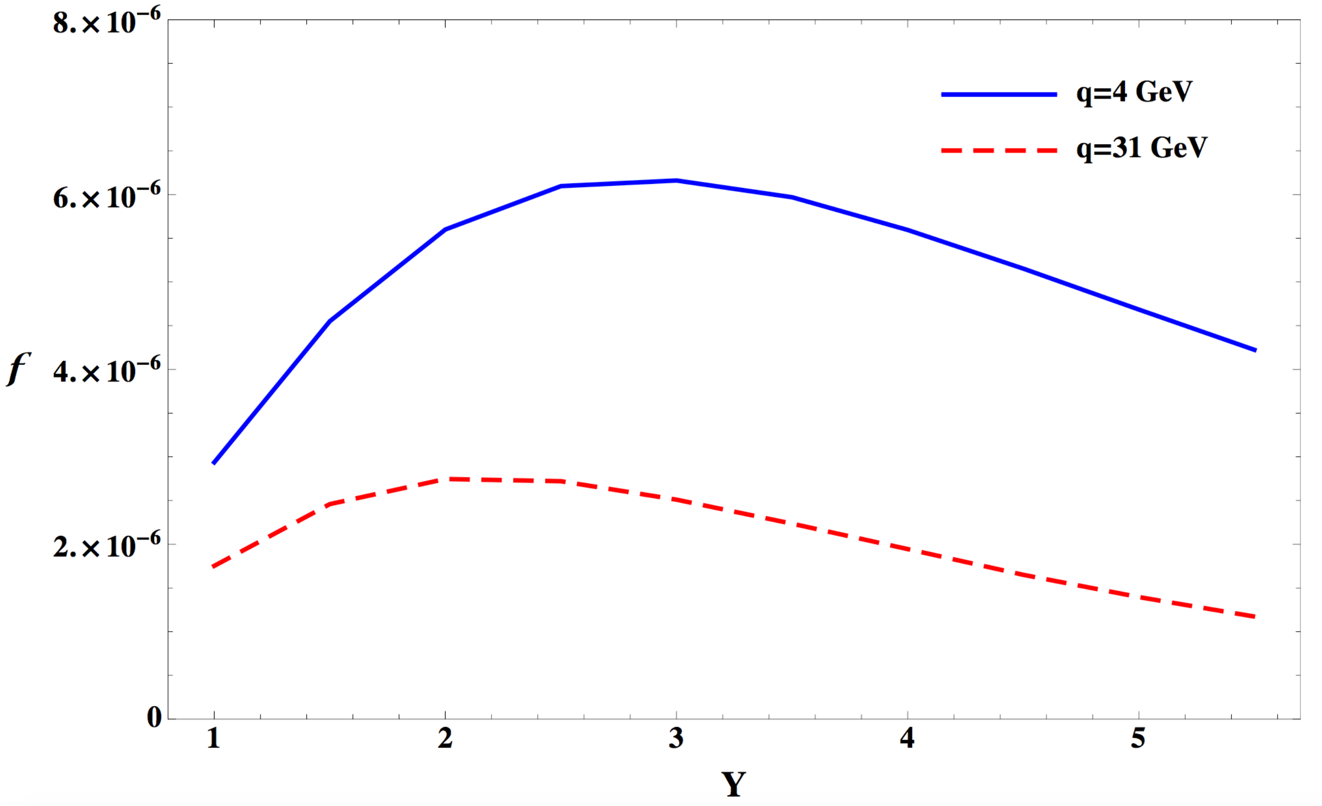


Open

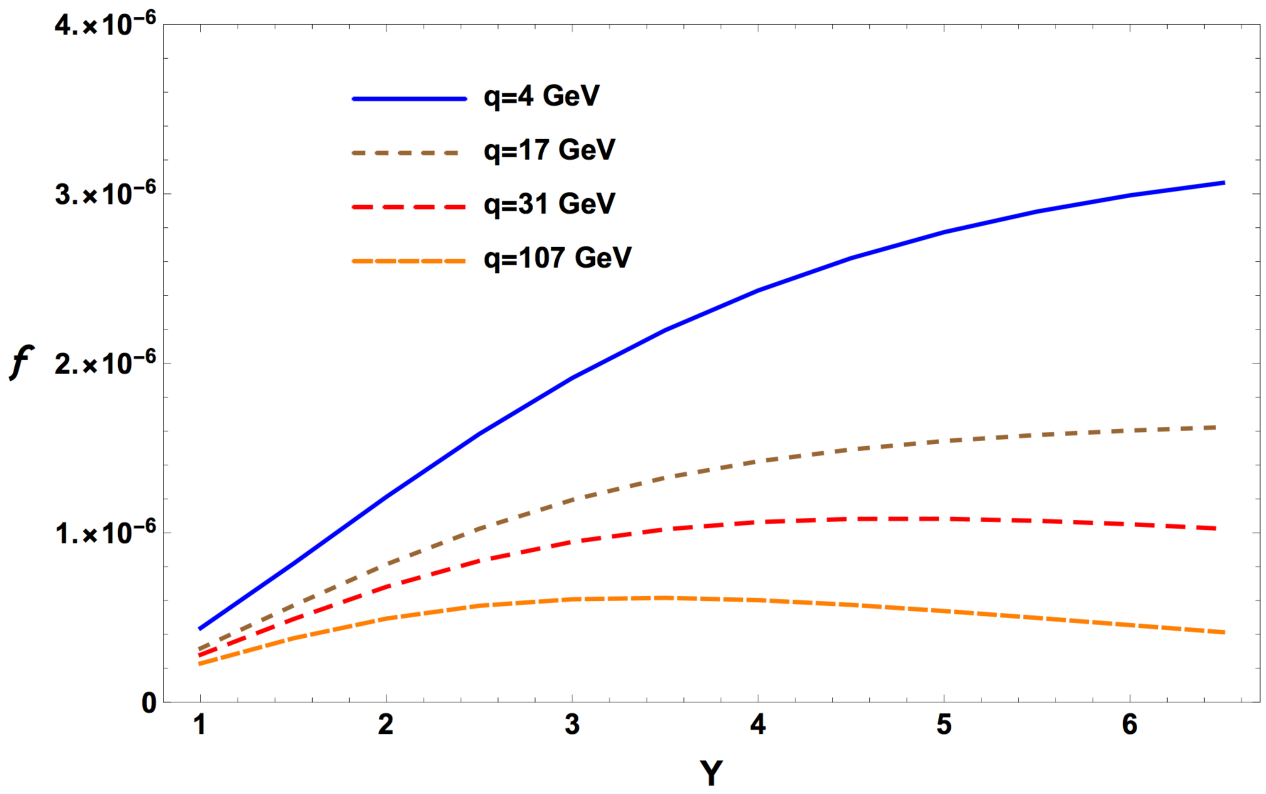


Results:
energy plots

Closed



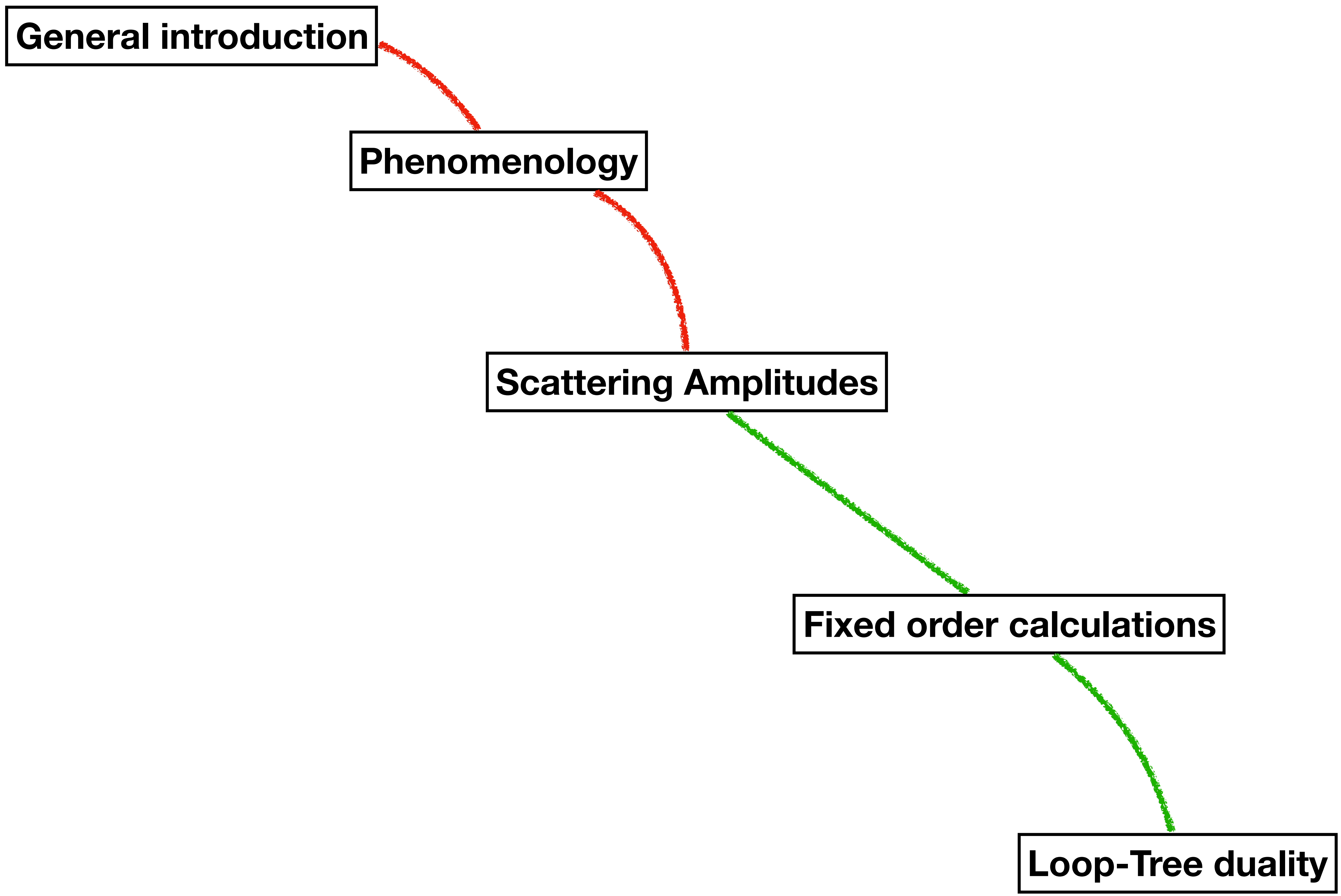
Open



G.C. A. Sabio Vera, Eur.Phys.J. C78 (2018) no.6, 496

Outlook

Use the Monte Carlo solution of the BKP equation (Odderon) for phenomenology


General introduction
Phenomenology
Scattering Amplitudes
Fixed order calculations
Loop-Tree duality

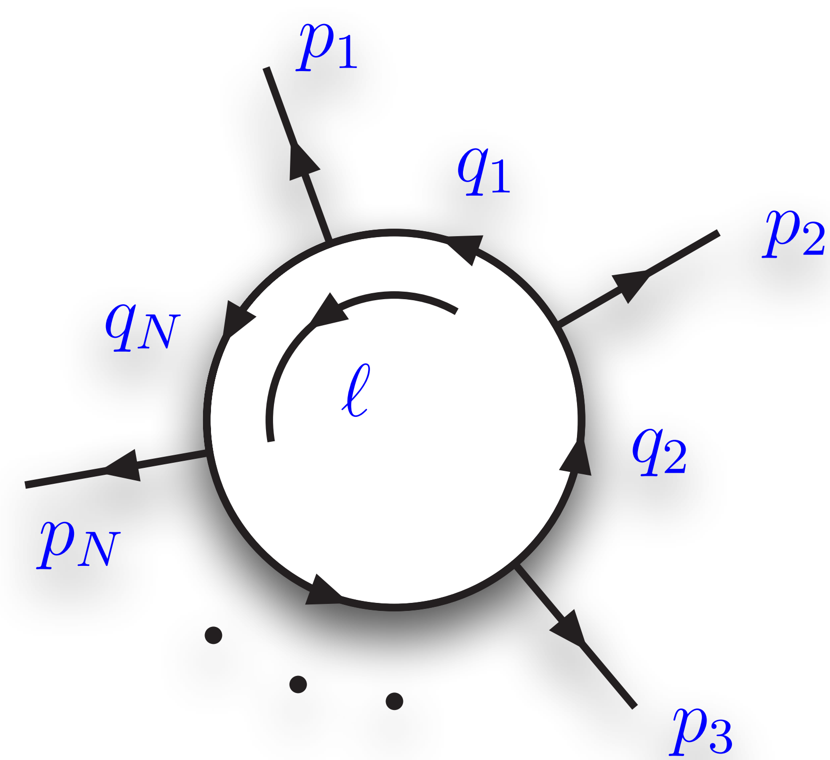
The constant need for higher order radiative corrections

- The LHC is a hadronic collider operating at high energies
  - higher multiplicities
  - proton structure
  - very large soft and collinear corrections
  - logarithms of ratios of very different scales
- Rule of thumb:
  - LO: order of magnitude evaluation
  - NLO: first reliable evaluation of the central value
  - NNLO: first reliable evaluation of the uncertainty
- The Loop-Tree Duality promises to deal with virtual and real corrections on equal footing. In this talk we will see how the method copes with the virtual corrections

A generic one-loop integral

Number of legs N, number of spacetime dimensions is D.

Assume that it is UV and IR finite.



$$L^{(1)}(p_1, p_2, \ldots, p_N) = -i \int \frac{d^d\ell}{(2\pi)^d} \prod_{i=1}^{N} \frac{1}{q_i^2 + i0}$$

ℓ^μ is the loop momentum and $q_i = \ell + \sum_{k=1}^{i} p_k$ are the momenta of the propagators.

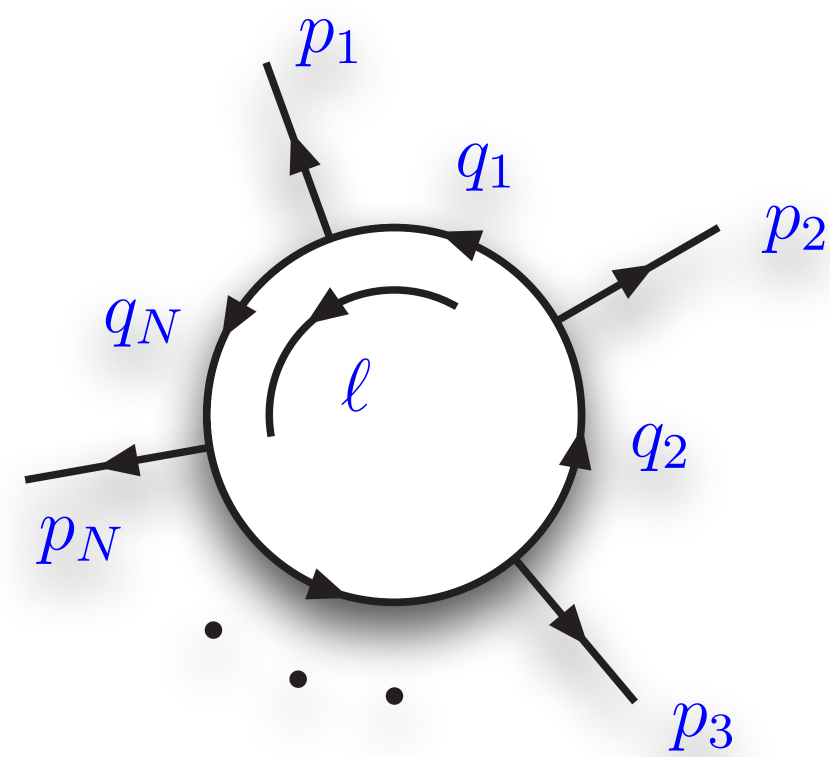
$G_F(q) \equiv \frac{1}{q^2 + i0}$ is the Feynman propagator.

Introduce the shorthand notation $-i \int \frac{d^d\ell}{(2\pi)^d} \bullet \equiv \int_\ell \bullet$, then

$$L^{(1)}(p_1, p_2, \ldots, p_N) = \int_\ell \prod_{i=1}^{N} G_F(q_i)$$

Feynman propagators

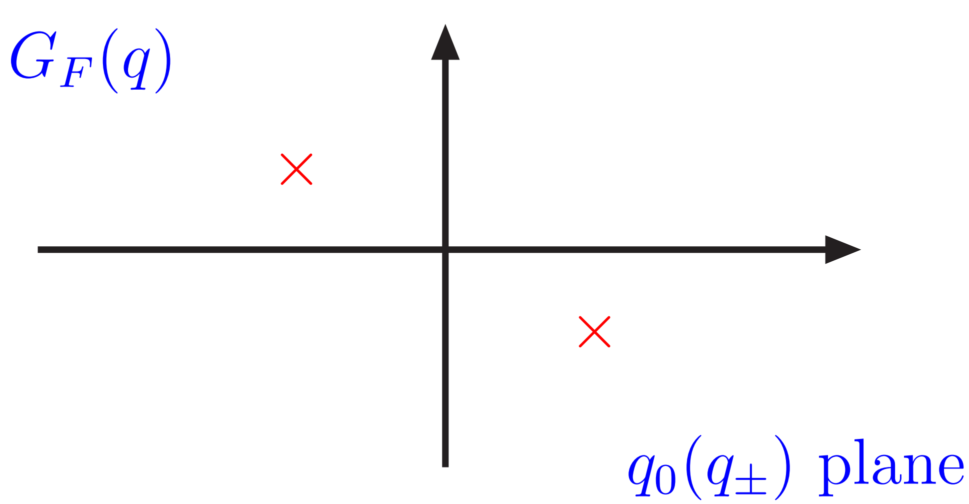
$$L^{(1)}(p_1, p_2, \ldots, p_N) = \int_\ell \prod_{i=1}^{N} G_F(q_i)$$



$$G_F(q) \equiv \frac{1}{q^2 + i0}$$

Feynman propagators have poles in the complex plane

$$[G_F(q)]^{-1} = 0 \quad \Longrightarrow \quad q_0 = \pm\sqrt{\mathbf{q}^2 - i0}$$



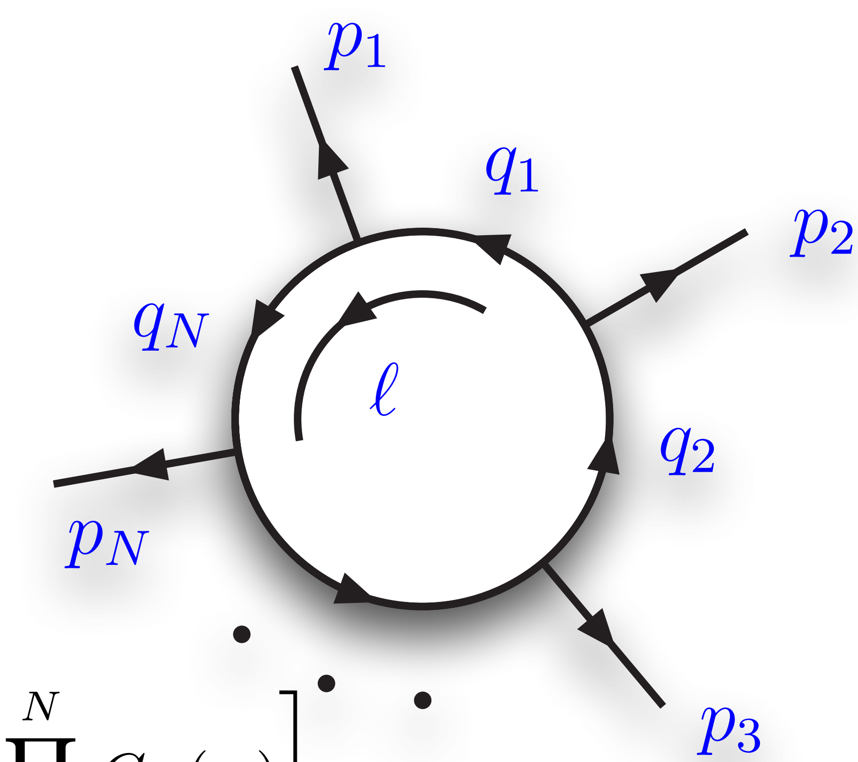
The Loop-Tree Duality

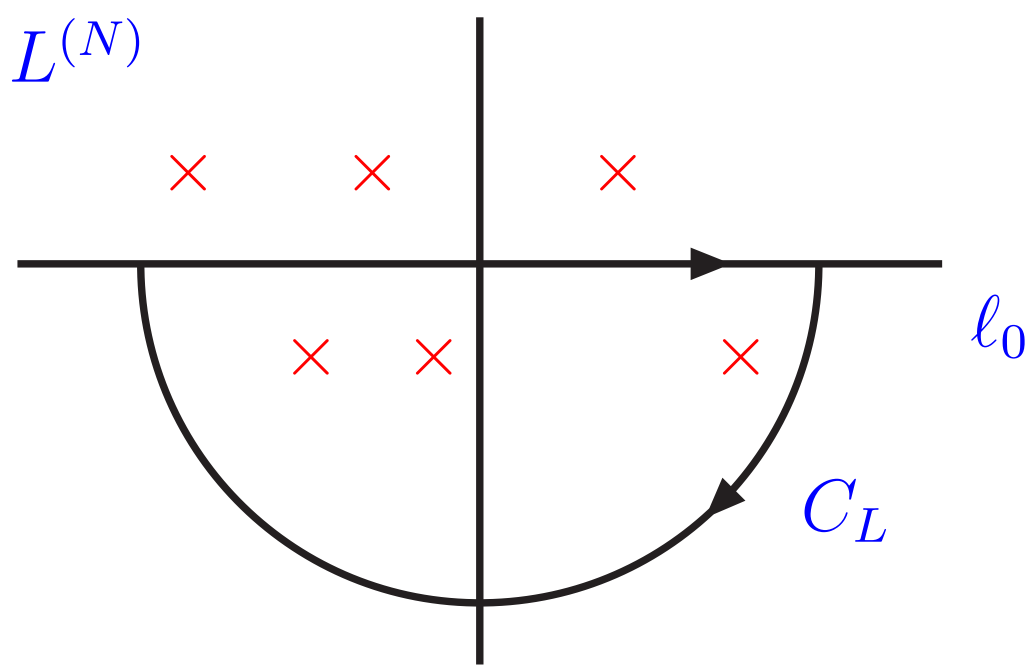
$$L^{(1)}(p_1, p_2, \dots, p_N) = \int_\ell \prod_{i=1}^{N} G_F(q_i)$$

$$G_F(q_i) = \frac{1}{q_i^2 - m_i^2 + i0}$$

$$L^{(1)}(p_1, p_2, \dots, p_N) = \int_{\vec{\ell}} \int d\ell_0 \prod_{i=1}^{N} G_F(q_i)$$

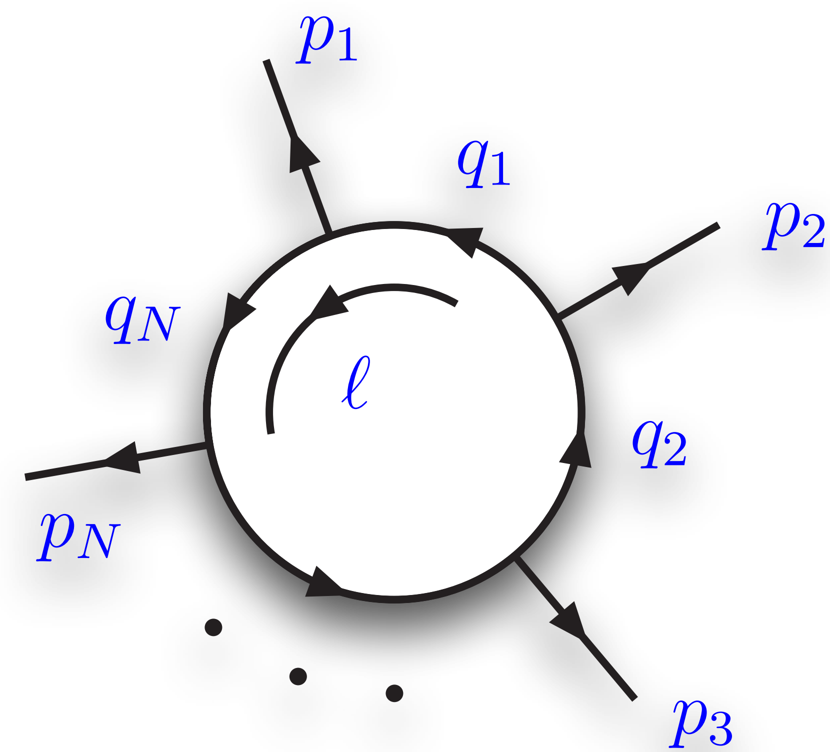
$$= \int_{\vec{\ell}} \int_{C_L} d\ell_0 \prod_{i=1}^{N} G_F(q_i) = -2\pi i \int_{\vec{\ell}} \sum \mathrm{Res}_{\{\mathrm{Im}\ \ell_0 < 0\}} \left[\prod_{i=1}^{N} G_F(q_i) \right]$$





The Loop-Tree Duality

$$L^{(1)}(p_1, p_2, \ldots, p_N) = \int_\ell \prod_{i=1}^{N} G_F(q_i)$$



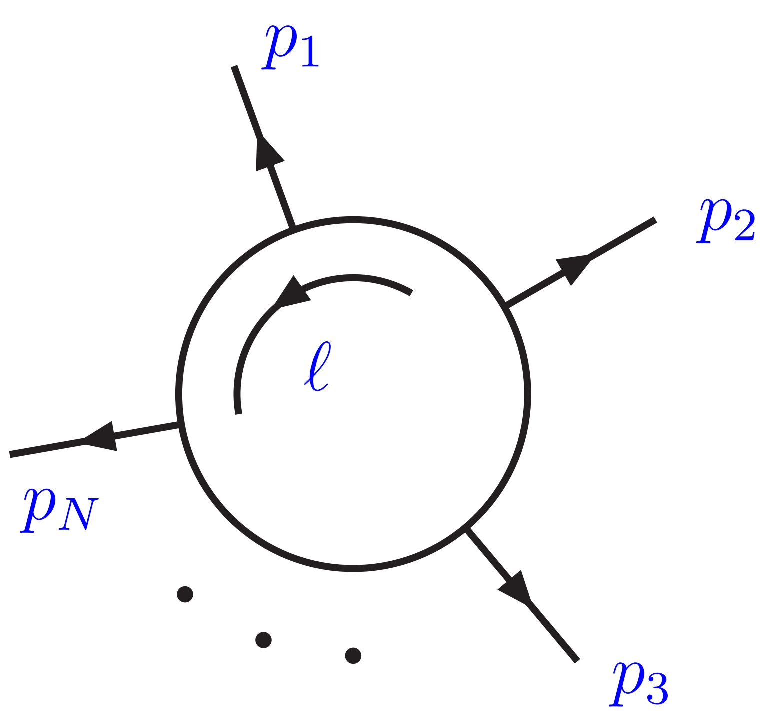
$$\boxed{L^{(1)}(p_1, p_2, \ldots, p_N) \;=\; -\sum \int_{\ell_1} \widetilde{\delta}(q_i) \prod_{\substack{j=1 \\ j\neq i}}^{N} G_D(q_i; q_j)}$$

$$G_D(q_i; q_j) = \frac{1}{q_j^2 - m_j^2 - i0\,\eta(q_j - q_i)}$$

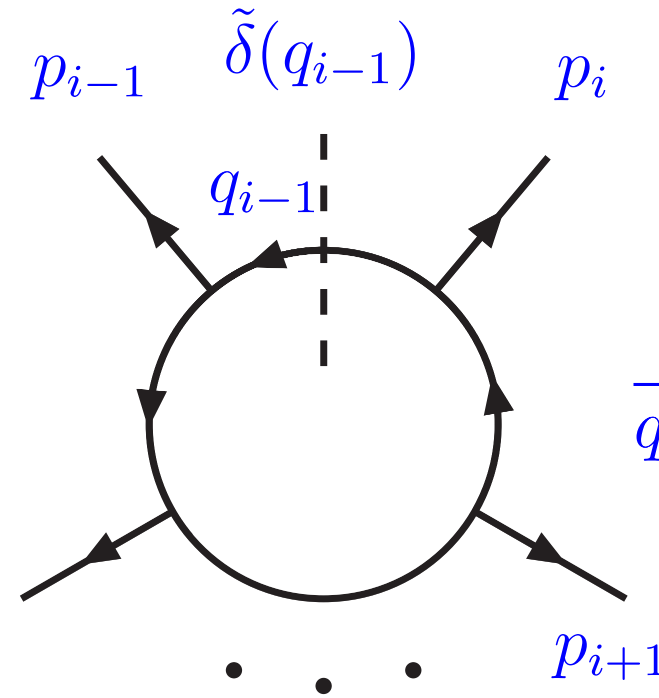
η is a future-like vector such that $\eta_\mu = (\eta_0, \boldsymbol{\eta})$, with $\eta_0 \geq 0$, $\eta^2 = \eta_\mu \eta^\mu \geq 0$

Dual propagator, keeps proper track of the small imaginary parts. Notice that $(q_j\text{-}q_i)$ does not depend on the loop momentum. It is $\widetilde{\delta}(q_i) \to \widetilde{\delta}(q_i) = 2\pi\, i\, \theta(q_{i,0})\, \delta(q_i^2 - m_i^2)$

A graphical representation of the Loop-Tree Duality



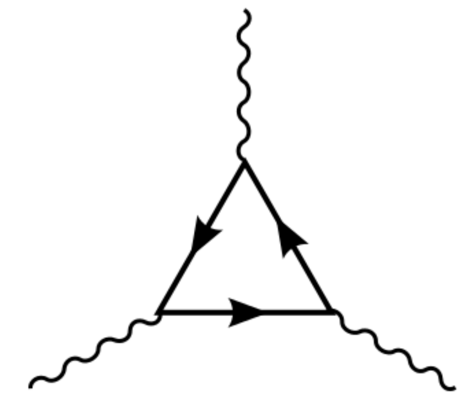
$$= \; - \sum_{i=1}^{N}$$



$$\frac{1}{q_i^2 - i0\,\eta p_i}$$

En explicit result

$$L^{(1)}(p_1,p_2,p_3) = \int_\ell G_F(q_1)G_F(q_2)G_F(q_3)$$



$$G_F(q_1) = \frac{1}{q_1^2 - m_1^2 + i0},\ G_F(q_2) = \frac{1}{q_2^2 - m_2^2 + i0},\ G_F(q_3) = \frac{1}{q_3^2 - m_3^2 + i0}$$

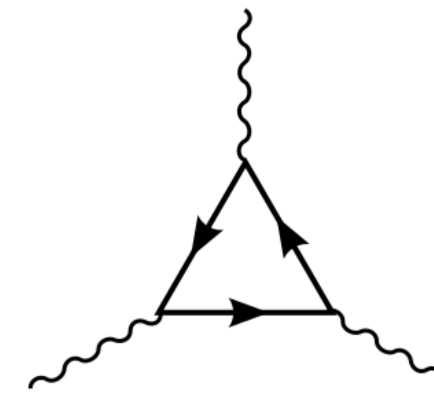
$$q_1 = \ell + p_1,\ q_2 = \ell + p_1 + p_2 = \ell,\ q_3 = \ell$$

Let us apply the Loop-Tree Duality

$$\begin{aligned} L^{(1)}(p_1,p_2,p_3) = & \int_\ell \widetilde{\delta}(q_1)G_D(q_1;q_2)G_D(q_1;q_3) && \text{first contribution} && (\mathrm{I}_1) \\ & + \int_\ell G_D(q_2;q_1)\widetilde{\delta}(q_2)G_D(q_2;q_3) && \text{second contribution} && (\mathrm{I}_2) \\ & + \int_\ell G_D(q_3;q_1)G_D(q_3;q_2)\widetilde{\delta}(q_3) && \text{third contribution} && (\mathrm{I}_3) \end{aligned}$$

En explicit result

$$
\begin{aligned}
L^{(1)}(p_1,p_2,p_3) &= \int_\ell \widetilde{\delta}(q_1) G_D(q_1;q_2) G_D(q_1;q_3) && \text{first contribution} \quad (\mathrm{I}_1) \\
&+ \int_\ell G_D(q_2;q_1) \widetilde{\delta}(q_2) G_D(q_2;q_3) && \text{second contribution} \quad (\mathrm{I}_2) \\
&+ \int_\ell G_D(q_3;q_1) G_D(q_3;q_2) \widetilde{\delta}(q_3) && \text{third contribution} \quad (\mathrm{I}_3)
\end{aligned}
$$



$$
\widetilde{\delta}(q_1) = \frac{\delta(\ell_0 - (-p_{1,0} + \sqrt{(\boldsymbol{\ell}+\mathbf{p}_1)^2 + m_1^2}))}{2\sqrt{(\boldsymbol{\ell}+\mathbf{p}_1)^2 + m_1^2)}} \,,
$$

$$
\widetilde{\delta}(q_2) = \frac{\delta(\ell_0 - (-p_{1,0} - p_{2,0} + \sqrt{(\boldsymbol{\ell}+\mathbf{p}_1+\mathbf{p}_2)^2 + m_2^2}))}{2\sqrt{(\boldsymbol{\ell}+\mathbf{p}_1+\mathbf{p}_2)^2 + m_2^2)}} \,,
$$

$$
\widetilde{\delta}(q_3) = \frac{\delta(\ell_0 - \sqrt{\boldsymbol{\ell}^2 + m_3^2})}{2\sqrt{\boldsymbol{\ell}^2 + m_3^2}}
$$

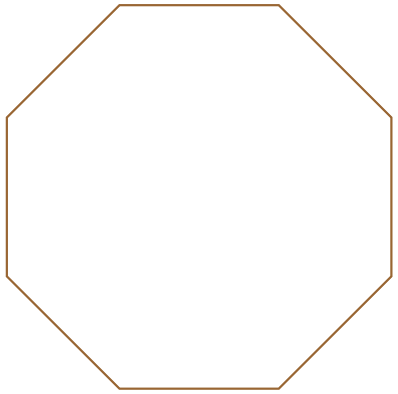
$$
\boxed{
\begin{aligned}
I_3 = -\int_{\boldsymbol{\ell}} & \frac{1}{2p_{1,0}\sqrt{\boldsymbol{\ell}^2+m_3^2} + 2\boldsymbol{\ell}\cdot\mathbf{p}_1 - m_1^2 + m_3^2 + p_1^2 - i0\eta k_{13}} \cdot \frac{1}{2\sqrt{\boldsymbol{\ell}^2+m_3^2}} \cdot \\
& \frac{1}{2(p_{1,0}+p_{2,0})\sqrt{\boldsymbol{\ell}^2+m_3^2} + 2\boldsymbol{\ell}\cdot(\mathbf{p}_1+\mathbf{p}_2) + (p_1+p_2)^2 - m_2^2 + m_3^2 - i0\eta k_{23}}
\end{aligned}
}
$$

The method in few words

- The Loop-Tree Duality is a Feynman integral transformation that maps loop integrals to a sum of phase-space (tree-) integrals.
- This transformation is achieved by performing the integration over the energy component of the loop integral. The resulting integration runs only over the three-momentum and is very similar to the real radiation corrections. Thus it encourages the idea of combination of the two, treating them simultaneously in a common Monte Carlo event generator.
- To do the integration over the energy component, one takes residues, one for each propagator. That leaves a sum of N contributions.
- Each summand is called Dual contribution and is constructed according to a special pattern: One of the internal lines gets cut, i.e. it is replaced by a Dual delta function, while all the other (non-cut) Feynman propagators are promoted to Dual propagators. This procedure is repeated for every internal line once and the results are added together yielding the Dual integral.

The implementation in C++ and Mathematica

- We have a fast C++ code implementation as well as a Mathematica one that uses the Tree-Loop Duality method to carry out one of the four integrations whereas different integration routines are used to perform the remaining three integrations.
- If there are still singularities on the real axes of the three momenta, the integration is carried out after a proper contour deformation such that the singularities on the real axes are avoided.
- The code, as it stands at the moment, can handle in principle any multi-leg diagram but it is only tested for diagrams with up to nine external particles. The ongoing optimization work is directed towards speed and improvement of the user interface,
- Apart from scalar integral, the code handles with no big additional effort tensor integrals as well.
- The progress from different Groups so far is impressive and it is expected that it will continue.



Octagons

SCALAR OCTAGON

numerator = 1

$p_1 = (-2.500000,\ 0,\ 0,\ -2.500000)$

$p_2 = (-2.500000,\ 0,\ 0,\ 2.500000)$

$p_3 = (-0.427656,\ 0.041109,\ -0.180818,\ 0.385362)$

$p_4 = (-0.907144,\ 0.289299,\ 0.859318,\ 2.805929)$

$p_5 = (-0.414246,\ 0.329547,\ 0.249476,\ -0.027570)$

$p_6 = (-1.907351,\ -0.950926,\ -1.460214,\ 0.775566)$

$p_7 = (-0.271157,\ 0.155665,\ 0.039639,\ -0.218456)$

$p_8 = - p_1 - p_2 - p_3 - p_4 - p_5 - p_6 - p_7$

$m_1 = m_2 = m_3 = m_4 = m_5 = m_6 = m_7 = m_8 = 4.506760$

LTD: REAL = $-2.079457\ 10^{-11}$ +/− $6.283601\ 10^{-15}$

LTD: IMAG = $9.439531\ 10^{-11}$ +/− $6.273917\ 10^{-15}$

TENSOR OCTAGON

numerator = $l.p_2 \times l.p_4$

$p_1 = (-2.500000,\ 0,\ 0,\ -2.500000)$

$p_2 = (-2.500000,\ 0,\ 0,\ 2.500000)$

$p_3 = (-0.427656,\ 0.041109,\ -0.180818,\ 0.385362)$

$p_4 = (-0.907144,\ 0.289299,\ 0.859318,\ 2.805929)$

$p_5 = (-0.414246,\ 0.329547,\ 0.249476,\ -0.027570)$

$p_6 = (-1.907351,\ -0.950926,\ -1.460214,\ 0.775566)$

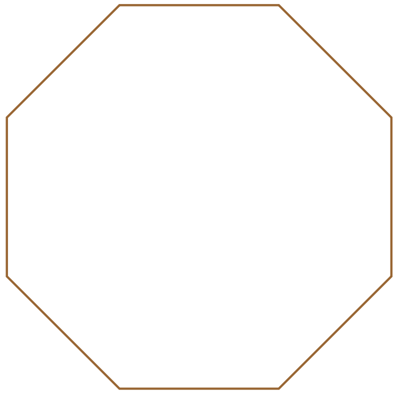
$p_7 = (-0.271157,\ 0.155665,\ 0.039639,\ -0.218456)$

$p_8 = - p_1 - p_2 - p_3 - p_4 - p_5 - p_6 - p_7$

$m_1 = m_2 = m_3 = m_4 = m_5 = m_6 = m_7 = m_8 = 4.506760$

LTD: REAL = $-3.774487\ 10^{-10}$ +/− $3.396473\ 10^{-14}$

LTD: IMAG = $2.827604\ 10^{-9}$ +/− $3.393935\ 10^{-14}$



Octagons

SCALAR OCTAGON – ALL MASSES DIFFERENT

numerator = 1

$p_1 = (-2.500000,\ 0,\ 0,\ -2.500000)$

$p_2 = (-2.500000,\ 0,\ 0,\ 2.500000)$

$p_3 = (-0.427656,\ 0.041109,\ -0.180818,\ 0.385362)$

$p_4 = (-0.907144,\ 0.289299,\ 0.859318,\ 2.805929)$

$p_5 = (-0.414246,\ 0.329547,\ 0.249476,\ -0.027570)$

$p_6 = (-1.907351,\ -0.950926,\ -1.460214,\ 0.775566)$

$p_7 = (-0.271157,\ 0.155665,\ 0.039639,\ -0.218456)$

$p_8 = -p_1 - p_2 - p_3 - p_4 - p_5 - p_6 - p_7$

$m_1 = 4.506760$

$m_2 = 2.814908$

$m_3 = 1.427626$

$m_4 = 7.621541$

$m_5 = 5.269166$

$m_6 = 3.521039$

$m_7 = 5.888145$

$m_8 = 4.422515$

LTD: REAL = $6.826303\ 10^{-10}$ +/– $3.731196\ 10^{-13}$

LTD: IMAG = $9.173787\ 10^{-10}$ +/– $3.701180\ 10^{-13}$

Outlook

- Automated code for NLO calculations in C++ and Mathematica
- Extension to 2-loops
- Merge virtual and real corrections