The information contained in the tides of merging neutron stars of nuclear matter equation of state

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Outline

- Introduction
- Background and Theory
    i) The nuclear matter equation of state.
    ii) The tidal deformation of NS.
    iii) The GW170817 event.
- Constraining the nuclear matter EOS.
- Deep learning.
- Constrain the NS tidal deformation from LAB experiment.
- Conclusions.

Introduction

"Neutron stars play a unique role in physics and astrophysics. On the one hand, they contain matter under extreme physical conditions, and their theories are based on risky and far extrapolations of what we consider reliable physical theories of the structure of matter tested in laboratory. On the other hand, their observations offer the unique opportunity to test these theories." – P. Hensel

- NSs are massive and compact astrophysical object.
- The coalescence of binary NS systems is one of the most promising sources of gravitational waves (GWs) observable by ground-based detectors.
- The GW signals emitted during a NS merger depends on the behavior of neutron star matter at high densities. Therefore, its detection opens the possibility of constraining the nuclear matter parameters characterizing the EoS.

History of Neutron star's Discovery

- In 1932, Sir James Chadwick discovered the neutron as an elementary particle, for which he was awarded the Nobel Prize in Physics in 1935.
- In 1933, Walter Baade and Fritz Zwicky proposed the existence of the neutron star, only a year after Chadwick's discovery of the neutron. In seeking an explanation for the origin of a supernova, they proposed that the neutron star is formed in a supernova.

- In 1967, Jocelyn Bell and Antony Hewish discovered radio pulses from a pulsar, later interpreted as originating from an isolated, rotating neutron star. The energy source is rotational energy of the neutron star. The largest number of known neutron stars are of this type.

Neutron star's properties in brief

- Central no. density 4 - 8 ρ_0
- Central mass density ~ 10^{15} g cm^{-3}
- Radius ~ 10 -14 km
- Mass ~ 10^{30} kg
- Asymmetry $$I = \frac{\rho_n - \rho_p}{\rho_n + \rho_p} \approx 0.7$$

Structure equations (Spherical case)

$$\frac{dP}{dr} = -\frac{G\epsilon(r)m(r)}{c^2r^2}\left[1+\frac{P(r)}{\epsilon(r)}\right]\left[1+\frac{4\pi r^3 P(r)}{m(r)c^2}\right]\left[1-\frac{2Gm(r)}{c^2r}\right]$$

$$\frac{dm}{dr} = 4\pi r^2\epsilon(r)$$

Boundary conditions

$$P_R \rightarrow 0$$

$$\mathcal{M}(0) = 0$$

- G : gravitational constant
- C : speed of light
- P : pressure
- $\mathcal{M}$: mass of the star
- $\mathcal{E}$: energy density
- r : radial coordinate

Physics input : Equation of State (EOS)

$$P = f(\varepsilon)$$

The equation of state (EoS)

$$p(\rho) = \rho^2 \frac{d}{d\rho}(e(\rho))$$

The energy per particle at a given density ρ and asymmetry δ can be decomposed, to a good approximation

$$e(\rho, \delta) = e(\rho, 0) + S(\rho)\,\delta^2$$

$$\rho = \rho_n + \rho_p \qquad \delta = \frac{\rho_n - \rho_p}{\rho}$$

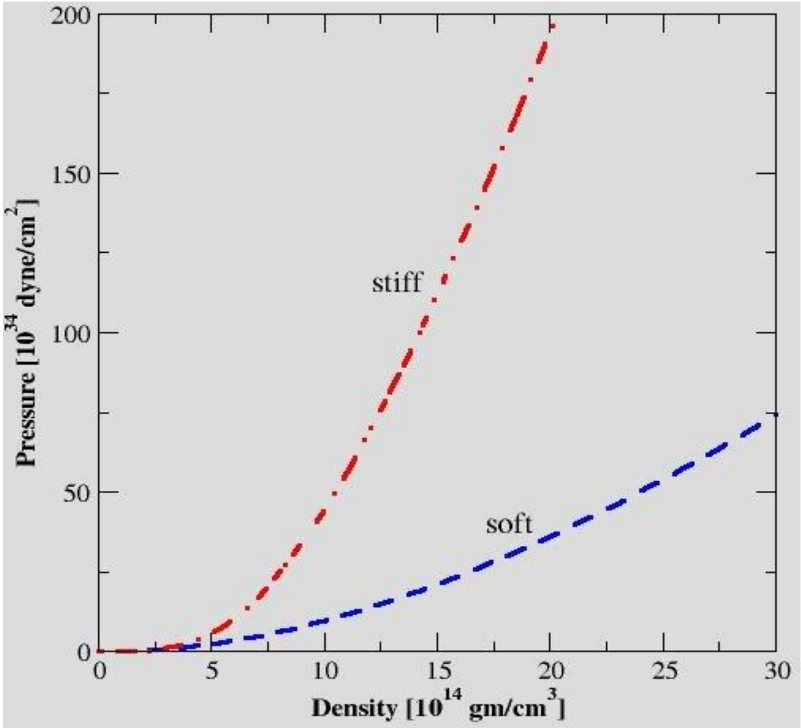
Symmetry Energy

SNM

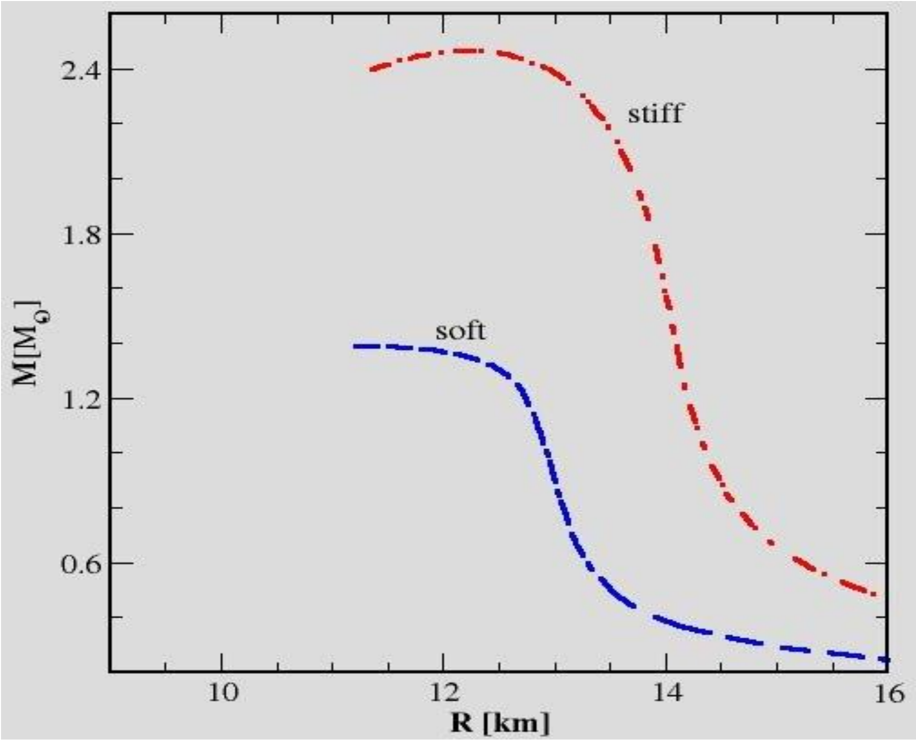
$$S(\rho) = J_0 + L_0\,\left(\frac{\rho - \rho_0}{3\,\rho_0}\right) + K_{sym,0}\,\left(\frac{\rho - \rho_0}{3\,\rho_0}\right)^2 + O(3)$$

$$e(\rho, 0) = e(\rho_0) + \frac{K_0}{2}\,\left(\frac{\rho - \rho_0}{3\rho_0}\right)^2 + \frac{Q_0}{6}\,\left(\frac{\rho - \rho_0}{3\rho_0}\right)^3 + O(4)$$

$$M_0 = Q_0 + 12K_0$$



stiff
soft
Pressure [10^{34} dyne/cm^2]
Density [10^{14} gm/cm^3]
0
50
100
150
200
0
5
10
15
20
25
30



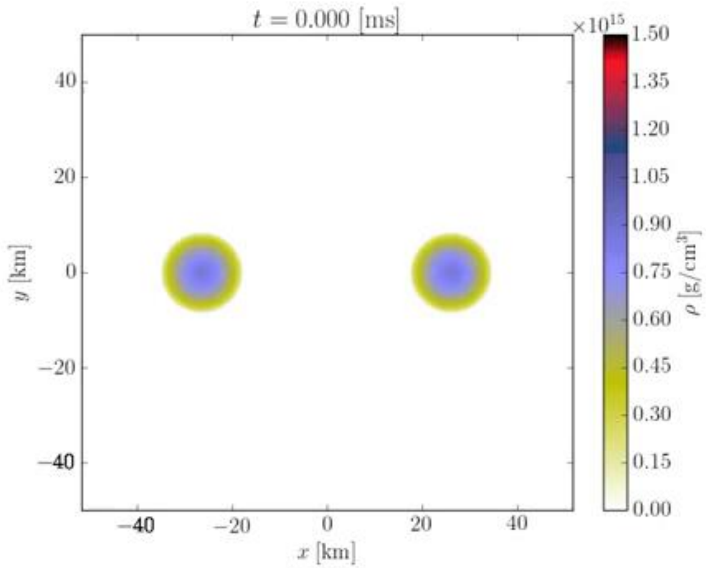
stiff
soft
M[$M_\odot$]
R [km]
0.6
1.2
1.8
2.4
10
12
14
16

Tidal deformation

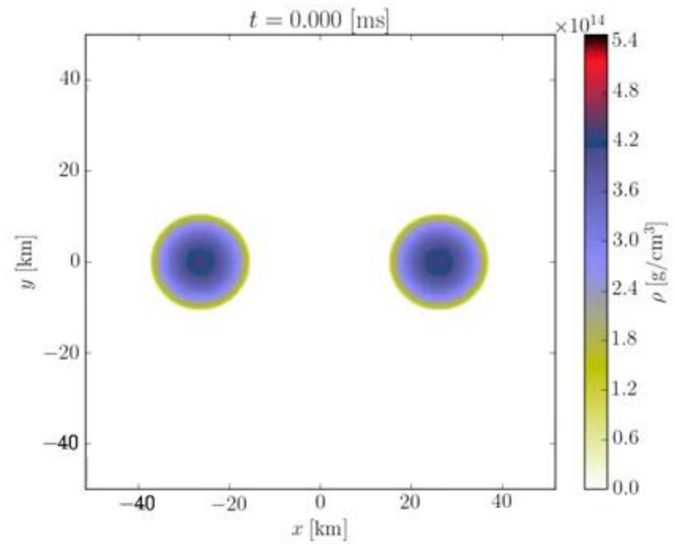
Binary NS system:

- Strong gravity of each of them induced a tidal deformation on each of them.
- This deformation is a decreasing function of NS mass.
- Correlates strongly with stiffness of equation of state.

Numerical Relativity Simulation (credit: David Radice)



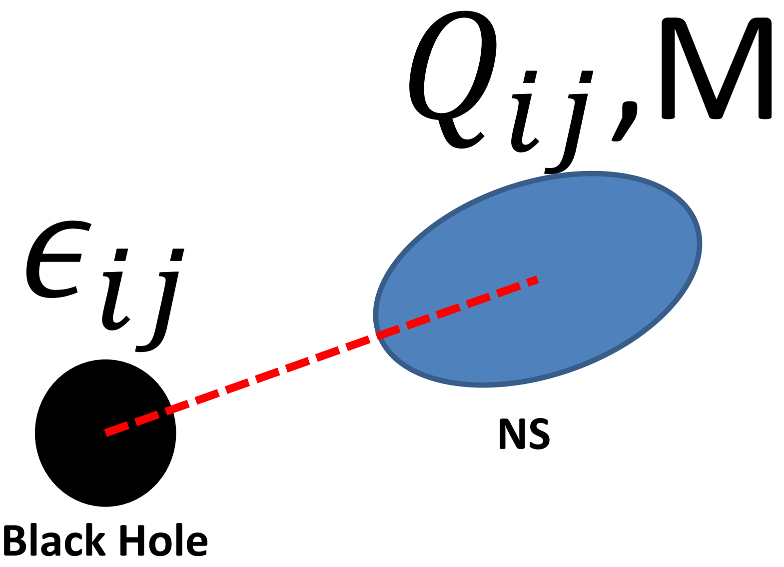
“Soft” EoS



“Stiff” EoS

[**David Radice** - Homepage](https://www.astro.princeton.edu/~dradice/research.html)
(https://www.astro.princeton.edu/~dradice/research.html)

Tidal deformability parameter



Quadrupole Moment

Tidal field

$$Q_{ij} = \lambda \, \epsilon_{ij}$$

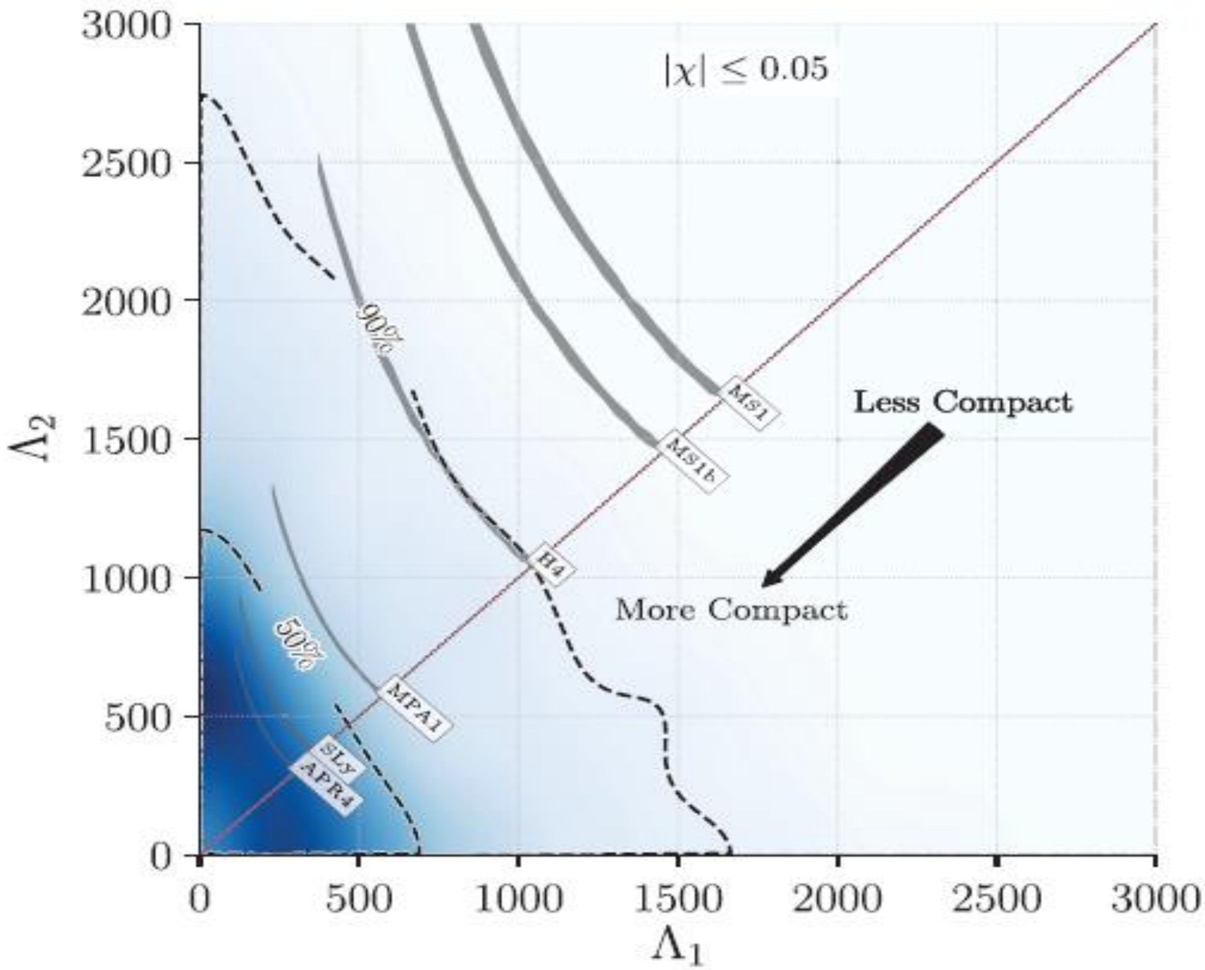
$$\lambda = \Lambda \, M^5$$

Second Love number

$$\Lambda = \frac{2}{3} \, k_2 \, (R/M)^5$$

k_2 depends on compactness of the NS

GW170817



@ PRL 119, 161101(2017)

Properties of the Binary Neutron Star Merger GW170817

B. P. Abbott *et al*: PRX 9, 011001 (2019)

Chirp Mass $\mathcal{M} = \dfrac{(m_1\, m_2)^{3/5}}{(m_1 + m_2)^{1/5}}$	$1.186^{+0.001}_{-0.001}\, M_\odot$
Mass Ratio $q = m_1/m_2$	0.73 – 1.00
m_1	(1.16 to 1.36) $M_\odot$
m_2	(1.36 to 1.60) $M_\odot$
Combined deformability $\tilde{\Lambda}$	300^{+420}_{-230}

Low-spin prior

$$\tilde{\Lambda} = \frac{16}{13}\frac{(12q+1)\Lambda_1 + (12+q)q^4\Lambda_2}{(1+q)^5}$$

Correlation

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GW170817: Constraining the nuclear matter equation of state from the neutron star tidal deformability

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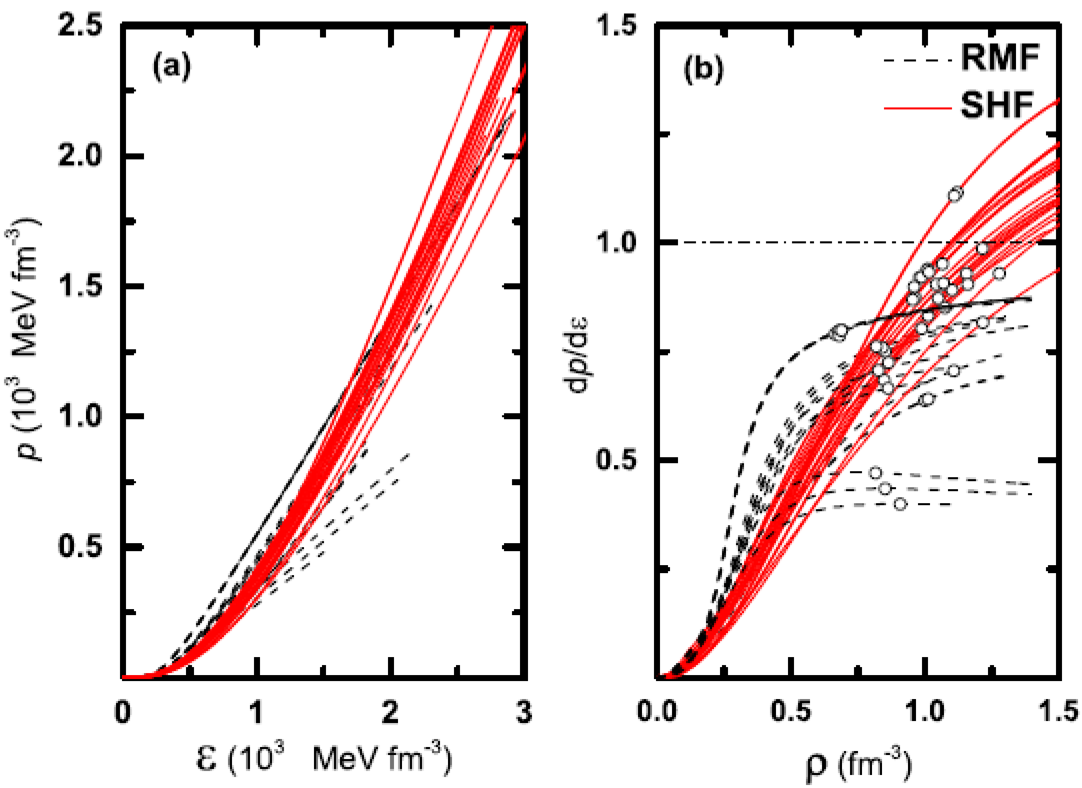
Constraints set on key parameters of the nuclear matter equation of state (EoS) by the values of the tidal deformability, inferred from GW170817, are examined by using a diverse set of relativistic and nonrelativistic mean-field models. These models are consistent with bulk properties of finite nuclei as well as with the observed lower bound on the maximum mass of neutron star $\approx 2M_\odot$. The tidal deformability shows a strong correlation with specific linear combinations of the isoscalar and isovector nuclear matter parameters associated with the EoS. Such correlations suggest that a precise value of the tidal deformability can put tight bounds on several EoS parameters, in particular on the slope of the incompressibility and the curvature of the symmetry energy. The tidal deformability obtained from the GW170817 and its UV, optical and infrared counterpart sets the radius of a canonical $1.4M_\odot$ neutron star to be $11.82 \leqslant R_{1.4} \leqslant 13.72$ km.

DOI: 10.1103/PhysRevC.98.035804

The equation of state



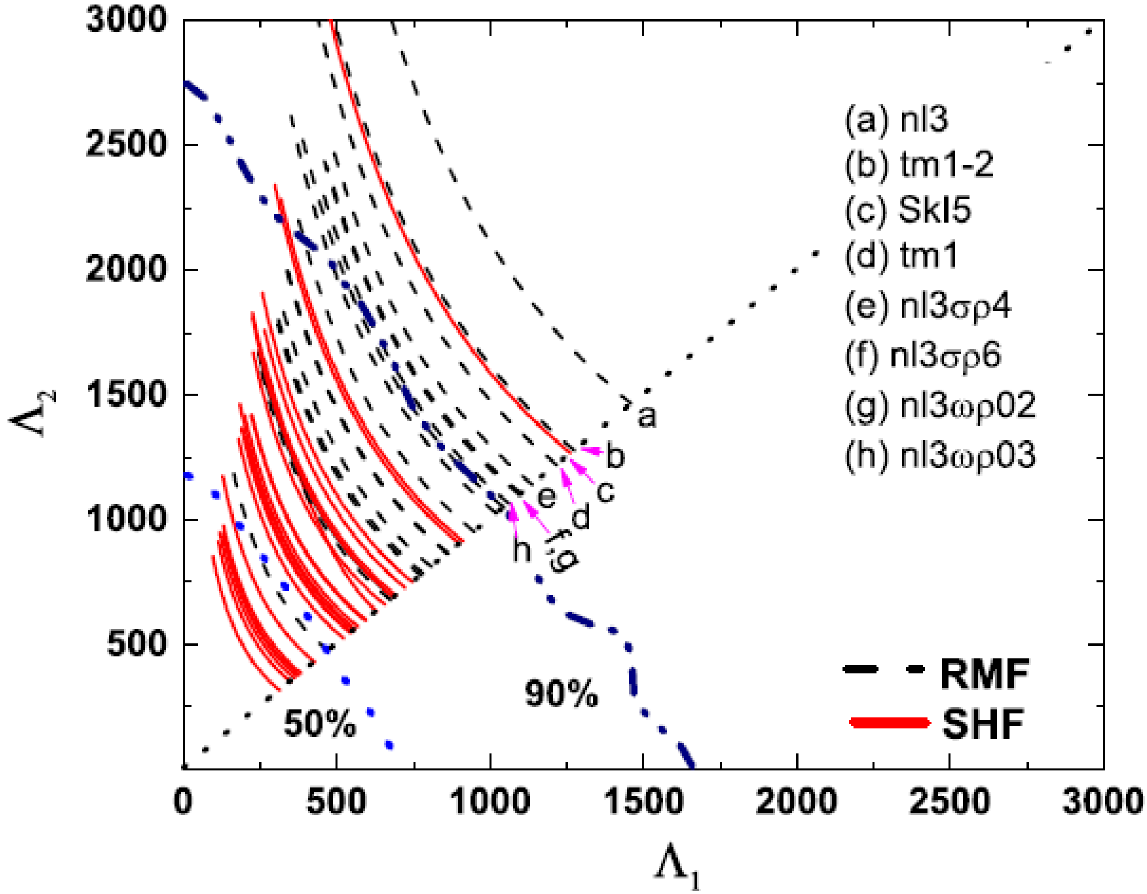
- ✓ The BSk20 and BSk26 EoSs are marginally acausal at the NS maximum masses ≈2.2 *Msolar*

- ❑ A set of eighteen relativistic and twenty-four nonrelativistic nuclear models.
- ❑ The RMF models employed are BSR2, BSR3, BSR6 , FSU2, GM1, NL3, NL3$\sigma\rho$4, NL3$\sigma\rho$6, NL3$\omega\rho$02, NL3$\omega\rho$03, TM1, TM1-2, DD2 , DDHδ, DDHδMod , DDME1, DDME2, and TW.
- ❑ The SHF models considered are the SKa, SKb, SkI2, SkI3, SkI4, SkI5, SkI6, Sly2, Sly9, Sly230a, Sly4, SkMP, SKOp, KDE0V1, SK255, SK272, Rs, BSk20, BSk21, BSk22, BSk23, BSk24, BSk25, and BSk26 .

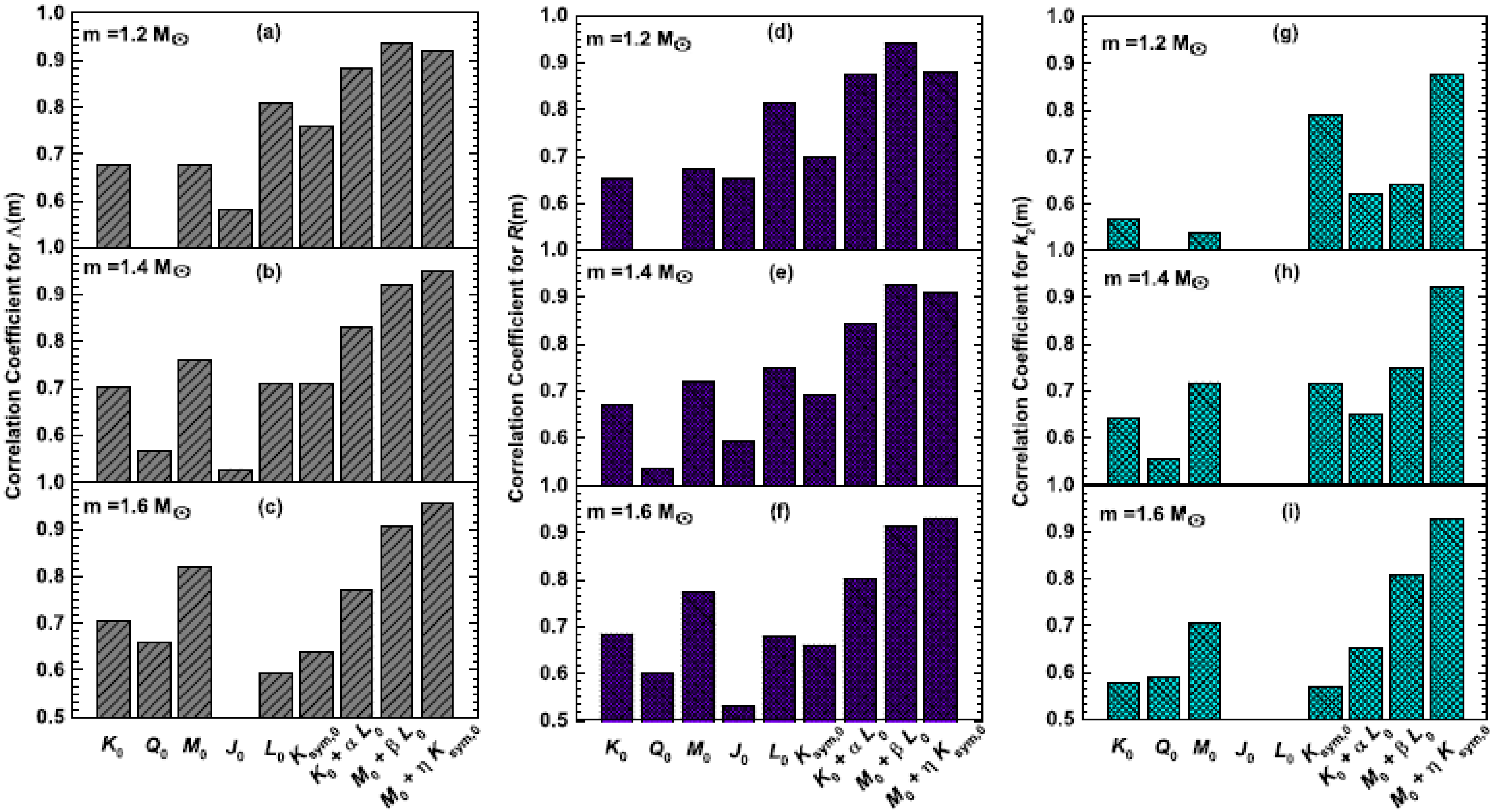
- The values of the nuclear matter properties, such as K_0, Q_0, M_0, J_0, L_0, and $K_{sym,0}$ vary over a wide range for our representative set of EoSs.
- Can be seen from the supplementary material of

@ PRC 94, 052801 (R) 2016

The tidal deformability

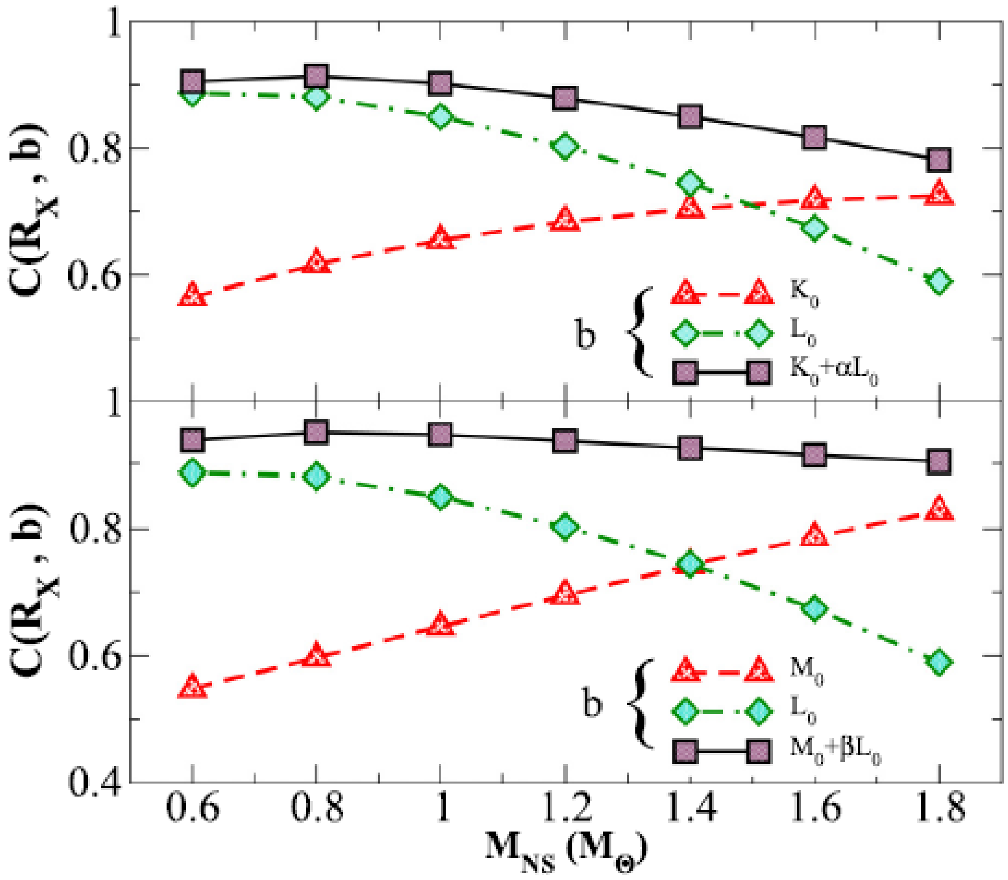


Correlation coefficients

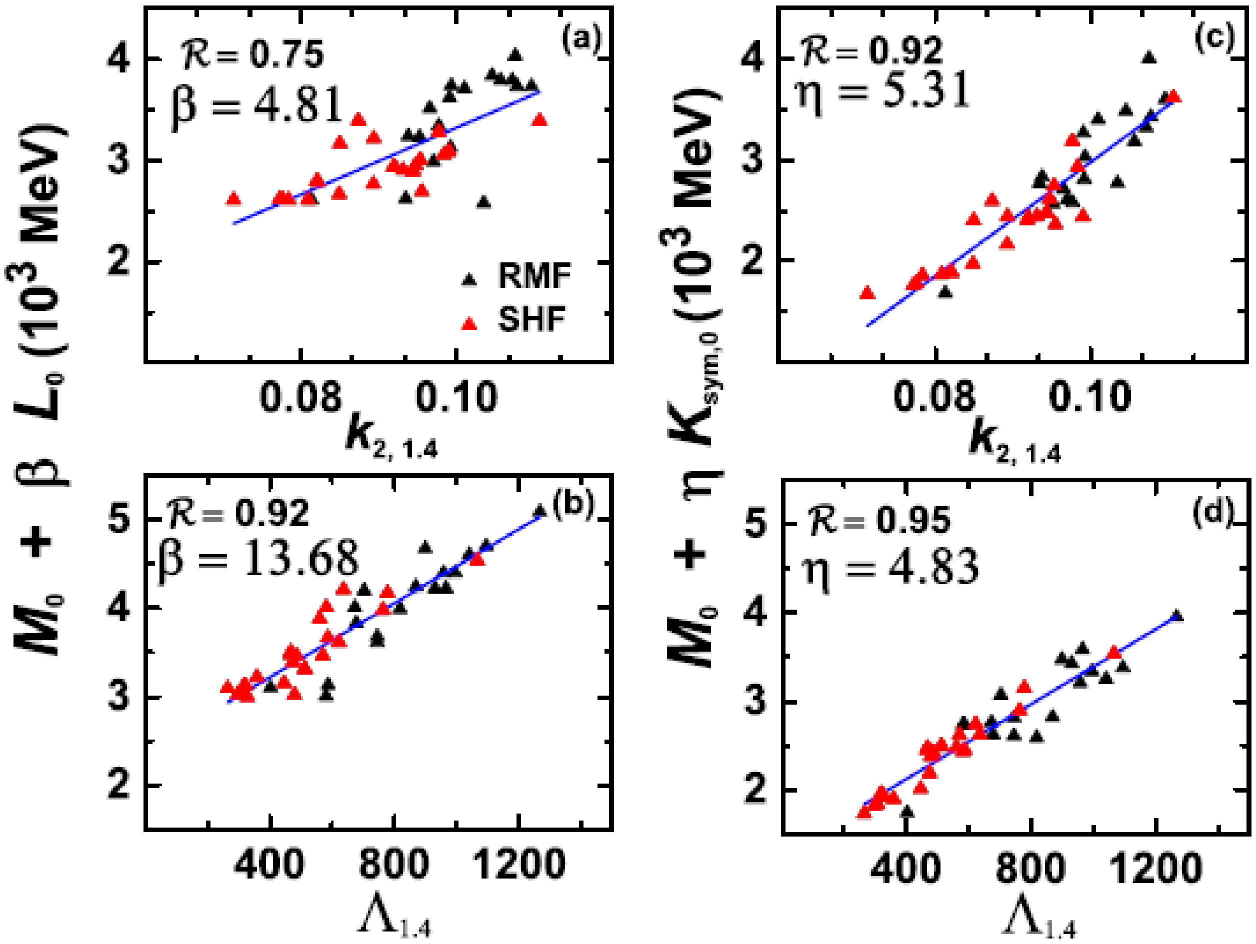


❑ Strong correlation with specific linear combinations of the isoscalar and isovector nuclear matter parameters.

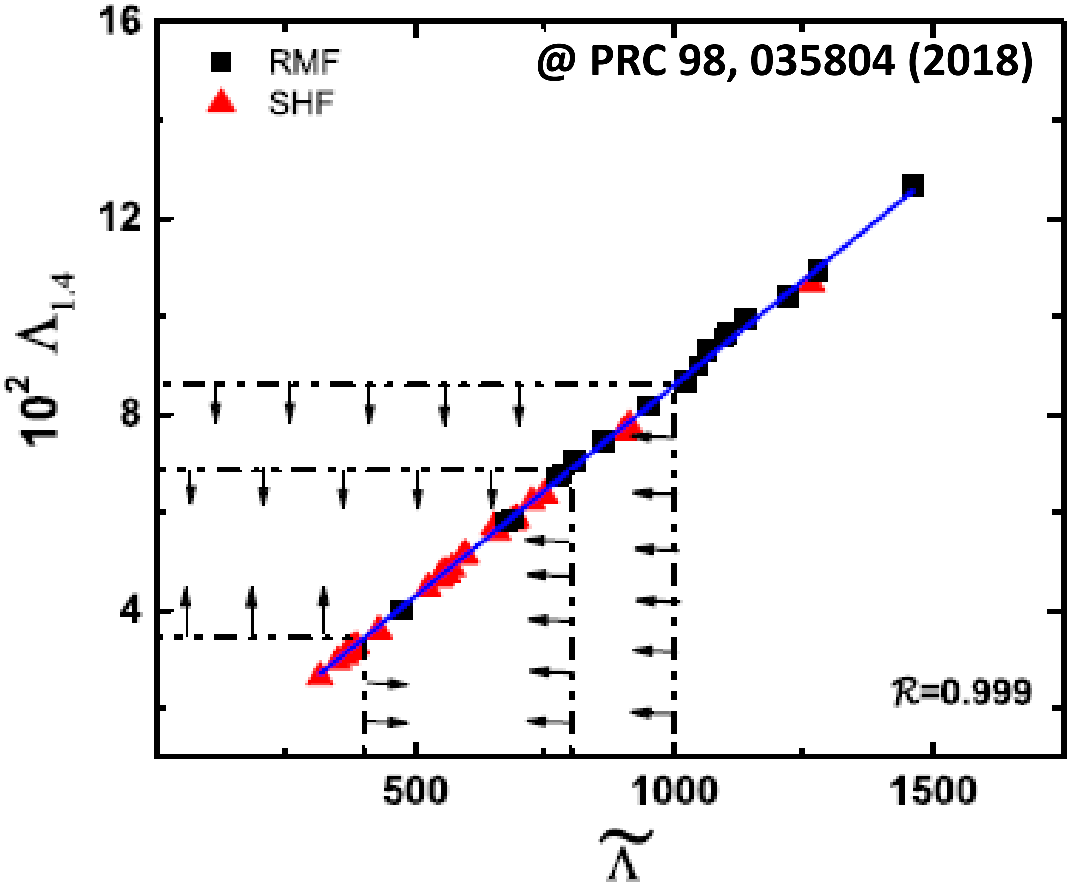
The neutron star radius



@ PRC 94, 052801 (R) 2016



$M_0 + \beta L_0$ (10^3 MeV)
$\mathcal{R} = 0.75$
$\beta = 4.81$
(a)
RMF
SHF
$k_{2,1.4}$
$\mathcal{R} = 0.92$
$\beta = 13.68$
(b)
$\Lambda_{1.4}$
$M_0 + \eta K_{sym,0}$ (10^3 MeV)
$\mathcal{R} = 0.92$
$\eta = 5.31$
(c)
$k_{2,1.4}$
$\mathcal{R} = 0.95$
$\eta = 4.83$
(d)
$\Lambda_{1.4}$



$$\tilde{\Lambda} = \frac{16}{13}\frac{(12q+1)\Lambda_1 + (12+q)q^4\Lambda_2}{(1+q)^5}$$

by replacing **PRL 121, 091102 (2018)**

$$\Lambda_2 = q^{-6}\Lambda_1$$

$$\Lambda_1 = \frac{13}{16}\tilde{\Lambda}\frac{q^2(1+q)^4}{12q^2 - 11q + 12},$$

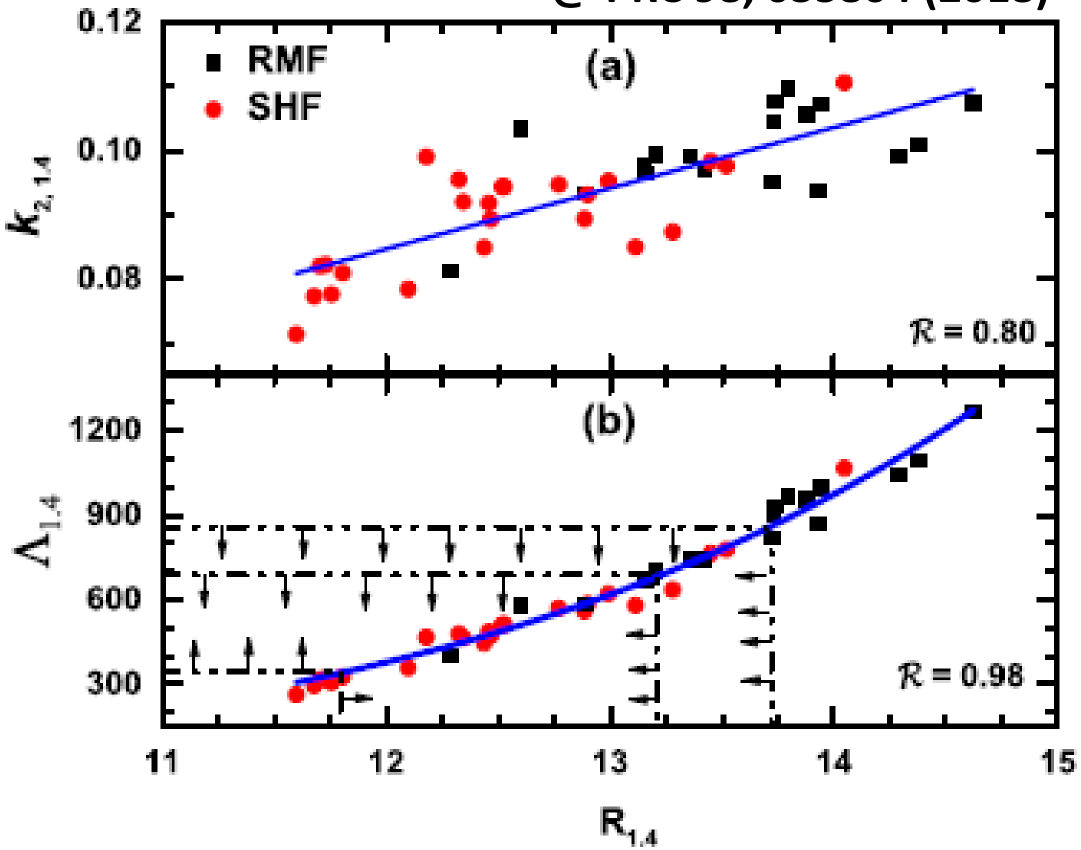
$$\Lambda_{1.4} = 0.856\,\tilde{\Lambda}$$

$$\Lambda_{1.4} = 0.859\tilde{\Lambda}$$

L_0 (MeV)	$\Lambda_{1.4}$	M_0 (MeV)	$K_{sym,0}$ (MeV)
40–62	344–687	2254–3272	−113– −52
	344–859	2254–3631	−112– −52
30–86	344–687	1926–3409	−141–16
	344–859	1926–3768	−140–16

Constraining neutron star radius

@ PRC 98, 035804 (2018)



$$\Lambda_{1.4} = 9.11 \times 10^{-5} \left(\frac{R_{1.4}}{km}\right)^{6.13}$$

$$\Lambda_{1.17} \propto \left(\frac{R_{1.4}}{km}\right)^{5.84}$$

$$\Lambda_{1.6} \propto \left(\frac{R_{1.4}}{km}\right)^{6.58}$$

$$R_{1.4} = 11.82\text{–}13.72 \text{ km.}$$

PHYSICAL REVIEW D **99**, 043010 (2019)

Constraining nuclear matter parameters with GW170817

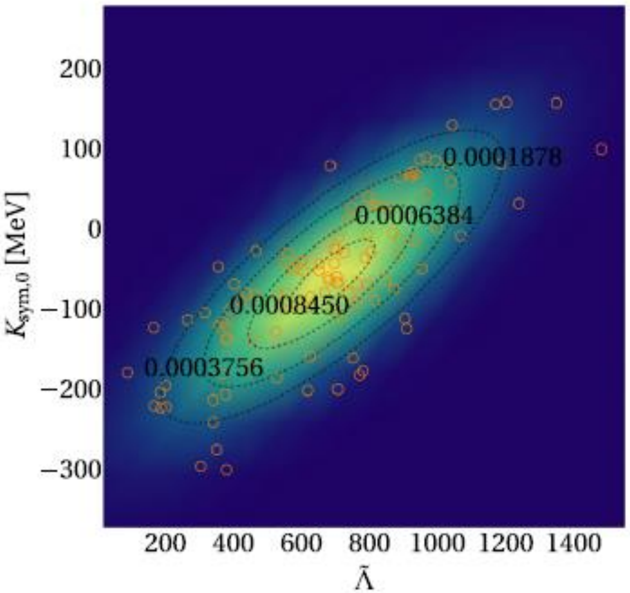
Zack Carson,[1] Andrew W. Steiner,[2,3] and Kent Yagi[1]

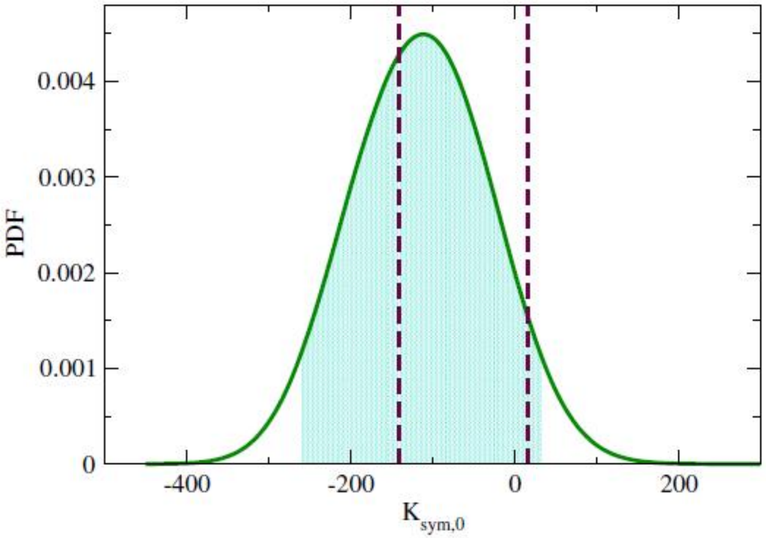
- **Use a broad set of EoSs to take into account systematic error.**
- **Find the correlation with directly $\tilde{\Lambda}$.**

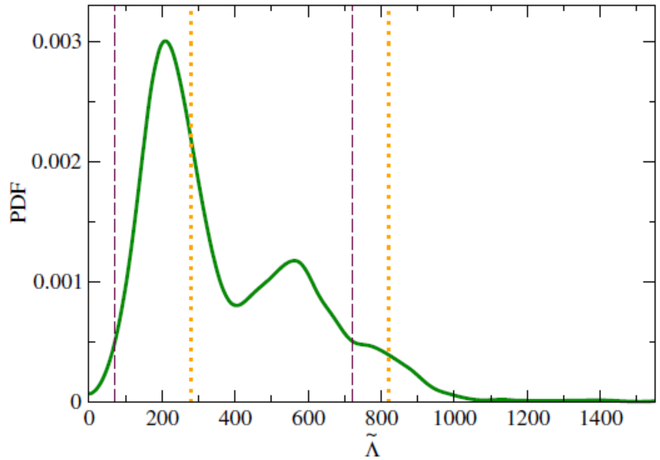
EOS used

a) Same set Malik *et al* (2018)

b) 88 Polytropes EOSs.



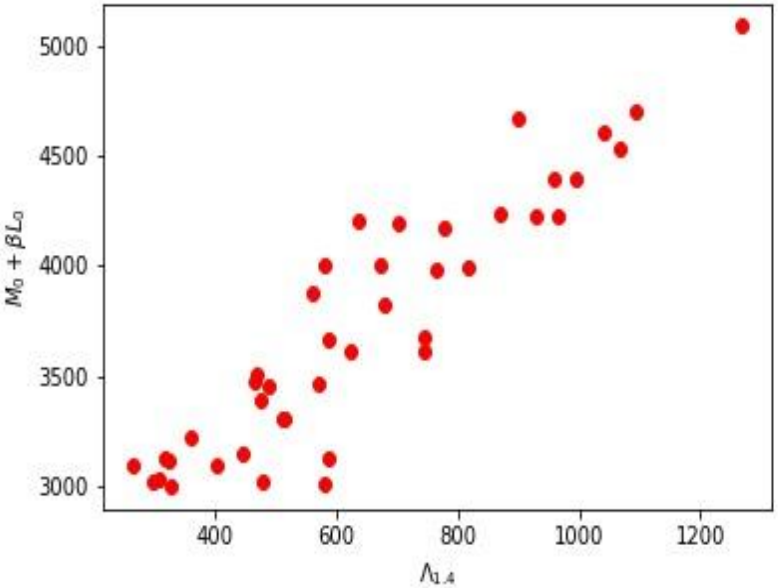


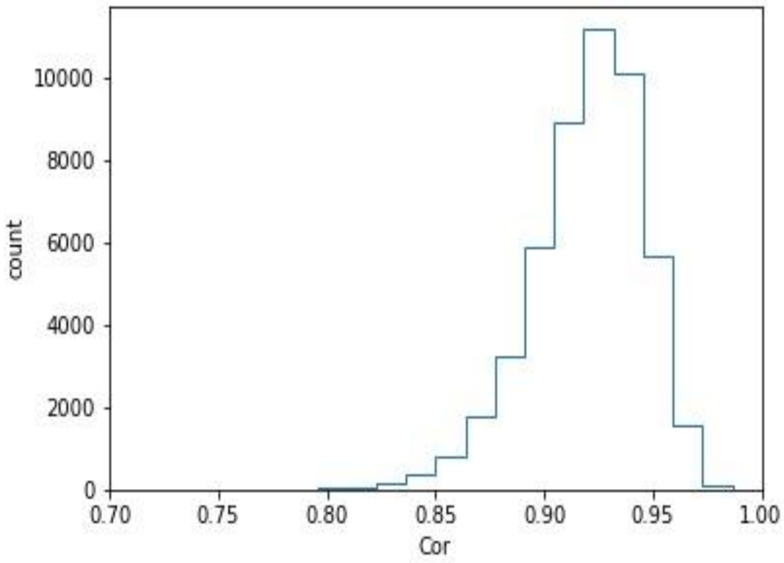


$$P(K_{sym,0}) = \int_{-\propto}^{+\propto} P(\tilde{\Lambda}, K_{sym,0}) P_{LIGO}(\tilde{\Lambda})\, d\tilde{\Lambda}$$

Work in progress.

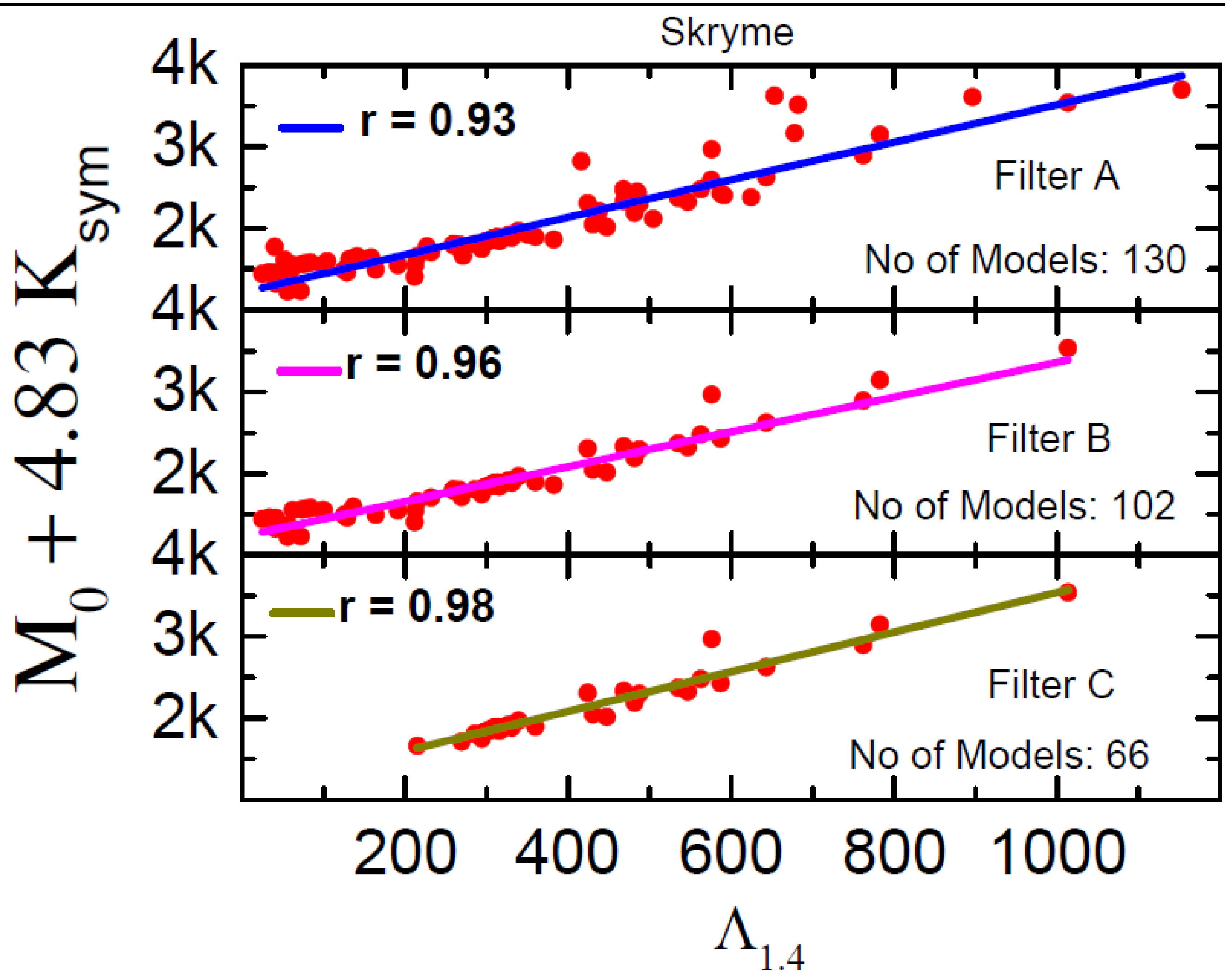
Bootstrap sampling





$$r = 0.92^{+0.04}_{-0.06}$$

Work in progress.



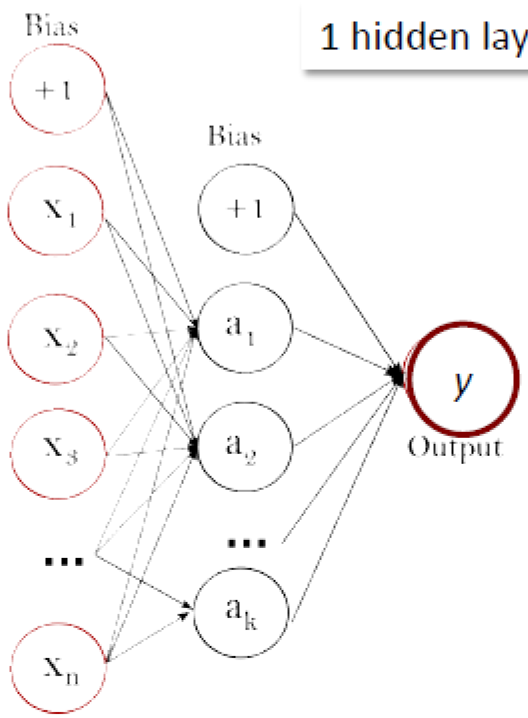
Deep learning

Artificial neural network:

Used when no physics principle, theory …. Helps, but have many data.

1 hidden layer neural network

features



$$a_1^{(2)} = g\left(w_{10}^{(1)}x_0 + w_{11}^{(1)}x_1 + w_{12}^{(1)}x_2 + w_{13}^{(1)}x_3\right)$$

$$a_2^{(2)} = g\left(w_{20}^{(1)}x_0 + w_{21}^{(1)}x_1 + w_{22}^{(1)}x_2 + w_{23}^{(1)}x_3\right)$$

$$\vdots$$

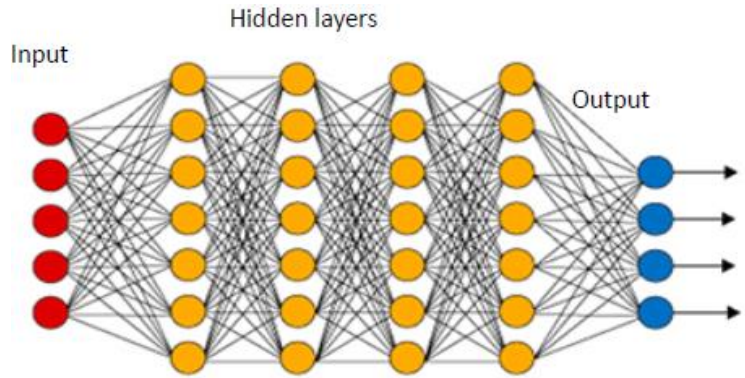
$$a_k^{(2)} = g\left(w_{k0}^{(1)}x_0 + w_{k1}^{(1)}x_1 + w_{k2}^{(1)}x_2 + w_{k3}^{(1)}x_3\right)$$

$$\hat{y}(w,\mathbf{x}) = a_1^3 = g^*\left(w_{10}^{(2)}a_0^{(2)} + w_{11}^{(2)}a_1^{(2)} + w_{12}^{(2)}a_2^{(2)} + w_{13}^{(2)}a_3^{(2)}\right)$$

$g^*(x)$ is activation function of output unit (e.g, identity)

NOTE: any function can be approximate by a sufficiently deep NN
K.-I. Funahashi, Neural Networks 2, 183-192 (1989).

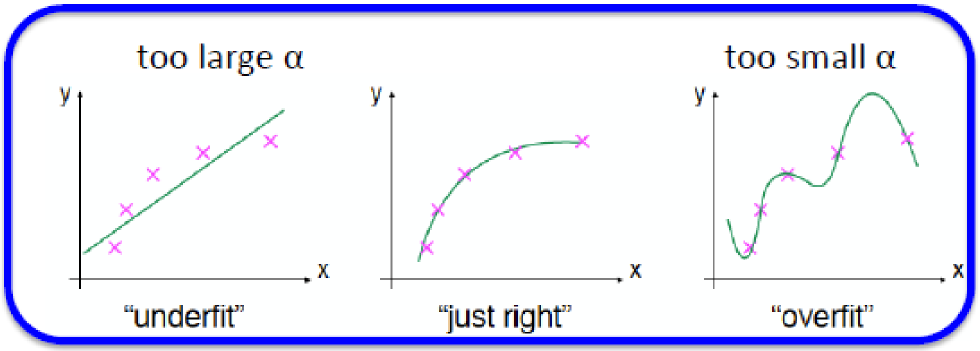
Deep neural network:



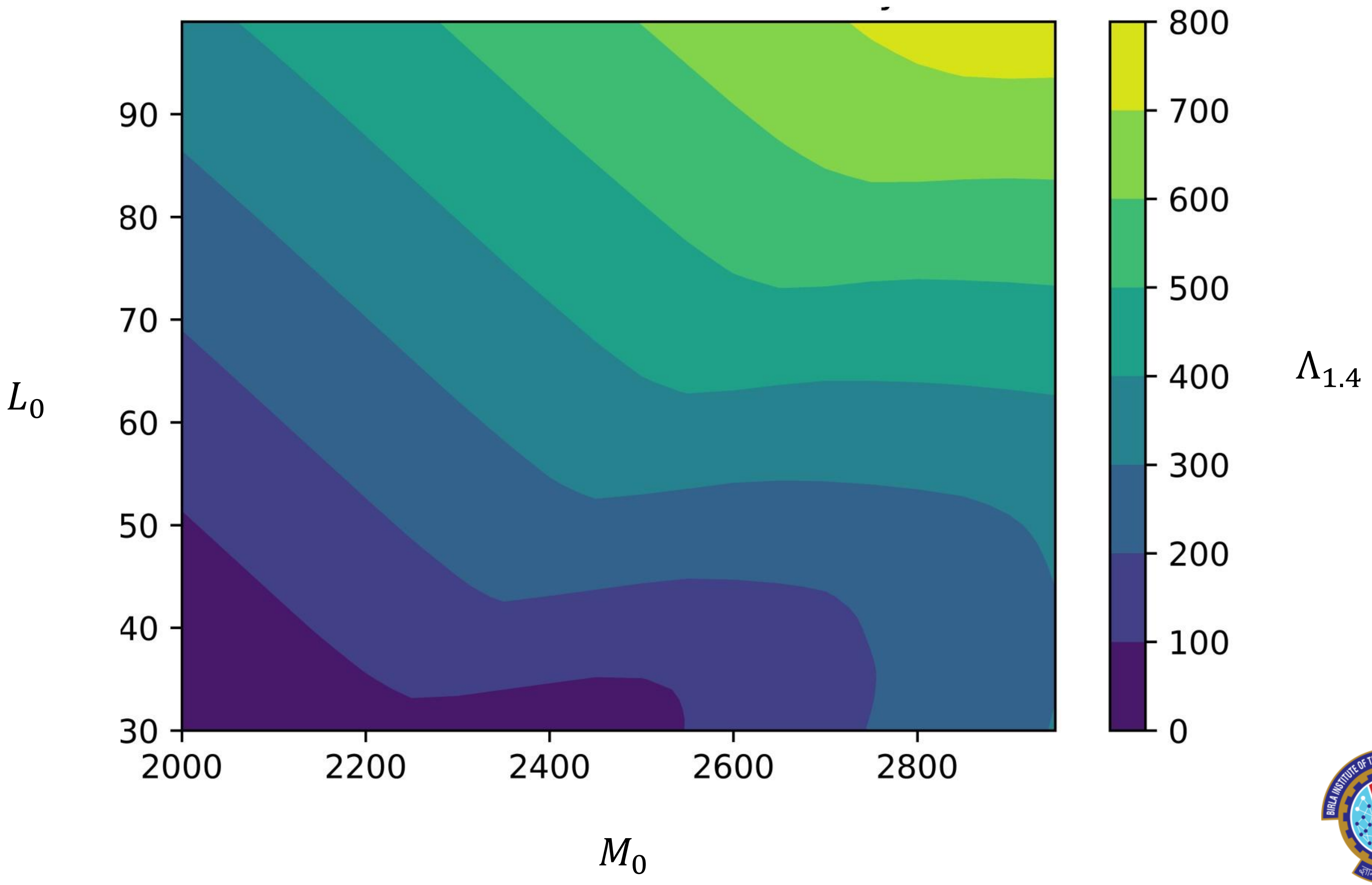
Loss function: $L(\mathbf{W}) = \frac{1}{N_{\text{train}}} \sum_{t=1}^{N_{\text{train}}} \left(F(\mathbf{x}_i) - y_t \right)^2 + \alpha \|\mathbf{W}\|_2^2$

mean squared error

regularization (penalizes large weights)



ANN in tidal deformation



Constraining Tidal deformability from Lab experiment !!

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Rapid Communications

Tides in merging neutron stars: Consistency of the GW170817 event with experimental data on finite nuclei

Tuhin Malik,[1,*] B. K. Agrawal,[2,3] J. N. De,[2] S. K. Samaddar,[2] C. Providência,[4] C. Mondal,[5] and T. K. Jha[1]
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The agreement of the nuclear equation of state (EOS) deduced from the GW170817-based tidal deformability with the one obtained from empirical data on microscopic nuclei is examined. It is found that suitably chosen experimental data on isoscalar and isovector modes of nuclear excitations together with the observed maximum neutron star mass constrain the EOS which displays a very good congruence with the GW170817 inspired one. The giant resonances in nuclei are found to be instrumental in limiting the tidal deformability parameter and the radius of a neutron star in somewhat narrower bounds. At the 1σ level, the values of the canonical tidal deformability $\Lambda_{1.4}$ and the neutron star radius $R_{1.4}$ come out to be 267 ± 144 and 11.6 ± 1.0 km, respectively.

DOI: 10.1103/PhysRevC.99.052801

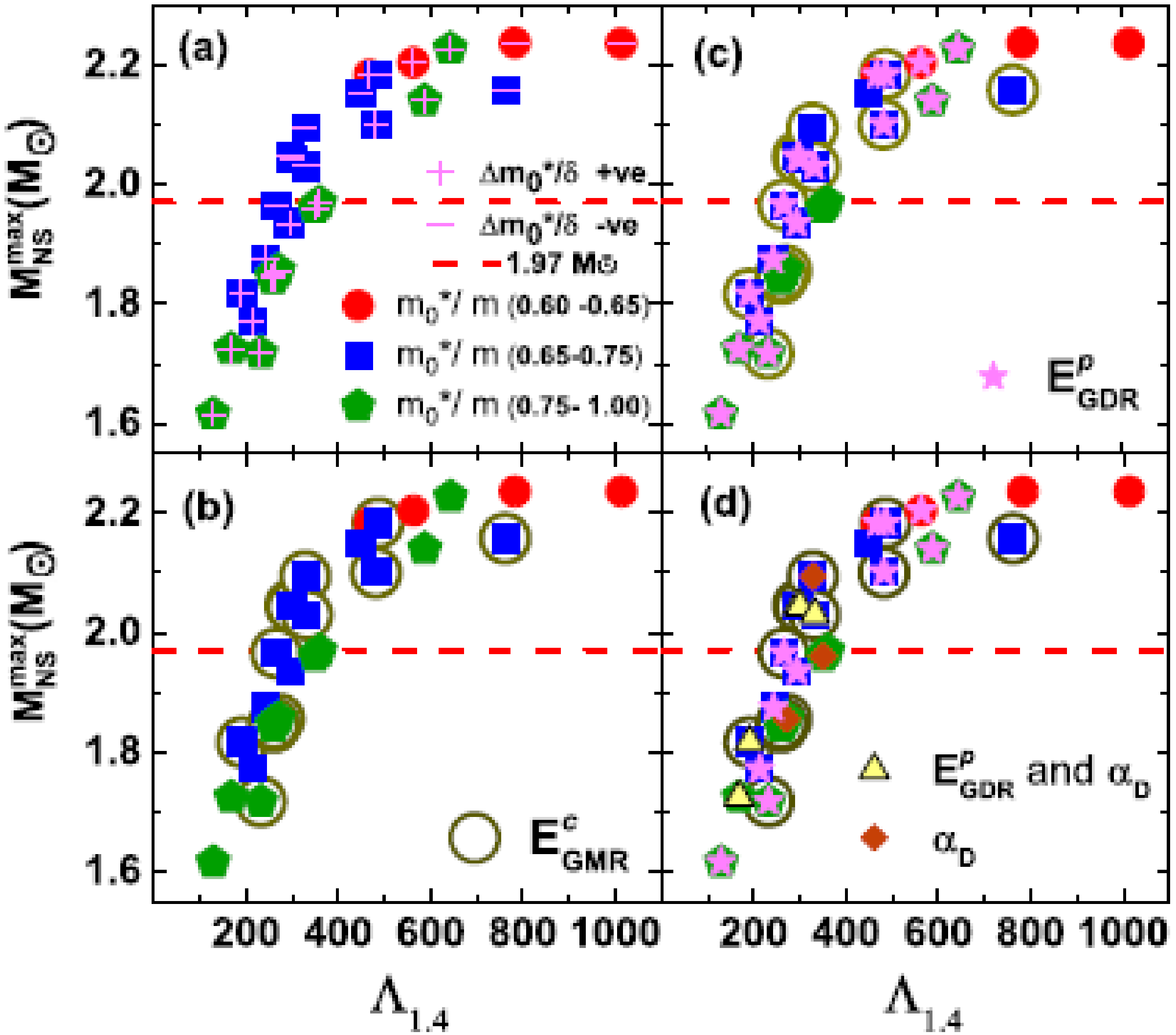
(a)
(b)
(c)
(d)
$M_{NS}^{max}(M_\odot)$
$\Lambda_{1.4}$
1.6
1.8
2.0
2.2
200
400
600
800
1000
$\Delta m_0^*/\delta$ +ve
$\Delta m_0^*/\delta$ -ve
1.97 M⊙
m_0^*/m (0.60 -0.65)
m_0^*/m (0.65-0.75)
m_0^*/m (0.75- 1.00)
E^c_{GMR}
E^p_{GDR}
E^p_{GDR} and α_D
α_D

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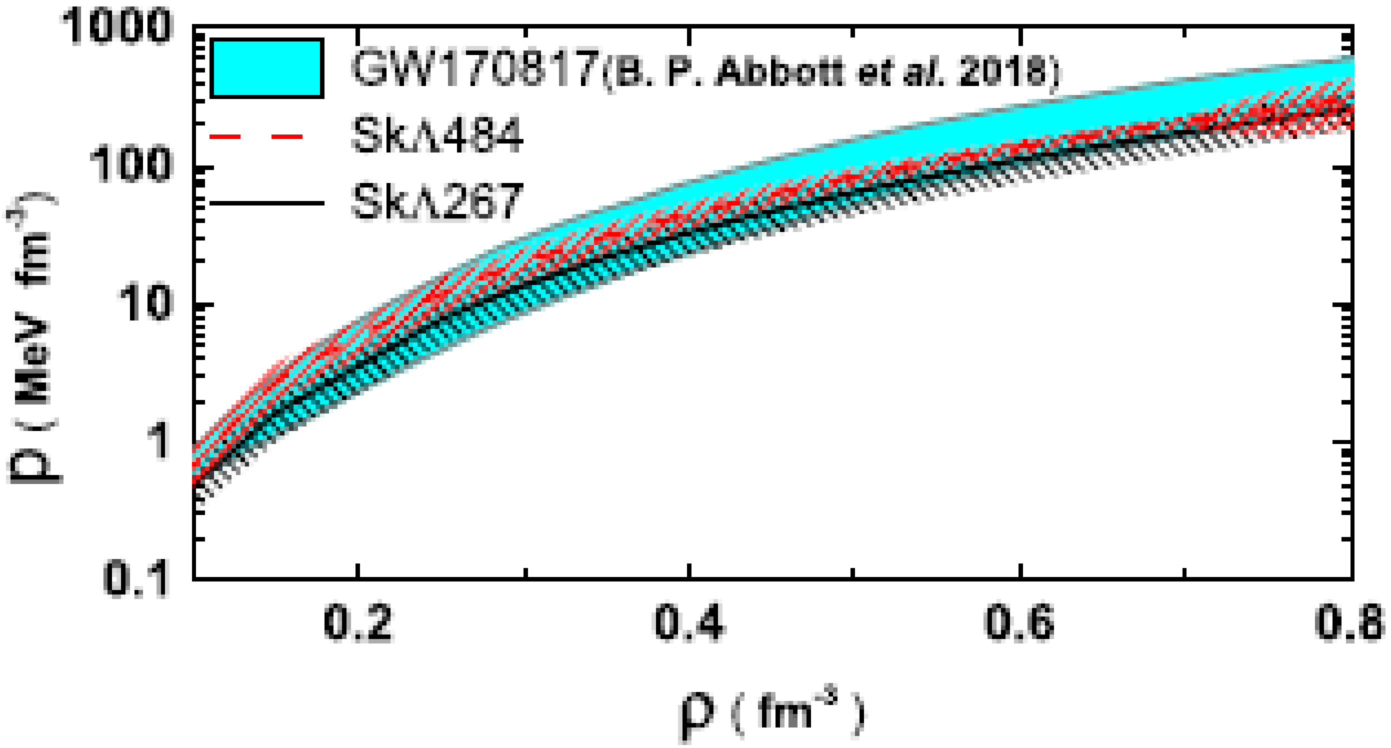
TABLE III. Dipole polarizability α_D, peak energy of IVGDR E^{p}_{GDR}, centroid energy of ISGMR E^{c}_{GMR} for different nuclei [5] and neutron skin thickness Δr_{np} of ^{208}Pb are listed for SkΛ267 and SkΛ484 models. The corresponding experimental values are also provided for comparison.

Quantity	Nuclei	Experiment	SkΛ267	SkΛ484
α_D (fm^3)	^{208}Pb	19.6(6)	19.42(50)	20.67(84)
	^{120}Sn	8.59(36)	8.99(25)	9.58(36)
	^{68}Ni	3.88(31)	3.97(12)	4.25(17)
	^{48}Ca	2.07(22)	2.36(16)	2.52(15)
E^{p}_{GDR} (MeV)	^{208}Pb	13.43	12.40	12.15
	^{120}Sn	15.00	14.30	14.00
	^{68}Ni	17.10	17.30	15.95
	^{48}Ca	18.90	18.20	16.90
E^{c}_{GMR} (MeV)	^{208}Pb	14.17(28)	14.04(11)	13.95(12)
	^{120}Sn	15.70(10)	16.46(13)	16.42(27)
	^{90}Zr	17.81(20)	18.36(12)	18.34(13)
Δr_{np} (fm)	^{208}Pb	0.16(03)*	0.15(05)	0.21(04)

*Obtained from the systematic analysis of measured electric dipole polarizability [6].


1000
100
10
1
0.1
p (MeV fm^{-3})
0.2
0.4
0.6
0.8
ρ (fm^{-3})
GW170817(B. P. Abbott et al. 2018)
SkΛ484
SkΛ267

Conclusions

- ✓ GW-based measurements of the macroscopic properties of neutron stars offer a very promising means of looking deeper into the nuclear microphysics.
- ✓ We establish a bridge to constrain NS radius, nuclear matter EOS with GW observation.
- ✓ The precise measurement of tidal deformability will provide an alternative and accurate estimate for M_0, $K_{sym,0}$, and $R_{1.4}$.
- ✓ We constrain NS tidal deformation parameter from LAB base nuclear experimental data.

Collaborators

B. K. Agrawal

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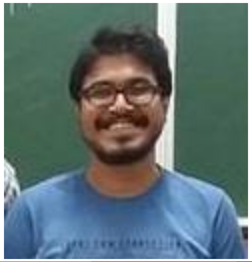
S. K. Patra

N Alam

C. Mondal



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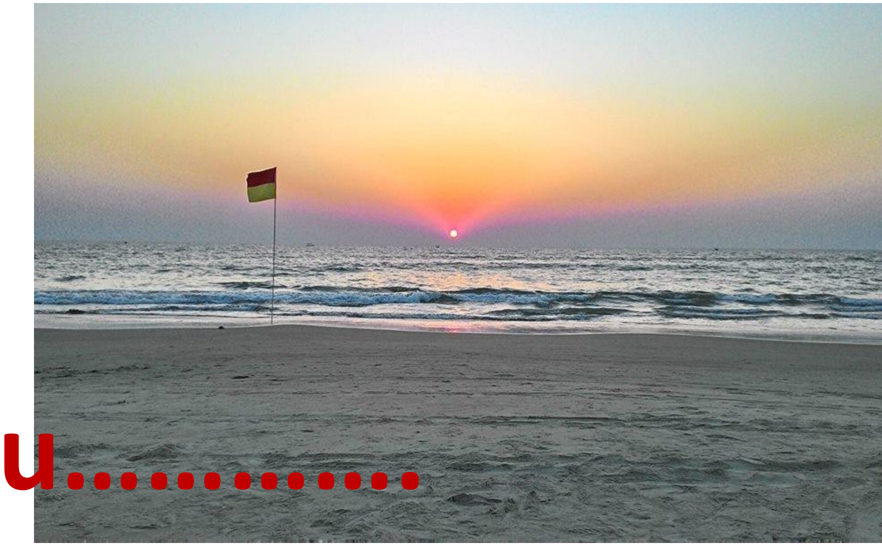

Thank You…………

Obrigado……………
Dziękuję Ci …………
Gracias ……………