THE OHIO STATE UNIVERSITY

Center for Frontiers
in Nuclear Science

The chiral anomaly and axion-like dynamic in polarized DIS

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Based on Andrey Tarasov and Raju Venugopalan
Phys. Rev. D 102 (2020) 11, 114022 (arXiv:2008.08104), and in preparation

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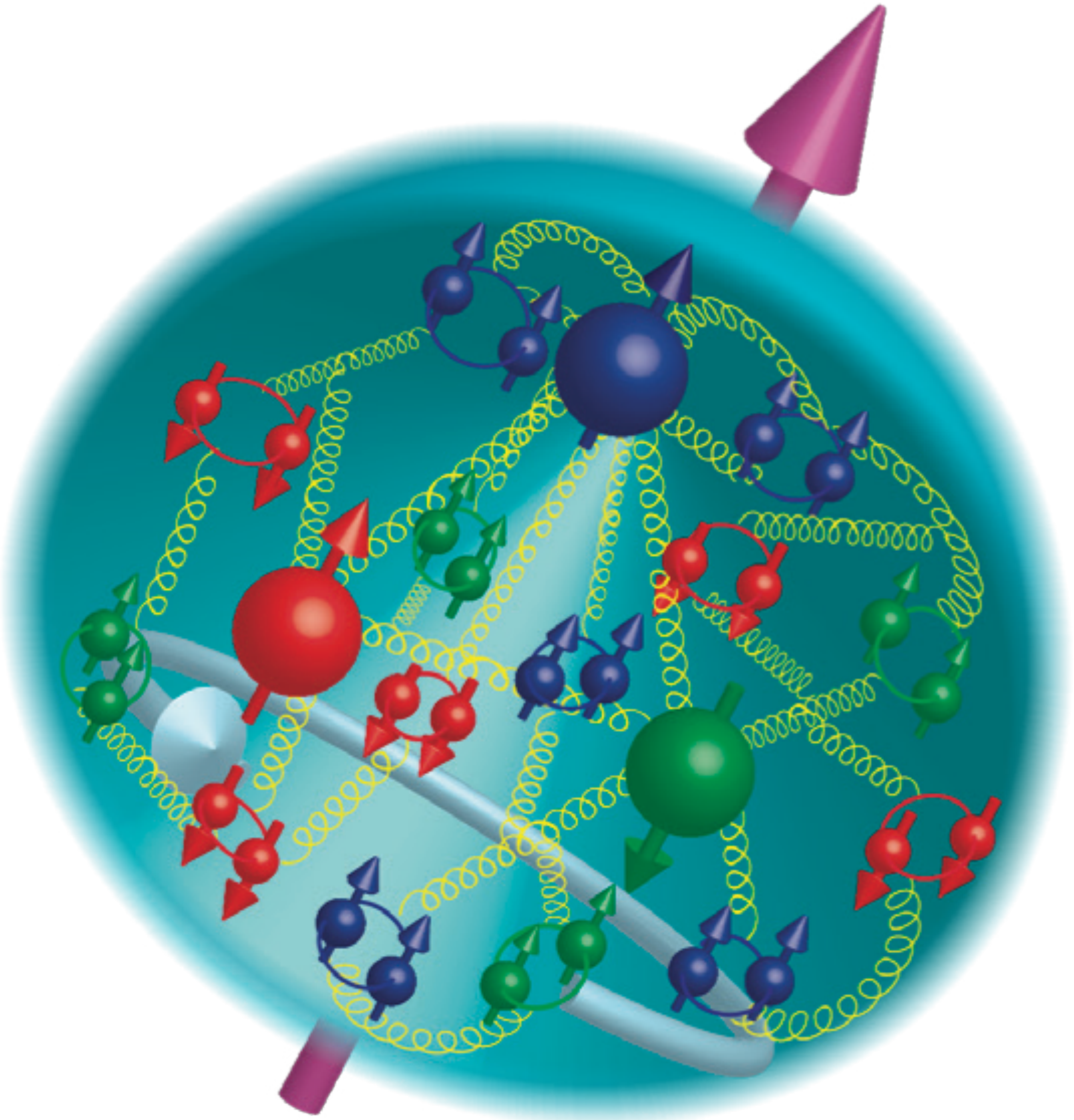
The proton’s spin puzzle



$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L_q + L_g$$

Spin

Quark helicity

Gluon helicity

Orbital angular
momentum

- DIS experiments showed that quarks carry only about 30% of the proton’s spin $\Delta\Sigma = 0.25 \sim 0.3$
- Failure of the constituent quark model to explain spin of the proton - *spin crisis*
- “Traditional” explanation in pQCD - interplay between quark and gluon contribution

Perturbative and non-perturbative interplay

$$S^\mu \Sigma(Q^2) = \frac{1}{M_N}\langle P, S | J_5^\mu(0) | P, S\rangle$$

R. L. Jaffe

A. Manohar

The key role of the anomaly is seen from the structure of the triangle graph in the off-forward limit. The current is not conserved due to presence of the chiral anomaly.

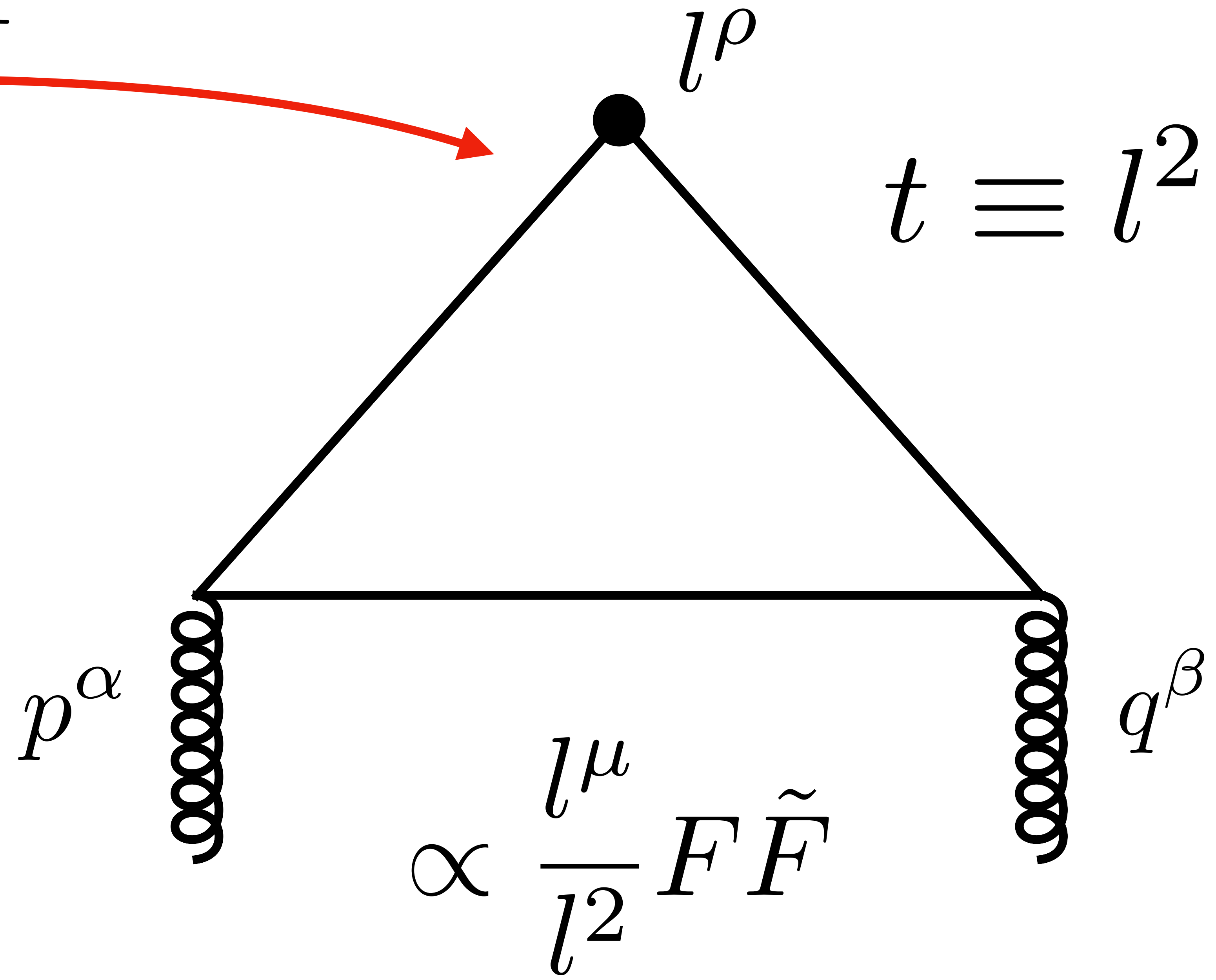
The triangle diagram has an infrared pole and $\kappa(t) \propto F\tilde{F}$

$$\frac{1}{M_N}\langle P', S | J_5^\mu(0) | P, S\rangle = \boxed{\frac{l\cdot S\, l^\mu}{l^2}\kappa(Q^2, t)}$$

Exact result!

R. L. Jaffe, A. Manohar
Nucl. Phys., B337:509–546, 1990

Shore, Veneziano (1990)
Narison, Shore, Veneziano, hep-ph/9812333
G. M. Shore, hep-ph/0701171
K.-F. Liu (1992)



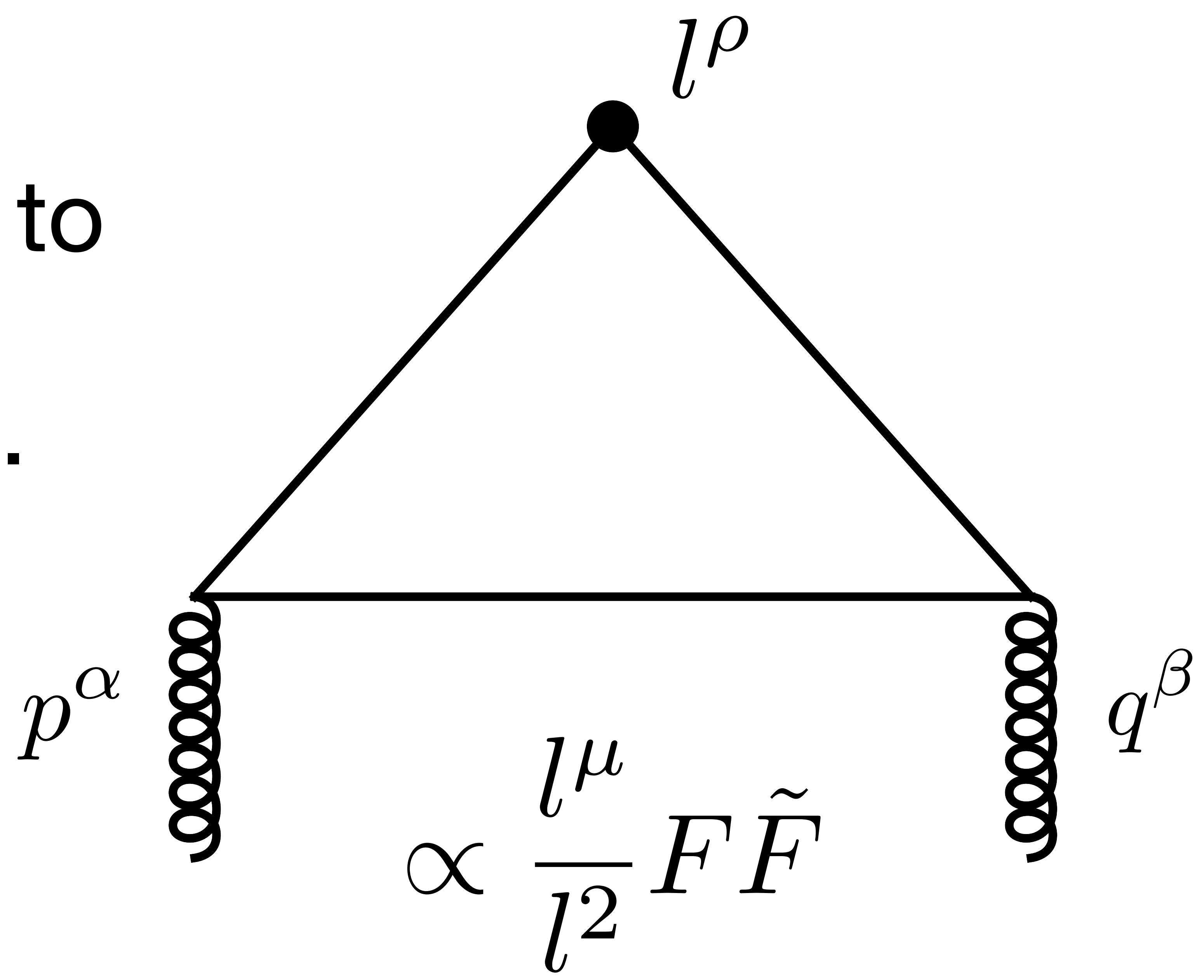
The existence of the infrared pole of the triangle diagram has not been addressed in pQCD calculations.

Perturbative and non-perturbative interplay

The infrared pole of the triangle diagram must be cancelled by a pole in the pseudo-scalar sector of the theory:

$$\Sigma(Q^2) = \frac{n_f\,\alpha_s}{2\pi\,M_N}\lim_{l_\mu\to 0}\langle P', S|\frac{1}{il\cdot s}\mathrm{Tr}\left(F\tilde{F}\right)(0)|P,S\rangle$$

The result is manifestly gauge invariant!



The presence of the pole in the triangle diagram is related to topological properties of QCD (measure of the QCD path integral), which are described by the chiral Ward identities. The triangle diagram is not local!

Generalization of this result to $g_1(x, Q^2)$, and interplay with non-perturbative physics can be explored efficiently in a worldline framework

The triangle anomaly in the worldline formalism

We consider a functional integral representation (worldline)
of the imaginary part (anomaly!) of the Dirac determinant

$$-\mathcal{W}[A,B] = \ln \mathrm{Det}\ \left[\not{p} - \not{A} - \gamma_5 \not{B}\right]$$

D.G.C. McKeon, C. Schubert, Phys. Lett. B 440 (1998) 101

The triangle anomaly:

$$\langle P', S | J_5^\kappa | P, S\rangle = \int d^4y\, \frac{\partial \mathcal{W}_I[A,B]}{\partial B_\kappa(y)}\Big|_{B_\kappa = 0} e^{ily} \equiv \Gamma_5^\kappa[l]$$

John S. Bell

We reproduce famous infrared pole of the anomaly

$$S^\mu \int_0^1 dx_B\, g_1(x_B, Q^2)\Big|_{Q^2\to\infty}$$

$$= \sum_f e_f^2 \frac{\alpha_S}{2i\pi M_N} \lim_{l_\mu \to 0} \boxed{\frac{l^\mu}{l^2}} \langle P', S | \mathrm{Tr_c} F_{\alpha\beta}(0) \tilde{F}^{\alpha\beta}(0) | P, S\rangle$$

Steven Adler

Roman Jackiw

The structure function g_1

infrared pole

non-local operator

non-pole contribution

$$S^\mu g_1(x_B, Q^2)\Big|_{Q^2\to\infty} = \sum_f e_f^2 \frac{\alpha_s}{i\pi M_N} \int_{x_B}^1 \frac{dx}{x} \left(1 - \frac{x_B}{x}\right) \int \frac{d\xi}{2\pi} e^{-i\xi x} \lim_{l_\mu \to 0} \frac{l^\mu}{l^2} \langle P', S | \mathrm{Tr_c} F_{\alpha\beta}(\xi n) \tilde{F}^{\alpha\beta}(0) | P, S\rangle + \mathcal{O}\left(\frac{\Lambda^2_{QCD}}{Q^2}\right)$$

The structure function g_1 is dominated by the triangle anomaly, hence g_1 is topological. The result is formally identical in both Bjorken and Regge limits. $g_1(x_B, Q^2)$ is topological in both asymptotic limits of QCD

Tarasov, Venugopalan (arXiv:2008.08104)

First moment of g_1:

$$S^\mu \int_0^1 dx_B \, g_1(x_B, Q^2)\Big|_{Q^2\to\infty} = \sum_f e_f^2 \frac{\alpha_S}{2i\pi M_N} \lim_{l_\mu \to 0} \frac{l^\mu}{l^2} \langle P', S | \mathrm{Tr_c} F_{\alpha\beta}(0) \tilde{F}^{\alpha\beta}(0) | P, S\rangle$$

matches calculation of the triangle diagram

The pole must be removed by contribution of the pseudo-scalar sector

Pseudoscalar contribution

To take into account contribution of the pseudoscalar sector we use the most general form of the imaginary part of the effective action

$$-\mathcal{W}[A,B,\Phi,\Pi] = \ln \mathrm{Det}\,\left[\not{p} - i\Phi - \gamma_5\,\Pi - \not{A} - \gamma_5\not{B}\right]$$

scalar field

pseudoscalar field

Functional integral representation of the effective action:

D'Hoker, Gagne,hep-th/9508131

$$\mathcal{W}_{\mathcal{I}} = -\frac{i}{32}\int_{-1}^{1} d\alpha \int_0^\infty dT\,\mathcal{N}\int_{\mathrm{PBC}} \mathcal{D}x\,\mathcal{D}\psi\ \mathrm{tr}\ \chi\,\bar{\omega}(0)\,\exp\left[-\int_0^T d\tau\,\mathcal{L}_{(\alpha)}(\tau)\right]$$

It generates the iso-singlet Wess-Zumino-Witten coupling $\propto \eta_0 F\tilde{F}$

Tarasov, Venugopalan (in preparation)

$$\mathcal{W}_{\mathcal{I}}[\Pi A^2] = \frac{ig^2\,2n_f}{16\pi^2}\,\frac{1}{\Phi}\,\mathrm{tr}_c\int d^4x\,\Pi(x)\,F_{\mu\nu}(x)\tilde{F}^{\mu\nu}(x)$$

in agreement with the corresponding term in $\mathscr{L}_{\mathrm{WZW}}$ which was derived from chiral perturbation theory

Leutwyler (1996); Herrara-Sikody et al (1997); Leutwyler-Kaiser (2000)

Imaginary part of the effective action

With this form of the effective action, we see explicitly how the pole of the anomaly is canceled by a massless $\bar{\eta}$ exchange ("primordial" ninth Goldstone boson)

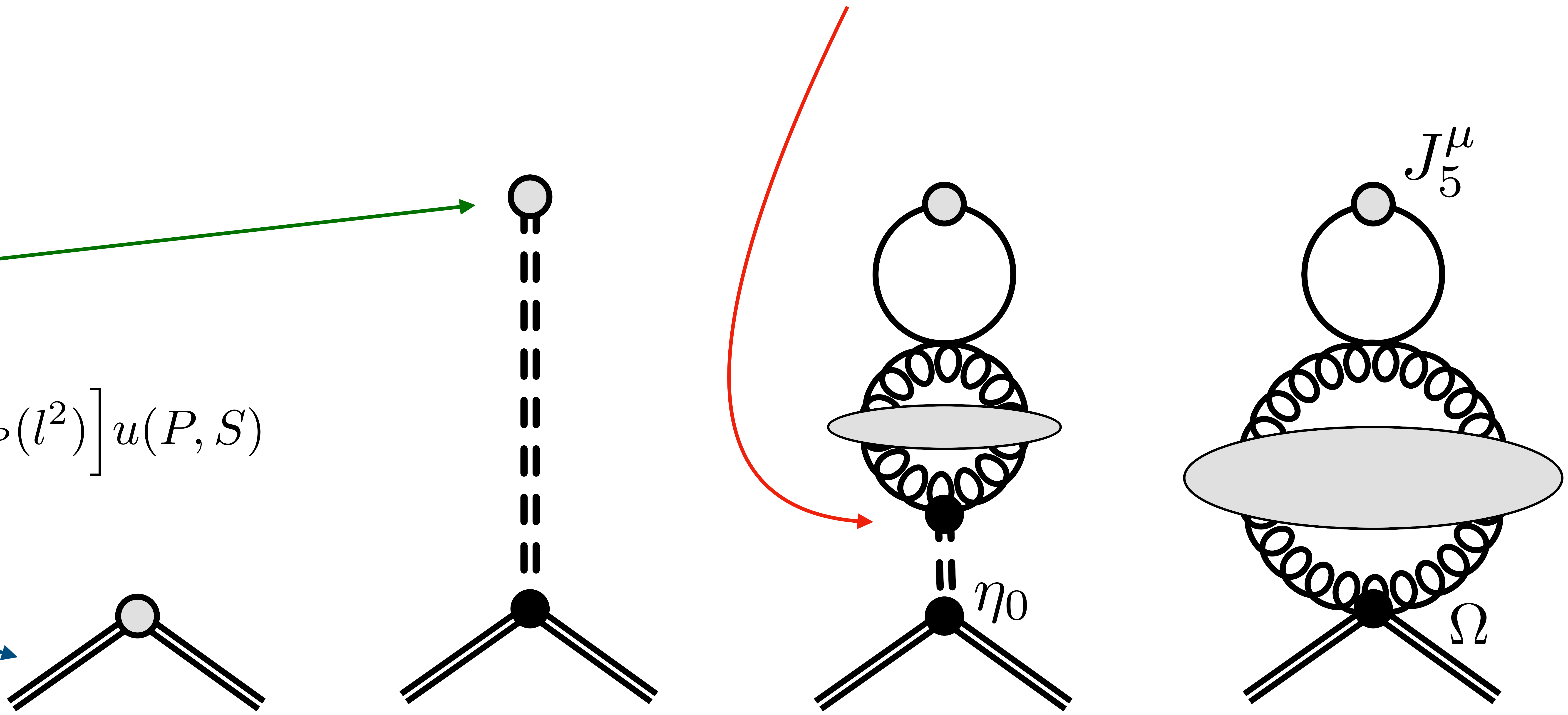
$$S^{\bar{\eta}}_{\mathrm{WZW}} = i\frac{\sqrt{2\,n_f}}{F_{\bar{\eta}}}\int d^4x\,\bar{\eta}\,\Omega$$

$$\Omega = \frac{\alpha_s}{4\pi}\mathrm{Tr}\left(F\tilde{F}\right)$$

$$\langle P', S|J_5^\mu|P, S\rangle$$

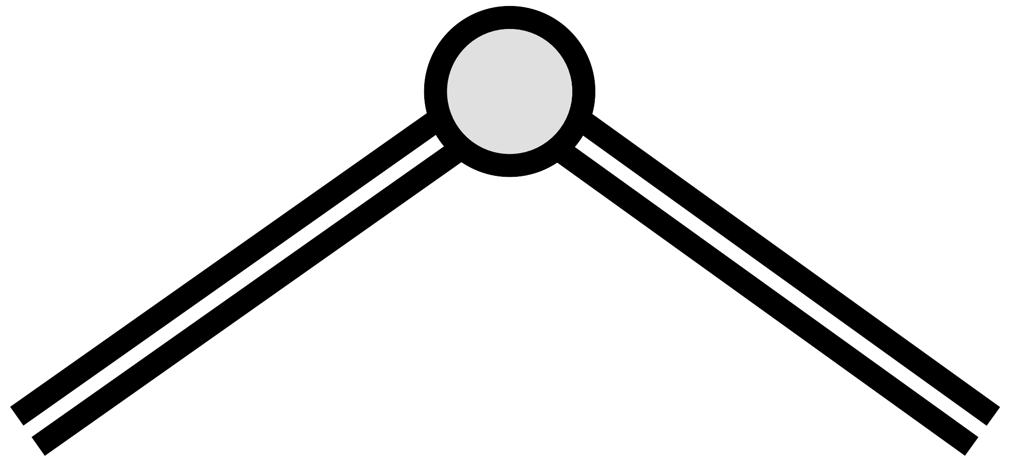
$$= \bar{u}(P', S)\Big[\gamma^\mu\gamma_5 G_A(l^2) + l^\mu\gamma_5 G_P(l^2)\Big]u(P, S)$$

We consider the matrix element of J_5^μ taking into account the WZW coupling



Imaginary part of the effective action

The diagram represents the direct coupling of the axial vector currents to the nucleon target. It is fundamentally different from other diagrams since it is the only diagram which contributes to the axial form factor $G_A(l^2)$



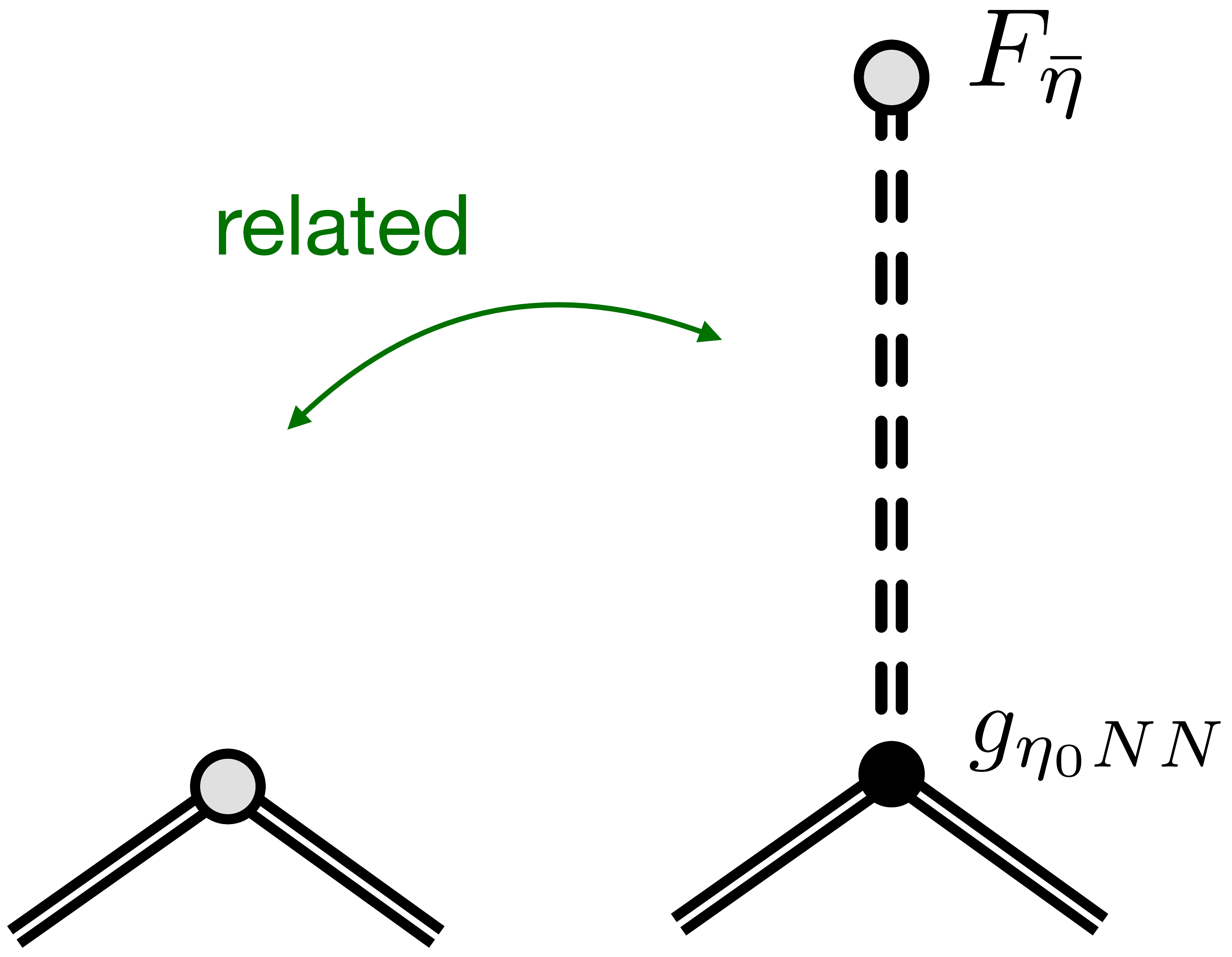
$$\langle P', S | J_5^\mu | P, S \rangle = G_A(l^2)\, \bar{u}(P', S)\, \gamma^\mu \gamma_5\, u(P, S)$$

$$\lim_{l \to 0} \langle P', S | J_5^\mu | P, S \rangle = 2M_N\, G_A(0)\, S^\mu$$

The anomaly equation for the divergence of the singlet axial current and the requirement of the infrared pole cancellation impose relation of this result to the contribution of other diagrams

Goldberger-Treiman relation

$$\langle P', S|\partial_\mu J_5^\mu|P, S\rangle = \langle P', S|2\, n_f\, \Omega|P, S\rangle$$



Using the anomaly equation we derive the generalization of the well-known Goldberger-Treiman relation to the isosinglet axial vector current

Shore and Veneziano (89-92)

$$G_A(0) = \frac{\sqrt{2\tilde{n}_f}}{2M_N}\, F_{\bar\eta}\, g_{\eta_0 NN}$$

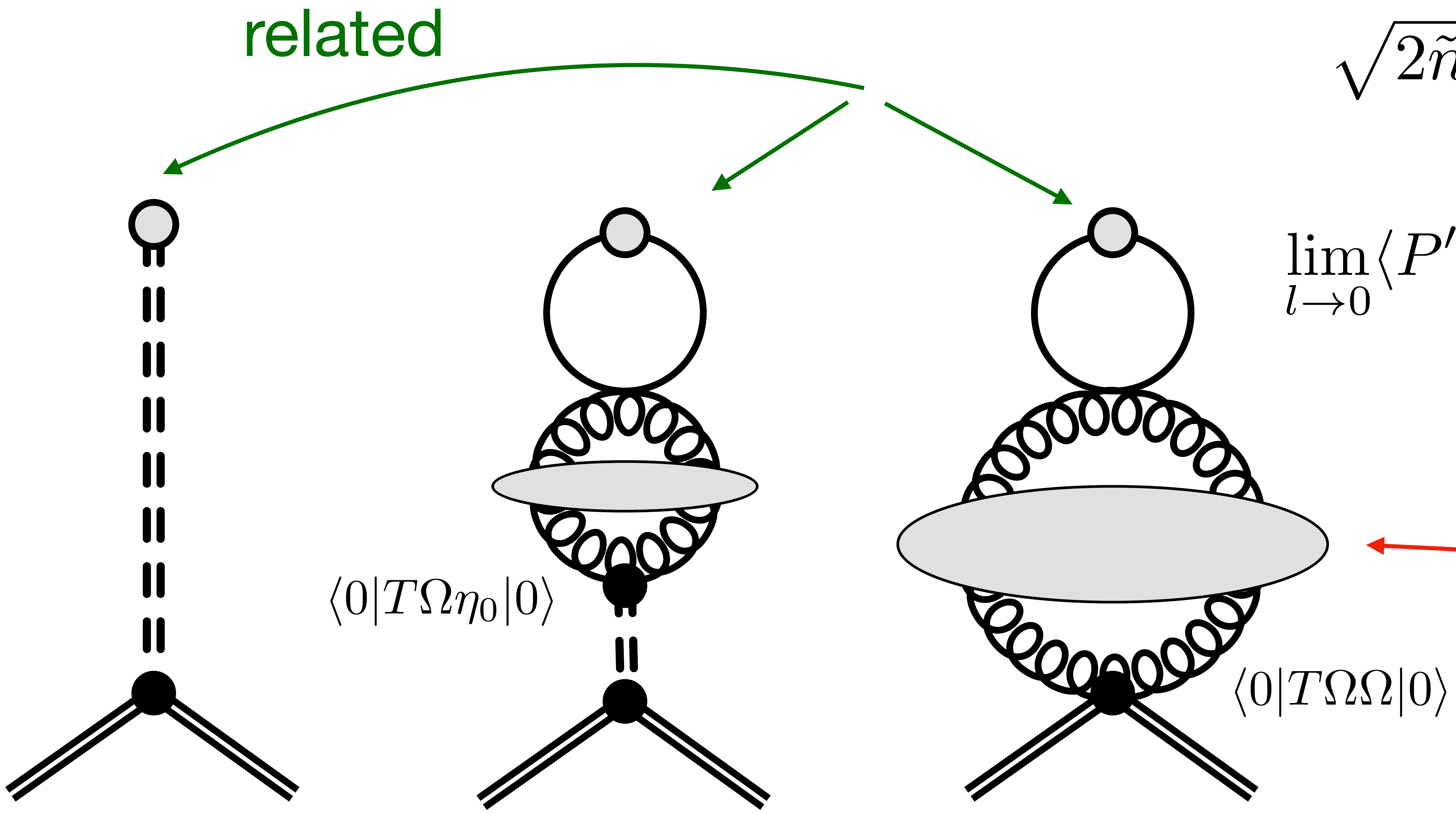
$$\lim_{l\to 0}\langle P', S|J_5^\mu|P, S\rangle = \sqrt{2\tilde{n}_f}\, F_{\bar\eta}\, g_{\eta_0 NN} S^\mu$$

decay constant

couling to the nucleon

Cancellation of the infrared pole

From requirement of the infrared pole cancellation we find a relation between the product of the η_0 decay constant and its coupling to the nucleon to the matrix element of the topological charge density in the forward limit



$$\sqrt{2\tilde{n}_f}\, F_{\bar{\eta}}\, g_{\eta_0 NN} = \lim_{l\to 0} \frac{1}{il\cdot S}\langle P', S|2n_f\Omega|P, S\rangle$$

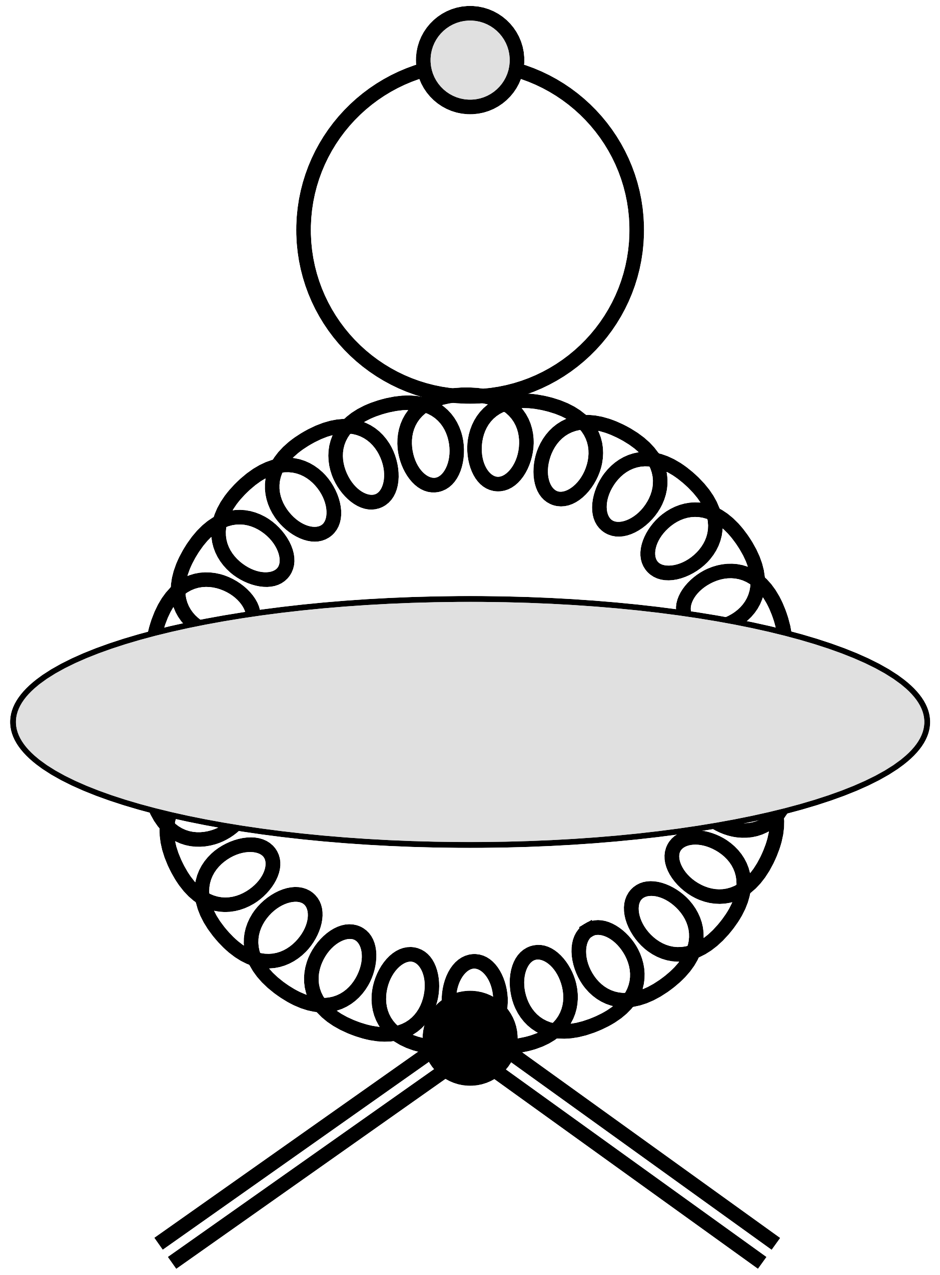
$$\lim_{l\to 0}\langle P', S|J_5^\mu|P, S\rangle = 2n_f \lim_{l\to 0}\langle P', S|\frac{1}{il\cdot S}\Omega|P, S\rangle S^\mu$$

We study the effect of the WZW coupling in the two-point Green functions and demonstrate how it generates a nonzero $m^2_{\eta'}$

The WZW-$\bar{\eta}$ term and the topological susceptiblity

The topological susceptibility: $\chi(l^2) = i\int d^4x\, e^{ilx}\langle 0|T\,\Omega(x)\Omega(0)|0\rangle$

Taking into account the coupling between $\bar{\eta}$ and Ω, as specified by the WZW action, it is easy to obtain



$$\chi(l^2) = l^2\frac{1}{l^2 - m^2_{\eta'}}\chi_{\mathrm{YM}}(l^2) \qquad m^2_{\eta'} \equiv -\frac{2\,n_f}{F^2_{\bar{\eta}}}\chi_{\mathrm{YM}}(0)$$

The topological susceptibility $\chi(l^2)$ vanishes in the forward limit because of topological mass generation

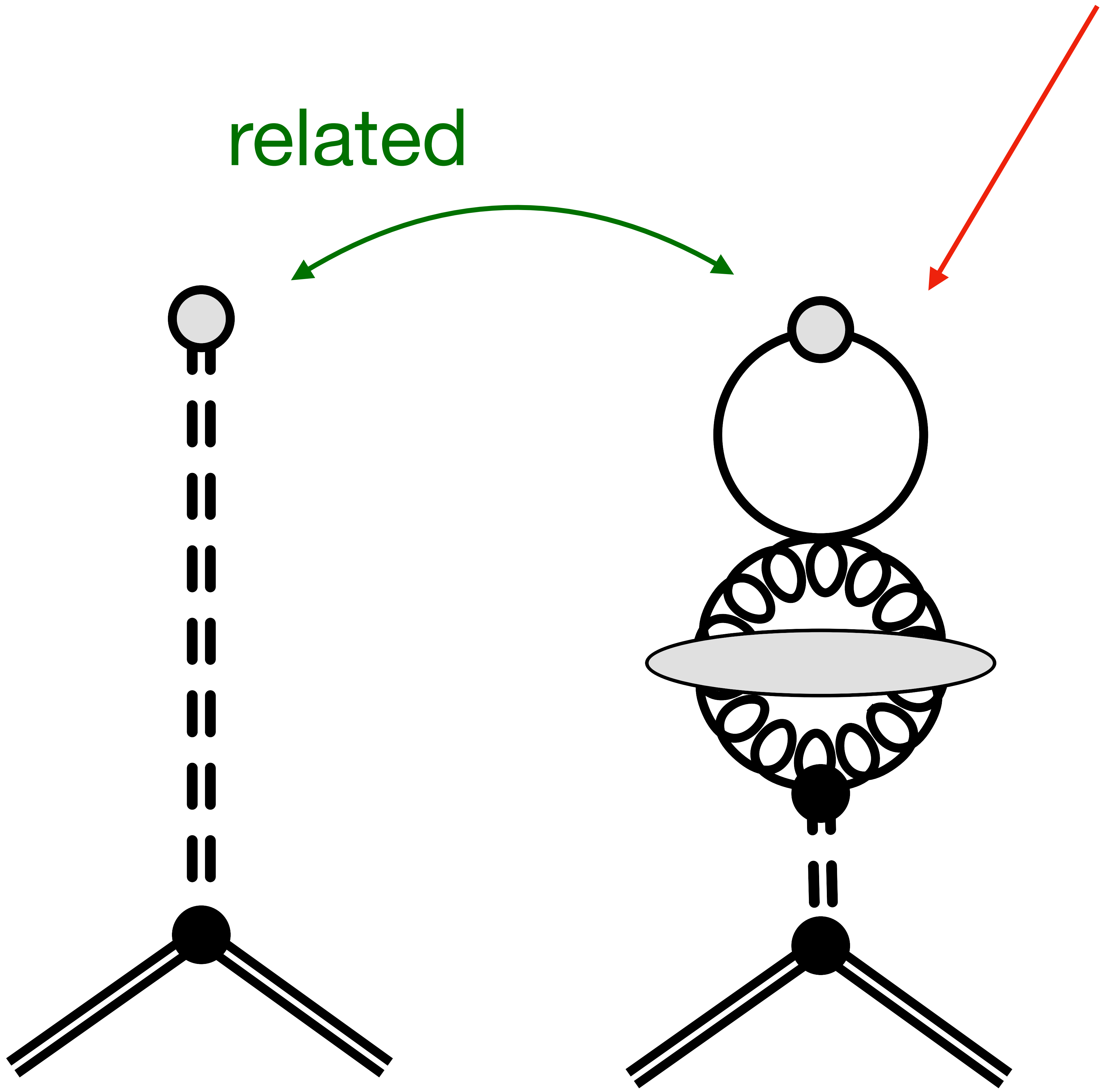
For the diagram we have:

$$\langle P', S|J_5^\mu|P, S\rangle = 2n_f\,\frac{l^\mu}{l^2 - m^2_{\eta'}}\chi_{\mathrm{YM}}(l^2)\cdot g_{\Omega NN}\,\bar{u}(P', S)\,\gamma_5\,u(P, S)$$

In the forward limit: $\lim_{l\to 0}\langle P', S|J_5^\mu|P, S\rangle = 0$

The WZW-$\bar{\eta}$ term and the topological susceptiblity

Contribution of the diagram is not zero even in the forward limit



From the infrared pole cancellation requirement we obtain a relation between the decay constant and the topological susceptibility

$$F_{\bar{\eta}}^2 = \lim_{l \to 0} \frac{2n_f}{l^2} \chi(l^2)$$

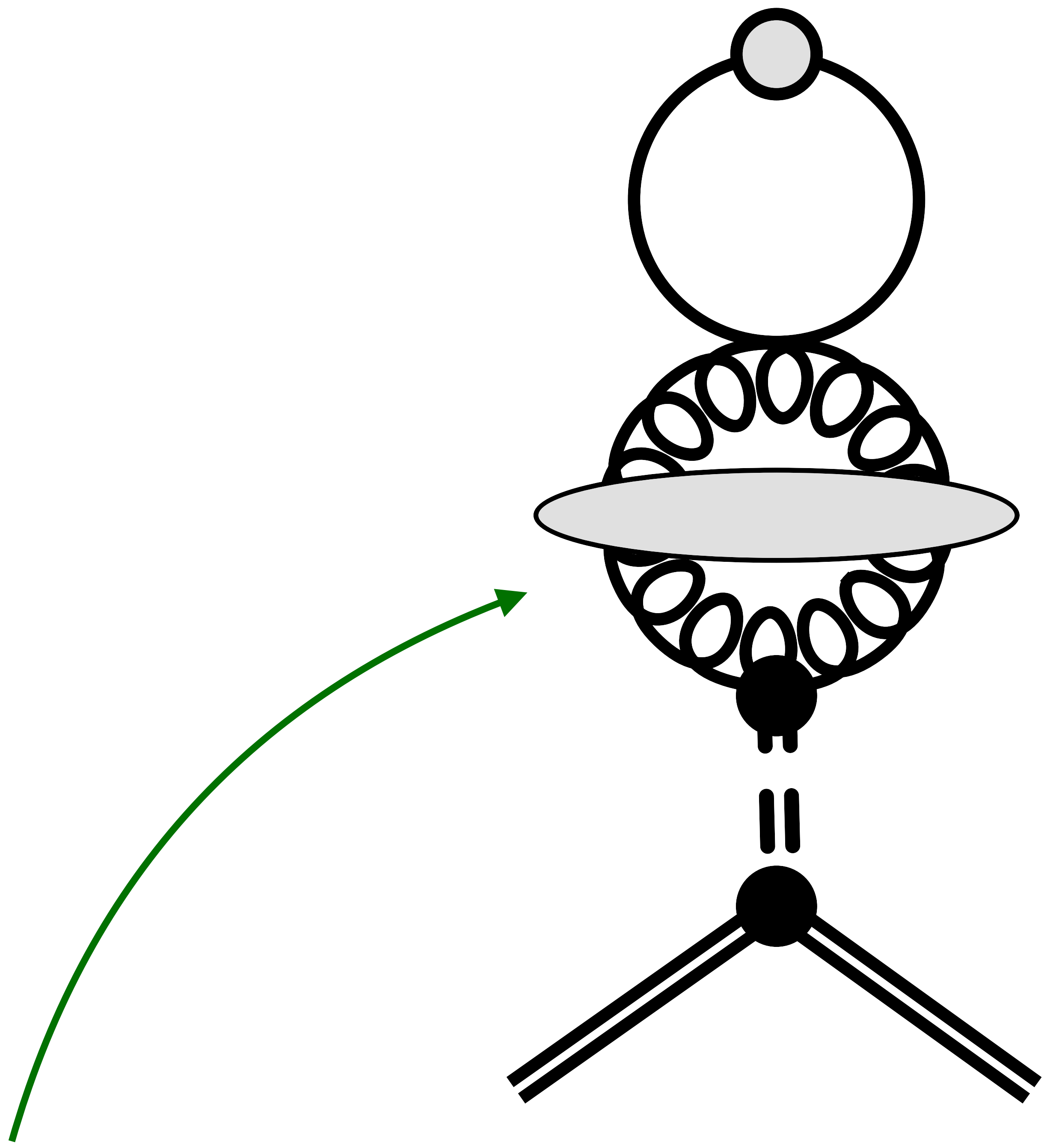
Expanding the topological susceptibility

$$\Sigma(Q^2) = \sqrt{\frac{2}{3}} \frac{2n_f}{M_N} g_{\eta_0 NN} \sqrt{\chi'(0)}$$

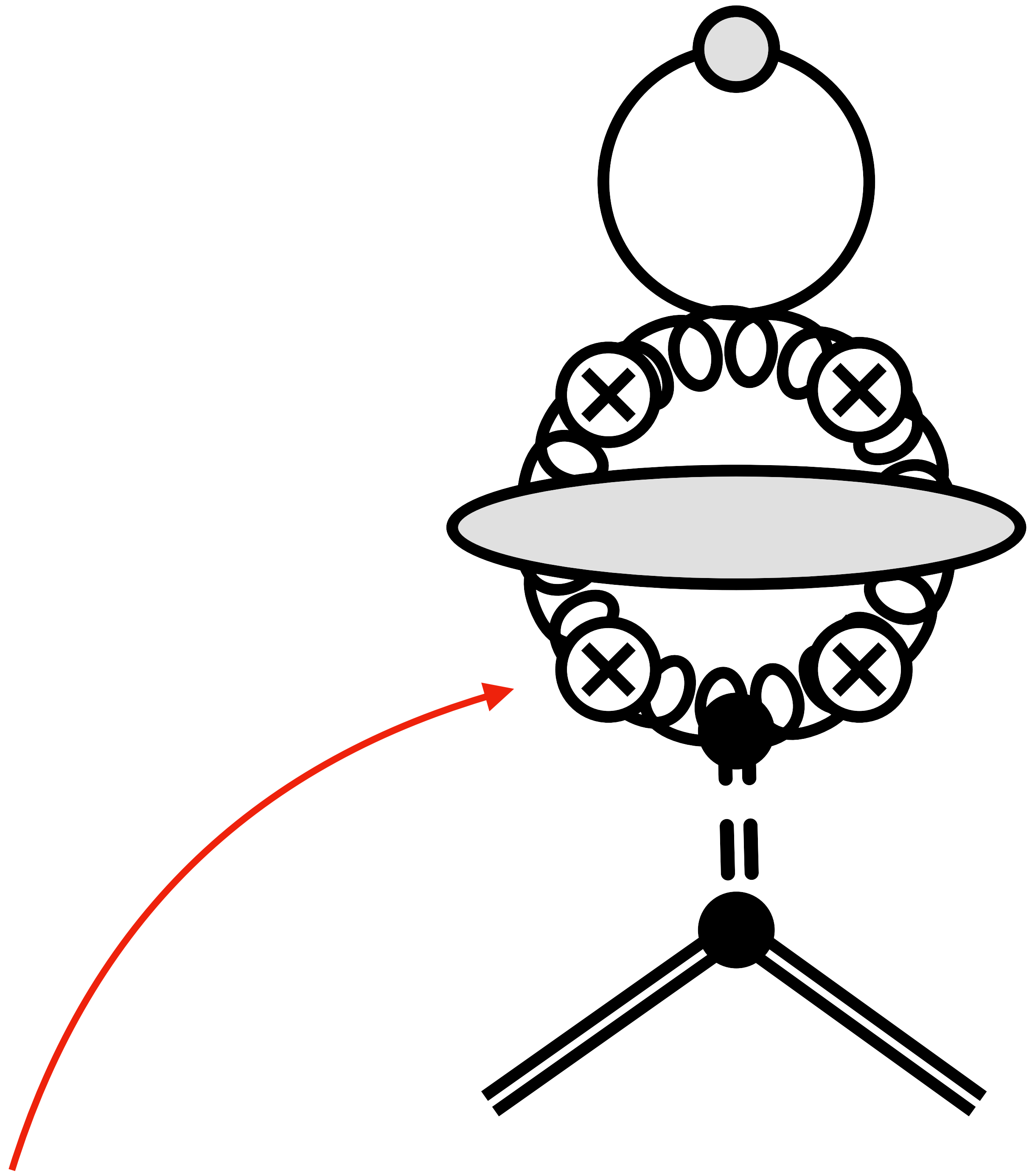
Shore, Veneziano (1992)

Effective action at small-x

Since the anomaly dominates the box diagram, the similar dynamics underlines the x_B dependence of $g_1(x_B, Q^2)$. Novel features emerge at small x_B.



At large-x the gluon field is dominated by the instanton configuration. The typical scale is $m_{\eta'}^2$.



At small-x there is a background of the small-x gluons characterized by the saturation scale Q_s^2.

Axion-like effective action

Veneziano, Mod. Phys. Lett. (1989)
Hatsuda, PLB (1990)

We construct an axion-like effective action at small x_B that describes the interplay between gluon saturation and the topology of the QCD vacuum. It contains the WZW coupling and a kinetic term for the $\bar{\eta}$ field. This dynamics is governed by the $m_{\eta'}^2$

Tarasov, Venugopalan (in preparation)

$$\langle \mathcal{O} \rangle = \int D\bar{\eta}\, \tilde{W}_{P,S}[\bar{\eta}] \int D\rho\, W_Y[\rho] \int [DA]$$

$$\times\, \mathcal{O} \exp\left(iS_{\mathrm{CGC}} + i \int d^4x \left[\frac{1}{2} \left(\partial_\mu \bar{\eta}\right) \left(\partial^\mu \bar{\eta}\right) + \frac{\sqrt{2n_f}}{F_{\bar{\eta}}} \bar{\eta}\, \Omega \right] \right)$$

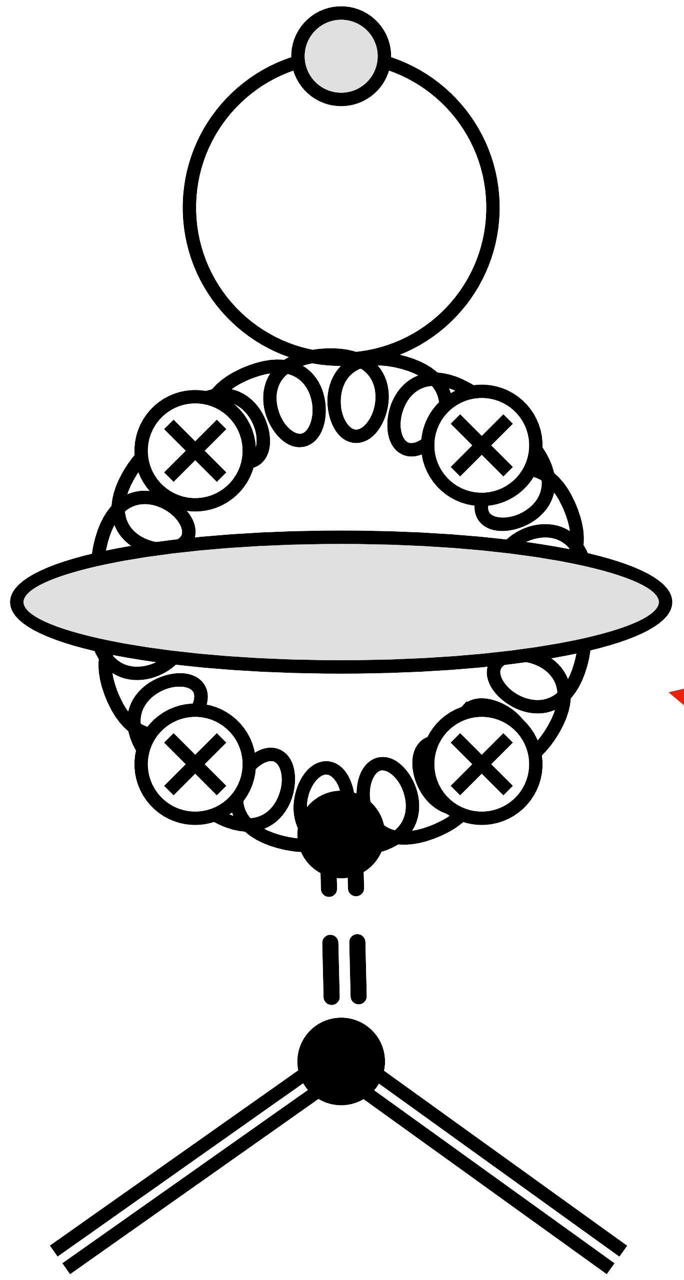
The background gauge configurations representing the saturated state are static classical configurations and their dynamics is described by the Color Glass Condensate (CGC) Effective Field Theory and controlled by the saturation scale Q_s^2

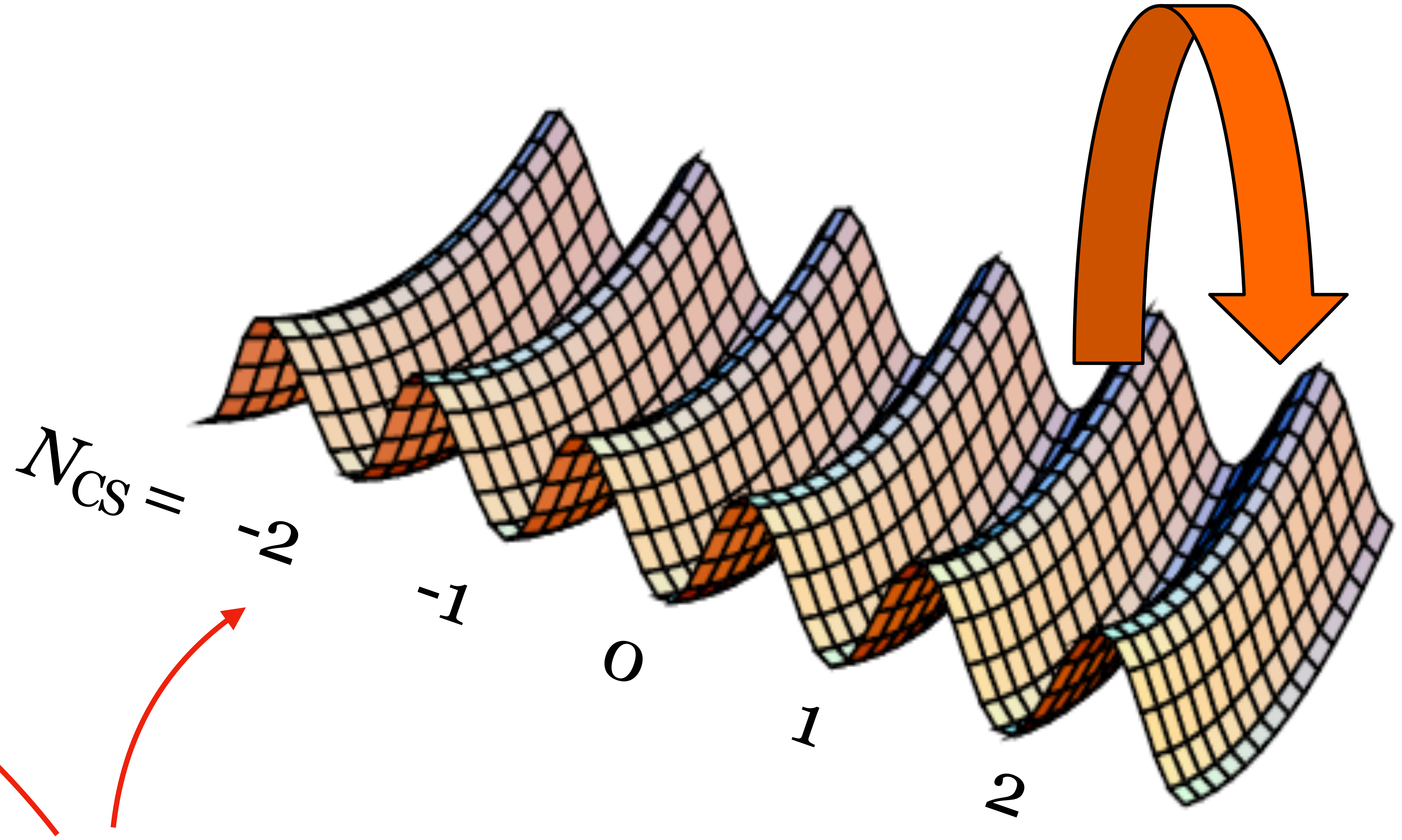
$$S_{\mathrm{CGC}}[A, \rho] = -\frac{1}{4} \int d^4x F_a^{\mu\nu} F^a_{\mu\nu} + \frac{i}{N_c} \int d^2x_\perp\ \mathrm{tr}_c \left[\rho(x_\perp) \ln \left(U_{[\infty, -\infty]}(x_\perp) \right) \right]$$

McLerran, Venugopalan (1994)

Topological transitions at small-x

Because of the presence of two scales - mass of the η' and saturation scale, g_1 can be sensitive to real-time topological transitions (sphaleron transitions)



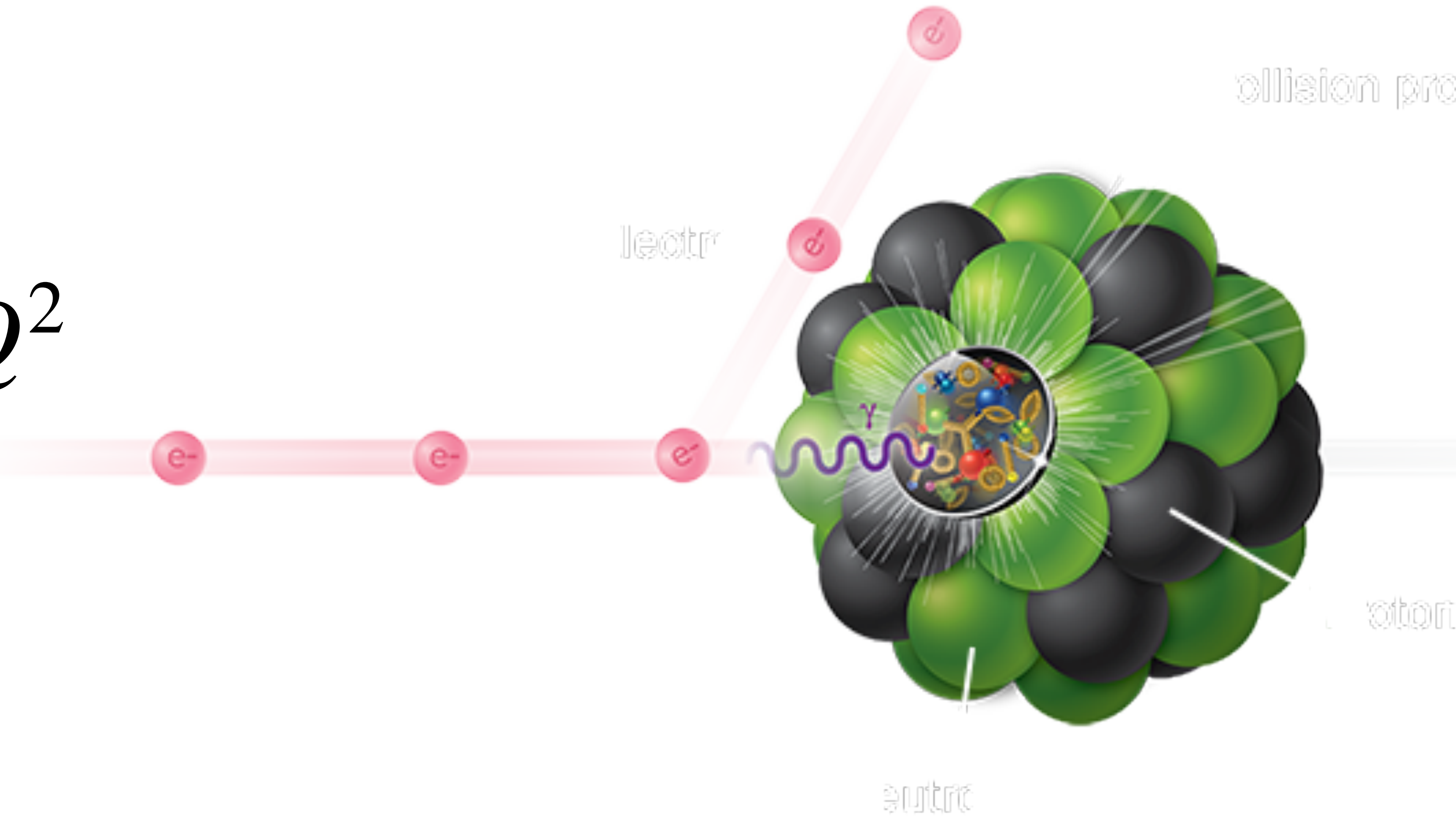


When $m_{\eta'}^2 \gg Q_s^2$ the gluon field is dominated by the instanton configurations

If $Q_s^2 \gg m_{\eta'}^2$ the CGC solution starts to dominate and we predict over-the-barrier sphaleron transitions between different topological sectors of QCD vacuum

Summary

- We show that the anomaly appears in both the Bjorken limit of large Q^2 and in the Regge limit of small x_B. We find that the infrared pole in the anomaly arises in both limits
- The cancellation of the pole involves a subtle interplay of perturbative and nonperturbative physics that is deeply related to the $U_A(1)$ problem in QCD
- We demonstrate the fundamental role of the WZW term both in topological mass generation of the η' and in the cancellation of the off-forward pole arising from the triangle anomaly in the proton's helicity. We recover the result by Shore and Veneziano that $\Sigma \propto \sqrt{\chi'(0)}$
- We introduce an axion-like effective action at small-x which describes the interplay between gluon saturation and the topology of the QCD vacuum
- We outline the role of “over-the-barrier” sphaleron-like transitions in spin diffusion at small x_B. Such topological transitions can be measured in polarized DIS at a future Electron-Ion Collider.



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