Gravitational form factors on the lattice

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Outline

Gravitational form factors

- Physics motivation
- Experimental accessibility

Gluon GFFs on the lattice [2107.10368]

- …for the pion, nucleon, ρ meson, Δ baryon
- …for an ensemble with $M_\pi = 450\ \mathrm{MeV}$
- **And:** spatial densities of energy, pressure, shear forces

Progress on total GFFs

- …for the nucleon, pion
- …for an ensemble with $M_\pi = 170\ \mathrm{MeV}$
- (Very) preliminary results

Gravitational form factors (GFFs)

$\{\ \} \equiv$ symmetrize e.g. $a^{\{\mu}b^{\nu\}} \equiv \frac{1}{2}(a^\mu b^\nu + a^\nu b^\mu)$

For (symmetric) EMT, $T^{\{\mu\nu\}} = T_g^{\{\mu\nu\}} + \sum_q T_q^{\{\mu\nu\}}$

$\overleftrightarrow{D} = (\overrightarrow{D} - \overleftarrow{D})/2$

Gluons $T_g^{\{\mu\nu\}} = 2\,\mathrm{Tr}[G^{\alpha\{\mu}G^{\nu\}\alpha}]$ Quarks $T_q^{\{\mu\nu\}} = \bar{\psi}\gamma^{\{\mu}i\overleftrightarrow{D}^{\nu\}}\psi$

Not conserved

$\sum_q \bar{c}_q + \bar{c}_g = 0$

GFFs decompose hadronic matrix elements of T, e.g. for nucleon:

$$\left\langle N(p',s')\middle|T_{g,q}^{\{\mu\nu\}}\middle|N(p,s)\right\rangle = \bar{u}(p',s')\left[A_{g,q}(t)\,\gamma^{\{\mu}P^{\nu\}} + B_{g,q}(t)\frac{i\,P^{\{\mu}\sigma^{\nu\}\rho}\Delta_\rho}{2M} + D_{g,q}(t)\frac{\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2}{4M} + \bar{c}_{g,q}(t)M g^{\mu\nu}\right]u(p,s)$$

$u, \bar{u}$ = Dirac spinors $\quad P = (p'+p)/2 \quad \Delta = p' - p \quad t = \Delta^2$

Physics:

$A_{q,g}(t)$ ~ momentum of constituents

→ Momentum fraction $A_{q,g}(0) = \langle x\rangle_{q,g}$

$J_{q,g}(t) = \frac{1}{2}(A_{q,g}(t) + B_{q,g}(t))$ ~ angular momentum

→ Total $J(0) = \frac{1}{2}$

$D_{q,g}(t)$ ~ pressure and shear forces

Total $D(0)$: "the last global unknown"

[from Polyakov & Schweitzer 1805.06596]

em:	$\partial_\mu J^\mu_{\rm em} = 0$	$\langle N'\vert J^\mu_{\bf em}\vert N\rangle$	$\longrightarrow$	$Q = 1.602176487(40)\times 10^{-19}$C $\mu = 2.792847356(23)\mu_N$
weak:	PCAC	$\langle N'\vert J^\mu_{\bf weak}\vert N\rangle$	$\longrightarrow$	$g_A = 1.2694(28)$ $g_p = 8.06(55)$
gravity:	$\partial_\mu T^{\mu\nu}_{\bf grav} = 0$	$\langle N'\vert T^{\mu\nu}_{\bf grav}\vert N\rangle$	$\longrightarrow$	$m = 938.272013(23)$ MeV$/c^2$ $J = \frac{1}{2}$ $\boldsymbol{D} = \boldsymbol{?}$

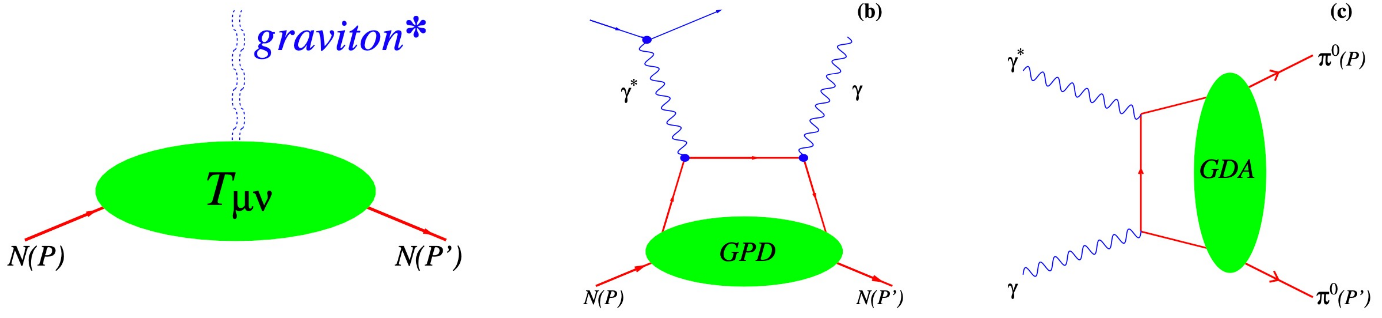
Table I. The global properties of the proton defined in terms of matrix elements of the conserved currents associated with respectively electromagnetic, weak, and gravitational interaction. Notice the weak currents include the partially conserved axial current, and g_A or g_p are strictly speaking defined in terms of transition matrix elements in the neutron β-decay or muon-capture. The values of the properties are from the particle data book [107] and [108] (for g_p) except for the unknown D-term.

Experimentally accessible?

[from Polyakov Schweitzer [1805.06596](https://arxiv.org/abs/1805.06596)]

Graviton colliders not presently feasible

but: GFFs ~ moments of (unpolarized) generalized parton distributions (GPDs), constrained by hard exclusive processes



Some extractions of quark GFFs from experiments:

JLAB: proton D term extracted from DVCS

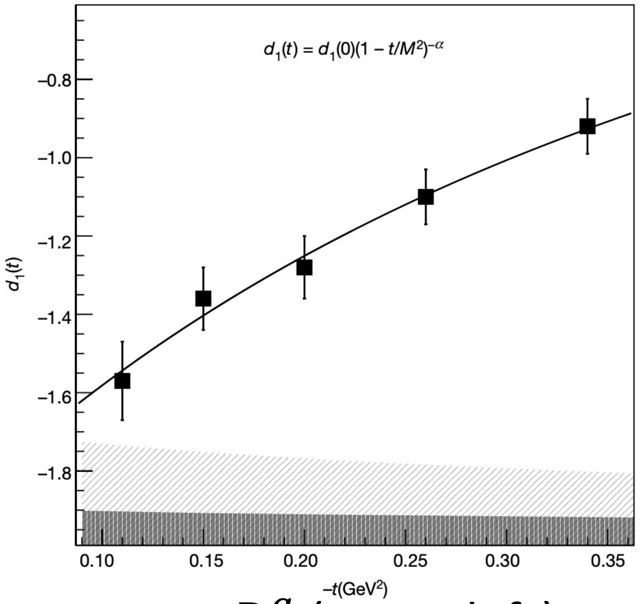
[Burkert Elouadrhiri Girod 2018]

Belle: pion GFFs extracted from $\gamma^*\gamma \to \pi^0\pi^0$

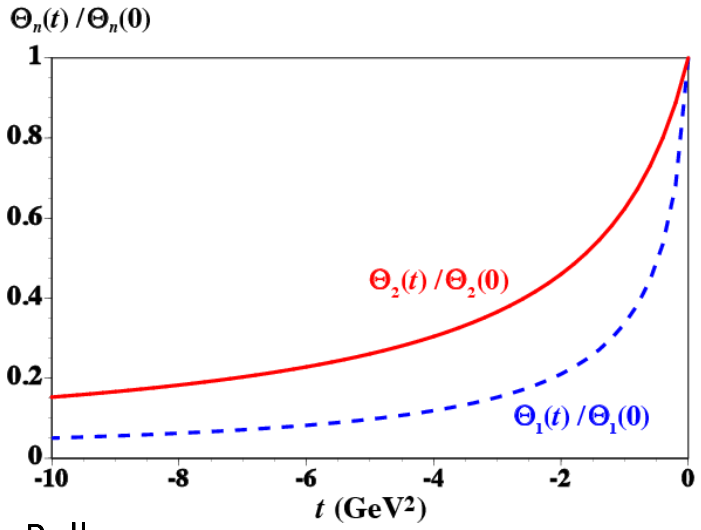
[Kumano Song Teryaev [1711.08088](https://arxiv.org/abs/1711.08088)]

No gluon GFFs ***yet***

Future: gluon GFFs from J/ψ and Υ leptoproduction at e.g. JLab, EIC



Proton D^q (up to defs)



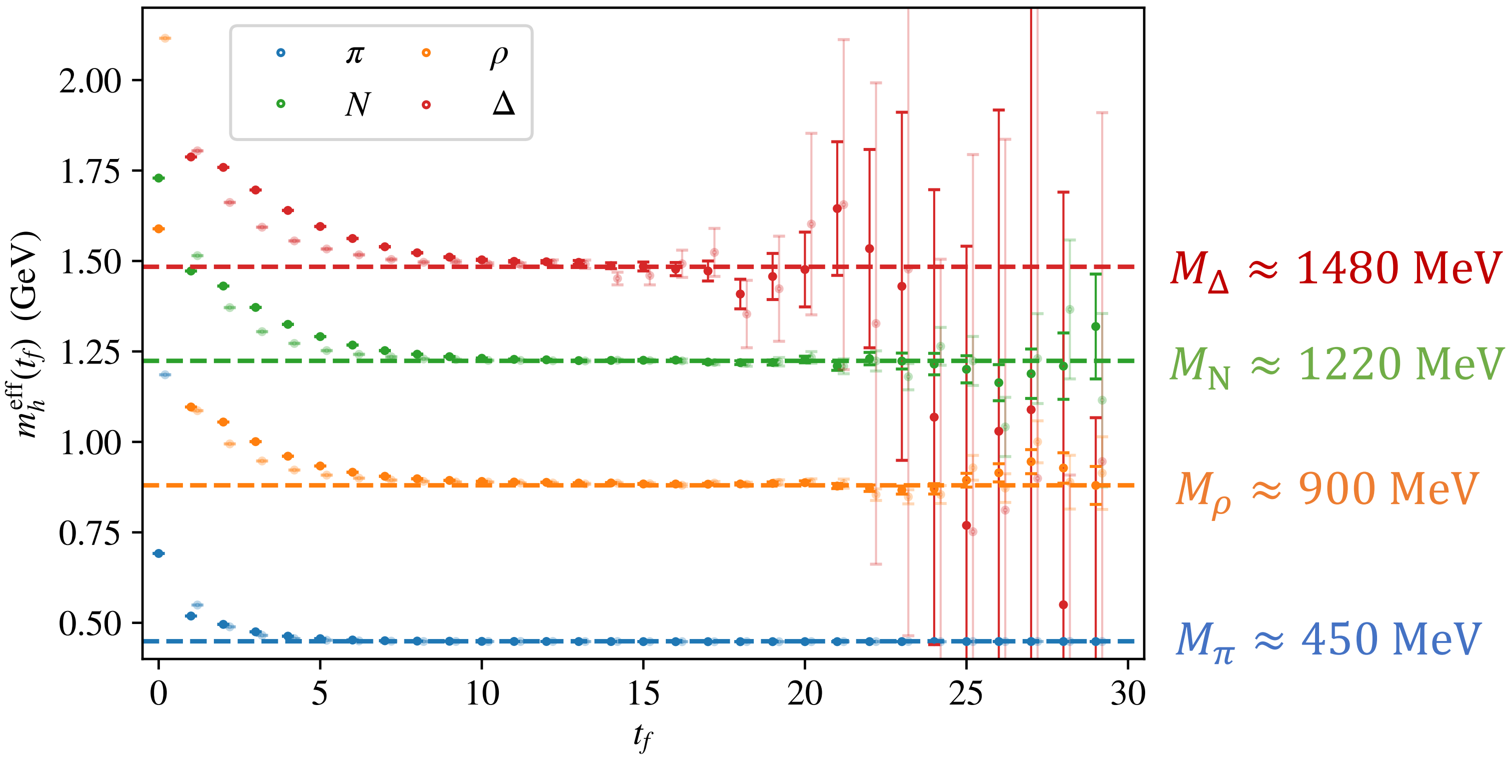
Belle

Pion D^q (up to defs)

Lattice calculation

Ensemble:

- $32^3{\times}96$ lattice, $M_\pi L \sim 8.5$
- Gauge action: Lüscher-Weisz
- Fermion action: 2+1 Wilson clover
  - Stout links, tree-level tadpole c_{sw}
- $a = 0.1167(16)$ fm
- $M_\pi = 450(5)$ MeV
  - $\rightarrow \rho, \Delta$ are stable
- 2820 configs, $\approx$ 235 sources/config
- Two source/sink smearings (SP, SS)



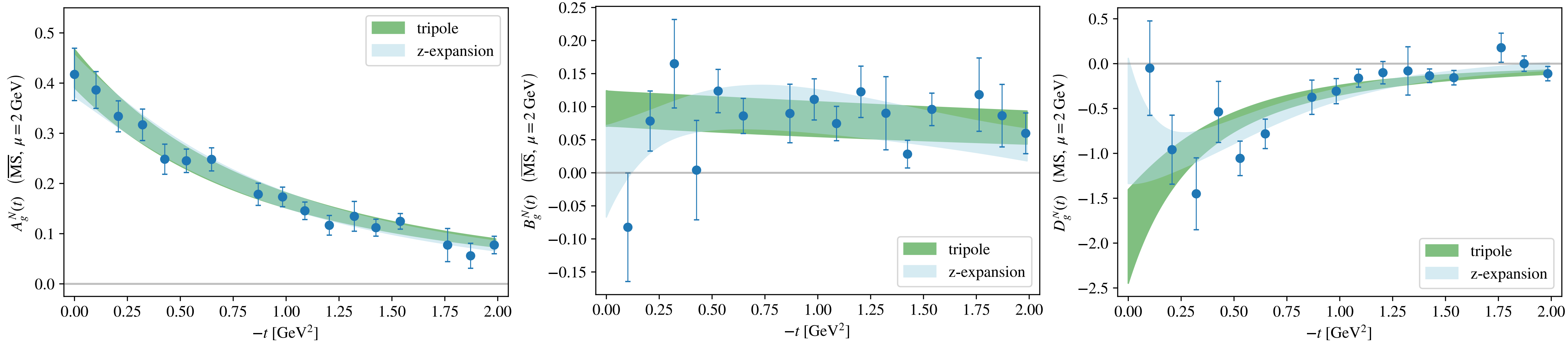
Compute glue GFFs only

- Neglect mixing w/ quark GFFs under renormalization – expected around few % level $\ll$ stat uncertainties

Results: nucleon

Not conserved $\sum_q \bar{c}_q + \bar{c}_g = 0$

$$\left\langle N(p',s') \middle| T_g^{\{\mu\nu\}} \middle| N(p,s) \right\rangle = \bar{u}(p',s') \left[A_g(t)\, \gamma^{\{\mu} P^{\nu\}} + B_g(t) \frac{i\, P^{\{\mu} \sigma^{\nu\}\rho} \Delta_\rho}{2M} + D_g(t) \frac{\Delta^\mu \Delta^\nu - g^{\mu\nu} \Delta^2}{4M} + \bar{c}_g(t) M g^{\mu\nu} \right] u(p,s)$$



Tripole:

$$\mathrm{G(t)} = \frac{\alpha}{\left(1 - \frac{t}{\Lambda^2}\right)^3}$$

(Modified) *z*-expansion:

$$\mathrm{G(t)} = \frac{1}{\left(1 - \frac{t}{\Lambda^2}\right)^3} \sum_{k=0}^{2} \alpha_k \, [z(t)]^k$$

$$z(t) = \frac{\sqrt{t_{\text{cut}} - t} - \sqrt{t_{\text{cut}} - t_0}}{\sqrt{t_{\text{cut}} - t} + \sqrt{t_{\text{cut}} - t_0}}$$

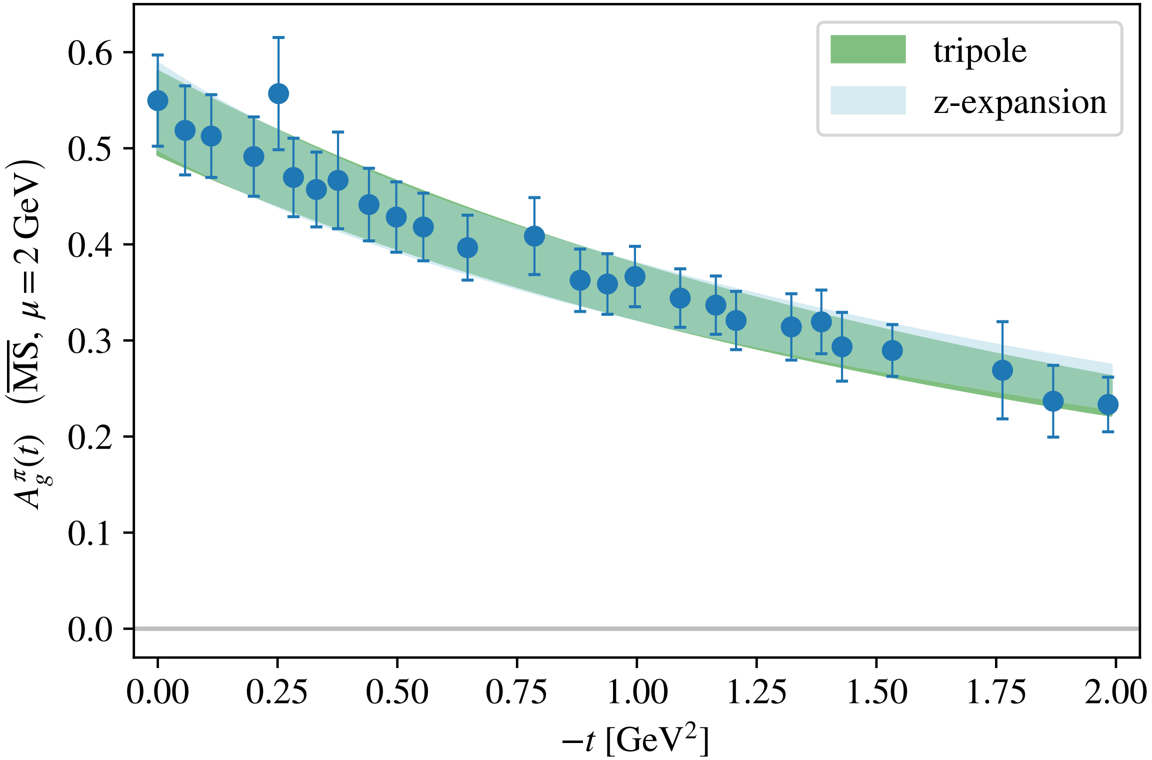
$$t_0 = t_{\text{cut}} \left(1 - \sqrt{1 + (2\ \text{GeV})^2 / t_{\text{cut}}}\right)$$

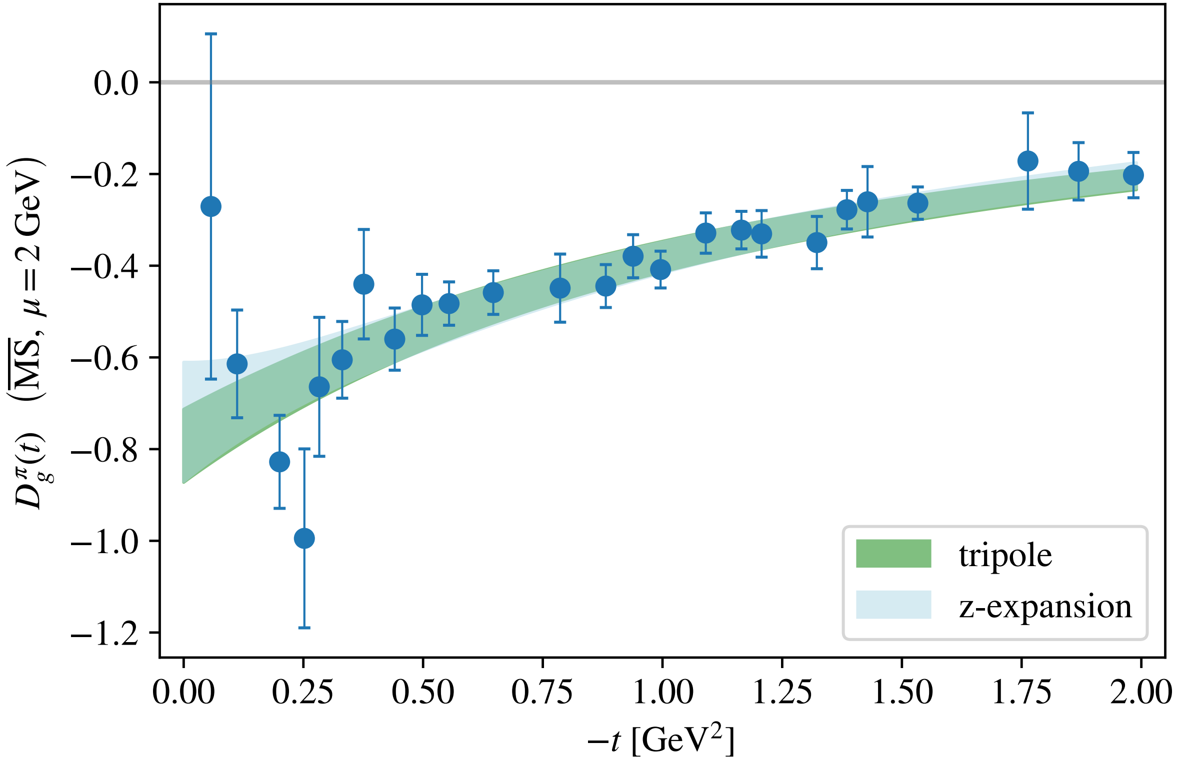
$$t_{\text{cut}} = 4 M_\pi^2$$

Results: pion

$\sim \langle x \rangle_g$ $[\sum_q A_{0q}(0) + A_{0g}(0) = 1]$

$$\left\langle \pi(p') \middle| T_g^{\{\mu\nu\}} \middle| \pi(p) \right\rangle = A_g(t)\, 2P^\mu P^\nu + D_g(t) \frac{1}{2}(\Delta^\mu \Delta^\nu - g^{\mu\nu}\Delta^2) + \bar{c}_g(t) 2M^2 g^{\mu\nu}$$





Results: ρ meson

$A(0) \sim \langle x \rangle_g$ i.e. $\sum_q A_{0q}(0) + A_{0g}(0) = 1$

$$\left\langle \rho(p',\lambda') \middle| T_g^{\{\mu\nu\}} \middle| \rho(p,\lambda) \right\rangle = E^*_{\alpha'}(p',\lambda') \Big\{ 2P^{\{\mu}P^{\nu\}} \Big[-g^{\alpha'\alpha} A_{0g}(t) + \frac{P^{\alpha'}P^{\alpha}}{M^2} A_{1g}(t) \Big] + \frac{1}{2}(\Delta^\mu \Delta^\nu - g^{\mu\nu}\Delta^2) \Big[g^{\alpha'\alpha} D_{0g}(t) + \frac{P^{\alpha'}P^{\alpha}}{M^2} D_{1g}(t) \Big]$$

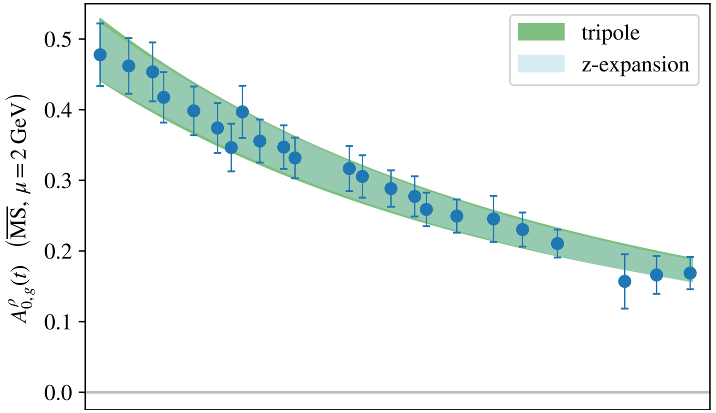
[Polarization vector]

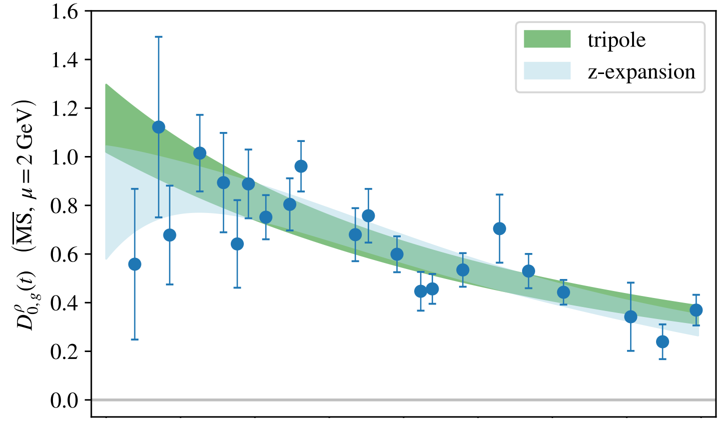
$$+ J_g(t)\, 8P^{\{\mu} g^{\nu\}\{\alpha'} P^{\alpha\}} + E_g(t) \left(g^{\alpha\{\mu} g^{\nu\}\alpha'} \Delta^2 - 2\, g^{\alpha'\{\mu} \Delta^{\nu\}} P^{\alpha} + 2\, g^{\alpha\{\mu} \Delta^{\nu\}} P^{\alpha'} - 4 g^{\mu\nu} P^{\alpha} P^{\alpha'} \right)$$

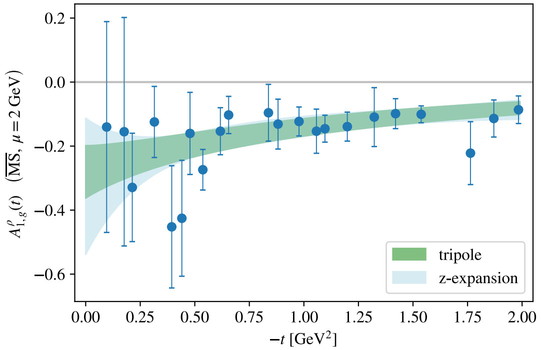
$\sum_q J_q(0) + J_g(0) = 1$

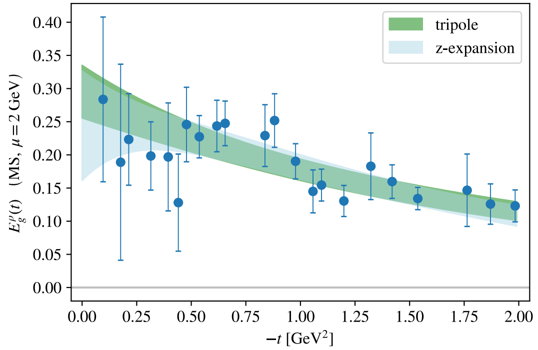
$$+ M^2 \left(2 g^{\alpha'\{\mu} g^{\nu\}\alpha} - \frac{1}{2} g^{\alpha\alpha'} g^{\mu\nu} \right) \bar{f}_g(t) + g^{\mu\nu} \left[g^{\alpha\alpha'} M^2 \bar{c}_{0g}(t) + P^{\alpha} P^{\alpha'} \bar{c}_{1g}(t) \right] \Big\} E^*_{\alpha}(p,\lambda)$$

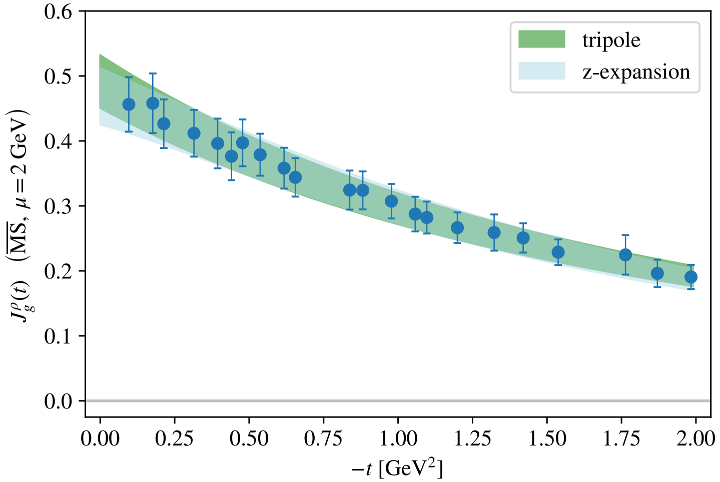
[Not conserved, $\sum_q \bar{f}_q + f_g = 0$]

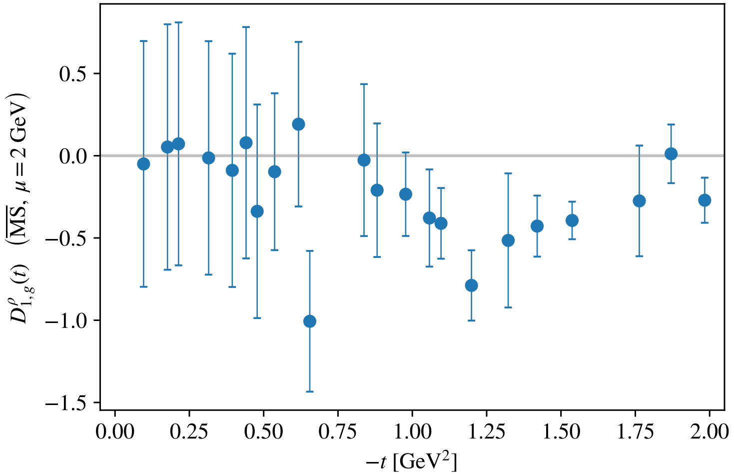


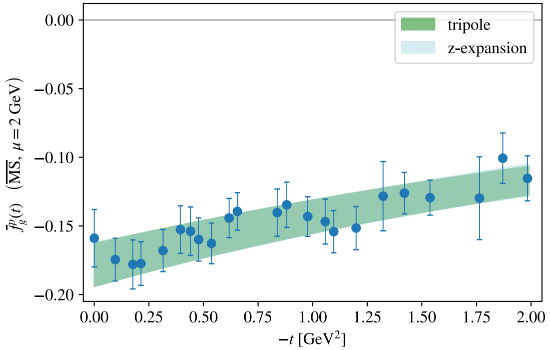












[conventions: Polyakov, Sun 1912.08749]

Results: Δ baryon

"$A(t)$" i.e. $A(0) \sim \langle x \rangle_g$

$$\langle \Delta(p',\xi') | T_g^{\mu\nu} | \Delta(p,\xi) \rangle = \bar{u}_{\alpha'}(p',\xi') \Bigg[\frac{P^\mu P^\nu}{M} \left(-g^{\alpha\alpha'} F_{10}^g(t) + \frac{\Delta^\alpha \Delta^{\alpha'}}{2M} F_{11}^g(t) \right) + \frac{\Delta^\mu \Delta^\nu - g^{\mu\nu}\Delta^2}{4M} \left(-g^{\alpha\alpha'} F_{20}^g(t) + \frac{\Delta^\alpha \Delta^{\alpha'}}{2M} F_{21}^g(t) \right)$$

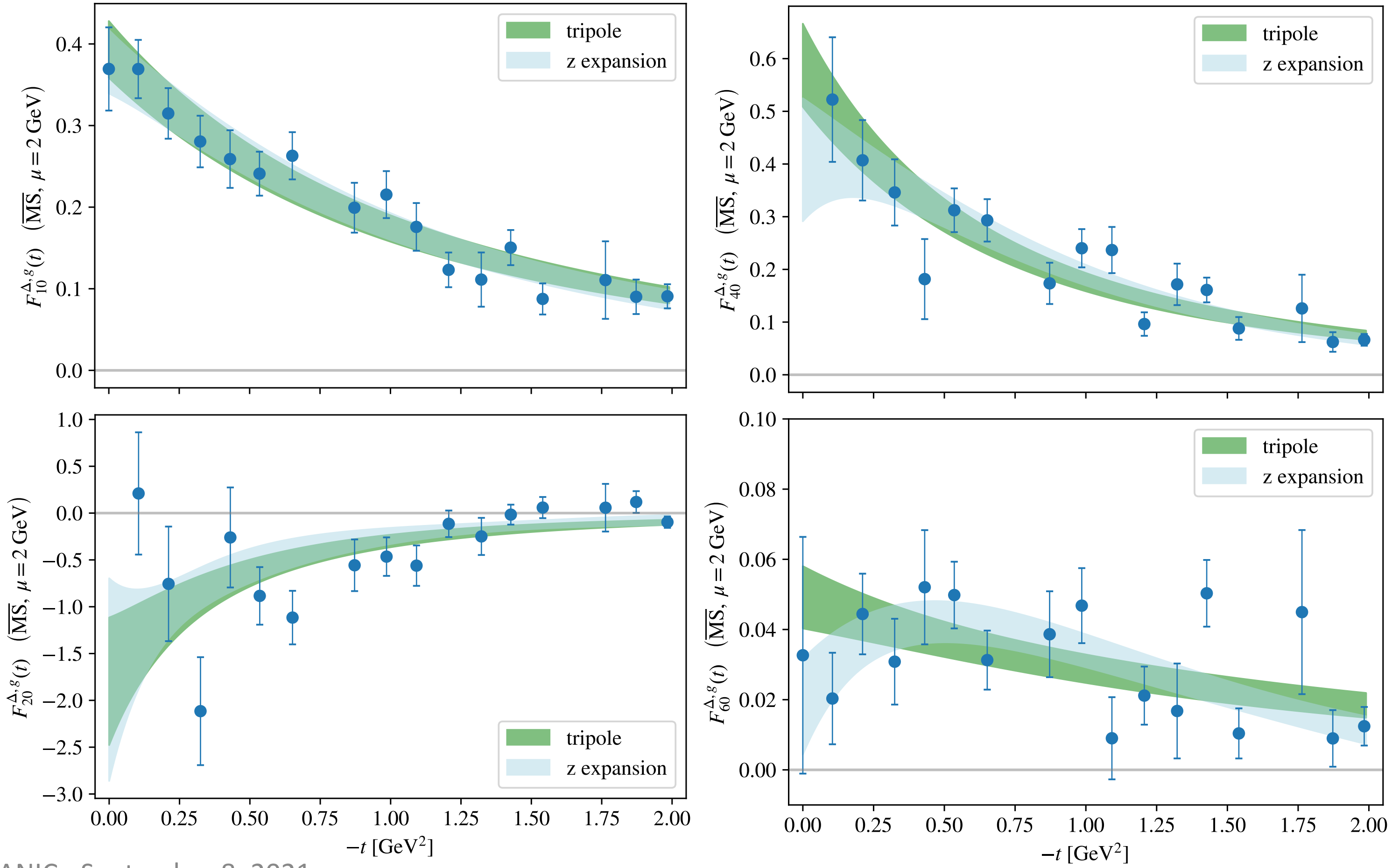
$$+ M g^{\mu\nu} \left(-g^{\alpha\alpha'} F_{30}^g(t) + \frac{\Delta^\alpha \Delta^{\alpha'}}{2M} F_{31}^g(t) \right) + \frac{i P^{\{\mu} \sigma^{\nu\}\rho} \Delta_\rho}{M} \left(-g^{\alpha'\alpha} F_{40}^g(t) + \frac{\Delta^{\alpha'} \Delta^\alpha}{2M^2} F_{41}^g(t) \right)$$

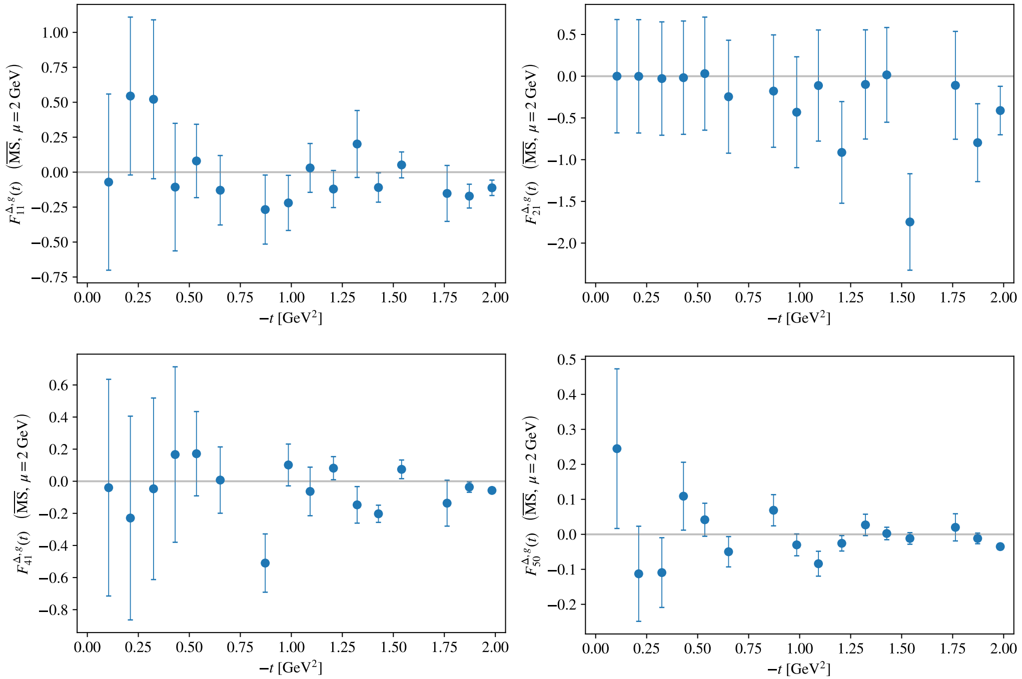
$$+ \frac{2}{M} \left(\Delta^{\{\mu} g^{\nu\}\{\alpha'} \Delta^{\alpha\}} - g^{\mu\nu} \Delta^\alpha \Delta^{\alpha'} - g^{\alpha'\{\mu} g^{\nu\}\alpha} \Delta^2 \right) F_{50}^g(t) - 2 g^{\alpha'\{\mu} g^{\nu\}\alpha} M F_{60}^g(t) \Bigg] u_\alpha(p,\xi)$$

[Rarita-Schwinger spinvector]

"$J(t)$", $\sum_q F_{40}^q + F_{40}^g = 3/2$

[Not conserved]





[conventions: Kim, Sun 2011.00292]

Energy, pressure, and shear force densities

Idea:

Fourier transform to get spatial EMT density $T_{\mu\nu}(r) = \mathrm{FT}[\, \mathrm{T}_{\mu\nu}(\Delta)\,]$

Identify: $T_{tt}(r) = \epsilon(r)$

$$T_{ij}(r) = \left(\frac{r_i r_j}{r^2} - \frac{1}{d}\delta_{ij}\right) s(r) + \delta_{ij} p(r)$$

→ Spatial densities of energy $\epsilon(r)$, pressure $p(r)$, shear forces $s(r)$

Complication 1: frame dependence. Consider three frames:

3D Breit frame (BF3): $\Delta^0 = 0, \mathbf{P} = \mathbf{0}$

"Traditional" frame, but recent work casts doubt on interpretation as spatial density [see e.g. Panteleeva Polyakov 2102.10902, Freese Miller 2102.01683, Jaffe 2010.15887, Lorce 2007.05318, Lorce Moutarde Trawinski 1810.09837 etc.]

2D Breit frame (BF2)

Infinite momentum frame (IMF): $\mathbf{\Delta} \cdot \mathbf{P} = 0$, $P_z \to \infty$

$$T^{\mu\nu}_{i,\mathrm{BF3}}(r) = \int \frac{d^3\Delta e^{-i\mathbf{\Delta}\cdot\mathbf{r}}}{2P^0(2\pi)^3} \langle h(p,s)|\, T^{\mu\nu}_i \,|h(p',s')\rangle\big|_{\mathbf{P}=0}$$

$$T^{\mu\nu}_{i,\mathrm{BF2}}(r) = \int \frac{d^2\Delta_\perp e^{-i\mathbf{\Delta}_\perp\cdot\mathbf{r}}}{2P^0(2\pi)^2} \langle h(p,s)|\, T^{\mu\nu}_i \,|h(p',s')\rangle\big|_{\mathbf{P}=0}$$

$$T^{\mu\nu}_{i,\mathrm{IMF}}(r) = \int \frac{d^2\Delta_\perp e^{-i\mathbf{\Delta}_\perp\cdot\mathbf{r}}}{2P^0(2\pi)^2} \langle h(p,s)|\, T^{\mu\nu}_i \,|h(p',s')\rangle\big|^{P_z\to\infty}_{\mathbf{P}\cdot\mathbf{\Delta}=0}$$

Different identifications of $\epsilon(r)$, $p(r)$, $s(r)$ with GFFs in each frame

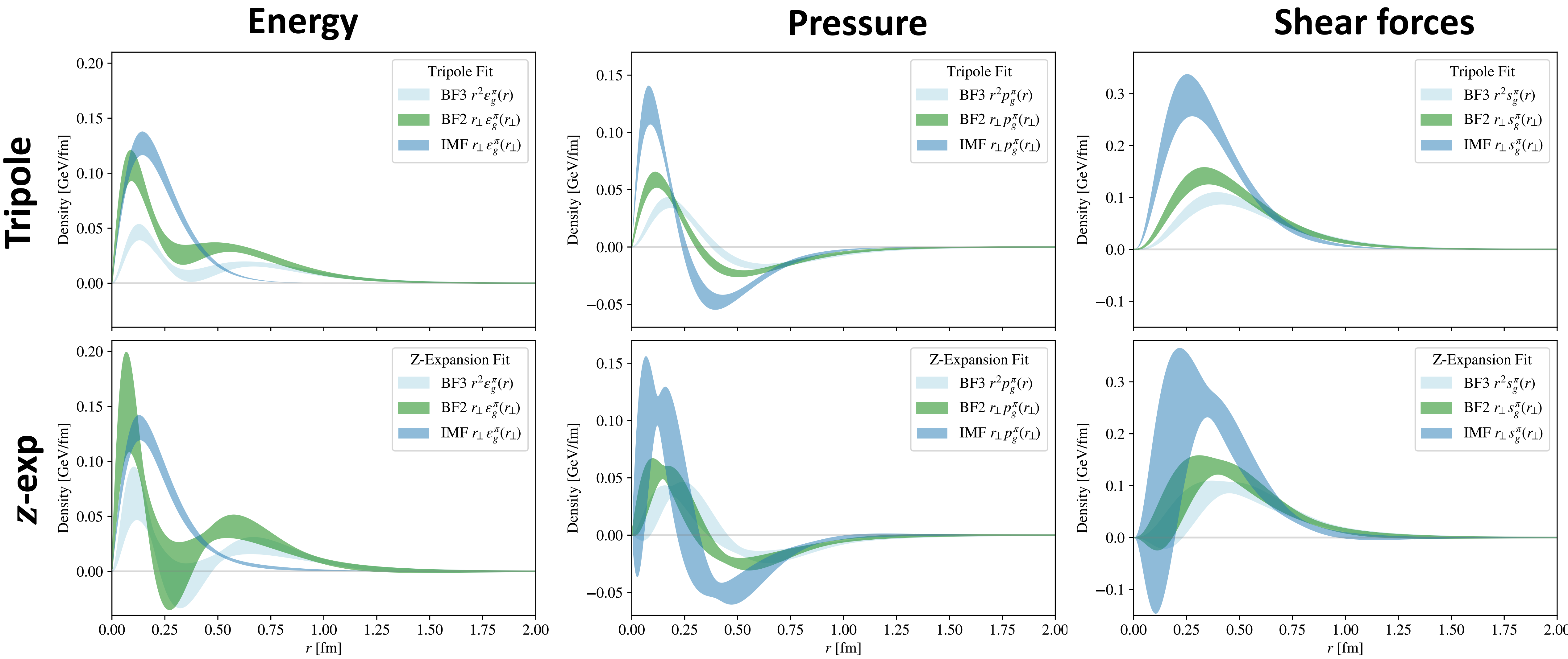
Complication 2: ρ, Δ not spherically symmetric → monopole and quadrupole densities

Complication 3: No trace GFFs $\sim \bar{c}$; not all GFFs fit for ρ, Δ

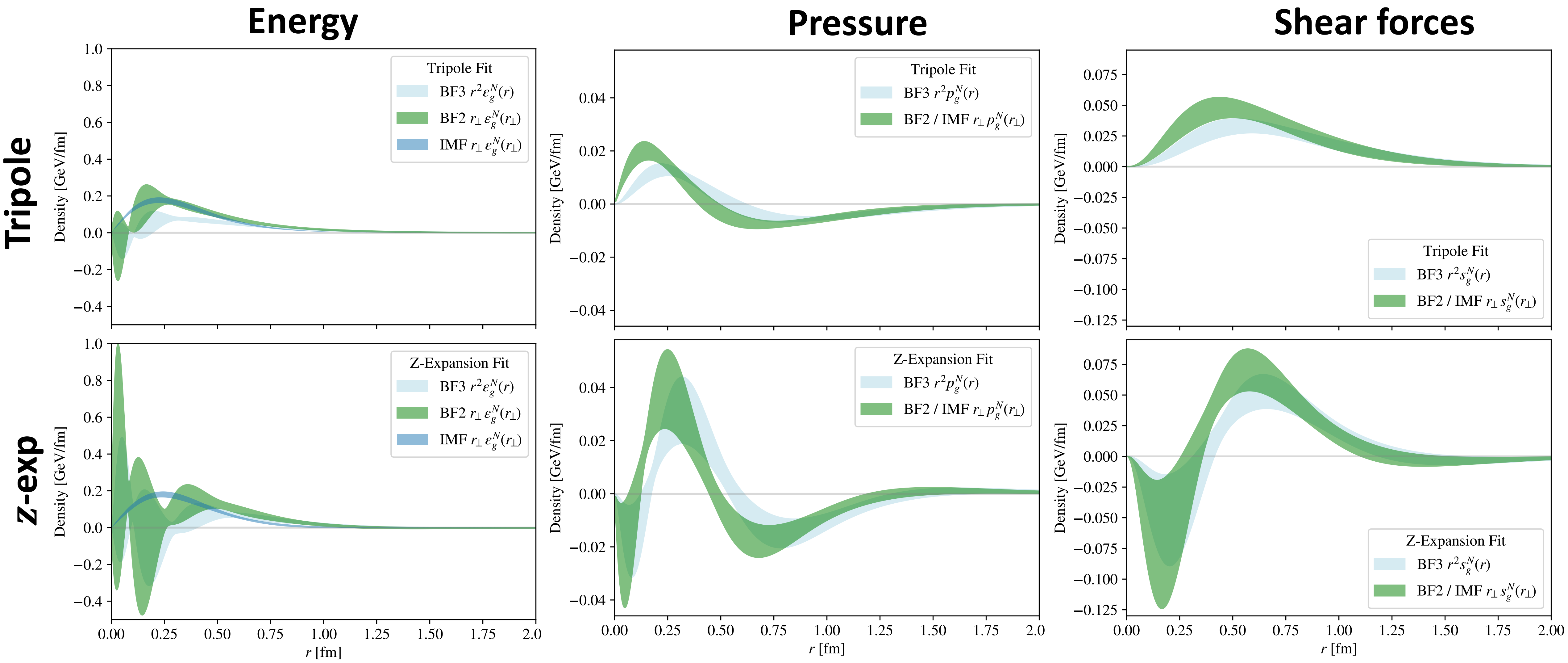
→ Partial densities

See [2107.10368] for details, expressions

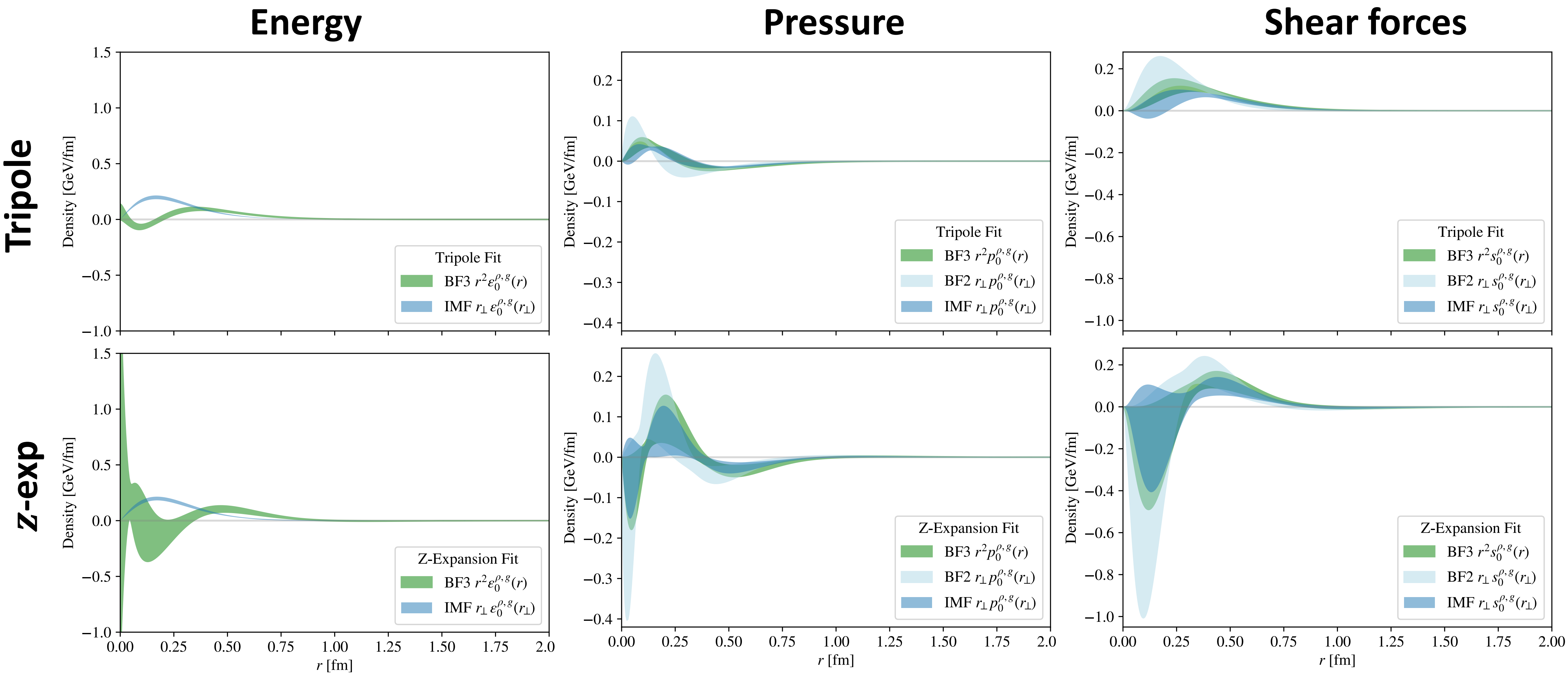
Results: pion densities



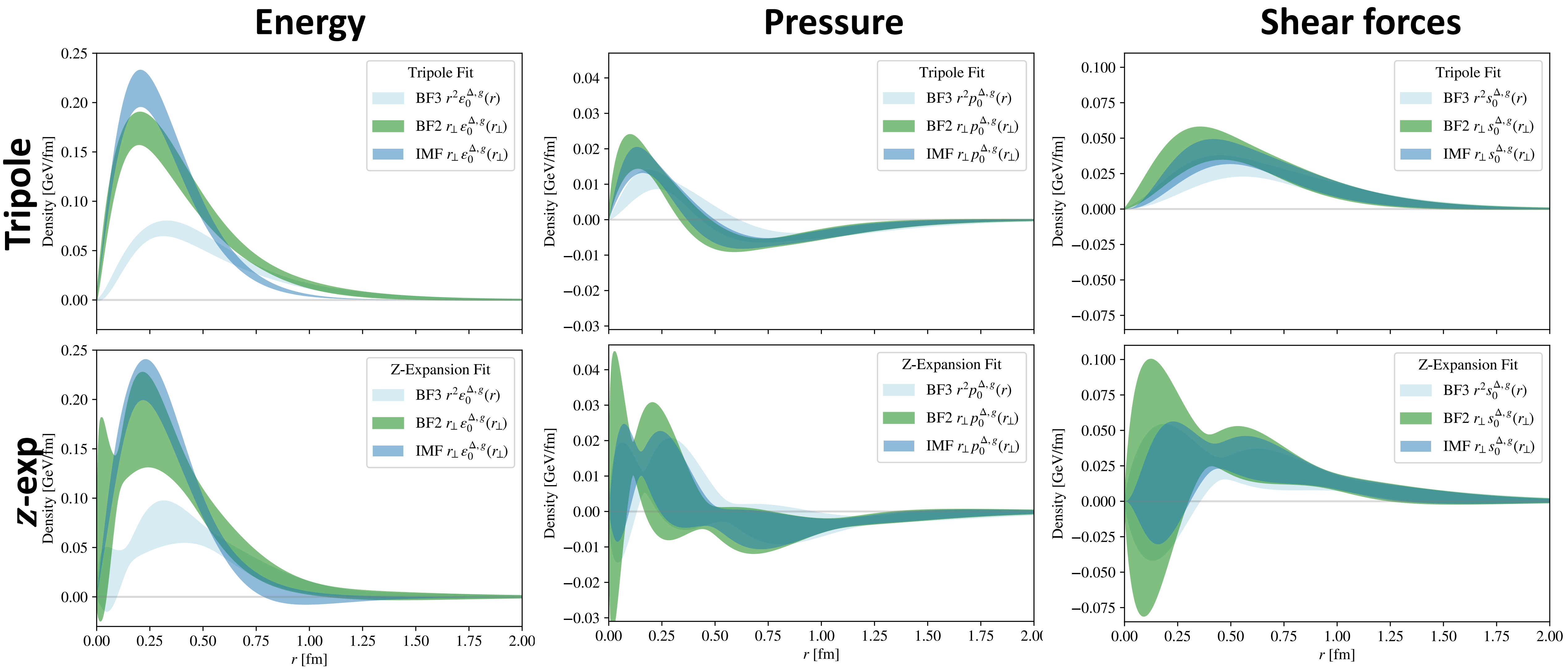
Results: nucleon densities



Results: (partial) ρ monopole densities



Results: (partial) Δ monopole densities



In progress

Compute both quark and glue GFFs on a different ensemble

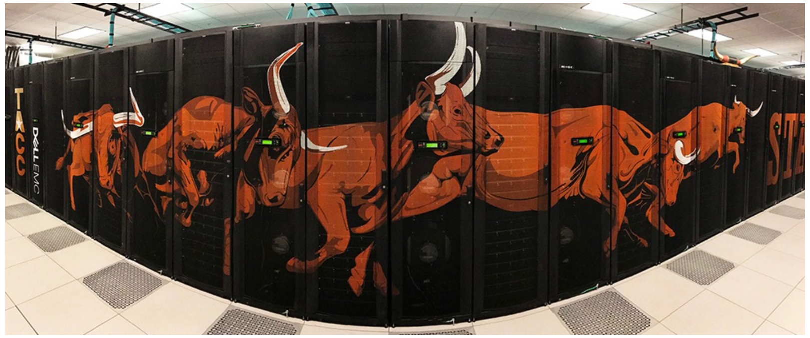
- Quantify systematics in glue GFFs due to $a \neq 0$, unphysical M_π, mixing with quark GFFs
- Compute total GFFs. Non-conserved trace GFFs cancel → can compute full, non-partial densities

Ensemble [“a091m170”]

- Gauge action: Tree-level tadpole-improved Symanzik
- Fermion action: 2+1 Wilson clover, stout links
- $M_\pi = 170\ \mathrm{MeV}$
- $a = 0.091\ \mathrm{fm}$ (from w_0)
- $48^3{\times}96$
- ρ, Δ unstable → N, π only

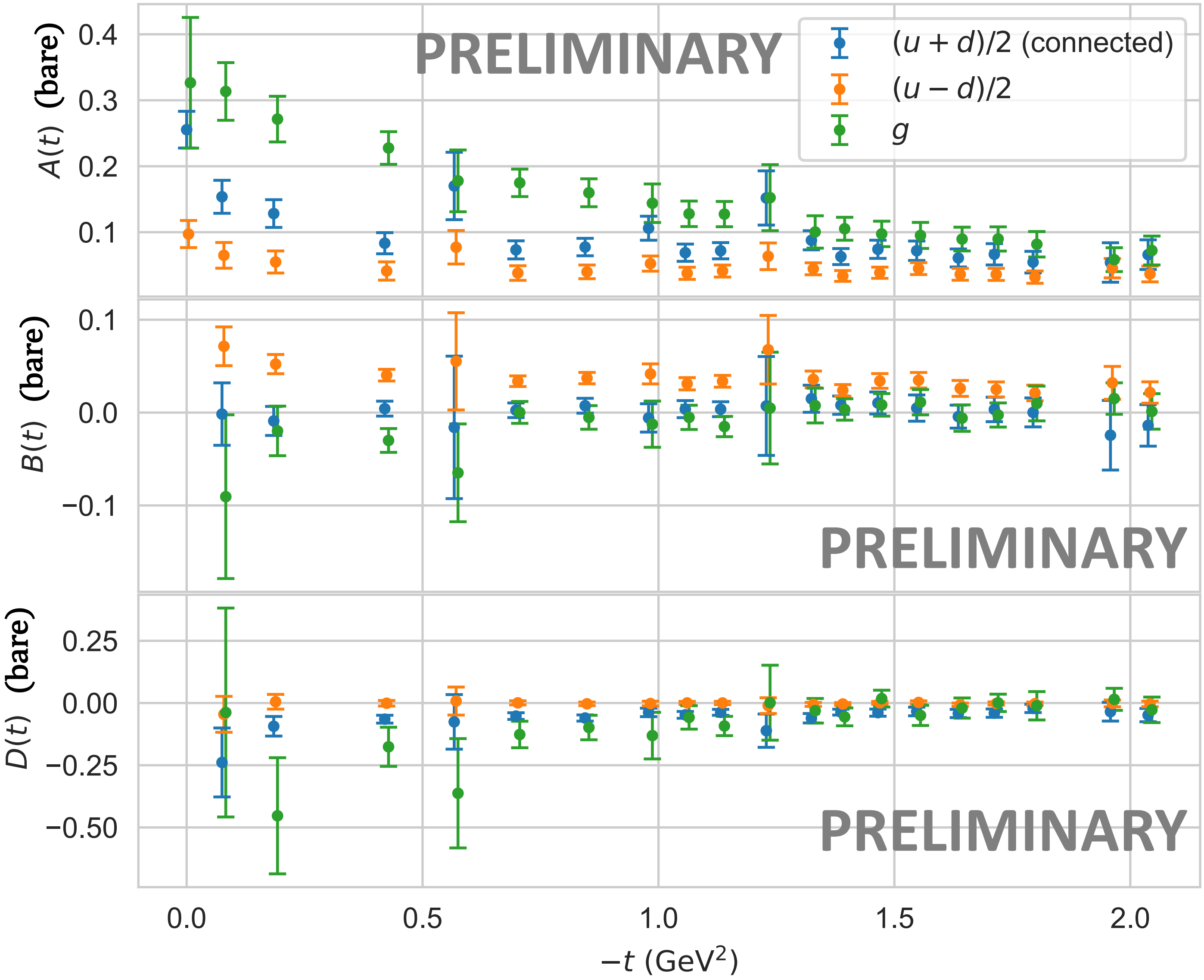
TODO:

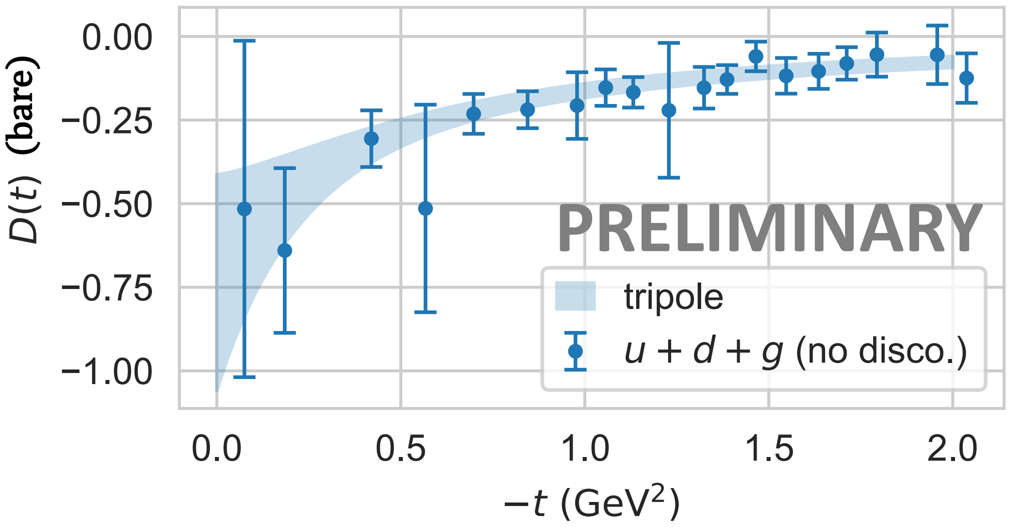
- Disconnected diagrams for quarks
- Non-perturbative renormalization
- Better treatment of excited state contamination

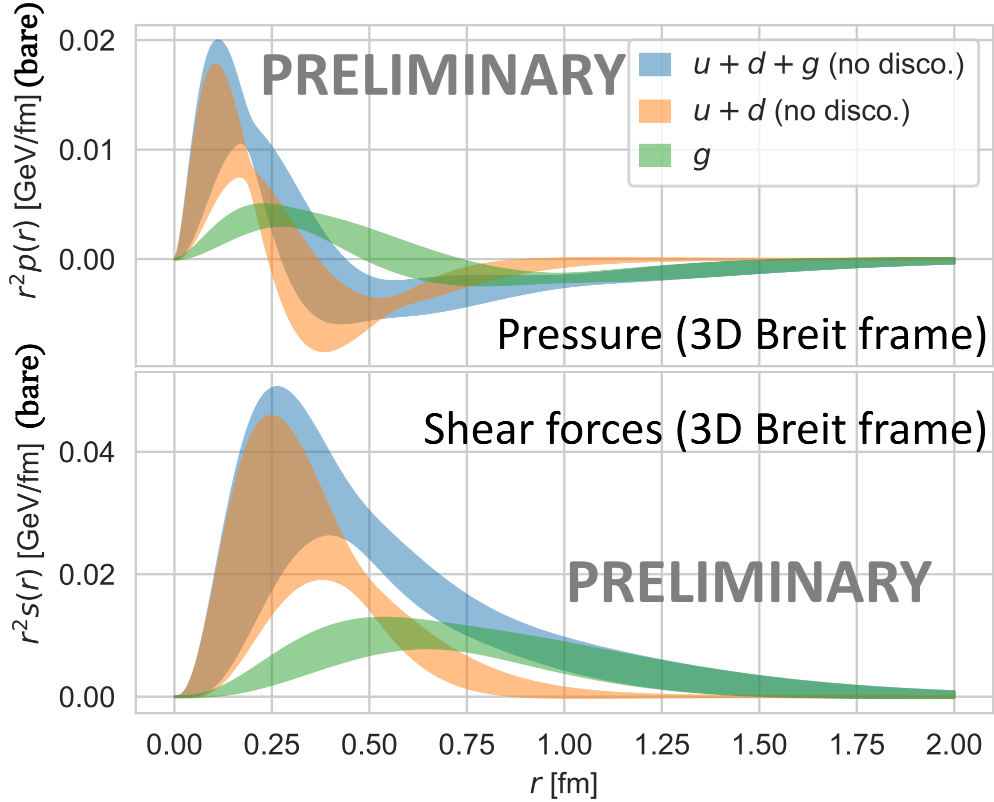


(Very preliminary) results

NOTE: Results are ~ bare GFFs







Conclusions/upcoming:

GFFs encode fundamental, global properties of hadrons

…including some that are presently only poorly constrained

GPDs are targets for near-future experiments

→ Lattice results on GFFs can inform kinematic regimes to target

→ Lattice results are necessary to test against experimental results

Computed π, N, ρ, Δ gluon GFFs

First-of-kind results for ρ, Δ

Calculations with higher stats, different ensembles necessary

Study of unstable ρ, Δ at physical masses harder, requires finite-volume Luscher method

New lattice calculation of quark+glue GFFs ongoing, early results promising.

Sketch of calculation [2107.10368]

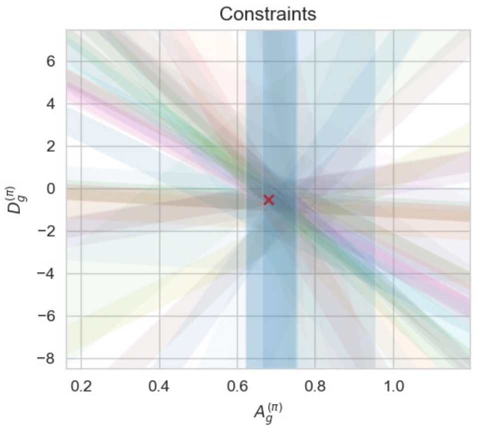
Compute hadronic two-point, three-point functions

Construct ratios of 3pts/2pts to isolate matrix element

$$R_{ss'}(p,p';\tau,t_f) = \frac{C^{3\text{pt}}_{ss'}(p,p';t_f,\tau)}{C^{2\text{pt}}_{ss'}(p',t_f)}\sqrt{\frac{C^{2\text{pt}}_{ss}(p,t_f-\tau)}{C^{2\text{pt}}_{s's'}(p',t_f-\tau)}\frac{C^{2\text{pt}}_{s's'}(p',t_f)}{C^{2\text{pt}}_{ss}(p,t_f)}\frac{C^{2\text{pt}}_{s's'}(p',\tau)}{C^{2\text{pt}}_{ss}(p,\tau)}}$$

$$\xrightarrow{t_f\gg\tau\gg 0} (\text{extra kinematic factors})\,\langle h(p',s')|T^g|h(p,s)\rangle$$

$$= (\text{kinematic coeffs})\cdot(\text{GFFs})(t)$$



Fit to extract GFFs

Result: GFFs for discrete values of t

Fit GFFs to model functions

[To extrapolate to $t=0$, and to do FTs for densities]

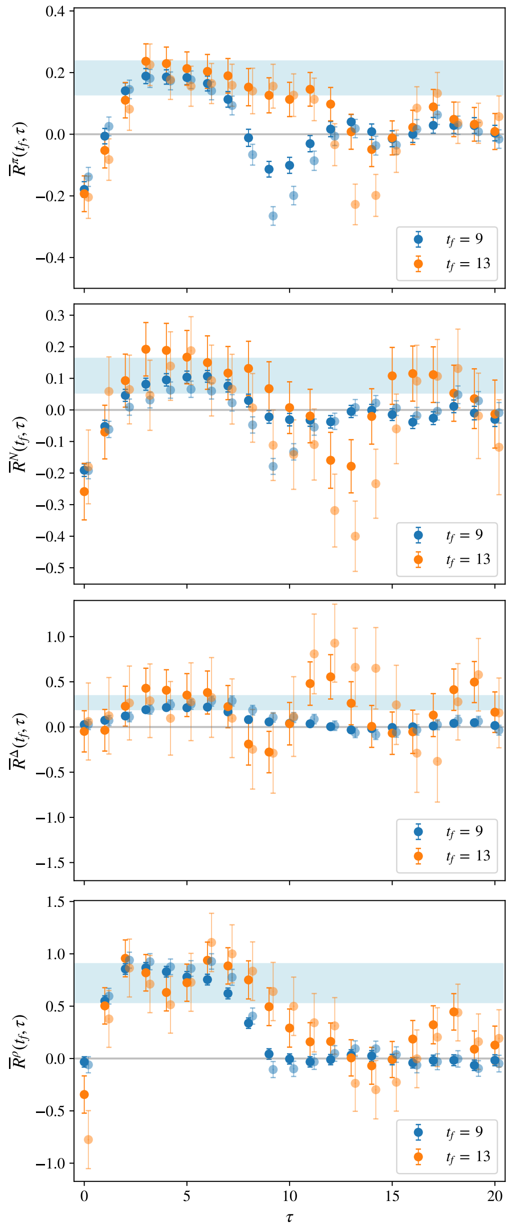
Tripole $\quad G(t) \sim \frac{\alpha}{(1-t/\Lambda^2)^3}$

Modified z-expansion $\quad G(t) \sim \frac{1}{(1-t/\Lambda^2)^3}\sum_{k=0}^{k_{\max}=2}\alpha_k[z(t)]^k$

$$z(t) = \frac{\sqrt{t_{\text{cut}}-t}-\sqrt{t_{\text{cut}}-t_0}}{\sqrt{t_{\text{cut}}-t}+\sqrt{t_{\text{cut}}-t_0}} \qquad t_{\text{cut}} = 4M_\pi^2 \qquad t_0 = t_{\text{cut}}\left(1-\sqrt{1+(2\text{ GeV}^2)/t_{\text{cut}}}\right)$$

Example ratios

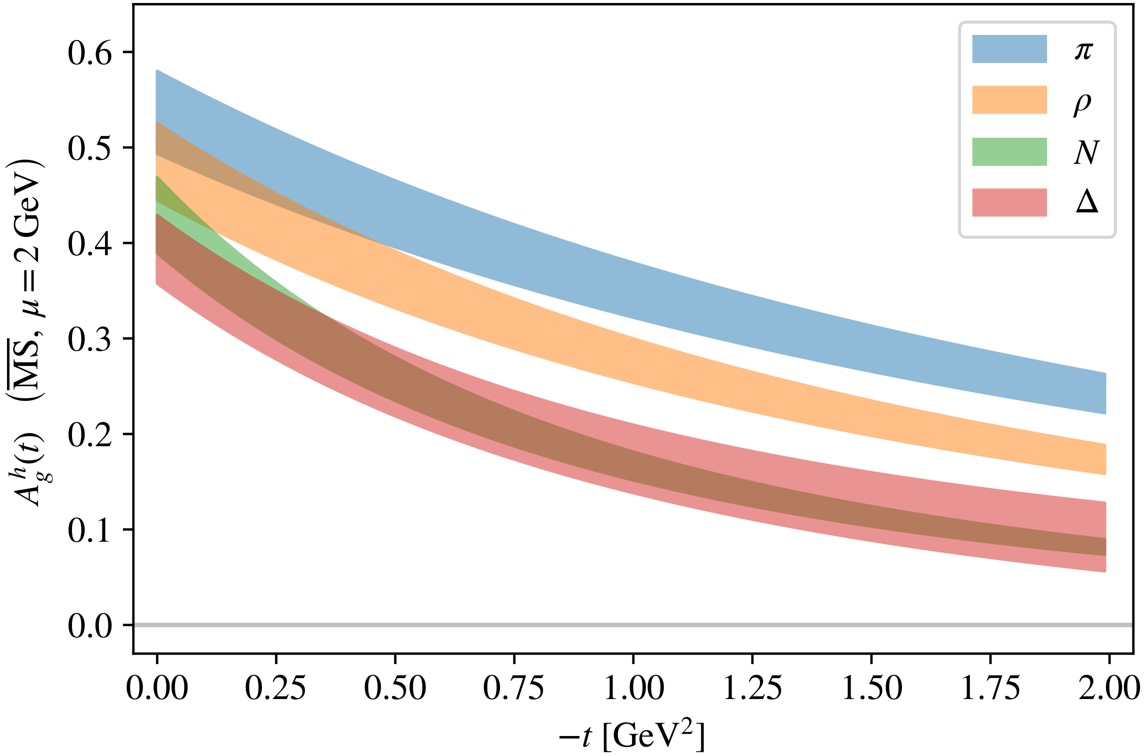
Note: plateaus

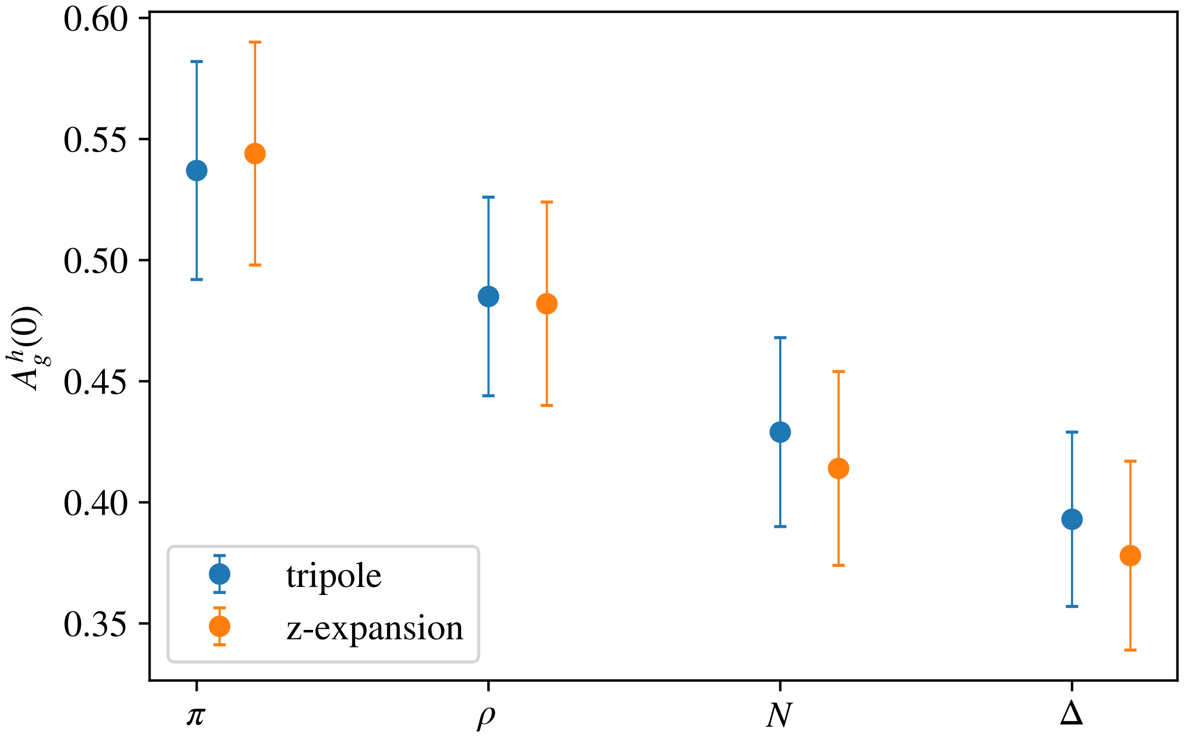


Comparison: glue momentum fraction

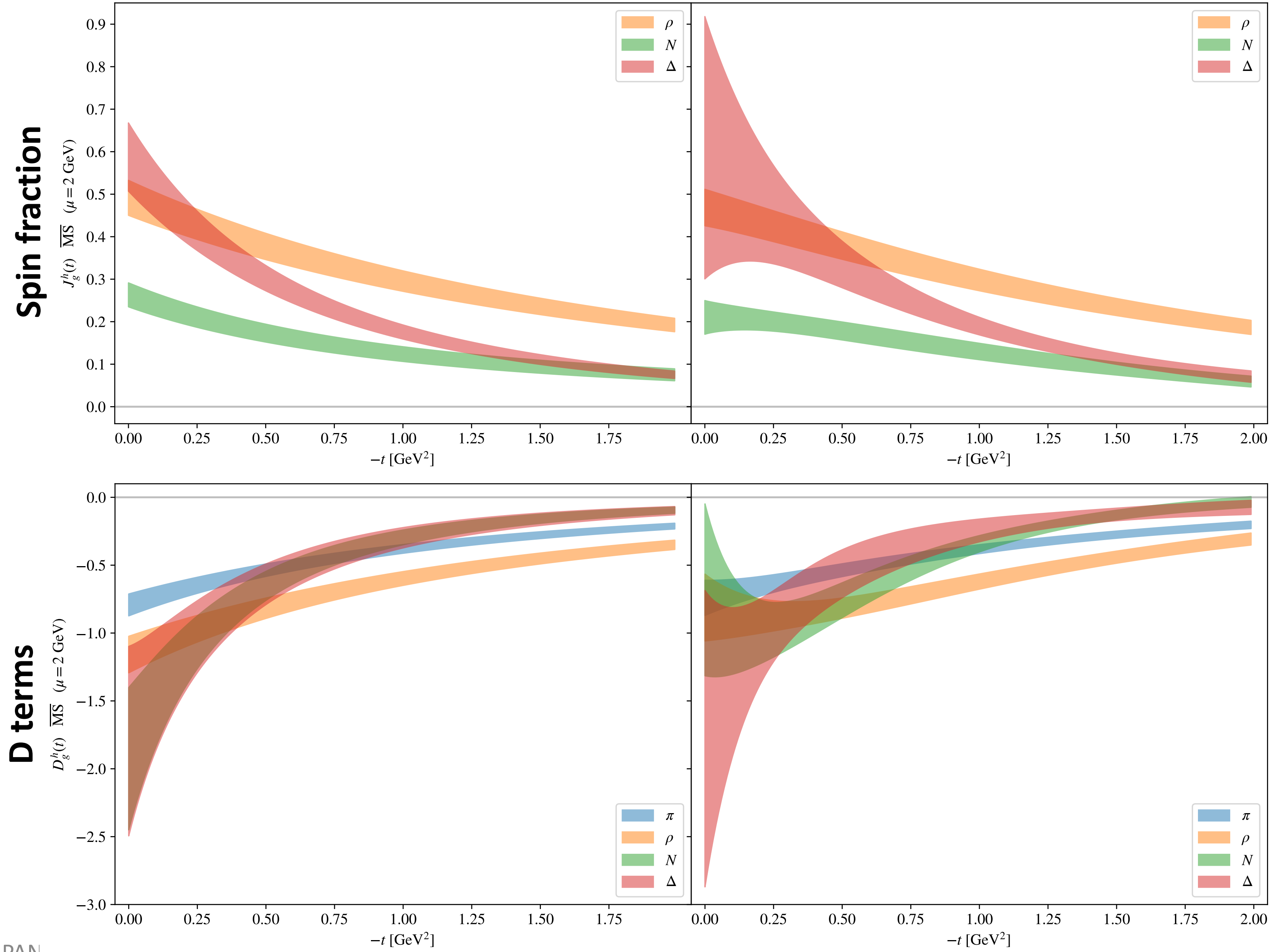
Tripole and z-expansion $A(t)$ same w/in error

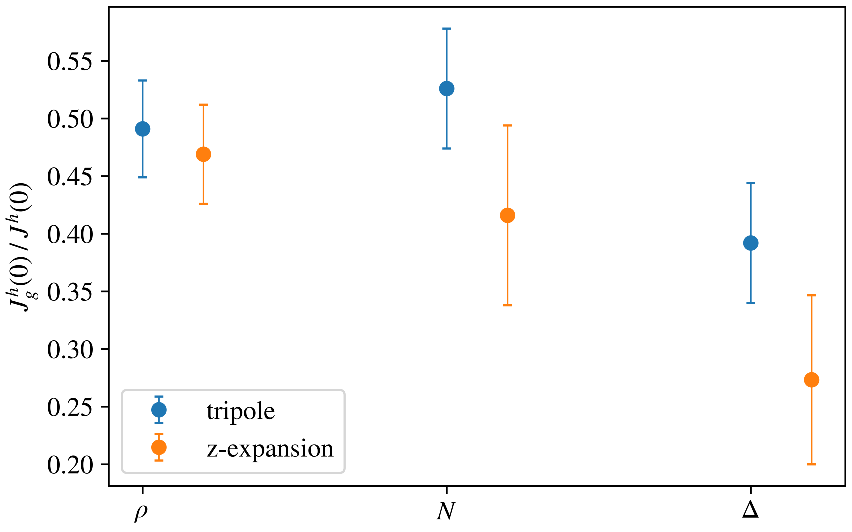
→ Little model dependence





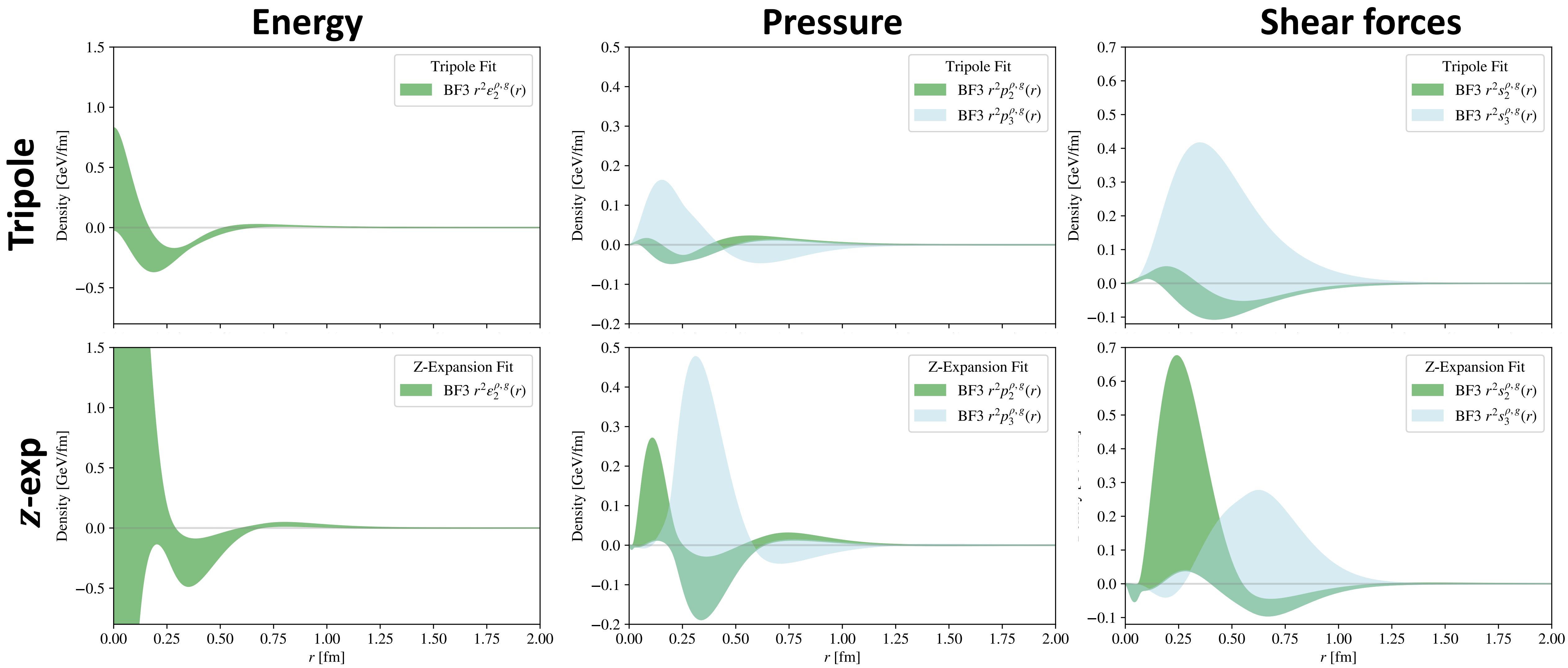
Comparison: glue spin fraction, D terms





Strong model dependence: nonmonotonic z-exp fits vs monotonic-by-construction tripole fits

Results: (partial) ρ quadrupole densities



Results: (partial) Δ quadrupole densities