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# Overview on progress in theoretical nuclear physics

Arthur B. McDonald Canadian Astroparticle Physics Research Institute

Jason D. Holt

Scientist, Theory Department PANIC 2021, Lisbon (I wish...) September 10, 2021









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## The Next Big Discovery? 0vββ-decay

Neutrino own antiparticle  $\iff 0\nu\beta\beta$  decay



#### **Tremendous impact on BSM physics:**

Lepton-number violating process

Majorana character of neutrino

**Absolute neutrino mass scale** 

$$\left(T_{1/2}^{0\nu\beta\beta}\right)^{-1} = G^{0\nu} \left|M^{0\nu}\right|^2 \left\langle m_{\beta\beta} \right\rangle^2 \left\langle m_{\beta\beta} \right\rangle = \left|\sum_{i=1}^3 U_{ei} m_i\right|$$

NME not observable: must be calculated

#### Nature of Neutrino: 0vββ Decay

## **% TRIUMF**



#### Spread from Nuclear Matrix Element; bands do not represent rigorous uncertainties

#### **Status of** 0vββ-**Decay Matrix Elements**

All calculations to date from extrapolated phenomenological models; large spread in mes of  $10^{10}$ 



All models missing essential physi $\mathfrak{A}^{0\nu}$   $0\nu\beta\beta$ Impossible to assign rigorous uncertainties

#### **Status of** 0vββ-**Decay Matrix Elements**

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Rethink approach to NME calculations...  $M^{0\nu} 0\nu\beta\beta$ Next-generation NMEs needed for next-generation searches!

#### **Dark Matter Direct Detection**



CDMS WIMPs and Neutrons scatter from the Atomic Nucleus Photons and Electrons scatter from the Atomic Electron production at colliders Observation of nuclear recoil

Direct detection:  $X \operatorname{SM} \to X \operatorname{SM}$ 

Leading candidates: neutralinos, ...?

Couple to scalar and axial-vector currents in atomic nuclei

#### **Dark Matter Direct Detection**

Exclusion plots for WIMP-nucleon total cross section require nuclear structure



Differential cross section: compare results from different target nuclei

 $\frac{\mathrm{d}\sigma}{\mathrm{d}p^2} = \frac{8G_F^2}{(2J_i+1)v^2} S_A(p)$ 

Structure functions required from nuclear theory

#### **Obligatory Super Technical Theory Slide**

$$H\psi_n = E_n\psi_n$$



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#### **Chronological Reach of Ab Initio Theory**

Moore's law: exponential growth in computing power

Methods for light nuclei (QMC, NCSM) scale exponentially with mass



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## Chronological Reach of Ab Initio Theory

Moore's law: exponential growth in computing power

2021: A>(>)100

Methods for light nuclei (QMC, NCSM) scale exponentially with mass



#### **Recent Breadth of Ab Initio Theory**

- Nuclear forces, electroweak physics
- Nuclear many-body problem

$$H\psi_n = E_n\psi_n$$



#### **Obligatory Super Technical Theory Slide**

$$H\psi_n = E_n\psi_n$$



#### **Today: Global Ab Initio Calculations**

- Nuclear forces, electroweak physics
- Nuclear many-body problem

$$H\psi_n = E_n\psi_n$$



#### **VS-IMSRG for Atomic Nuclei**



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#### **Dripline Predictions to Fe Isotopes**

#### First predictions of proton and neutron driplines from first principles



Known drip lines largely predicted within uncertainties (artifacts at shell closures) Provide ab initio predictions for neutron-rich region

### **Nuclei in BSM Physics Searches**

- Nuclear forces, electroweak physics
- Nuclear many-body problem



#### **VS-IMSRG for Atomic Nuclei**



## **∂**TRIUMF

#### **Ab Initio Calculations of Heavy Nuclei**

Improvements in (clever) storage of 3N MEs greatly expands reach of ab initio theory

Previous limit:  $e_1 + e_2 + e_3 \le E_{3max} = 18$ 



## Convergence in Heavy Nuclei: <sup>132</sup>Sn

Improvements in (clever) storage of 3N MEs greatly expands reach of ab initio theory

Previous limit: E<sub>3max</sub>=18 not converged



First converged ground-state properties of <sup>132</sup>Sn Opens path to heavy nuclei



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First converged ground-state properties of <sup>132</sup>Sn

#### **Opens path to heavy nuclei!**

### **Towards Neutrinoless Double Beta Decay**

- Nuclear forces, electroweak physics
- Nuclear many-body problem



#### **VS-IMSRG for Atomic Nuclei**



## **TRIUMF** Laser Spectroscopy: Charge Radii Across Chains

Study odd-even staggering of charge radii across Cu isotopic chain



Cu isotopes, odd-even staggering well reproduced

Ab initio competitive with DFT (fit to reproduce odd-even staggering)

#### **TRIUMF** Laser Spectroscopy: Charge Radii Across Chains

Study charge radii systematics across Ni isotopic chain



Multiple ab-initio methods generally agree within uncertainties Ab initio (again) competitive with DFT

#### **EM Moments in Heavy Nuclei: In**

Improvements in storage of 3N matrix elements greatly expands reach of ab initio theory!

Calculate radii, moments of entire In chain – focus on odd-even

Reproduces trends of new measurements



Neglected physics: two-body meson-exchange currents

#### **VS-IMSRG for Atomic Nuclei**



## Strategy I: Study Simpler, Relevant Cases



## **TRIUMF** Large-Scale Efforts for Ab Initio GT Transitions

#### Calculate large GT matrix elements

$$M_{\rm GT} = g_A \left\langle f | \mathcal{O}_{\rm GT} | i \right\rangle$$
$$\mathcal{O}_{\rm GT} = \mathcal{O}_{\sigma\tau}^{\rm 1b} + \mathcal{O}_{2BC}^{\rm 2b}$$

- Light, medium, and heavy regions
- Benchmark different ab initio methods
- Wide range of NN+3N forces
- Consistent inclusion of 2BC



Discrepancy between experimental and theoretical  $\beta$ -decay rates resolved from first principles

P. Gysbers<sup>1,2</sup>, G. Hagen<sup>®3,4</sup>\*, J. D. Holt<sup>®1</sup>, G. R. Jansen<sup>®3,5</sup>, T. D. Morris<sup>3,4,6</sup>, P. Navrátil<sup>®1</sup>, T. Papenbrock<sup>®3,4</sup>, S. Quaglioni<sup>®7</sup>, A. Schwenk<sup>8,9,10</sup>, S. R. Stroberg<sup>1,11,12</sup> and K. A. Wendt<sup>7</sup>

#### NUCLEAR PHYSICS

#### Beta decay gets the ab initio treatment

One of the fundamental radioactive decay modes of nuclei is  $\beta$  decay. Now, nuclear theorists have used first-principles simulations to explain nuclear  $\beta$  decay properties across a range of light- to medium-mass isotopes, up to <sup>100</sup>Sn.


# **∂**TRIUMF

# Solution g<sub>A</sub>-Quenching Problem

Comparison to standard phenomenological shell model

Ab initio calculations across the chart explain data with unquenched g<sub>A</sub>



Refine results with improvements in forces and many-body methods

#### Ab Initio 2vββ Decay: <sup>48</sup>Ca

Consistent many-body wfs/operators from chiral NN+3N forces (with 2b currents)



#### **VS-IMSRG: decrease in final matrix element**

Potential issues: limited 1<sup>+</sup> states, missing IMSRG(3)... Benchmarks with CC underway

# % TRIUMF Strategy II: Benchmark NMEs in Light Nuclei

Benchmark with quasi-exact NCSM, IT-NCSM, IM-GCM, and CC in light systems: A=6-22



#### **Reasonable to good agreement in all cases!**

Pursue true double-beta decay nuclei

#### **Strategy III: Final Predictions <sup>48</sup>Ca**

**Ab initio**: consistent many-body wfs/operators from chiral NN+3N forces (**no 2b currents**) Small uncertainty from NO reference indicated; well converged in  $e_{max}$ ,  $E_{3max}$ 



#### **Strategy III: Final Predictions** <sup>76</sup>**Ge**

#### Key nucleus in worldwide searches

NME smaller than all previous calculations...



#### **Strategy III: Final Predictions** <sup>82</sup>**Se**

**Ab initio**: consistent many-body wfs/operators from chiral NN+3N forces (**no 2b currents**) Small uncertainty from NO reference indicated; well converged in  $e_{max}$ ,  $E_{3max}$ 



# **Ab Initio** 0vββ **Decay: Multiple Interactions**

Study uncertainty from input NN+3N forces with **5 chiral Hamiltonians** 

Convergence in virtually all cases: globally smaller than phenomenology, much less spread



# **Ab Initio** 0vββ **Decay: Multiple Interactions**

Study uncertainty from input NN+3N forces with **5 chiral Hamiltonians** 

Convergence in virtually all cases: globally smaller than phenomenology, much less spread In agreement with other ab initio approaches in <sup>48</sup>Ca, <sup>76</sup>Ge (preliminary)



All ab initio calculations agree within uncertainties!

### **TRIUMF** Strategy III: Final Predictions (with uncertainties!)

#### Converged predictions for all major players in global searches: <sup>76</sup>Ge, <sup>130</sup>Te, <sup>136</sup>Xe

Significant differences from nuclear model/phenomenological calculations



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#### **TRIUMF** The Year(s) We Lost Hope: Leading-Order Contact

#### Proper renormalization requires short-range contact term at leading order

Physics Cirigliano et al. PRL (2018)

#### A Missing Piece in the Neutrinoless Beta-Decay Puzzle

May 16, 2018 • Physics 11, s58

The inclusion of short-range interactions in models of neutrinoless double-beta decay could impact the interpretation of experimental searches for the elusive decay.





New paradigm for  $0\nu\beta\beta$  decay: must include long- and short-range terms

$$M^{0\nu} = M_{\rm GT} + \frac{M_{\rm F}}{g_A^2} + M_{\rm T} \to M_L + M_S = M_{\rm GT} + \frac{M_{\rm F}}{g_A^2} + M_{\rm T} + M_{c.t}$$

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**Huge problem**: unclear how to fix undetermined coupling constant  $C_1$ 

#### **TRIUMF** The Year We Regained Hope: Coupling Constant Fit

Cirigliano et al. propose new method to estimate the size of the contact term

First ab initio calculations with short-range term from IM-GCM, VS-IMSRG



#### Overall increase of 40% in <sup>76</sup>Ge to 60% in <sup>82</sup>Se!

<sup>130</sup>Te, <sup>136</sup>Xe waiting in computing queue... results in days

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#### **TRIUMF** Structure Functions from Large-Scale Shell Model

Best to date: phenomenological wfs + bare operator (axial currents)



#### 

Use consistent NN+3N forces and 2-body currents

Natural orbitals basis (NAT) displays much more rapid convergence in <sup>129</sup>Xe



Hu et al. arXiv: 2109.00193

# **TRIUMF** Ab Initio SD WIMP-Nucleus Response Overview

Use three NN+3N chiral interactions with consistent chiral currents

Overall similar to phenomenology at low q, discrepancies in 127I



Provide new structure functions for all SD direct-detection candidates

### **Present and Future for Theory**

M. Martin

140

MINES K. G. Leach

Aim of modern nuclear theory: Develop unified *first-principles* picture of structure and reactions

#### **Nuclear Structure**

Development of forces and currents Dripline predictions up to Fe Evolution of magic numbers: masses, radii, spectra, EM transitions Multi-shell theory: Islands of inversion Forbidden decays

Atomic systems



#### **Fundamental Symmetries/BSM Physics**

**EW operators: GT quenching, muon capture** 0vββ **decay matrix elements WIMP-Nucleus/neutrino scattering Superallowed Fermi transitions Symmetry-violating moments [molecules]** 

#### Work in progress

Higher-order many-body effects: IMSRG(3) Monte Carlo shell model diagonalization Extension to superheavy nuclei Precision data on GT transitions

UNIVERSITAT DE - L. Jokiniemi

120

BARCELONA\_J\_Menéndez

J.M. Yao

100

H. Hergert

#### Valence-Space IMSRG

Explicitly construct unitary transformation from sequence of rotations

$$U = e^{\Omega} = e^{\eta_n} \dots e^{\eta_1} \quad \eta = \frac{1}{2} \arctan\left(\frac{2H_{\text{od}}}{\Delta}\right) - \text{h.c.}$$
$$\tilde{H} = e^{\Omega} H e^{-\Omega} = H + [\Omega, H] + \frac{1}{2} [\Omega, [\Omega, H]] + \dots$$

All operators truncated at two-body level IMSRG(2) IMSRG(3) in progress

**Step 1: Decouple core** 



Tsukiyama, Bogner, Schwenk, PRC 2012 Morris, Parzuchowski, Bogner, PRC 2015

excluded

valence

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Step 1: Decouple core Step 2: Decouple valence space

Can we achieve accuracy decouple of large-space methods?

 $| | | = |^{16} O \rangle$ 

 $\langle \tilde{\Psi}_n | P \tilde{H} P \mid \tilde{\Psi}_n \rangle \approx \langle \Psi_i | H | \Psi_i \rangle$ 

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$$\tilde{\Psi}_n | P\tilde{H}P | | \tilde{\Psi}_n \rangle \approx \langle \Psi_i | H | \Psi_i \rangle$$

$$\langle \tilde{\Psi}_n | P\tilde{M}_{0\nu}P | | \tilde{\Psi}_n \rangle \approx \langle \Psi_i | M_{0\nu} | \Psi_i \rangle$$

$$\text{Careful benchmarking essential}$$

$\langle P H P angle$	$\langle P H Q\rangle  ightarrow 0$
$\langle Q H P angle  ightarrow 0$	$\langle Q H Q angle$

4

2

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#### **Convergence of N=82 Gap**

Size of N=70 gap not converged at  $E_{3max}$ =18: for neutron-rich Sn, In, Cd...

000000

HFB31

UNEDF

 $DD-ME\delta$ 

42 44 46 48 50 52 54 56 58 60 62 64 66 68 70 **Proton number** 

Theory

00

SLy4-MF

-V- SLy4-GCM

- IMSRG



Resorted to unreliable extrapolations...

New capabilities: converged spectra in N=82 region

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excluded

valence

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#### Global Trends in Absolute B(E2): sd Shell

Study charge E2 transitions across sd-shell



USDB with effective charges typically reproduces absolute values well VS-IMSRG (**no effective charges**) typically underpredicts experiment Trends well reproduced in both...

#### **Global Trends in B(E2): IS/IV Components**

Study charge E2 transitions across sd-shell: IS (M<sub>0</sub>) and IV (M<sub>1</sub>)



# **Origin of E2 Puzzle <sup>14</sup>C in psd Shell**

Perform CC and VS-IMSRG calculations of <sup>14</sup>C in toy p-sd space with phenomenological potential



Energies well converged all around

p/n amplitudes increase with p/h ex.

Only converged at ~6 Nph

Not possible to capture fully in CC or IMSRG

# TRIUMF Systematic Studies in Heavy Region: In/Sn

Improvements in storage of 3N matrix elements greatly expands reach of ab initio theory!

Calculate radii, EM moments of entire In chain!

Reproduces trends of new measurements – correlates with B(E2) in Sn chain



#### Systematic studies in Sn region: Sb

Improvements in storage of 3N matrix elements greatly expands reach of ab initio theory!

Calculate radii, moments of entire Sb chain... explore new physics



### GT Transitions in Light nuclei and <sup>100</sup>Sn

**NCSM** in light nuclei, **CC** calculations of GT transition in <sup>100</sup>Sn from different forces



Large quenching effect from correlations

Gysbers et al., Nature Phys. (2019)

### GT Transitions in Light nuclei and <sup>100</sup>Sn

Ce l

NCSM in light nuclei, CC calculations of GT transition in <sup>100</sup>Sn from different forces



Addition of 2BC further quenches and reduces spread in results

### GT Transitions in Light nuclei and <sup>100</sup>Sn

NCSM in light nuclei, CC calculations of GT transition in <sup>100</sup>Sn from different forces



#### **Spectroscopy**

**Ab initio**: Consistent many-body wfs/operators from chiral NN+3N forces + 2b currents Calculated spectroscopy (generally) well reproduces experiment



Hu, Padua, in prep.
**Extent to Ab Initio Atomic Systems**  
Motivation: isotof  

$$\delta E_{i,MS}^{A,A'} = \left(\frac{M'-M}{MM'}\right)(K_{i,NMS} + K_{i,SMS}) = \left(\frac{M'-M}{MM'}\right)K_{i,MS},$$
  
riments  
**Mass Shift:**  $\delta E_{i,MS}^{A,A'} = \left(\frac{M'-M}{MM'}\right)(K_{i,NMS} + K_{i,SMS}) = \left(\frac{M'-M}{MM'}\right)K_{i,MS}$   
**Field Shift:**  $\delta E_{i,HS}^{A,A'} = \left(\frac{M'-M}{MM'}\right)(K_{i,NMS} + K_{i,SMS}) = \left(\frac{M'-M}{MM'}\right)K_{i,MS}$   
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 $\int E_{i,HS}^{A,A'} = -\int_{R^2} \left[V^A(\mathbf{r}) - V^{A'}(\mathbf{r})\right]_{A} g_{i}^{A}(\mathbf{r}) g_{2M'}^{A}(\mathbf{r}) g_{2M'}^{A}(\mathbf{r}) g_{2M'}^{A,A'} \int_{T_{i,R}} E_{i,R}^{A,A'} \delta(r^{n+2})^{A,A'}$   
 $\int \delta E_{i,HS}^{(1)A,A'} = -\int_{R^2} \left[V^A(\mathbf{r}) - V^{A'}(\mathbf{r})\right] \rho_i^{e}(\mathbf{r}) d^3\mathbf{r} \cdot \frac{A \cdot A'}{2\pi} \int_{T_{i,R}} E_{i,R} \delta(r^{n+2})^{A,A'}$   
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 $\int E_{i,R}^{A,A'} = -\int_{R^2} \left[V^A(\mathbf{r}) - V^{A'}(\mathbf{r})\right] \rho_i^{A,A'} d_A + F \delta(r^2)^{A,A'} \int_{T_{i,R}} E_{i,R} \delta(r^{n+2})^{A,A'} \int_{T_{i,R}} E_{i,R} \delta(r^{n+2})^{A,A'} \int_{T_{i,R}} E_{i,R} \delta(r^{n+2})^{A,A'} \int_{T_{i,R}}$ 

# **% TRIUMF**

## **IMSRG for Closed-Shell Atoms**

### Apply single-reference IMSRG for closed-shell systems



Converges to "best" coupled-cluster results Good experimental agreement for light atoms... Discrepancies point to neglected relativistic effects

# **% TRIUMF**

## **VS-IMSRG for Open-Shell Atoms**

### Apply VS-IMSRG for binding energies of systems: Mg, Ca



Converges to best available many-body results Good(?) experimental agreement in both cases

# **%TRIUMF**

## **VS-IMSRG for Open-Shell Atoms**

#### Apply VS-IMSRG for ionization energies of systems



No ab initio calculations available

Good(at times) experimental agreement in both cases

# **% TRIUMF**

## **VS-IMSRG for Isotope Shifts**

#### VS-IMSRG calculations of isotope shifts in light atoms



Results comparable to currently used code

Extension to Dirac-Hartree-Fock in progress to access heavy atomic systems!