Theoretical understanding of hadrons in hot/dense media

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PANIC2021 Conference, Online
September 09, 2021.

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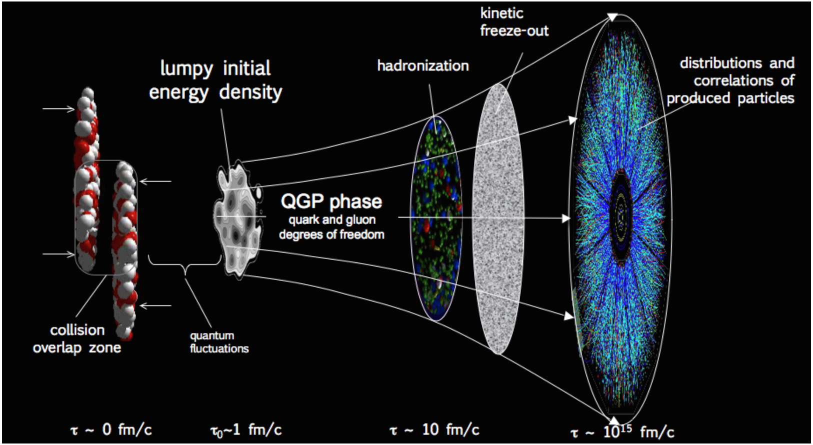
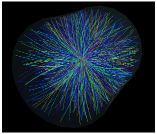
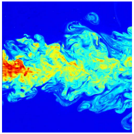
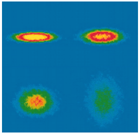
High energy nuclear collisions: QCD fluid



Fluid dynamics

- long distances, long times or strong enough interactions
- quantum fields form a fluid!
- needs **macroscopic** fluid properties
  - thermodynamic equation of state $p(T, \mu)$
  - shear + bulk viscosity $\eta(T, \mu)$, $\zeta(T, \mu)$
  - heat conductivity $\kappa(T, \mu)$, ...
  - relaxation times, ...
  - electrical conductivity $\sigma(T, \mu)$
- fixed by **microscopic** properties encoded in Lagrangian $\mathscr{L}_{\text{QCD}}$
- old dream of condensed matter physics: understand the fluid properties!

Relativistic fluid dynamics

Energy-momentum tensor and conserved currents

$$T^{\mu\nu} = \epsilon\, u^\mu u^\nu + (p + \pi_{\text{bulk}})\Delta^{\mu\nu} + \pi^{\mu\nu}$$
$$N_j^\mu = n_j\, u^\mu + \nu_j^\mu$$

- tensor decomposition using fluid velocity u^μ, $\Delta^{\mu\nu} = g^{\mu\nu} + u^\mu u^\nu$
- thermodynamic equation of state $p = p(T, \mu_j)$

Covariant **conservation laws** $\nabla_\mu T^{\mu\nu} = 0$ and $\nabla_\mu N_j^\mu = 0$ imply

- equation for energy density ϵ
- equation for fluid velocity u^μ
- equations for particle number densities n_j

Need in addition **constitutive relations** [e.g Israel & Stewart]

- equation for shear stress $\pi^{\mu\nu}$
- equation for bulk viscous pressure π_{bulk}

$$\tau_{\text{bulk}}\, u^\mu \partial_\mu \pi_{\text{bulk}} + \ldots + \pi_{\text{bulk}} = -\zeta\, \nabla_\mu u^\mu$$

- equations for diffusion currents ν_j^μ

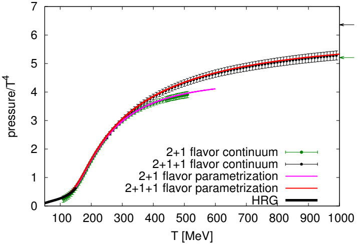
Thermodynamic equation of state

- describes volume V with temperature T and chemical potentials μ_B, μ_C and μ_S associated with conserved baryon, charge and strangeness numbers
- exchange of energy and particles with heat bath
- can be simulated with Lattice QCD
- all thermodynamic properties follow from

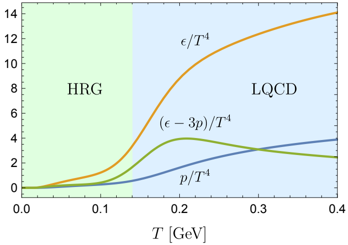
$$p(T, \mu_B, \mu_Q, \mu_S)$$

- chemical potentials
  - μ_B for (net) baryon number
  - μ_Q for (net) electric charge
  - μ_S for (net) strangeness

Thermodynamics of QCD



[Borsányi *et al.* (2016)], similar Bazavov *et al.* (2014)



[Floerchinger, Grossi & Lion (2018)]

- thermodynamic equation of state $p(T)$ rather well understood now
- used for fluid dynamics at LHC energies

Moments and cumulants at equilibrium

- mean value of net baryon number

$$\bar{N}_B = \langle N_B \rangle = V \frac{\partial}{\partial \mu_B} p(T, \mu_B, \mu_Q, \mu_S)$$

- variance in terms of $\delta N_B = N_B - \bar{N}_B$

$$\sigma_B^2 = \langle \delta N_B^2 \rangle = TV \frac{\partial^2}{\partial \mu_B^2} p(T, \mu_B, \mu_Q, \mu_S)$$

- skewness

$$S_B = \frac{\langle \delta N_B^3 \rangle}{\sigma_B^3} = \frac{1}{\sigma_B^3} T^2 V \frac{\partial^3}{\partial \mu_B^3} p(T, \mu_B, \mu_Q, \mu_S)$$

- kurtosis

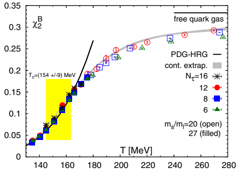
$$\kappa_B = \frac{\langle \delta N_B^4 \rangle - 3\langle \delta N_B^2 \rangle^2}{\sigma_B^4} = \frac{1}{\sigma_B^4} T^3 V \frac{\partial^4}{\partial \mu_B^4} p(T, \mu_B, \mu_Q, \mu_S)$$

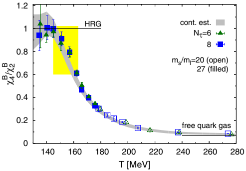
- similar for mixed derivatives

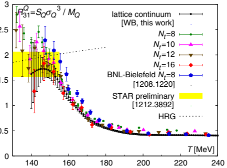
Lattice QCD results for cumulants

- lattice QCD results for

$$\chi_2^B = \frac{\sigma_B^2}{VT^3} = \frac{\langle \delta N_B^2 \rangle}{VT^3} \qquad \frac{\chi_4^B}{\chi_2^B} = \frac{\langle \delta N_B^4 \rangle}{\langle \delta N_B^2 \rangle} \qquad R_{31}^Q = \frac{\chi_3^Q}{\chi_1^Q} = \frac{\langle \delta N_Q^3 \rangle}{\langle \delta N_Q \rangle}$$







[Bazavov *et al.* (2017), similar Bellwied *et al.* (2015)] [Borsányi *et al.* (2013)]

- hadron resonance gas (HRG) approximation works at small temperatures

Correlation functions as generalized moments / cumulants

- correlation function of baryon number density

$$C_2^{(B,B)}(t,\vec{x};t',\vec{x}') = \langle n_B(t,\vec{x})\, n_B(t',x')\rangle - \langle n_B(t,\vec{x})\rangle\langle n_B(t',\vec{x}')\,\rangle$$

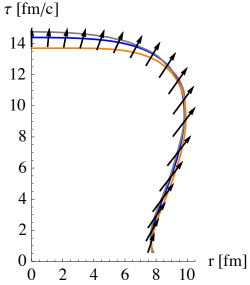
- integral over equal time correlation gives variance

$$\sigma_B^2 = \langle \delta N_B^2 \rangle = \int_V d^3x \int_V d^3x'\; C_2^{(B,B)}(t,\vec{x};t,\vec{x}')$$

- similar for higher order correlation functions
- thermodynamic variables can be traded

$$(\epsilon, n_B, n_Q, n_S) \quad \leftrightarrow \quad (T, \mu_B, \mu_Q, \mu_S)$$

Cooper-Frye freeze-out



- single particle distribution [Cooper & Frye (1974)]

$$E\frac{dN_i}{d^3p} = -p^\mu \int_{\Sigma_f} \frac{d\Sigma_\mu}{(2\pi)^3}\, f_i(p;x)$$

  with close-to equilibrium distribution

$$f_i(p;x) = f_i(p; T(x), \mu_i(x), u^\mu(x), \pi^{\mu\nu}(x), \varphi(x), \dots)$$

- precise position of freeze-out surface is unknown, usual assumption

$$\langle T(x)\rangle = T_{\text{fo}} = \text{const}$$

Particle correlations from fluid field correlations

[Aasen, Floerchinger, Giacalone, Guenduez & Masciocchi, work in progress]

- can be used for expectation values...

$$\left\langle E \frac{dN_i}{d^3p} \right\rangle = \left\langle -p_\mu \int_{\Sigma_f} \frac{d\Sigma^\mu}{(2\pi)^3} \, f_i(p;x) \right\rangle$$

- ... but also for correlation functions

$$\left\langle E \frac{dN_i}{d^3p} E' \frac{dN_j}{d^3p'} \right\rangle = p_\mu p'_\nu \int_{\Sigma_f} \frac{d\Sigma^\mu}{(2\pi)^3} \frac{d\Sigma'^\nu}{(2\pi)^3} \left\langle f_i(p;x) \, f_j(p';x') \right\rangle$$

- the right hand side involves correlation functions

$$\left\langle f_i(p;x) \, f_j(p';x') \right\rangle$$

  between different points x and x' on the freeze-out surface.
- works similar for higher order correlation functions.
- thermal fluctuations and initial state fluctuations contribute to correlations

Particle correlations from fluid field correlations

[Aasen, Floerchinger, Giacalone, Guenduez & Masciocchi, work in progress]

- one can decompose

$$T(x) = \bar{T}(x) + \delta T(x), \qquad \mu(x) = \bar{\mu}(x) + \delta\mu(x)$$

and expand the distribution functions

$$\begin{aligned} f_i(p;x) =& f_i(p;\bar{T}(x),\bar{\mu}_i(x),\ldots) \\ &+ \delta T(x)\frac{\partial}{\partial T} f_i(p;\bar{T}(x),\bar{\mu}(x),\ldots) \\ &+ \delta\mu(x)\frac{\partial}{\partial \mu} f_i(p;\bar{T}(x),\bar{\mu}(x),\ldots) + \ldots \end{aligned}$$

- two-particle correlation function governed by integral over

$$\begin{aligned} \big\langle f_i(p;x)\, f_j(p';x')\big\rangle =& f_i(p;\bar{T}(x),\ldots)\, f_j(p';\bar{T}(x'),\ldots) \\ &+ \big\langle \delta T(x)\delta T(x')\big\rangle \frac{\partial}{\partial T} f_i(p;\bar{T}(x),\ldots)\frac{\partial}{\partial T} f_j(p;\bar{T}(x'),\ldots) \\ &+ \big\langle \delta\mu(x)\delta\mu(x')\big\rangle \frac{\partial}{\partial \mu} f_i(p;\bar{T}(x),\ldots)\frac{\partial}{\partial \mu} f_j(p;\bar{T}(x'),\ldots) \\ &+ \ldots \end{aligned}$$

Cooper-Frye freeze-out with resonance decays

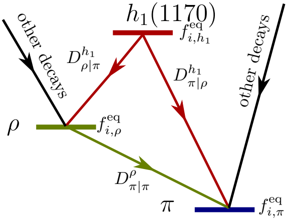
[Mazeliauskas, Floerchinger, Grossi & Teaney, EPJC 79, 2842019 (2019)]

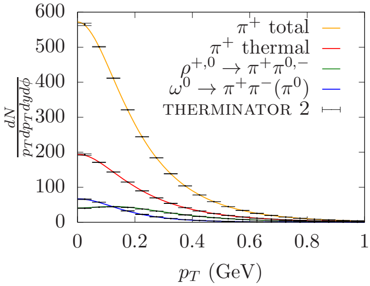
- decay map relates spectra before and after resonance decays

$$E_p \frac{dN_b}{d^3p} = \int_q D_b^a(p,q)\, E_q \frac{dN_a}{d^3q}$$

- Cooper-Frye with resonance decays

$$E_p \frac{dN_a}{d^3p} = -\frac{1}{(2\pi)^3} \int d\Sigma_\mu\, g_a^\mu(x,p), \qquad g_b^\mu(x,p) = \int_q D_b^a(p,q) f_a(x,q) q^\mu$$



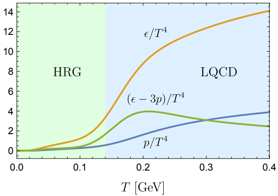


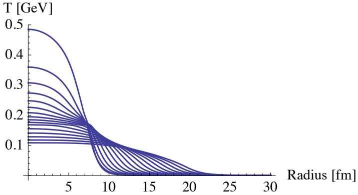
Flow and fluctuations in heavy ion collisions

Fluid*u*M: Fluid dynamics of heavy ion collisions with Mode expansion

[Floerchinger & Wiedemann, PLB 728, 407 (2014), PRC 88, 044906 (2013), 89, 034914 (2014)]

[Floerchinger, Grossi & Lion, PRC 100, 014905 (2019)]

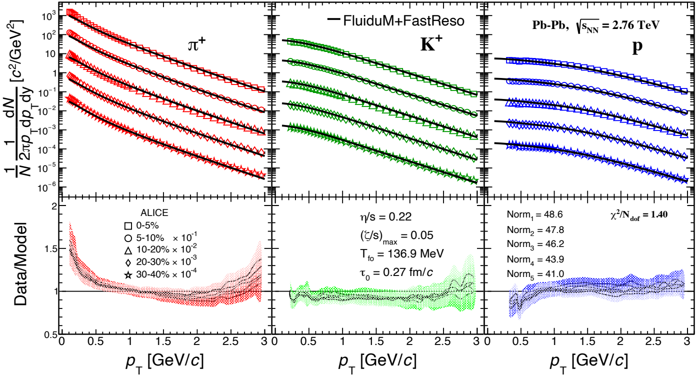


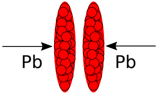


- background-fluctuation splitting + mode expansion
- analogous to cosmological perturbation theory
- substantially improved numerical performance (pseudospectral method)
- resonance decays included
  [Mazeliauskas, Floerchinger, Grossi & Teaney, EPJC 79, 284 (2019)]
- allows fast and precise comparison between theory and experiment

Particle production at the Large Hadron Collider

[Devetak, Dubla, Floerchinger, Grossi, Masciocchi, Mazeliauskas & Selyuzhenkov, 1909.10485]

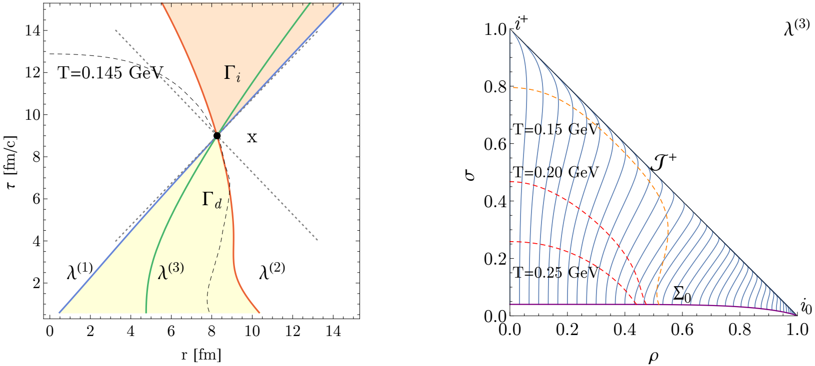




- data are very precise now - high quality theory development needed!
- next step: include coherent fields / condensates

Causality

[Floerchinger & Grossi, JHEP 08 (2018) 186]



- inequalities for relativistic causality
- dissipative fluid equations *can* be of hyperbolic type
- characteristic velocities depend on fluid fields
- need $|\lambda^{(j)}| < c$ for relativistic causality

Entropy current, local dissipation and unitarity

- local dissipation = local entropy production

$$\nabla_\mu s^\mu(x) \geq 0$$

- e. g. from analytically continued quantum effective action
  [Floerchinger, JHEP 1609, 099 (2016)]
- fluid dynamics in Navier-Stokes approximation

$$\nabla_\mu s^\mu = \frac{1}{T}\left[2\eta\sigma_{\mu\nu}\sigma^{\mu\nu} + \zeta(\nabla_\rho u^\rho)^2\right] \geq 0$$

- unitary time evolution conserves von-Neumann entropy

$$S = -\text{Tr}\{\rho \ln \rho\} = -\text{Tr}\{(U\rho U^\dagger)\ln(U\rho U^\dagger)\} \qquad \Rightarrow \qquad \frac{d}{dt}S = 0$$

quantum information is globally conserved

What is local dissipation in isolated quantum systems ?

Classical statistics

- consider system of two random variables x and y
- joint probability $p(x, y)$, joint entropy

$$S = -\sum_{x,y} p(x, y) \ln p(x, y)$$

- reduced or marginal probability $p(x) = \sum_y p(x, y)$
- reduced or marginal entropy

$$S_x = -\sum_{x} p(x) \ln p(x)$$

- one can prove: **joint entropy is greater than** or equal to **reduced entropy**

$$S \geq S_x$$

- **globally pure** state $S = 0$ is also **locally pure** $S_x = 0$

Quantum statistics

- consider system with two subsystems A and B
- combined state ρ , combined or full entropy

$$S = -\text{Tr}\{\rho \ln \rho\}$$

- reduced density matrix $\rho_A = \text{Tr}_B\{\rho\}$
- reduced or **entanglement entropy**

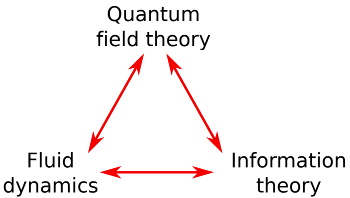
$$S_A = -\text{Tr}_A\{\rho_A \ln \rho_A\}$$

- pure **product** state $\rho = \rho_A \otimes \rho_B$ leads to $S_A = 0$
- pure **entangled** state $\rho \neq \rho_A \otimes \rho_B$ leads to $S_A > 0$
- for quantum systems **entanglement makes a difference**

$$S \ngeq S_A$$

- **coherent information** $I_{B\rangle A} = S_A - S$ can be **positive**!
- **globally pure** state $S = 0$ can be **locally mixed** $S_A > 0$

Quantum field dynamics

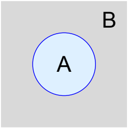


- new hypothesis

local dissipation = *quantum entanglement generation*

- quantum information is spread
- locally, quantum state approaches mixed state form
- full loss of *local* quantum information = *local* thermalization

Entanglement entropy in quantum field theory



- entanglement entropy of region A is a local notion of entropy

$$S_A = -\text{tr}_A\{\rho_A \ln \rho_A\} \qquad \rho_A = \text{tr}_B\{\rho\}$$

- however, it is infinite already in vacuum state

$$S_A = \frac{\text{const}}{\epsilon^{d-2}} \int_{\partial A} d^{d-2}\sigma\sqrt{h} + \text{subleading divergences} + \text{finite}$$

- UV divergence proportional to entangling surface
- quantum fields are very strongly entangled already in vacuum

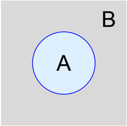
Relative entropy

- **relative entropy** of two density matrices

$$S(\rho|\sigma) = \text{tr}\left\{\rho\left(\ln\rho - \ln\sigma\right)\right\}$$

- measures how well state ρ can be distinguished from a model σ
- Gibbs inequality: $S(\rho|\sigma) \geq 0$
- $S(\rho|\sigma) = 0$ if and only if $\rho = \sigma$
- quantum generalization of Kullback-Leibler divergence
- Thermodynamics can be formulated with relative entropy!
  [Floerchinger & Haas, PRE 102, 052117 (2020)]

Relative entanglement entropy



- consider now reduced density matrices

$$\rho_A = \text{Tr}_B\{\rho\}, \qquad \sigma_A = \text{Tr}_B\{\sigma\}$$

- define **relative entanglement entropy**

$$S_A(\rho|\sigma) = \text{Tr}\left\{\rho_A\left(\ln\rho_A - \ln\sigma_A\right)\right\} = -\text{Tr}\left\{\rho_A \ln\Delta_A\right\}$$

  with relative modular operator Δ_A
- measures how well ρ is represented by σ locally in region A
- UV divergences cancel: contains real physics information
- well defined in algebraic quantum field theory [Araki (1977)]
  [see also works by Casini, Myers, Lashkari, Witten, Liu, ...]

Local equilibrium description

[Dowling, Floerchinger & Haas, PRD 102 (2020) 10, 105002]

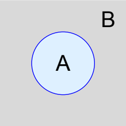
- consider non-equilibrium situation with
  - true density matrix ρ
  - local equilibrium approximation

$$\sigma = \frac{1}{Z} e^{-\int d\Sigma_\mu \{\beta_\nu(x) T^{\mu\nu} + \alpha(x) N^\mu\}}$$

- reduced density matrices $\rho_A = \text{Tr}_B\{\rho\}$ and $\sigma_A = \text{Tr}_B\{\sigma\}$
- σ is very good model for ρ in region A when

$$S_A = \text{Tr}_A\{\rho_A(\ln \rho_A - \ln \sigma_A)\} \to 0$$

- does *not* imply that globally $\rho = \sigma$



Monotonicity of relative entropy

- monotonicity of relative entropy [Lindblad (1975)]

$$S(\mathcal{N}(\rho)|\mathcal{N}(\sigma)) \leq S(\rho|\sigma)$$

  with $\mathcal{N}$ completely positive, trace-preserving map

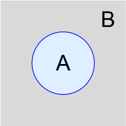
- $\mathcal{N}$ unitary evolution

$$S(\mathcal{N}(\rho)|\mathcal{N}(\sigma)) = S(\rho|\sigma)$$

- $\mathcal{N}$ open system evolution with generation of entanglement to environment

$$S(\mathcal{N}(\rho)|\mathcal{N}(\sigma)) < S(\rho|\sigma)$$

- local, second law of thermodynamics in terms of relative entropy
  [Dowling, Floerchinger & Haas, PRD 102 (2020) 10, 105002]



Remarks on status of relativistic fluid dynamics

- derivation from quantum effective action $\Gamma[\phi]$ wanted
  [Floerchinger, JHEP 1205, 021 (2012); JHEP 1609, 099 (2016)]
- expectation values *and* correlation functions of interest
- underlying principle: most excitations or modes relax quickly
  [Kadanoff & Martin (1963)]
- exception: conserved quantities like energy, momentum or particle density ("hydrodynamic modes")
- but: some non-hydrodynamic modes are needed for causality
- how to obtain additional equations of motion for them?

Covariant energy-momentum conservation

- quantum effective action $\Gamma[\phi, g]$ at stationary matter fields

$$\frac{\delta}{\delta\phi(x)}\Gamma[\phi, g] = 0$$

- diffeomorphism is gauge transformation of metric

$$g_{\mu\nu}(x) \to g_{\mu\nu}(x) + \nabla_\mu \varepsilon_\nu(x) + \nabla_\nu \varepsilon_\mu(x)$$

- energy-momentum tensor defined by

$$\delta\Gamma[\phi, g] = \frac{1}{2}\int d^d x \sqrt{g}\, T^{\mu\nu}(x)\delta g_{\mu\nu}(x)$$

- from invariance of $\Gamma[\phi, g]$ under diffeomorphisms

$$\nabla_\mu T^{\mu\nu}(x) = 0$$

- worked here in Riemann geometry with Levi-Civita connection

$$\delta\Gamma_\mu{}^\rho{}_\nu = \frac{1}{2}g^{\rho\lambda}\left(\nabla_\mu \delta g_{\nu\lambda} + \nabla_\nu \delta g_{\mu\lambda} - \nabla_\lambda \delta g_{\mu\nu}\right)$$

Hypermomentum current

[von der Heyde, Kerlick & Hehl (1976)] [Floerchinger & Grossi, arXiv:2102.11098]

- connection can be varied independent of the metric

$$\delta\Gamma = \int d^d x \sqrt{g} \left\{ \frac{1}{2} \mathscr{U}^{\mu\nu}(x) \delta g_{\mu\nu}(x) - \frac{1}{2} \mathscr{S}^{\mu}{}_{\rho}{}^{\sigma}(x) \delta \Gamma_{\mu}{}^{\rho}{}_{\sigma}(x) \right\}$$

with new symmetric tensor $\mathscr{U}^{\mu\nu}$ and *hypermomentum* current $\mathscr{S}^{\mu}{}_{\rho}{}^{\sigma}$

- hypermomentum current can be decomposed further

$$\mathscr{S}^{\mu}{}_{\rho}{}^{\sigma} = Q^{\mu}{}_{\rho}{}^{\sigma} + W^{\mu}\,\delta_{\rho}{}^{\sigma} + S^{\mu}{}_{\rho}{}^{\sigma} + S^{\sigma\mu}{}_{\rho} + S_{\rho}{}^{\mu\sigma}$$

with

- spin current $S^{\mu\rho\sigma} = -S^{\mu\sigma\rho}$
- dilatation current W^{μ}
- shear current $Q^{\mu\rho\sigma} = Q^{\mu\sigma\rho}, \qquad Q^{\mu\rho}{}_{\rho} = 0$

Equations of motion for dilatation and shear current

[Floerchinger & Grossi, arXiv:2102.11098]

- variation of connection contains Levi-Civita part and non-Riemannian part

$$\delta\Gamma_\mu{}^\rho{}_\sigma = \frac{1}{2} g^{\rho\lambda} \left(\nabla_\mu \delta g_{\sigma\lambda} + \nabla_\sigma \delta g_{\mu\lambda} - \nabla_\lambda \delta g_{\mu\sigma} \right) + \delta C_\mu{}^\rho{}_\sigma + \delta D_\mu{}^\rho{}_\sigma$$

- variation at $\delta C_\mu{}^\rho{}_\sigma = \delta D_\mu{}^\rho{}_\sigma = 0$ gives energy-momentum tensor

$$T^{\mu\nu} = \mathscr{U}^{\mu\nu} + \frac{1}{2} \nabla_\rho \left(Q^{\rho\mu\nu} + W^\rho g^{\mu\nu} \right)$$

- new equation of motion for dilatation or Weyl current

$$\nabla_\rho W^\rho = \frac{2}{d} (T^\mu{}_\mu - \mathscr{U}^\mu{}_\mu)$$

- new equation of motion for shear current

$$\nabla_\rho Q^{\rho\mu\nu} = 2 \left[T^{\mu\nu} - \mathscr{U}^{\mu\nu} - \frac{g^{\mu\nu}}{d} (T^\sigma{}_\sigma - \mathscr{U}^\sigma{}_\sigma) \right]$$

- non-conserved Noether currents

Spin current

[..., Floerchinger & Grossi, arXiv:2102.11098]

- tetrad formalism: vary tetrad ${V_\mu}^A$ and spin connection ${\Omega_\mu}^{AB}$

$$\delta\Gamma = \int d^dx\sqrt{g}\left\{ \mathscr{T}^\mu{}_A(x)\delta {V_\mu}^A(x) - \frac{1}{2}S^\mu{}_{AB}(x)\delta{\Omega_\mu}^{AB}(x)\right\}$$

with
  - canonical energy-momentum tensor $\mathscr{T}^\mu{}_A$
  - spin current $S^\mu{}_{AB}$
- symmetric energy-momentum tensor in Belinfante-Rosenfeld form

$$T^{\mu\nu}(x) = \mathscr{T}^{\mu\nu}(x) + \frac{1}{2}\nabla_\rho\left[S^{\rho\mu\nu}(x) + S^{\mu\nu\rho}(x) + S^{\nu\mu\rho}(x)\right]$$

- equation of motion for spin current

$$\nabla_\mu S^{\mu\rho\sigma} = \mathscr{T}^{\sigma\rho} - \mathscr{T}^{\rho\sigma}$$

- non-conserved Noether current

Conclusions

- hot and dense media well described by relativistic fluid dynamics
- free streaming hadrons emerge at freeze-out / after resonance decays
- information about fluid phase encoded in various correlation functions
- relativistic fluid dynamics has a foundation in quantum information theory
- proper description of local thermalization in terms of relative entanglement
- quantum field theoretic description with two density matrices:
  - true density matrix ρ evolves unitary
  - fluid model σ agrees locally but evolves non-unitary
- new geometric foundation in terms of dilatation current, shear current and spin current