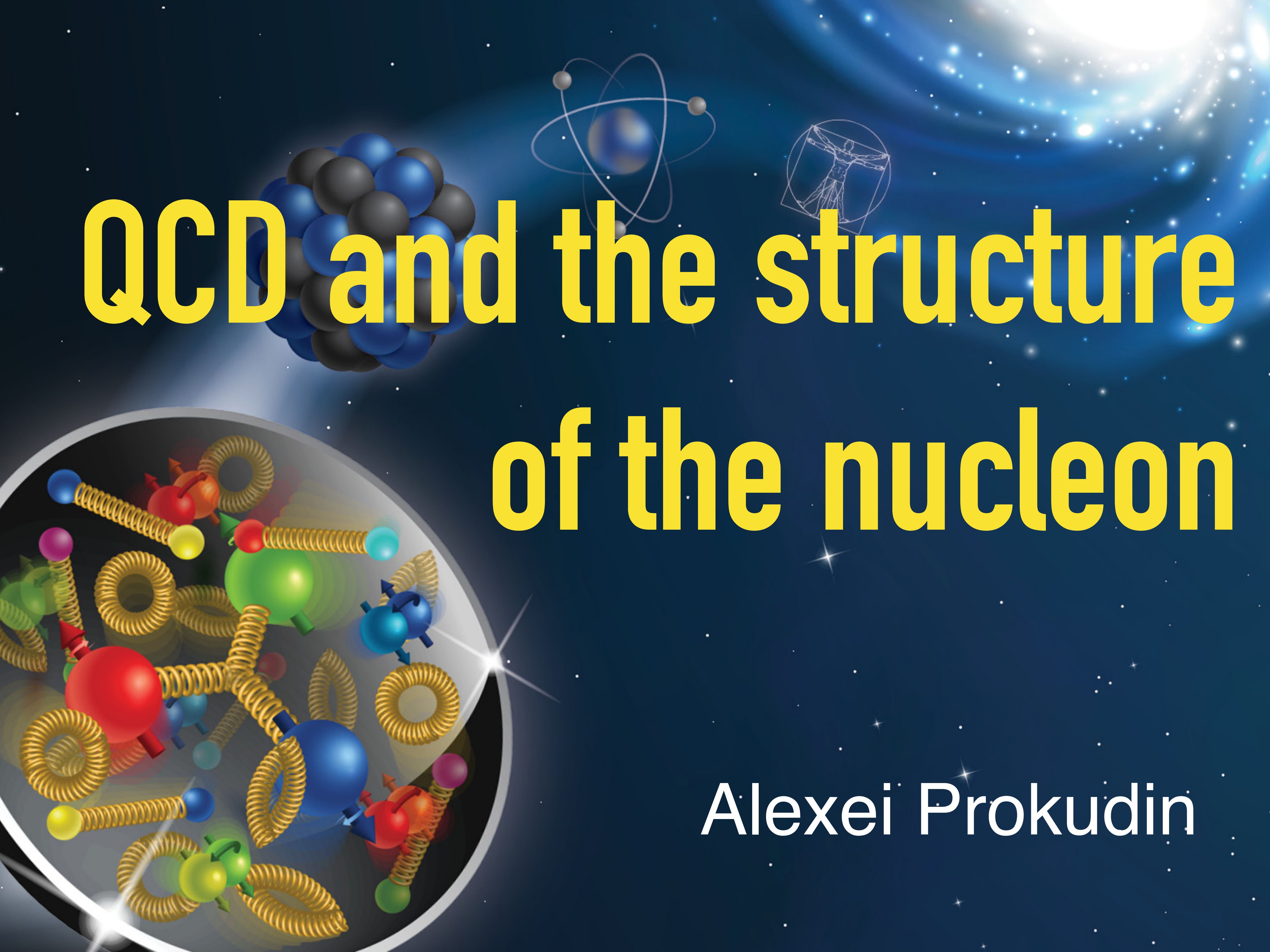


QCD and the structure of the nucleon

Alexei Prokudin

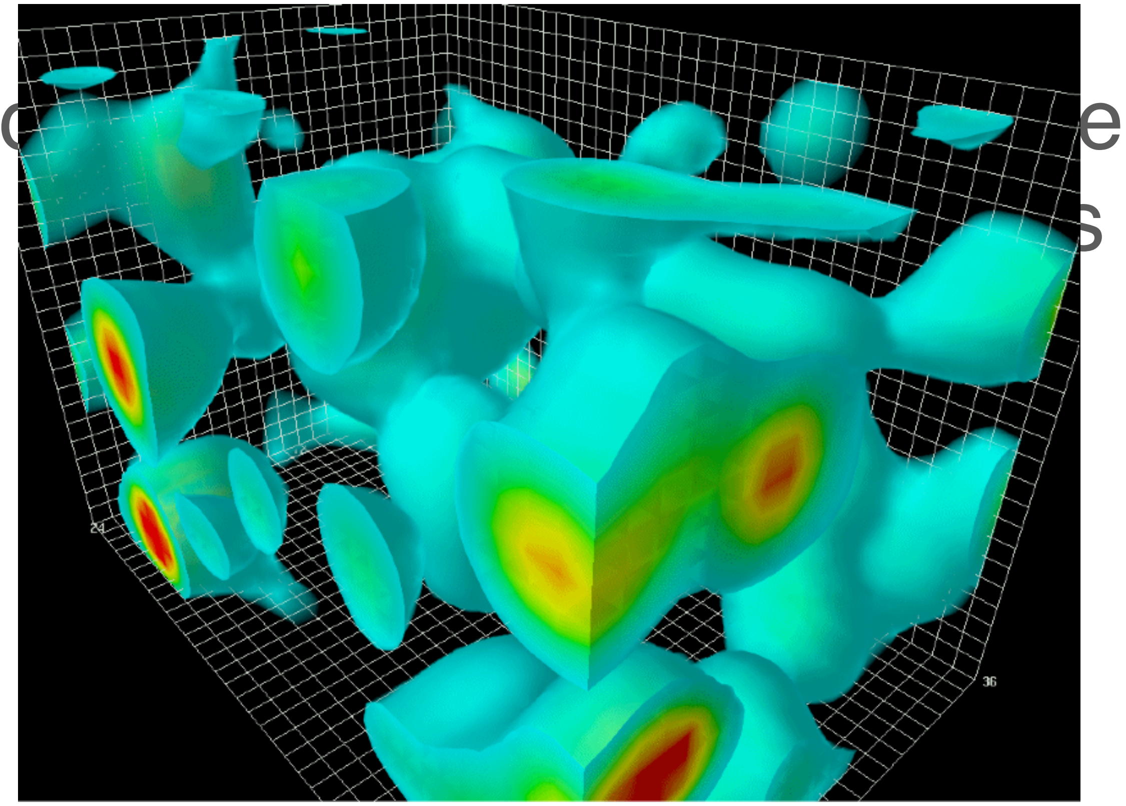
THEORETICAL FOUNDATIONS OF QCD

- QCD is a fundamental field theory that encodes nucleon structure and dynamics

$$\mathcal{L}_{QCD} = \sum_{j=1}^{n_f} \bar{\psi}_j \left(iD_\mu \gamma^\mu - m_j \right) \psi_j - \frac{1}{4} \operatorname{Tr} G^{\mu\nu} G_{\mu\nu}$$

- Interactions arise from fundamental symmetries, properties such as mass and the spin are emergent through complex structure of QCD

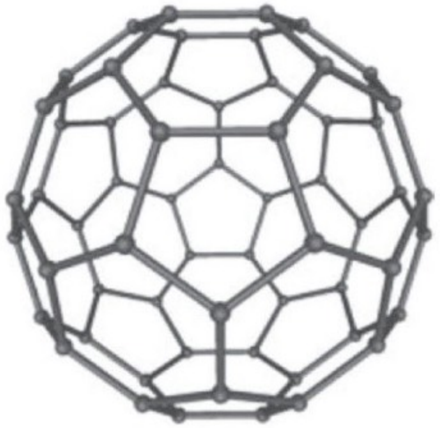
- Major goal is to unc erties of QCD by exploring structure s and nuclear matter

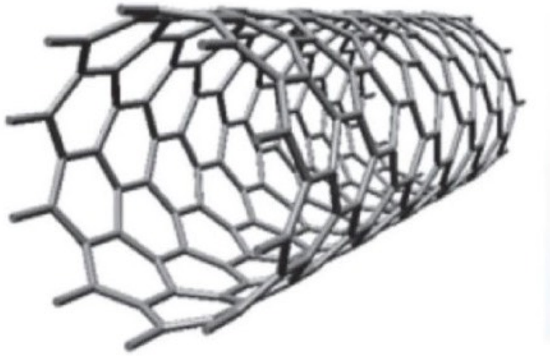


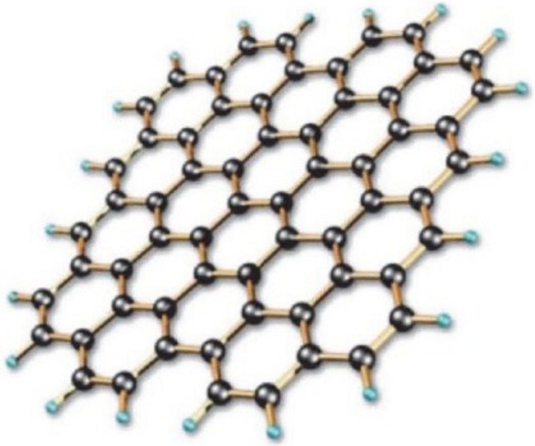
"Visualizations of the QCD Vacuum." lattice QCD calculation by by Derek Leinweber

HOW TO QUANTIFY THE STRUCTURE?

Solid state physics: relatively simple Lagrangian - complex structures.
If motion of nuclei is much slower than the speed of light:
structure = “still picture”. Allows one to create new materials

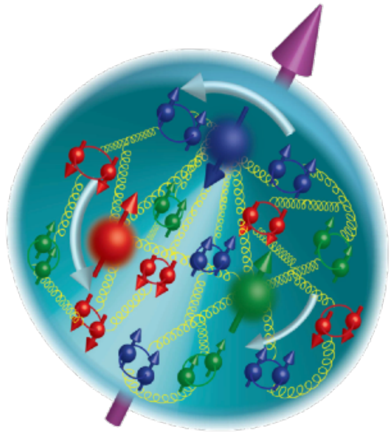

fullerene


nanotube


graphene


Geim, Novoselov, Nobel Prize 2010

No still picture for the proton’s structure: Quarks and gluons are moving relativistically, their number changes. The language of partonic structure is the language of quantum probabilities

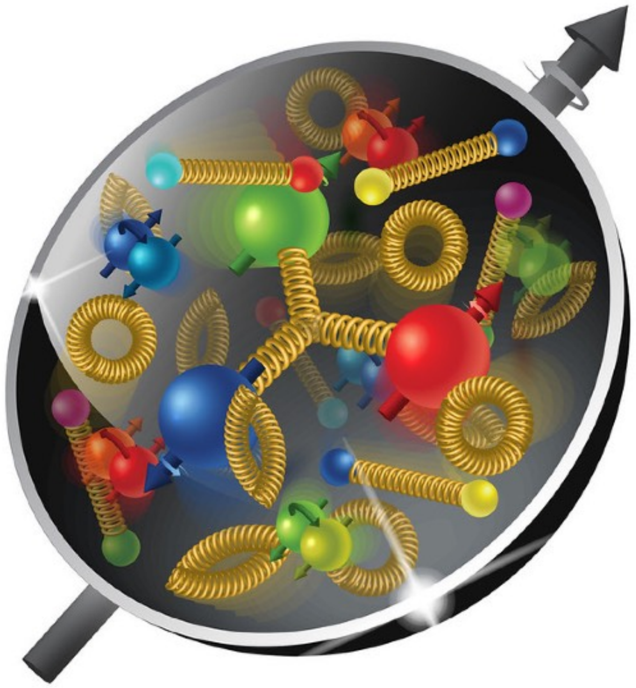


$$\langle P,S|\mathcal{O}(\bar{\Psi},\Psi,A^{\mu})|P,S\rangle$$

WHY SPIN?

Spin is a fundamental degree of freedom originated from the space-time symmetry

Spin plays a critical role in determining the basic structure of fundamental interactions

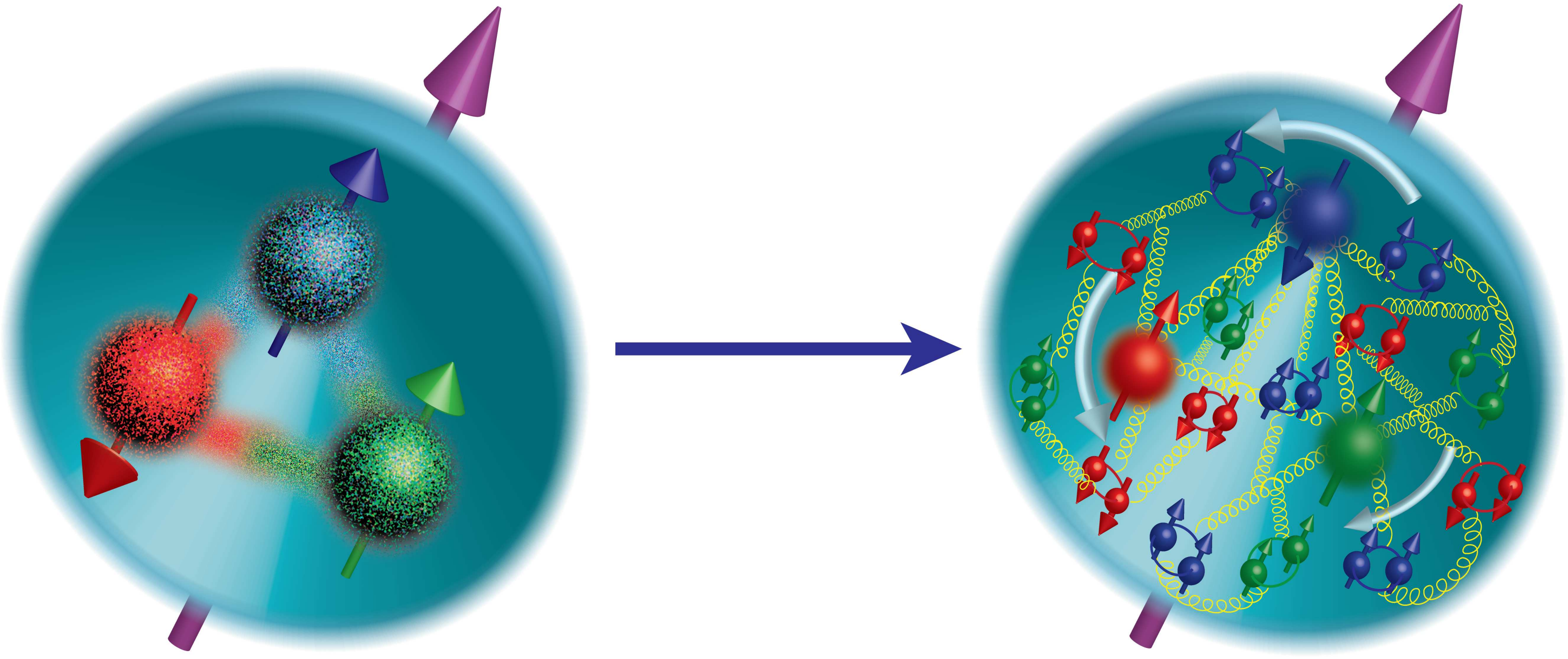


Spin provides a unique opportunity to probe the inner structure of a composite system (such as the proton) and hence to test our ability to understand the working of non-perturbative QCD

Test of a theory is not complete without a full exploration of spin-dependent decays and scatterings

Xiangdong Ji, DIS08

EVOLUTION OF OUR UNDERSTANDING OF THE SPIN STRUCTURE

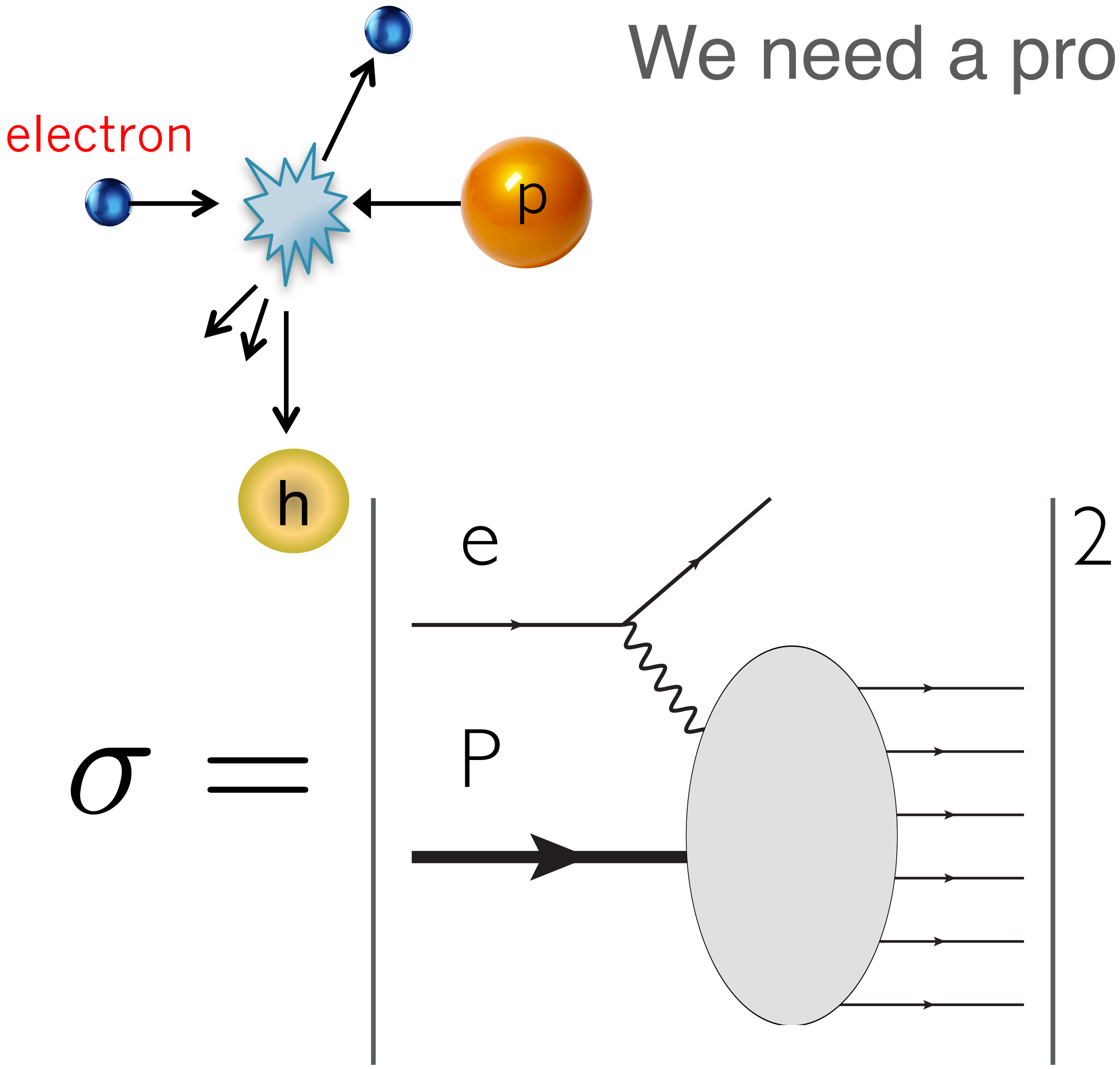


1980’ - the spin of the nucleon
is due to the valence quarks

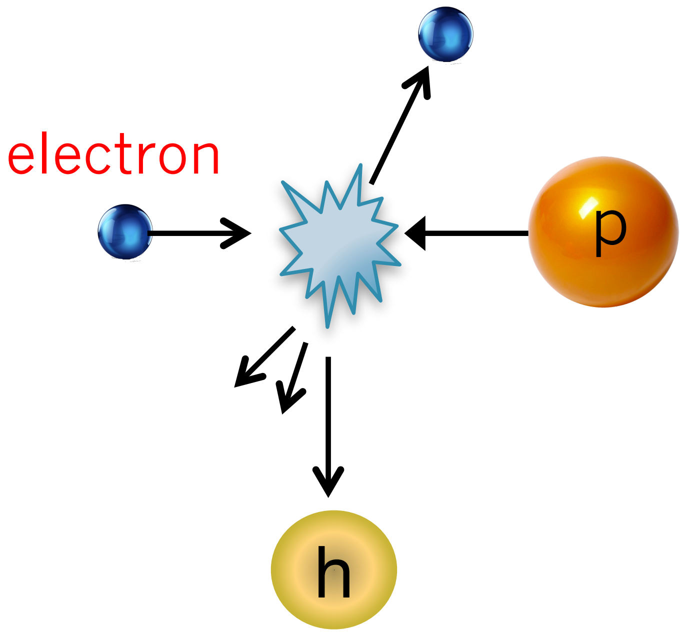
Modern concept: valence quarks, sea quarks, and gluons together with orbital angular momentum are contributing

QCD FACTORIZATION IS THE KEY!

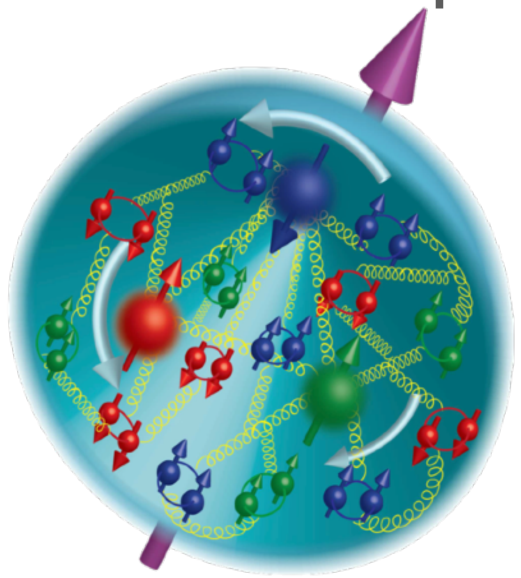
We need a probe to “see” quarks and gluons



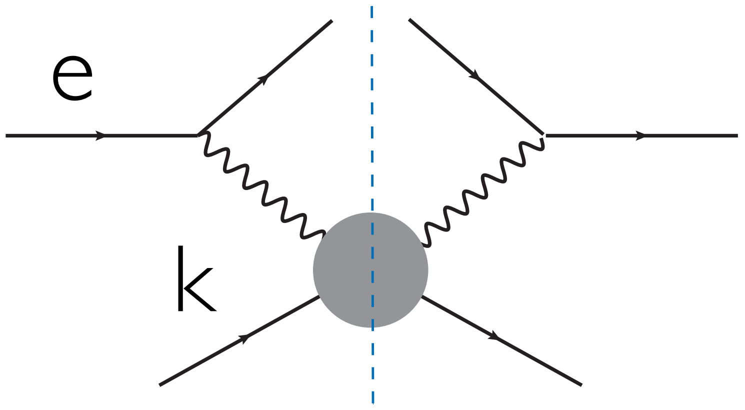
QCD FACTORIZATION IS THE KEY!



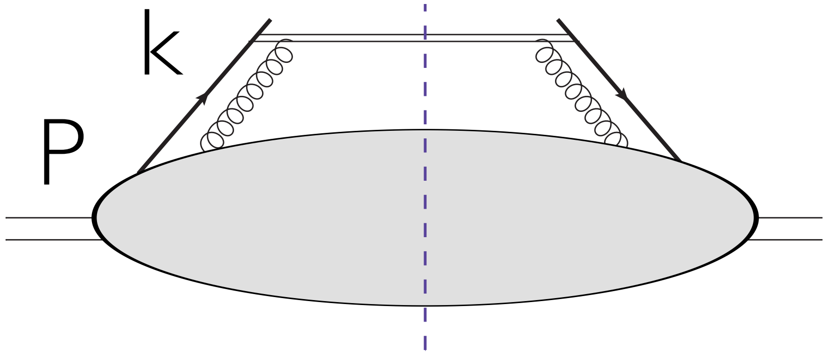
We need a probe to “see” quarks and gluons



$$\sigma \approx$$



$$\otimes$$

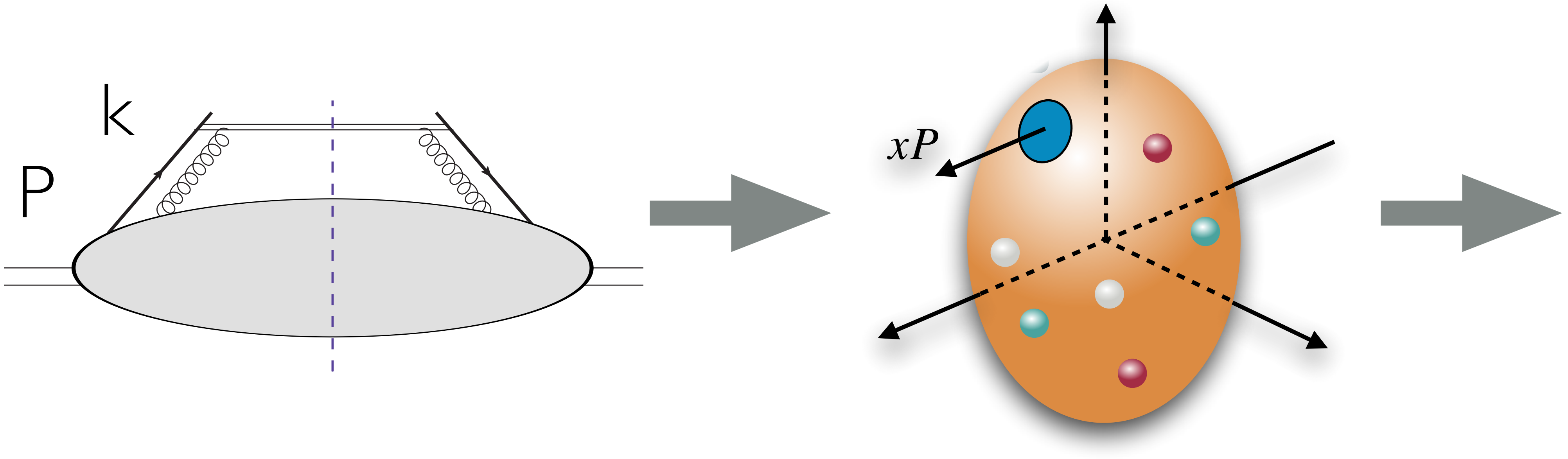


$$+\mathcal{O}\left(\frac{M^2}{Q^2}\right)$$

Factorization Probe Structure Power corrections

HADRON'S PARTONIC STRUCTURE

Collinear Parton Distribution Functions



$f_{q/P}(x)$

longitudinal

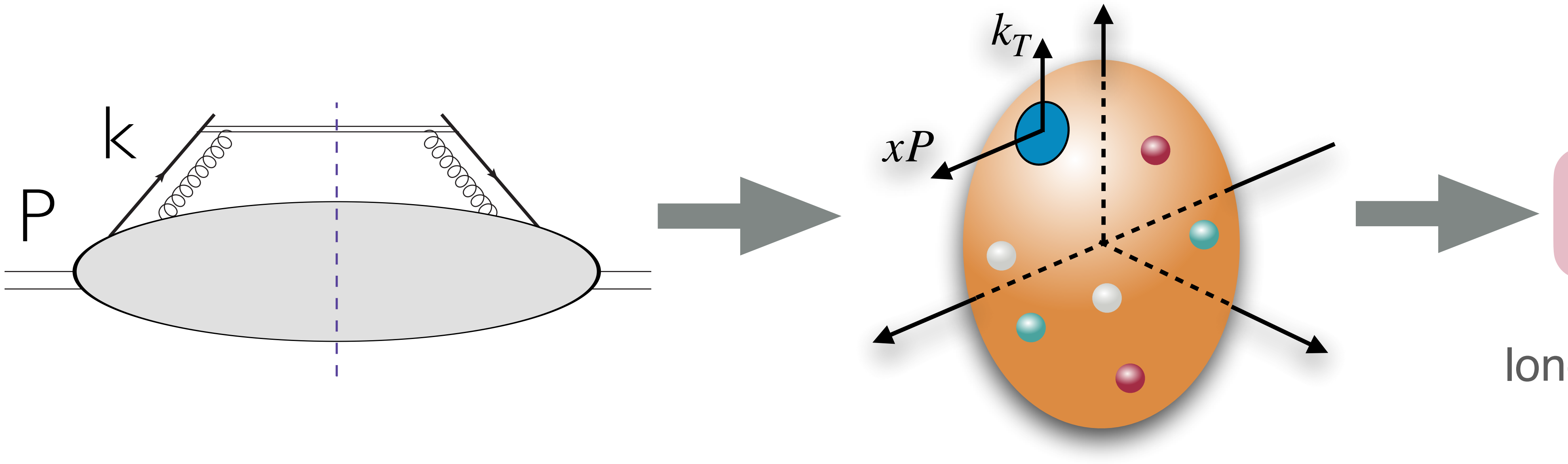
Probability density to find a quark with a momentum fraction x

Hard probe resolves the particle nature of partons, but is not sensitive to hadron's structure at ~fm distances.

HADRON'S PARTONIC STRUCTURE

To study the physics of *confined motion of quarks and gluons* inside of the proton one needs a new type "hard probe" with two scales.

Transverse Momentum Dependent functions



$$f_{q/P}(x, k_T)$$

longitudinal & transverse

One large scale (Q) sensitive to particle nature of quark and gluons
One small scale (k_T) *sensitive to how QCD bounds partons* and to the detailed structure at ~fm distances.

COLLINEAR SPIN STRUCTURE

COLLINEAR SPIN STRUCTURE

Spin-1/2 nucleon can be described by three collinear parton distribution functions (pdf)

N \ q	U	L	T
U	f_1		
L		g_1	
T			h_1

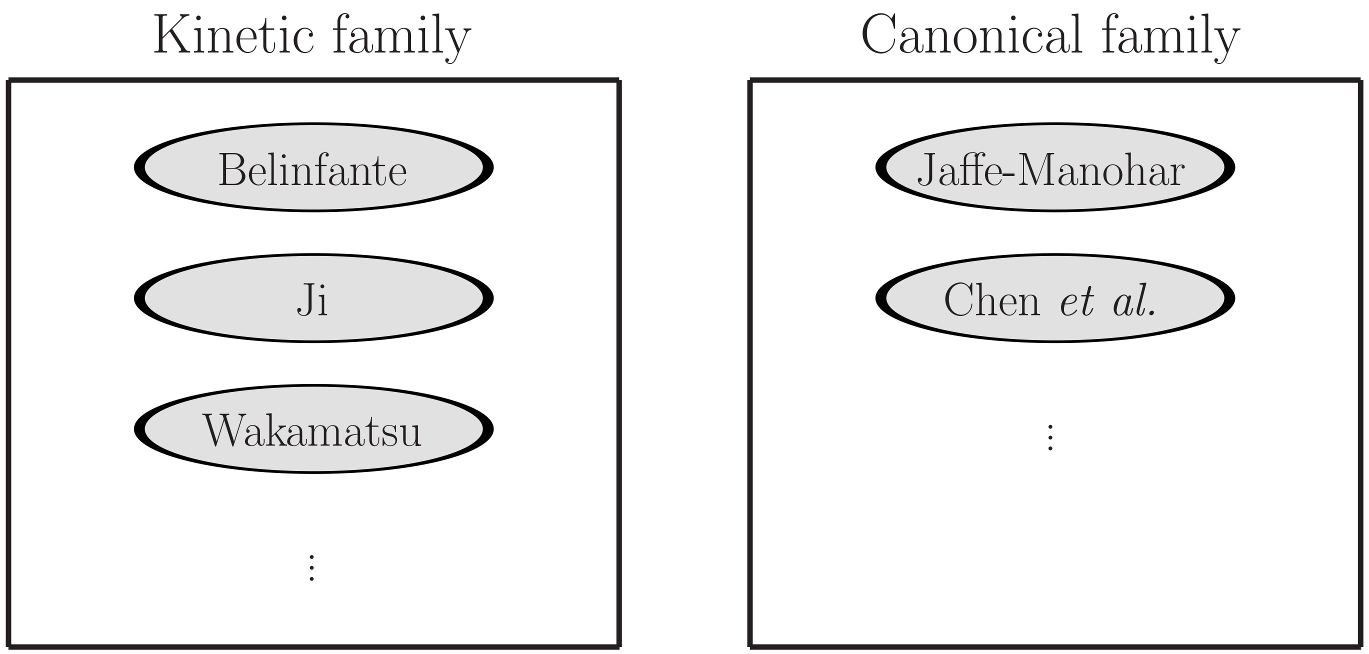
unpolarized pdf

helicity pdf

transversity pdf

SPIN DECOMPOSITION

- The nucleon is a composite system. The spin is carried by its constituents: quarks, anti-quarks and gluons and the angular momentum generated by their motion.
- The nucleon at rest has spin 1/2, however its decomposition in terms of spin and orbital contributions associated with quarks and gluons is not unique.
- There are two types of decompositions of the proton spin operator: kinetic (also known as mechanical) and canonical. These two types differ by how the OAM operator is split into the quark and gluon contributions. They share the same quark spin operator.



R. L. Jaffe and A. Manohar, Nucl. Phys. B337, 509 (1990)

S. Bashinsky and R. L. Jaffe, Nucl. Phys. B 536 (1998)

X. Ji, Phys. Rev. Lett. 78 (1997)

X. -S. Chen, X. -F. Lu, W. -M. Sun, F. Wang and T. Goldman, Phys. Rev. Lett. 100 (2008)

M. Wakamatsu, Phys. Rev. D 83 (2011)

Y. Hatta, Phys. Rev. D 84 (2011)

E. Leader and C. Lorce, Phys.Rept. 541, 163 (2014)

C. Lorcé and B. Pasquini, JHEP 09 (2013)

C. Lorcé and B. Pasquini, Phys. Rev. D 84, (2011)

C. Lorcé, B. Pasquini, X. Xiong and F. Yuan, Phys. Rev. D 85, (2012)

L. Adhikari and M. Burkard, Phys.Rev.D 94 (2016)

- Kinetic family is related to Generalized Parton Distributions, while canonical in light cone gauge is related to collinear helicity distribution functions

LONGITUDINAL SPIN

When the proton or the neutron are polarized, quarks and gluons are polarized as well. Helicity distribution functions: number of quarks/gluons with spin parallel to the nucleon momentum minus the number of quarks/gluons with the spin opposite to the nucleon momentum

$$\Delta f(x,Q^2) = g_1(x,Q^2) \equiv f^+(x,Q^2) - f^-(x,Q^2)$$

The relevant spin decomposition is by Jaffe and Manohar

R. L. Jaffe and A. Manohar, Nucl. Phys. B337, 509 (1990)

$$\frac{1}{2} = S_q + \mathcal{L}_q + S_g + \mathcal{L}_g$$

Related to measured observables:

Quark spin contribution

$$S_q = \frac{1}{2}\int_0^1 \Delta\Sigma(x,Q^2)dx \equiv \frac{1}{2}\int_0^1 (\Delta u + \Delta\bar{u} + \Delta d + \Delta\bar{d} + \Delta s + \Delta\bar{s})(x,Q^2)dx$$

Gluon spin contribution

$$S_g = \int_0^1 \Delta g(x,Q^2)dx$$

Difficult to measure in experiment:

$$\mathcal{L}_q + \mathcal{L}_g$$

quark and gluon orbital angular momenta (OAM) via twist-3 GPDs, Wigner functions

D.V. Kiptily, M.V. Polyakov, Eur. Phys. J. C 37 (2004
A. Courtoy, G. R. Goldstein, J. O. Gonzalez Hernandez, S. Liuti, A. Rajan, PLB 731 (2014)
Y. Hatta, Phys. Lett. B 708 (2012);
Y. Hatta, S. Yoshida, J. High Energy Phys. 1210 (2012

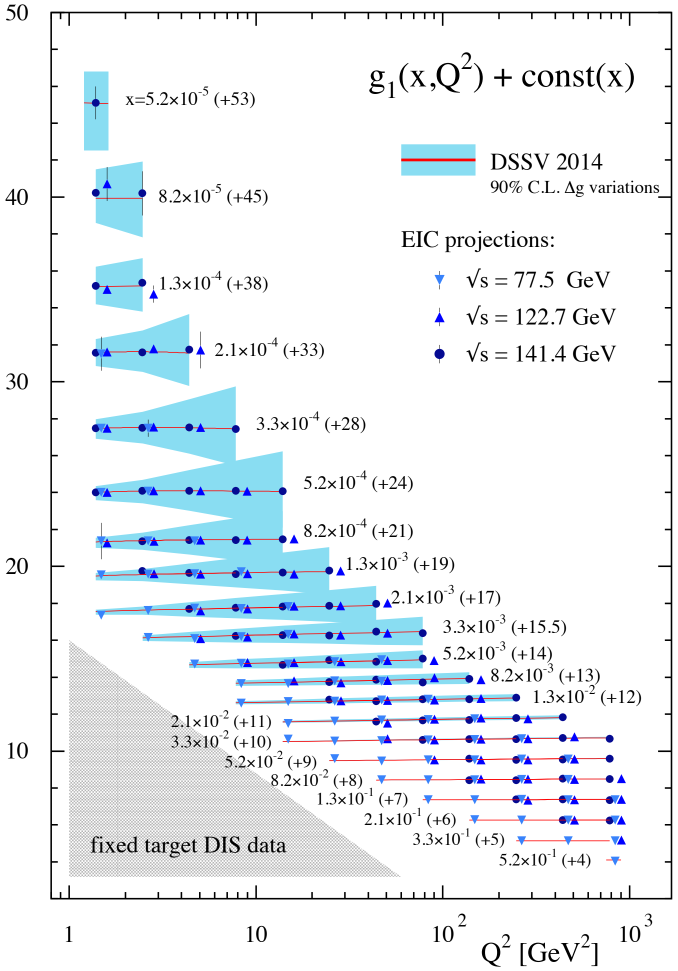
LONGITUDINAL SPIN

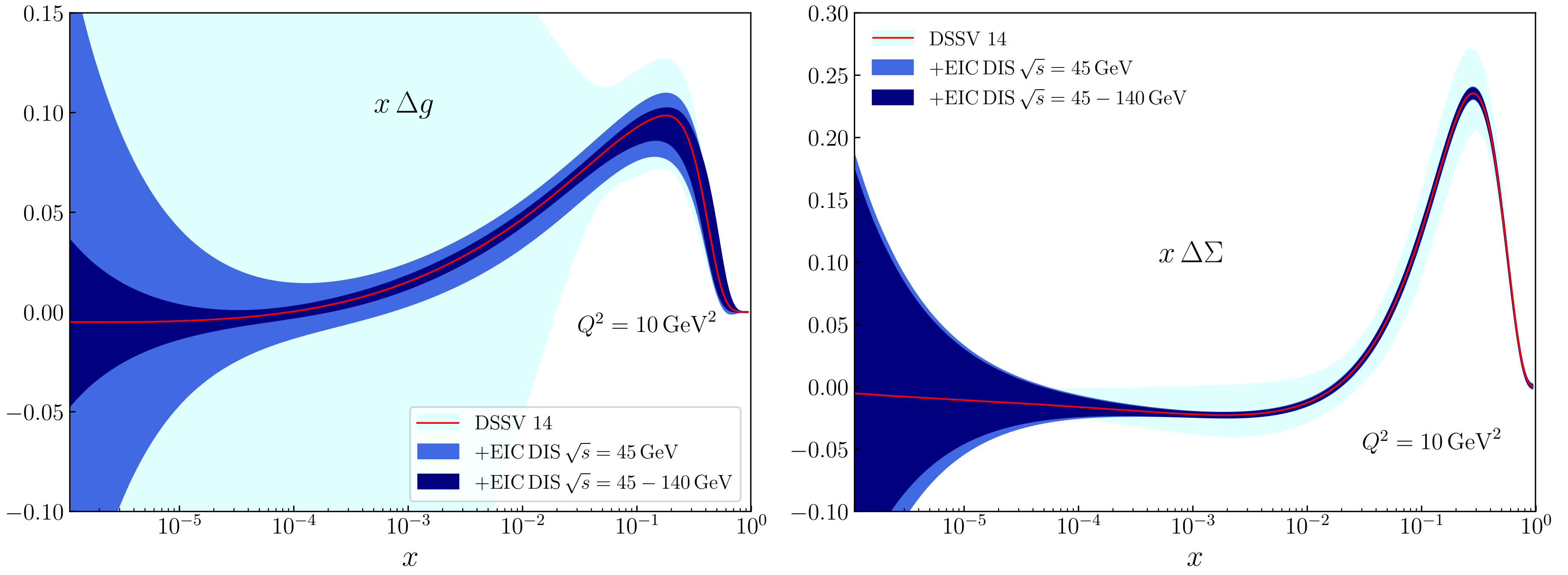
- Global QCD analyses are performed to extract helicity pdfs:

  DSSV: D. de Florian, R. Sassot, M. Stratmann and W. Vogelsang, Phys. Rev. Lett. 113 (2014)
  NNPDFpol: E. R. Nocera, R. D. Ball, S. Forte, G. Ridolfi, J. Rojo, Nucl. Phys. B 887 (2014)
  JAM: J. J. Ethier, N. Sato, W. Melnitchouk, Phys. Rev. Lett. 119 (13) (2017)

- At present around $25\ \%$ of the spin is attributed to quarks and anti-quarks.
- The evidence for non-zero gluon contribution, around $30\ \%$, is mainly due to RHIC spin program E. R. Nocera, Impact of Recent RHIC Data on Helicity-Dependent Parton Distribution Functions (2017). arXiv:1702.05077.

The impact of the EIC on determination of the quark and gluon contributions





Yellow Report (2021) arXiv:2103.05419

Aschenauer , Sassot, Stratmann, Phys.Rev.D 92 (2015)

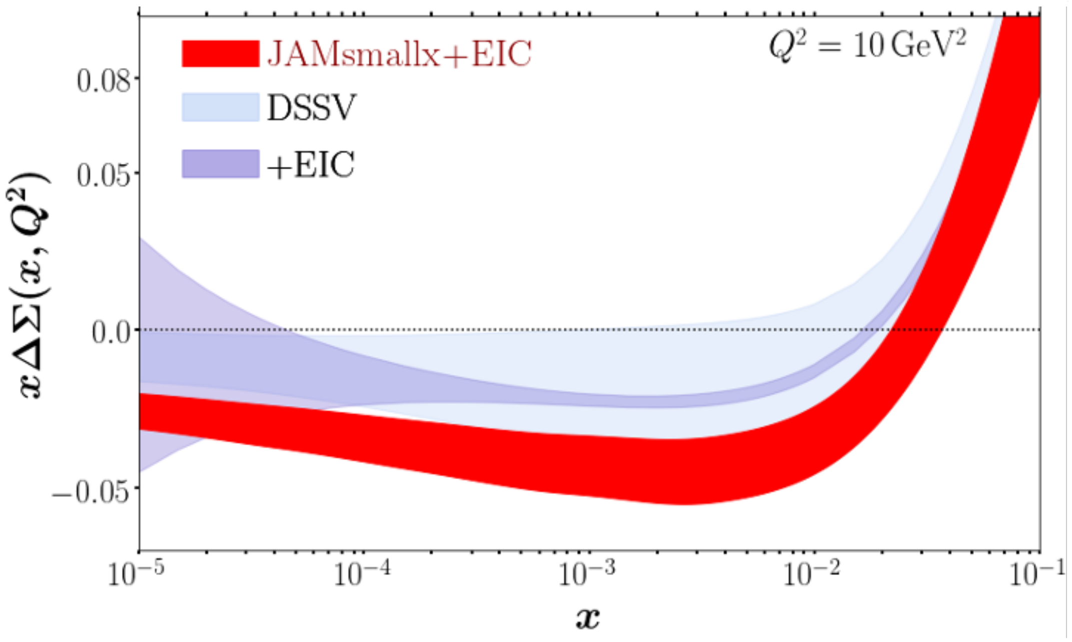
SMALL-x SPIN CHALLENGE

- Can we constrain theoretically the amount of proton spin and OAM coming from small x?
- Existing and future experiments probe the helicity distributions and OAM down to some min x

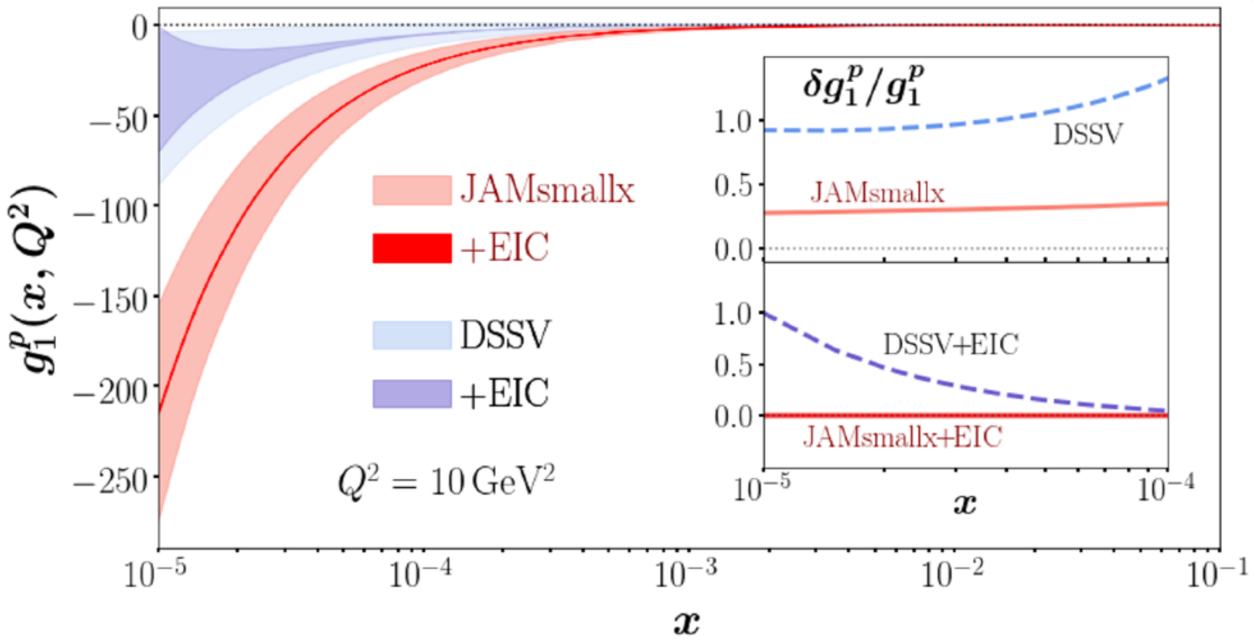
$$S_g = \int_{\textcircled{0}}^{1} \Delta g(x, Q^2) dx \qquad S_q = \frac{1}{2} \int_{\textcircled{0}}^{1} \Delta\Sigma(x, Q^2) dx$$

- If we want to predict helicity PDFs at small x, we need a different evolution equation similar to BK/JIMWLK evolving in x starting from some value of $x \approx 10^{-3}$. Pitonyak, Sievert, Kovchegov (15), (18)
- Potentially negative 10-20% of the proton spin may be carried by small-x quarks helicity (JAMsmallx, preliminary)

JAMsmallx: Adamiak, Melnitchouk, Pitonyak, Sato, Sievert, Kovchegov (2102.06159)



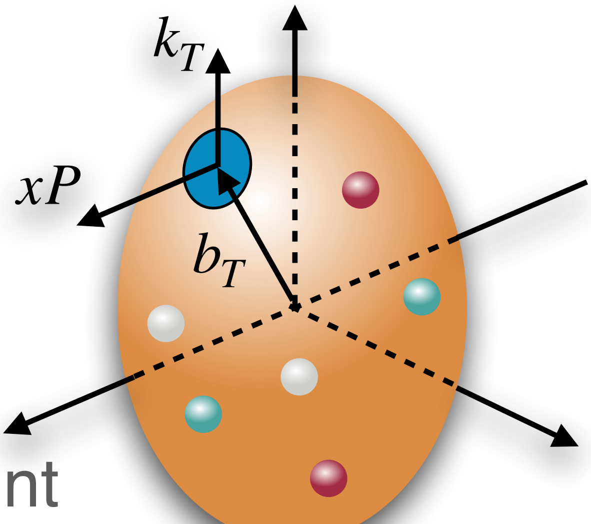
Better constraint at small-x achieved compared to the traditional approaches



$$g_1(x, Q^2) = \frac{1}{2} \sum_f e_f^2 \left[\Delta q_f + \Delta q_{\bar{f}}\right]$$

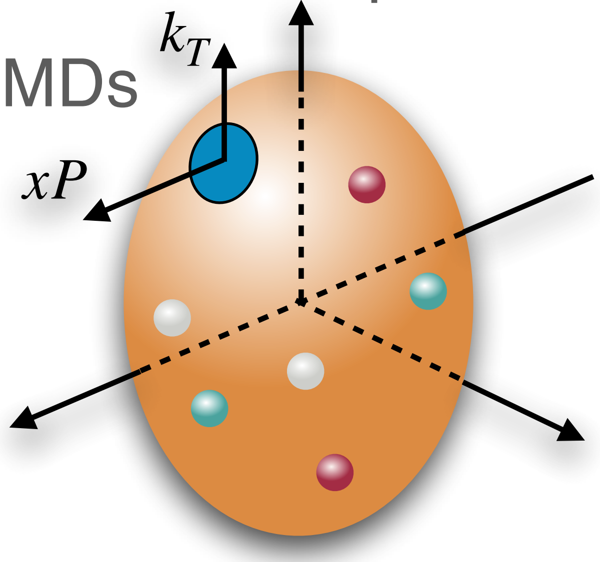
BEYOND THE COLLINEAR PICTURE

Wigner distributions
(Fourier transform of GTMDs =
Generalized Transverse
Momentum Distributions)



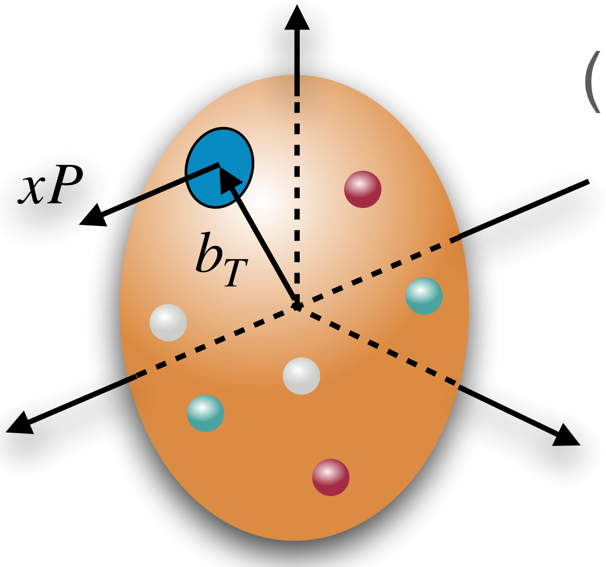
Transverse Momentum Dependent

Distributions TMDs

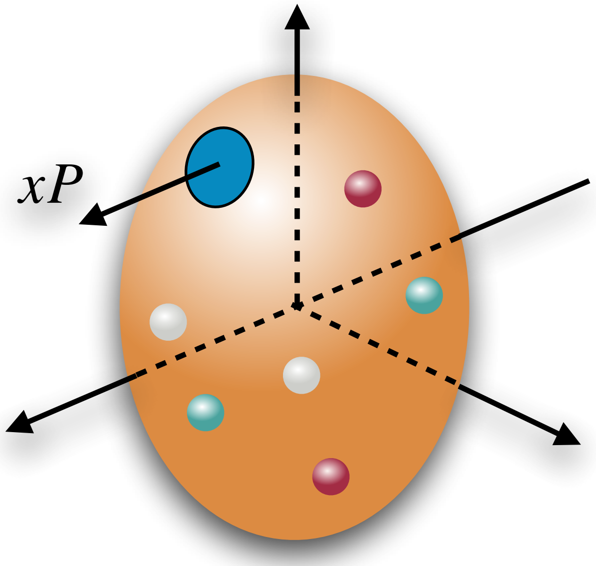


Fourier transform
of Generalized Parton Distributions

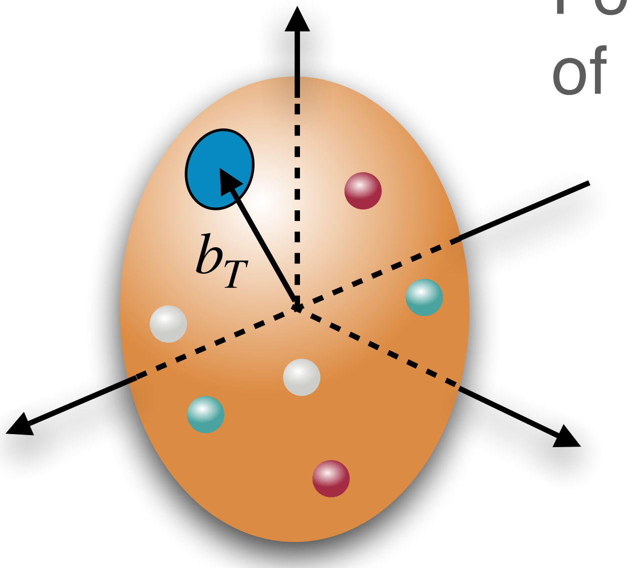
(GPDs)



PDFs



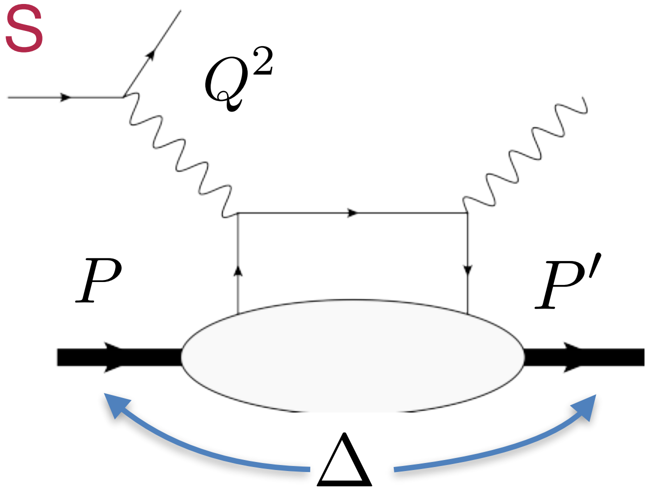
Fourier transform
of Form Factors



see, e.g., C. Lorcé, B. Pasquini, M. Vanderhaeghen, JHEP 1105 (11)

Generalized Parton Distributions

DVCS



Ji (1997)
Radyushkin (1997)

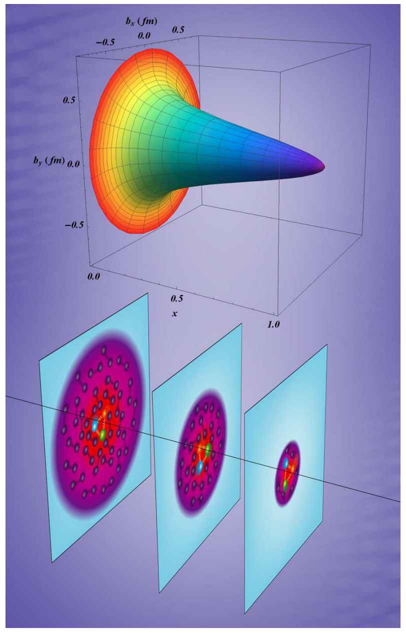
Q^2 ensures hard scale, pointlike interaction

$\Delta = P' - P$ momentum transfer can be varied independently

Connection to 3D structure

Burkardt (2000)
Burkardt (2003)

$$\rho(x,\vec{b}_\perp) = \int \frac{d^2\vec{\Delta}_\perp}{(2\pi)^2} e^{-i\vec{\Delta}_\perp\cdot\vec{b}_\perp} H_q(x,\xi=0,t=-\vec{\Delta}_\perp^2)$$

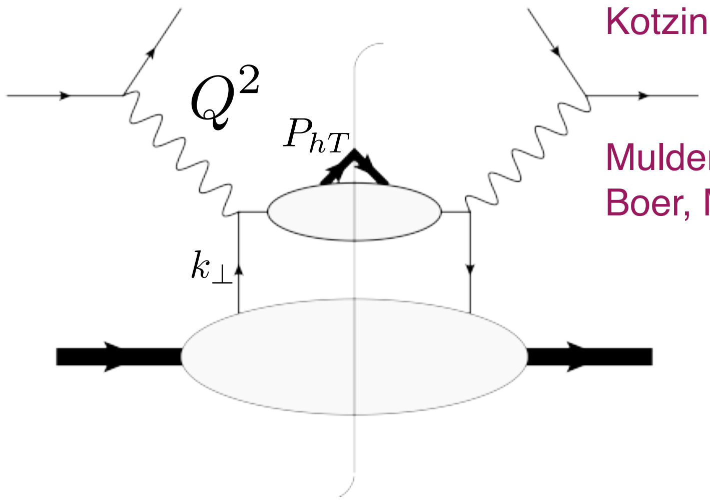


Drell-Yan frame $\Delta^+ = 0$

Dupré, Guidal, Niccolai, Vanderhaeghen, arXiv:1704.07330

Transverse Momentum Dependent distributions

SIDIS



Kotzinian (1995)

Mulders, Tangerman (1995)
Boer, Mulders (1998)

Q^2 ensures hard scale, pointlike interaction

P_{hT} final hadron transverse momentum can be varied independently

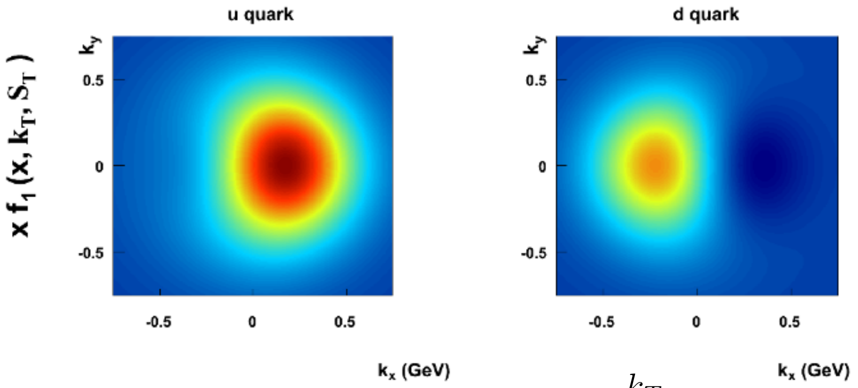
Connection to 3D structure

Ji, Ma, Yuan (2004)
Collins (2011)

$$f(x,\vec{k}_T) = \int \frac{d^2\vec{b}_T}{(2\pi)^2} e^{i\vec{k}_T\cdot\vec{b}_T} \tilde{f}(x,\vec{b}_T)$$

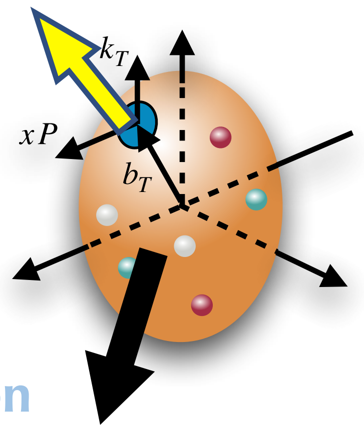
$\vec{b}_T$ is the transverse separation of parton fields in configuration space

White Paper (2012) Accardi et al, arXiv:1212:1701



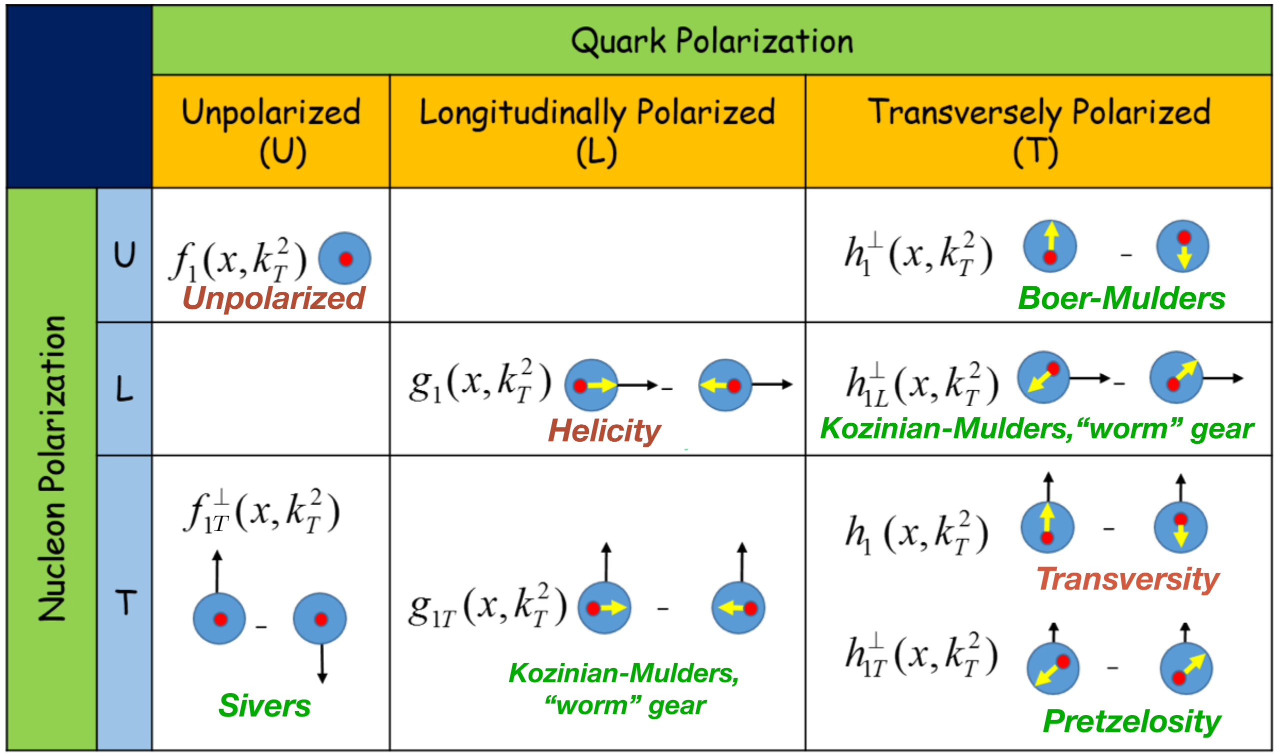
$$\rho_{1;q\leftarrow h^\uparrow}(x,\boldsymbol{k}_T,\boldsymbol{S}_T,\mu) = f_{1;q\leftarrow h}(x,k_T;\mu,\mu^2) - \frac{k_{Tx}}{M} f^{\perp}_{1T;q\leftarrow h}(x,k_T;\mu,\mu^2)$$

Our understanding of the hadron evolves: SPIN



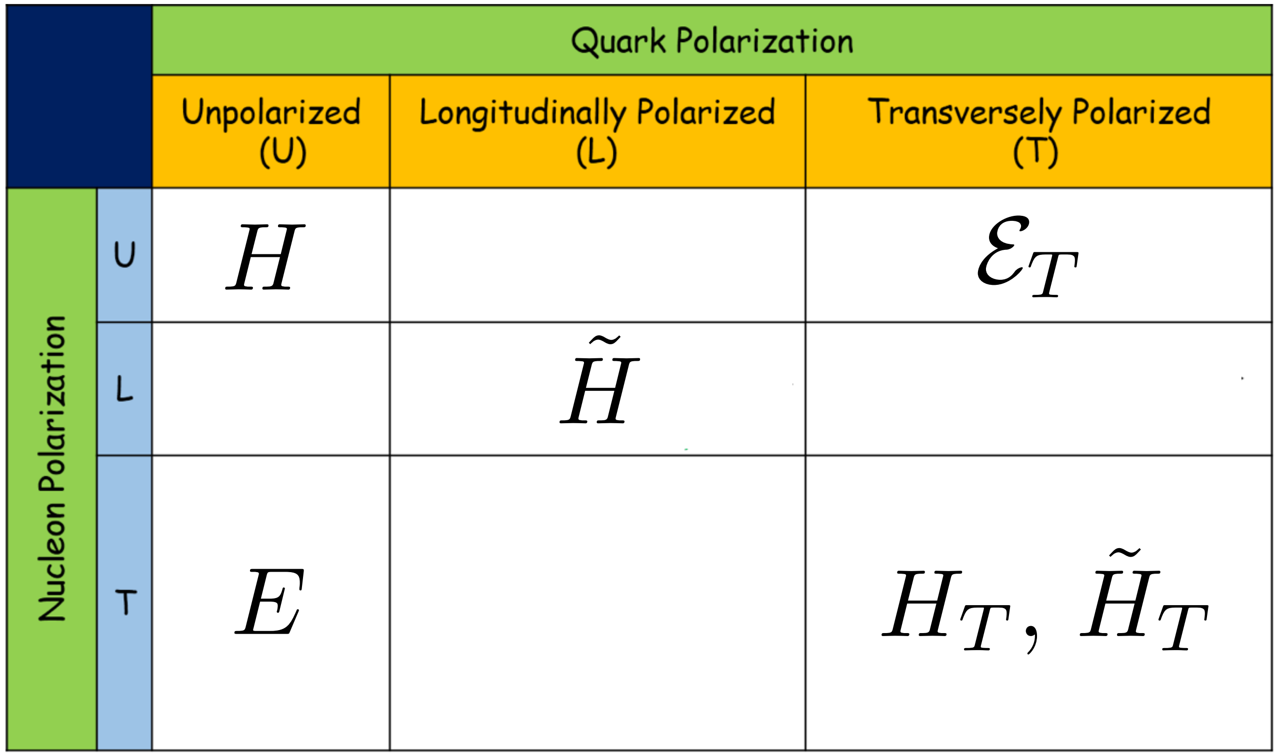
Nucleon emerges as a strongly interacting, relativistic bound state of quarks and gluons

TMDs



		Quark Polarization		
		Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	$f_1(x,k_T^2)$ ***Unpolarized***		$h_1^{\perp}(x,k_T^2)$ ***Boer-Mulders***
	L		$g_1(x,k_T^2)$ ***Helicity***	$h_{1L}^{\perp}(x,k_T^2)$ ***Kozinian-Mulders,"worm" gear***
	T	$f_{1T}^{\perp}(x,k_T^2)$ ***Sivers***	$g_{1T}(x,k_T^2)$ ***Kozinian-Mulders, "worm" gear***	$h_1(x,k_T^2)$ ***Transversity*** $h_{1T}^{\perp}(x,k_T^2)$ ***Pretzelosity***

GPDs



		Quark Polarization		
		Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	H		$\mathcal{E}_T$
	L		$\tilde{H}$	
	T	E		$H_T,\ \tilde{H}_T$

Many TMDs and GPDs cannot exist without OAM.

Examples: TMD Sivers function $f_{1T}^{\perp}$ and GPD *E*

Generalized Parton Distributions

LONGITUDINAL SPIN

Studies of DVCS process were highly motivated by Ji decomposition

$$\frac{1}{2} = S_q + L_q + J_g$$

X. Ji, Phys. Rev. Lett. 78 (1997) 610

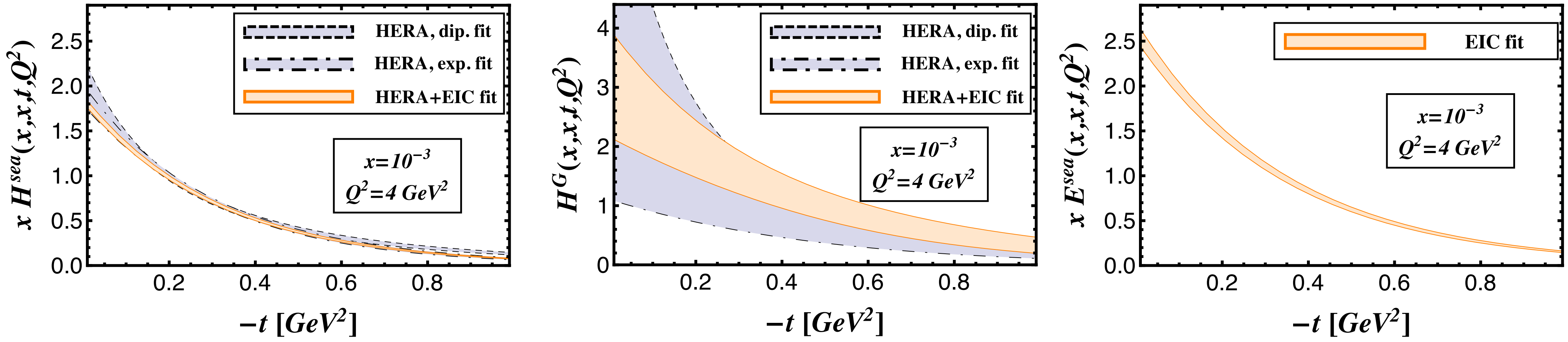
Related to twist-2 GPDs:

$$J_q \equiv S_q + L_q = \frac{1}{2}\int_0^1 \Delta\Sigma(x,Q^2)dx + L_q = \frac{1}{2}\int_{-1}^1 dx x\left(H^q(x,\xi=0,t=0) + E^q(x,\xi=0,t=0)\right)$$

$$J_g = \frac{1}{2}\int_{-1}^1 dx x\left(H^g(x,\xi=0,t=0) + E^g(x,\xi=0,t=0)\right)$$

These quantities can be computed also in lattice simulations

The impact of the EIC on determination of the sea-quark and gluon contributions



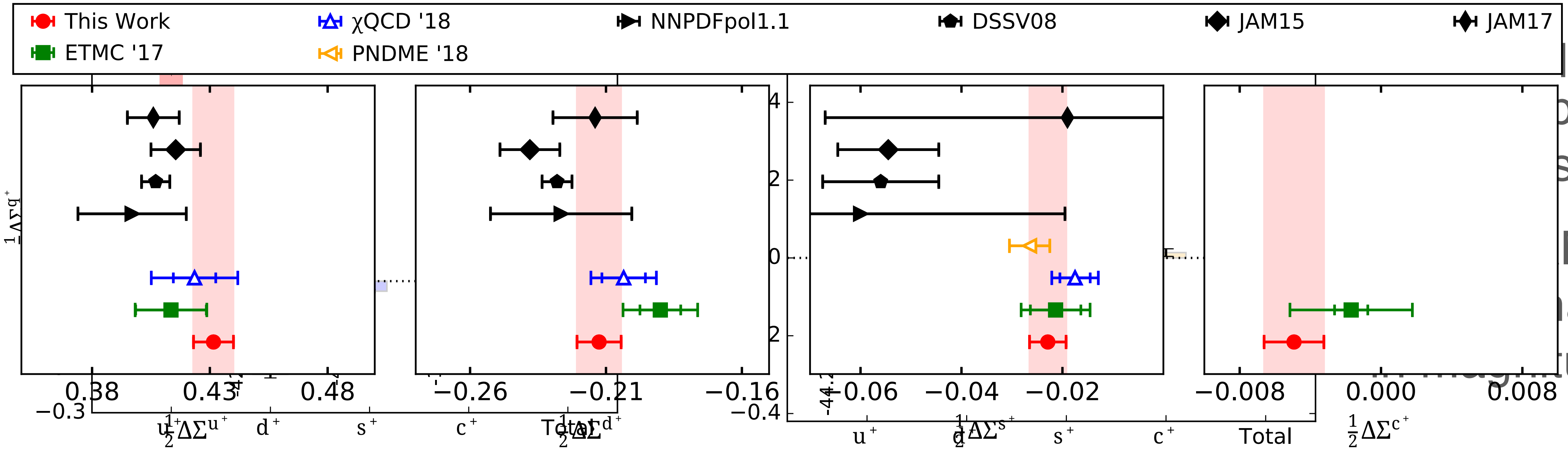
E. Aschenauer, S. Fazio, K. Kumericki, D. Mueller, JHEP 09 (2013) 093

LONGITUDINAL SPIN AND LATTICE QCD

- Many methods are explored by lattice QCD to calculate the spin and OAM, subtraction, direct computations

See e.g. review of Keh-Fei Liu and Cédric Lorcé, Eur.Phys.J.A 52 (2016)

- Spin contributions can be computed at physical pion mass. An example from ETMC Collaboration C. Alexandrou et al, Phys.Rev.D 101 (2020)



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Ms are
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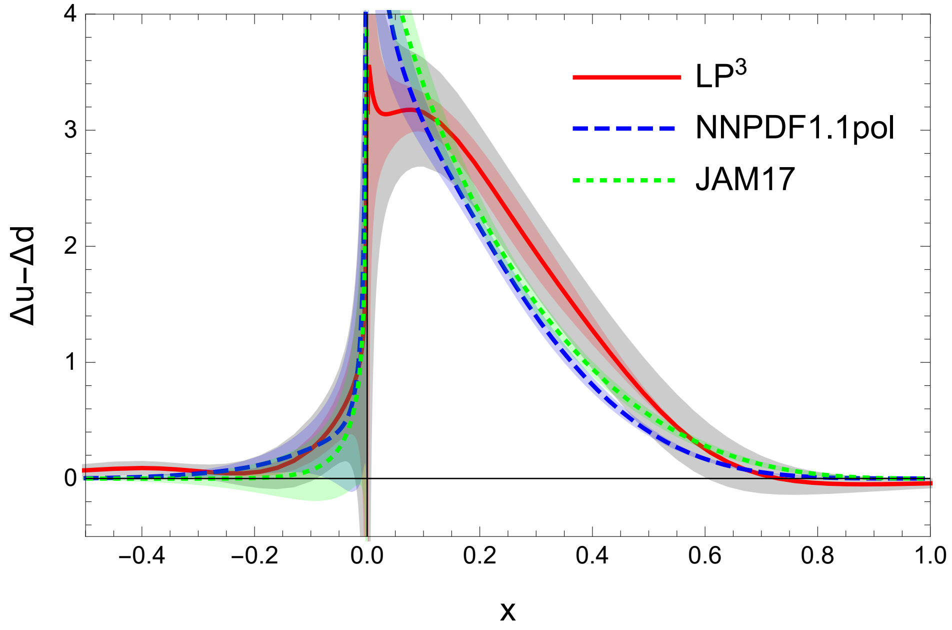
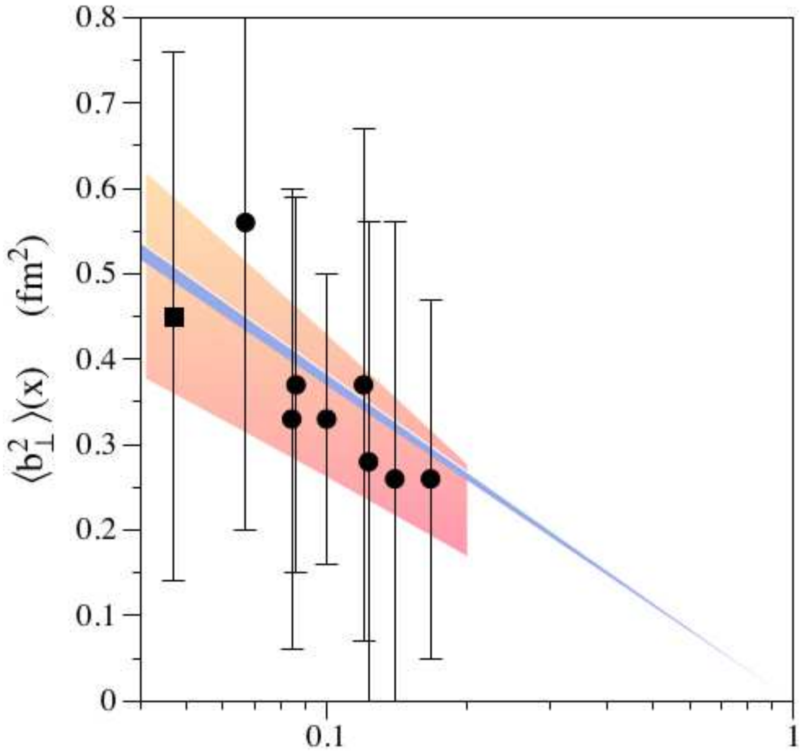
- Lattice QCD computes also the shape of pdfs, GPDs, TMDs using various approaches. An example from LP3 Collaboration

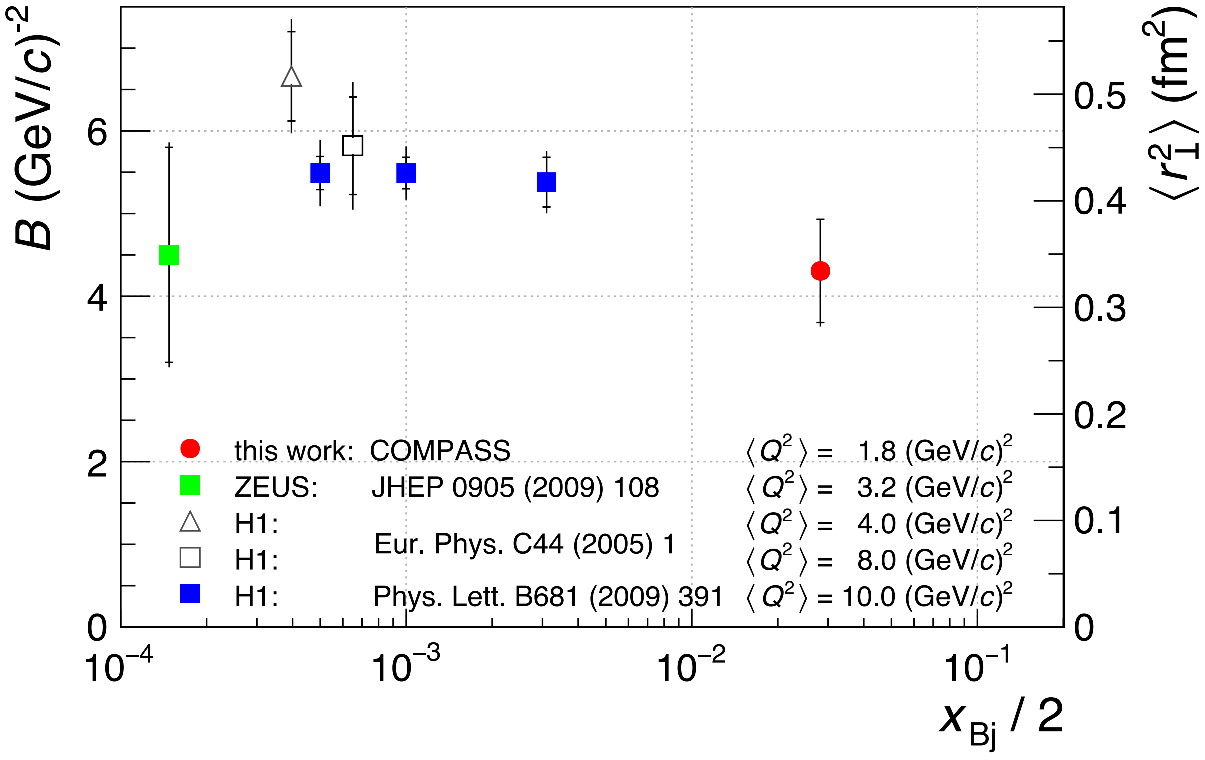
H. -W. Lin et al Phys.Rev.Lett. 121 (2018)



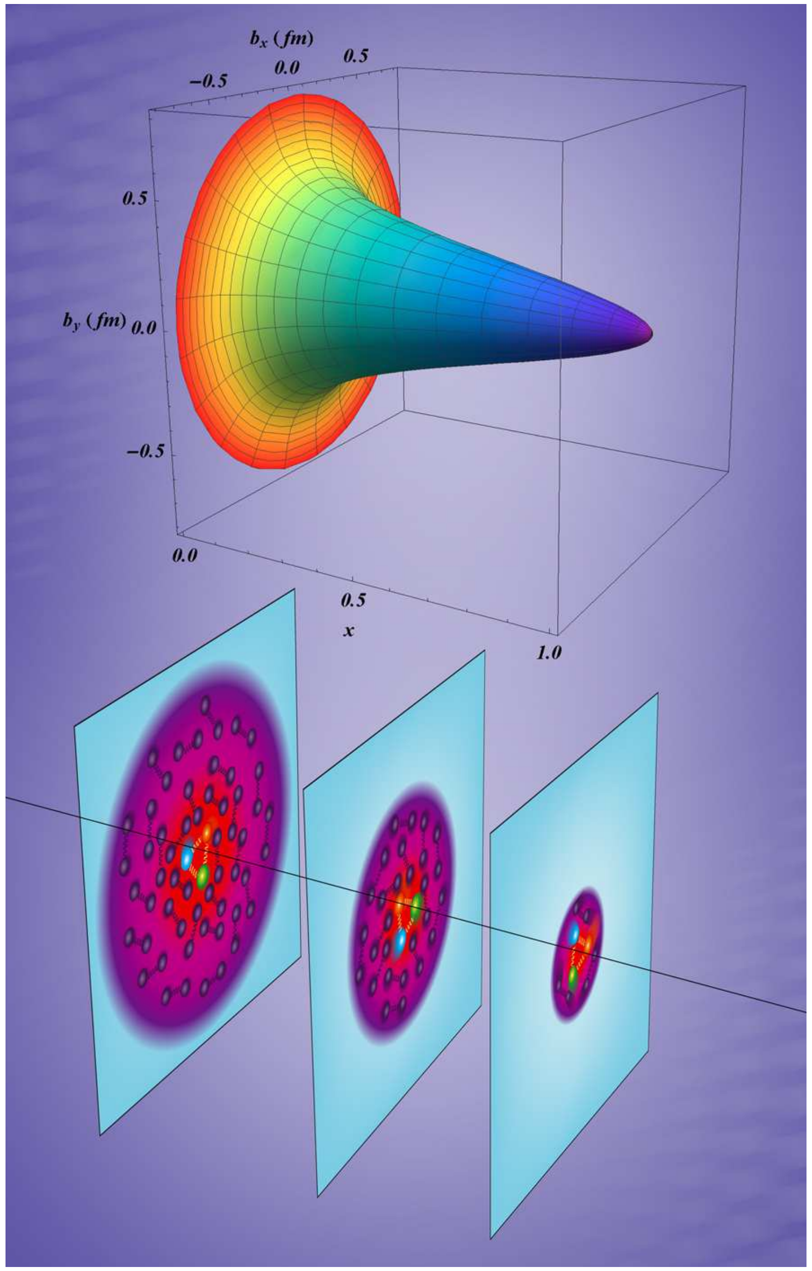
IMPACT PARAMETER DISTRIBUTIONS

Dupré, Guidal, Niccolai, Vanderhaeghen, arXiv:1704.07330





COMPASS coll., arXiv:1802.02739

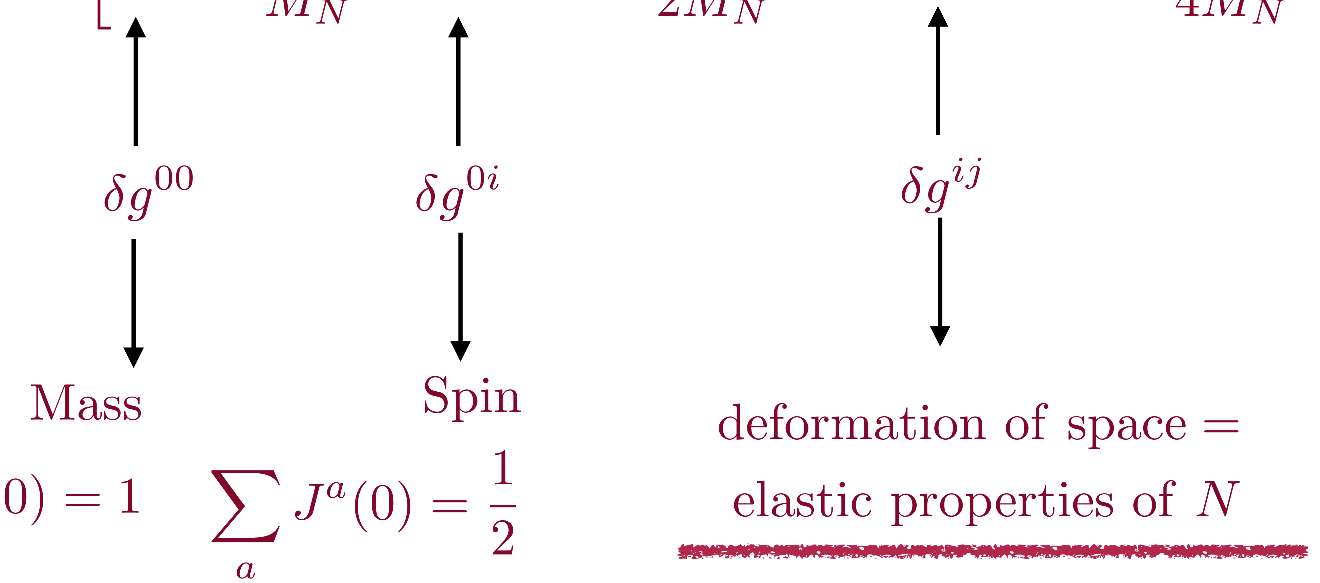


PRESSURE DISTRIBUTION IN THE PROTON

The study of the multidimensional structure of the proton can in principle allow us to access the proton energy-momentum tensor

Kobazarev, Okun (1963)

$$\langle p'|T^a_{\mu\nu}(0)|p\rangle = \bar{u}'\left[A^a(t)\,\frac{P_\mu P_\nu}{M_N} + J^a(t)\,\frac{i\,P_{\{\mu}\sigma_{\nu\}\rho}\Delta^\rho}{2M_N} + D^a(t)\,\frac{\Delta_\mu\Delta_\nu - g_{\mu\nu}\Delta^2}{4M_N} + \ldots\right]u$$

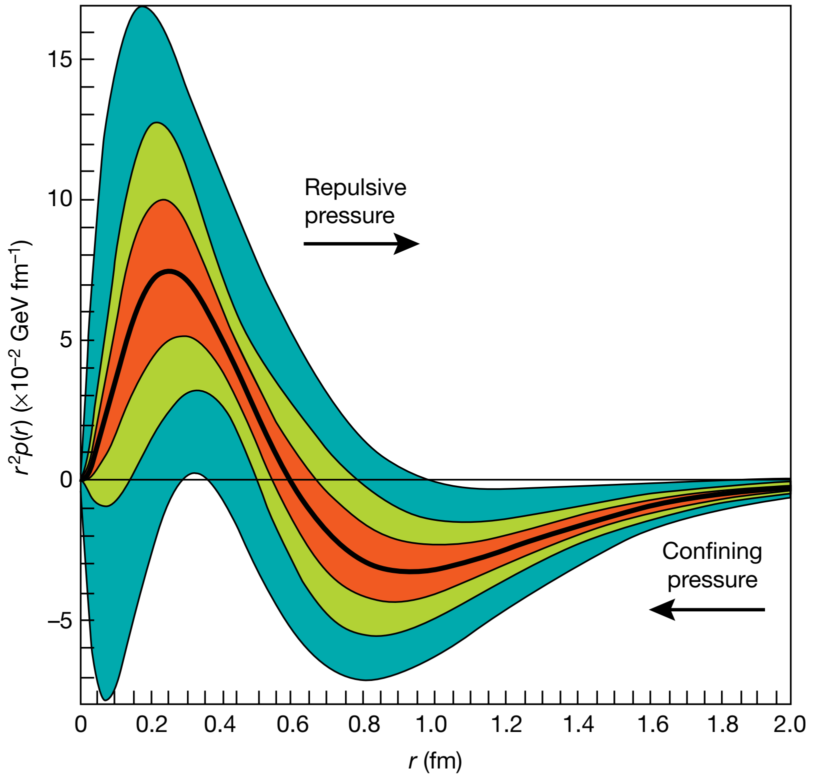


δg^{00} — Mass — $\sum_a A^a(0) = 1$

δg^{0i} — Spin — $\sum_a J^a(0) = \frac{1}{2}$

δg^{ij} — deformation of space = elastic properties of N

M.V. Polyakov (03)

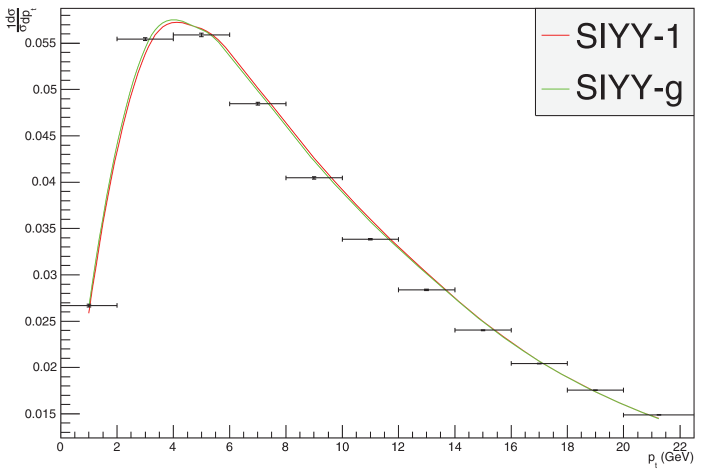


Burkert, Elouadrhiri, Girod, Nature 577 (18)

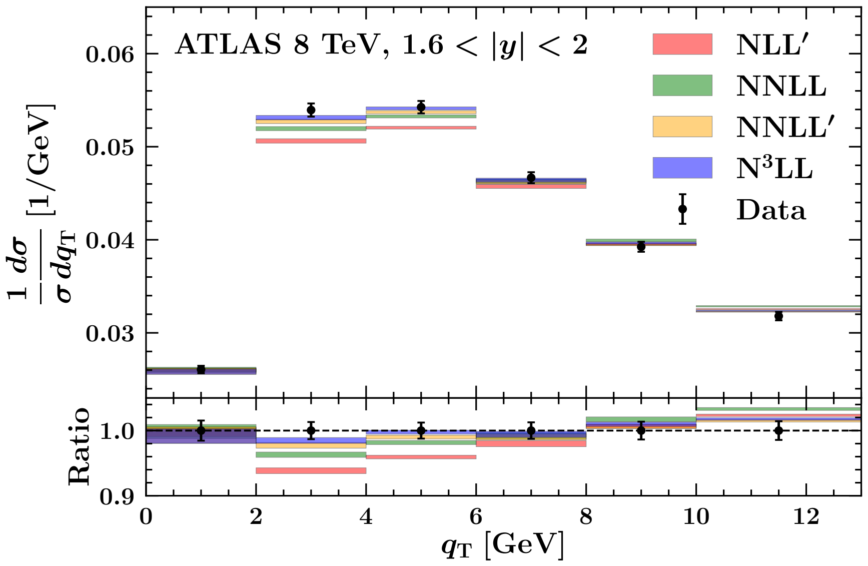
Tantalizing results. Need more solid underpinning.

Transverse Momentum Dependent distributions

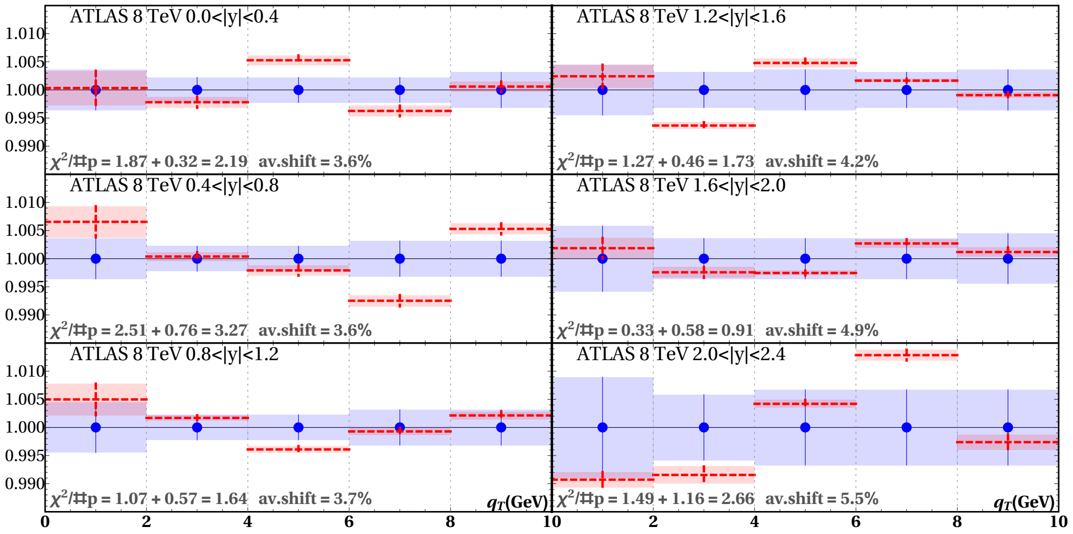
SUCCESS OF TMD FACTORIZATION PREDICTIVE POWER



Sun, Isaacson, Yuan, Yuan Int.J.Mod.Phys.A 33 (2018)



Bacchetta et al, JHEP 07 (2020) 117



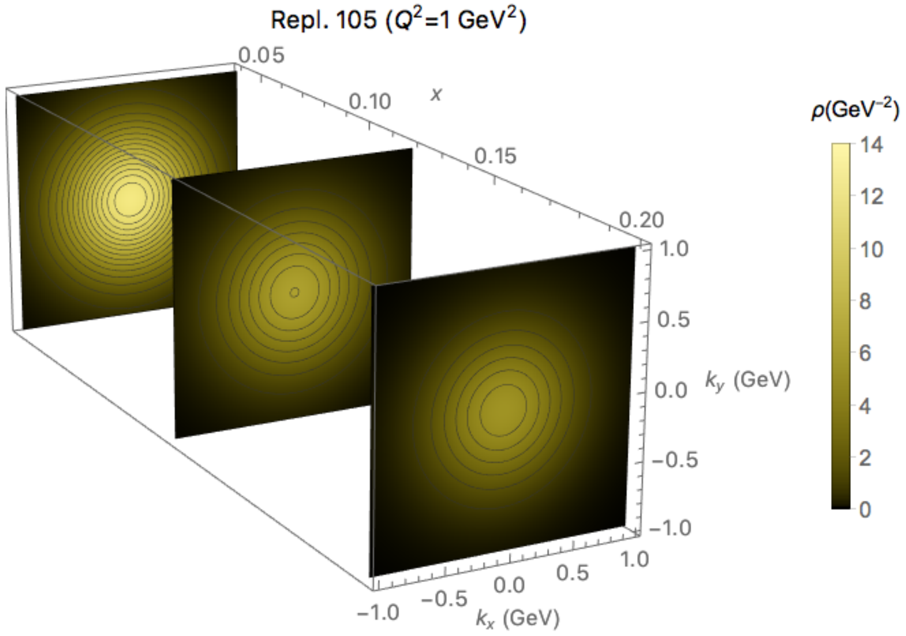
Bertone, Scimemi, Vladimirov JHEP 06 (2019) 028

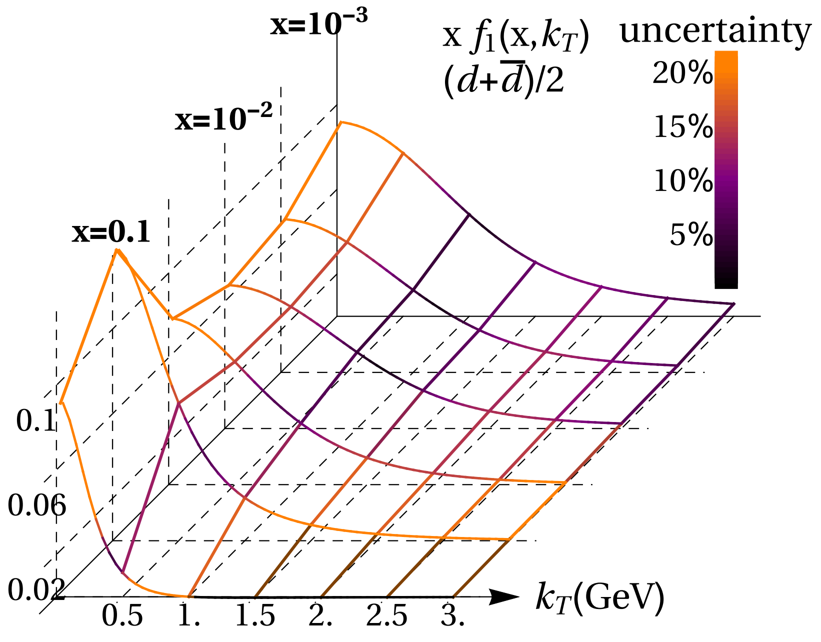
Z boson production at the LHC

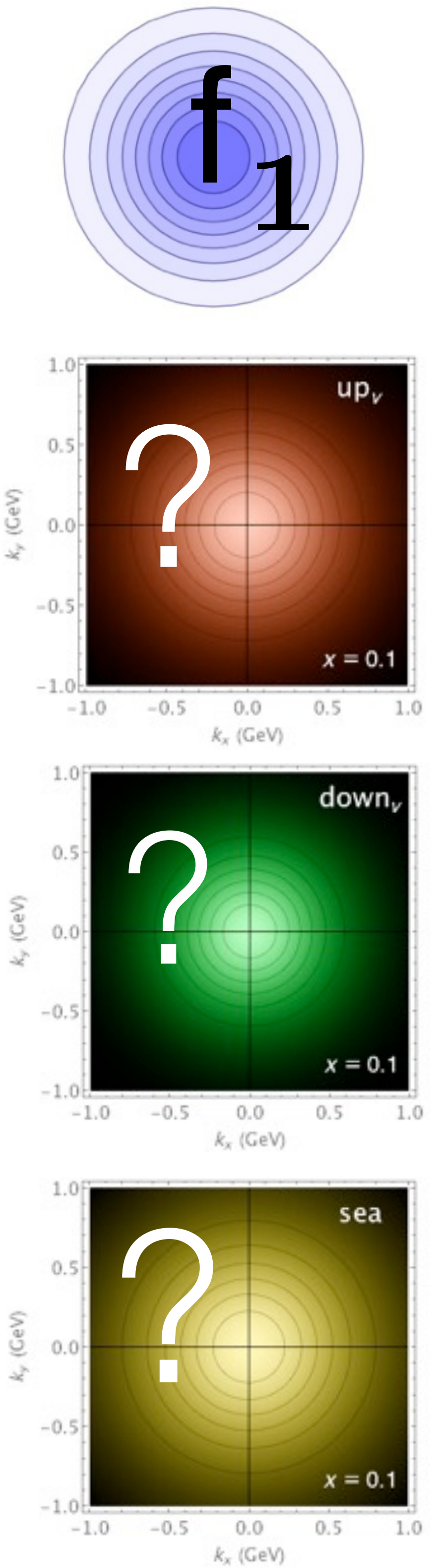
- TMD factorization (with an appropriate matching to collinear results) aims at an accurate description (and prediction) of a differential in q_T cross section in a wide range of q_T
- LHC results at 7 and 13 TeV are accurately predicted from fits of lower energies

UNPOLARIZED TMD MEASUREMENTS

Unpolarized cross section







Bacchetta, Delcarro, Pisano, Radici, Signori, arXiv:1703.10157

Bertone, Scimemi, Vladimirov, arXiv:1902.08474

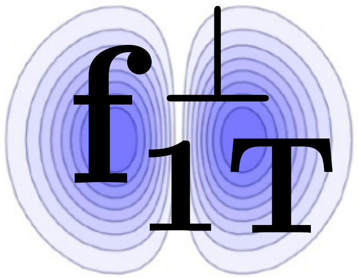
- Addresses the question of partonic confined motion
- Evolution with x and Q^2
- Flavor dependence of unpolarized TMDs
- Interplay with collinear QCD at large q_T

TMD FITS OF UNPOLARIZED DATA

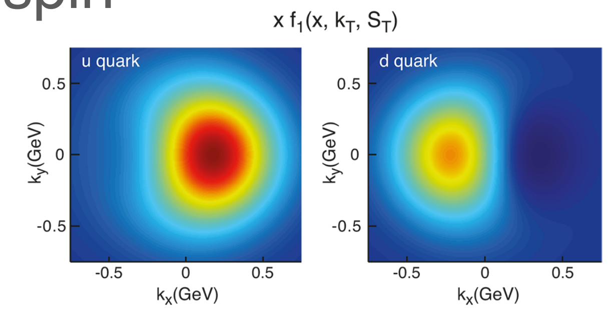
	Framework	W+Y	HERMES	COMPASS	DY	Z boson	N of points
KN 2006 hep-ph/0506225	LO-NLL	W	✘	✘	✔	✔	98
QZ 2001 hep-ph/0506225	NLO-NLL	W+Y	✘	✘	✔	✔	28 (?)
RESBOS resbos@msu	NLO-NNLL	W+Y	✘	✘	✔	✔	>100 (?)
Pavia 2013 arXiv:1309.3507	LO	W	✔	✘	✘	✘	1538
Torino 2014 arXiv:1312.6261	LO	W	✔ (separately)	✔ (separately)	✘	✘	576 (H) 6284 (C)
DEMS 2014 arXiv:1407.3311	NLO-NNLL	W	✘	✘	✔	✔	223
EIKV 2014 arXiv:1401.5078	LO-NLL	W	1 (x,Q^2) bin	1 (x,Q^2) bin	✔	✔	500 (?)
SIYY 2014 arXiv:1406.3073	NLO-NLL	W+Y	✘	✔	✔	✔	200 (?)
Pavia 2017 arXiv:1703.10157	LO-NLL	W	✔	✔	✔	✔	8059
SV 2017 arXiv:1706.01473	NNLO-NNLL	W	✘	✘	✔	✔	309
BSV 2019 arXiv:1902.08474	NNLO-NNLL	W	✘	✘	✔	✔	457
Pavia 2019 arXiv:1912.07550	NNLO-N3LL	W	✘	✘	✔	✔	353
SV 2019 arXiv:1912.06532	NNLO-N3LL	W	✔	✔	✔	✔	1039

POLARIZED TMD FUNCTIONS

Sivers function

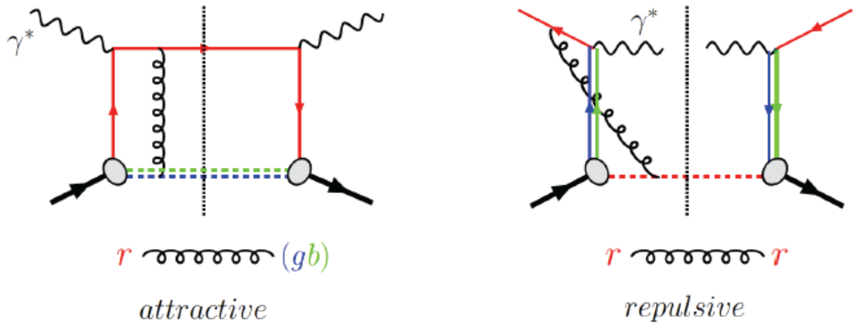


- Describes unpolarized quarks inside of transversely polarized nucleon
- Encodes the correlation of orbital motion with the spin

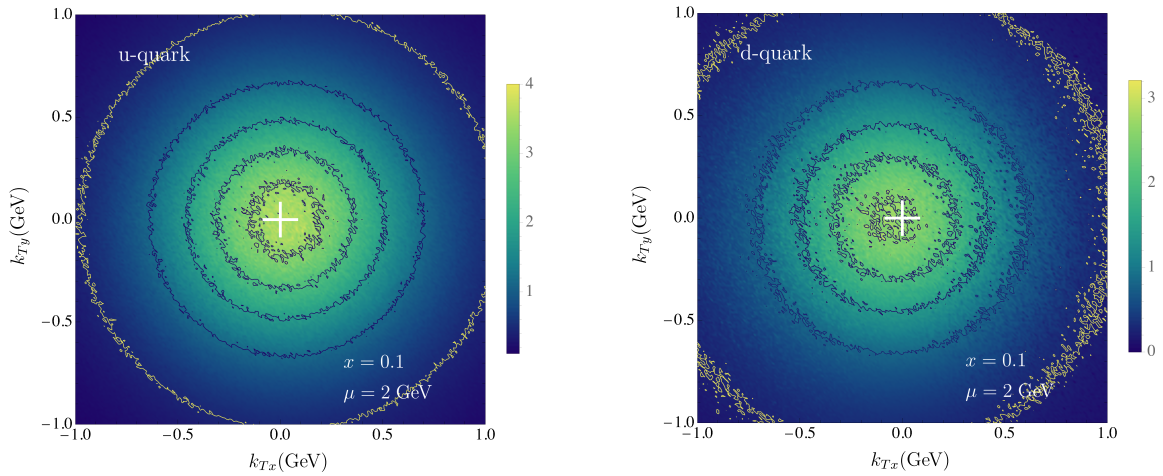


- Sign change of Sivers function is fundamental consequence of QCD

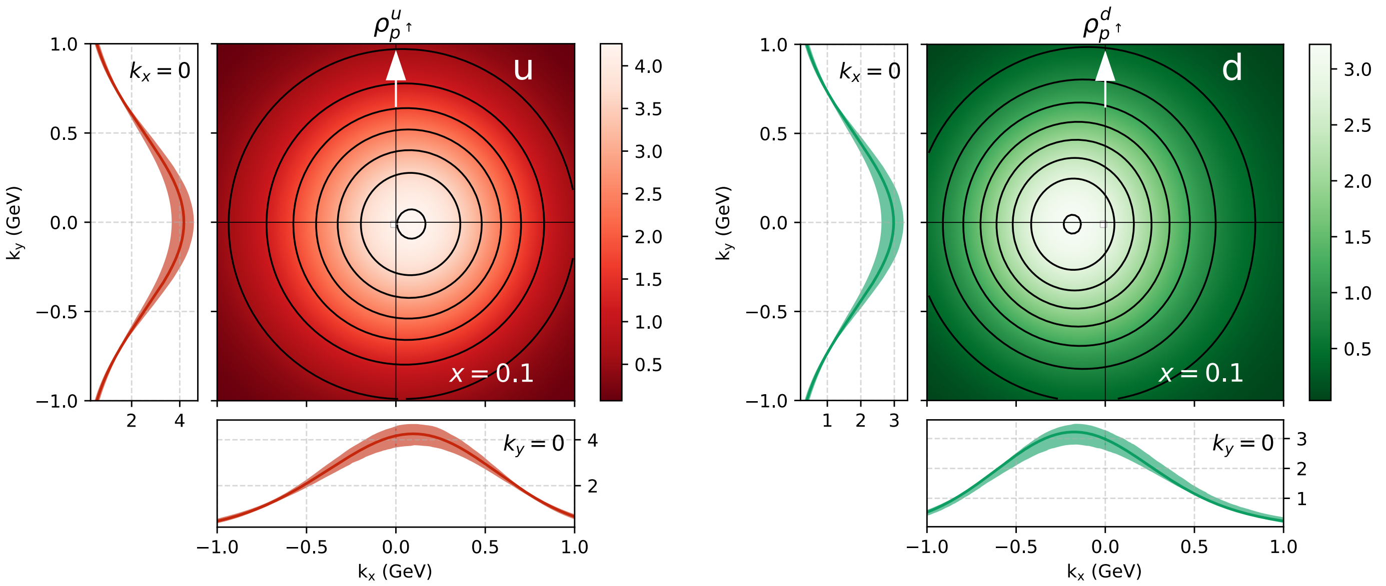
Brodsky, Hwang, Schmidt (2002), Collins (2002)



$$f_{1T}^{\perp SIDIS} = -f_{1T}^{\perp DY}$$



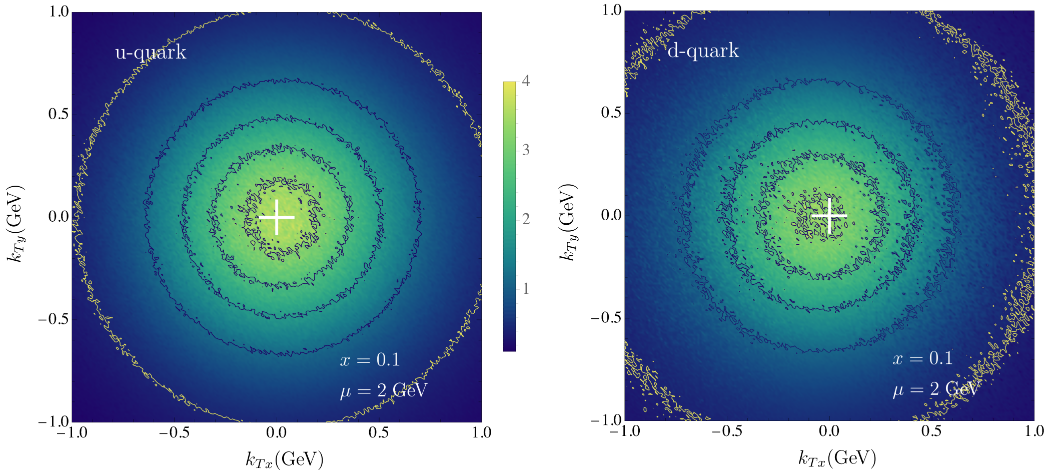
M. Bury, A. Prokudin, A. Vladimirov, Phys.Rev.Lett. 126 (2021)



A. Bacchetta, F. Delcarro, C. Pisano, M. Radici (2020)

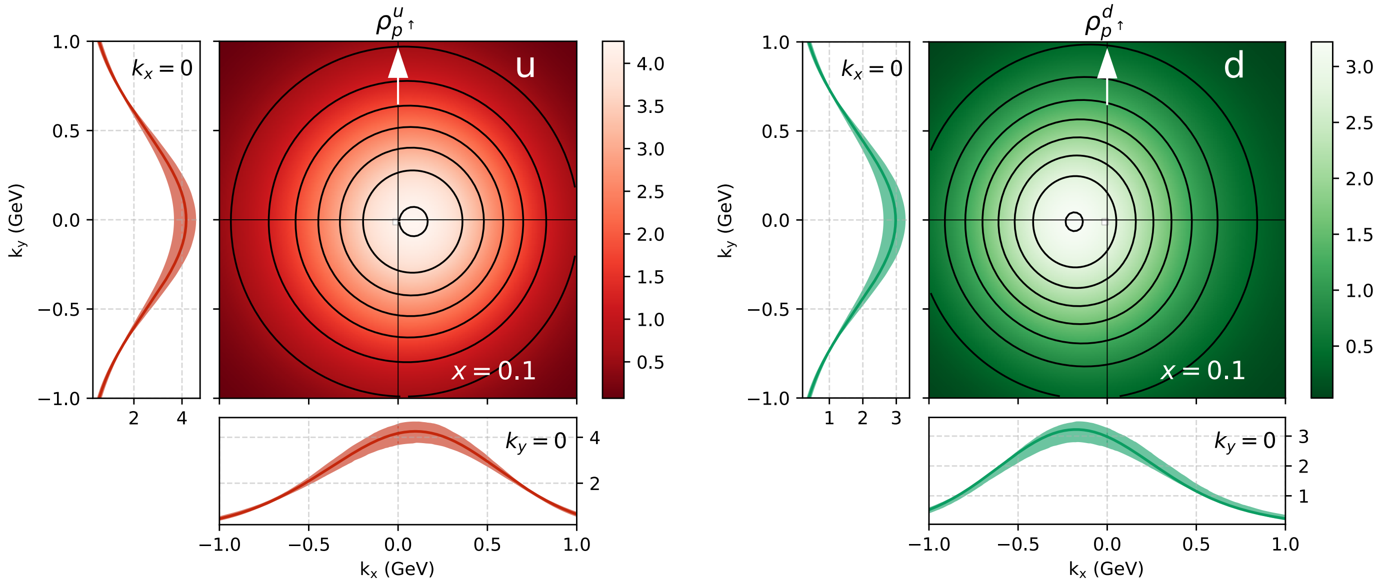
NUCLEON TOMOGRAPHY – THE FINAL GOAL OF THE EIC

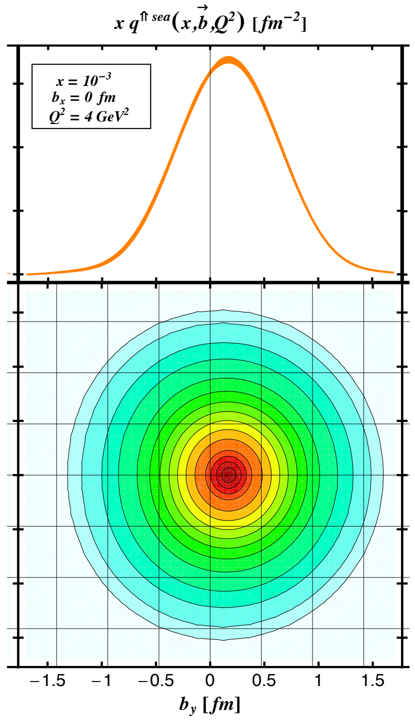
$$\rho_{1;q\leftarrow h^{\uparrow}}(x,\boldsymbol{k}_T,\boldsymbol{S}_T,\mu) = f_{1;q\leftarrow h}(x,k_T;\mu,\mu^2) - \frac{k_{Tx}}{M} f_{1T;q\leftarrow h}^{\perp}(x,k_T;\mu,\mu^2)$$



M. Bury, A. Prokudin, A. Vladimirov, Phys.Rev.Lett. 126 (2021)

A. Bacchetta, F. Delcarro, C. Pisano, M. Radici (2020)





$$q^{\Uparrow}(x,\vec{b},\mathcal{Q}^2) = q(x,\vec{b},\mathcal{Q}^2) - \frac{1}{2M_p}\frac{\partial}{\partial b_y}E(x,\vec{b},\mathcal{Q}^2)$$

E. Aschenauer, S. Fazio, K. Kumericki, D. Mueller, JHEP 09 (2013) 093

- The shift in the transverse plane is generated by the Sivers function and GPD *E* that cannot exist without OAM
- The opposite signs of the shift is consistent with lattice QCD findings on the opposite signs of the OAM for u and d quarks

TRANSVERSITY

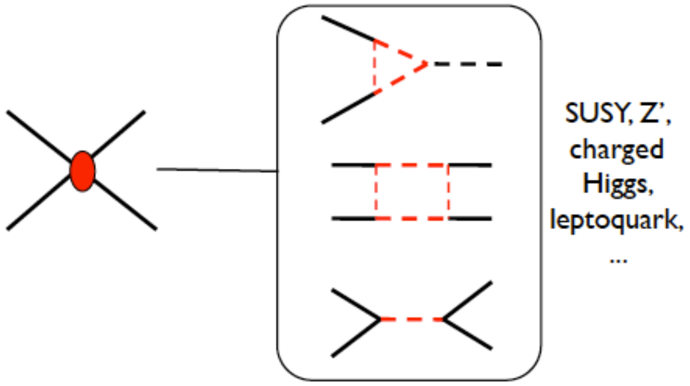
- The only source of information on tensor charge of the nucleon

$$\delta q \equiv g_T^q = \int_0^1 dx \ \left[h_1^q(x,Q^2) - h_1^{\bar{q}}(x,Q^2)\right]$$

- Tensor couplings, not present in the SM Lagrangian, could be the footprints of new physics at higher scales

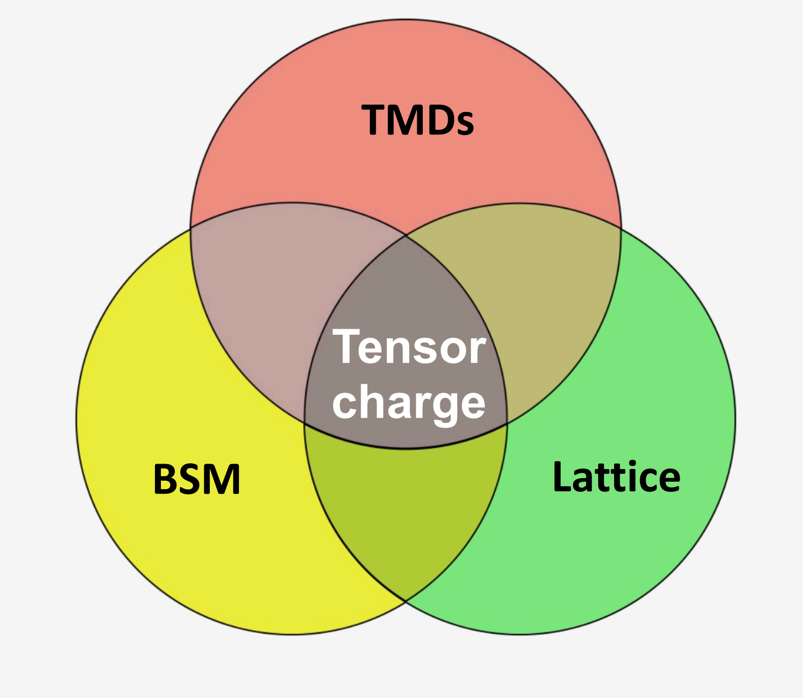
Bhattacharya et al, PRD 85 (12)
Pattie et al., P.R. C88 (13)

$\varepsilon_T \, g_T \approx M_W^2 / M_{BSM}^2$



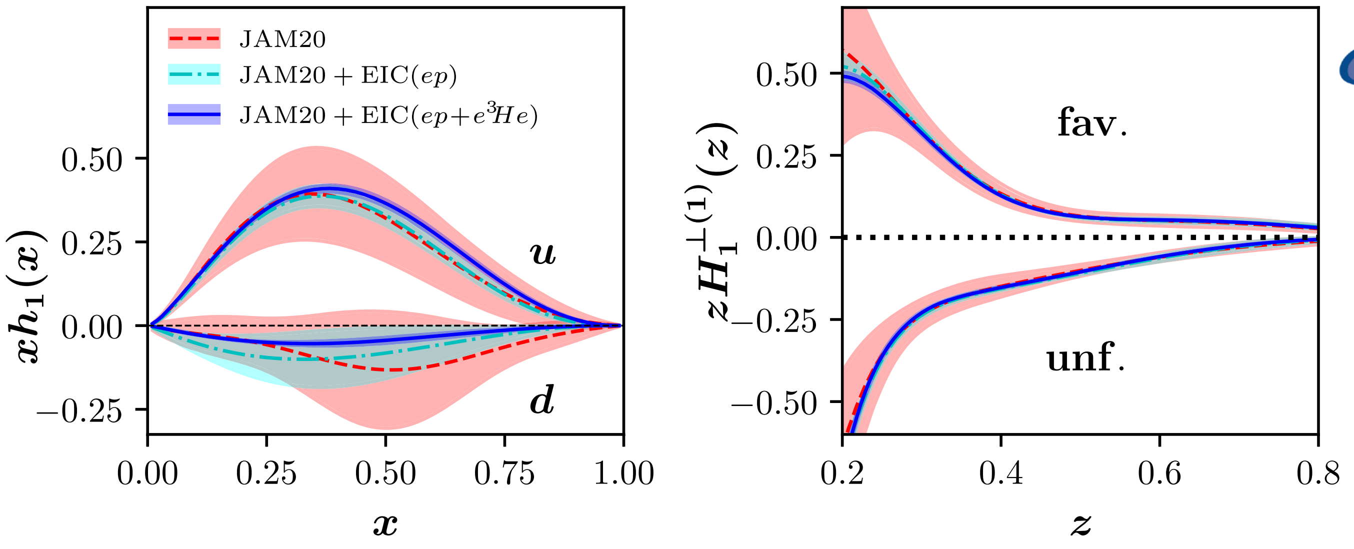
- Tensor charge is extensively studied on the lattice

Gupta et al, (18), Alexandrou et al., (19)

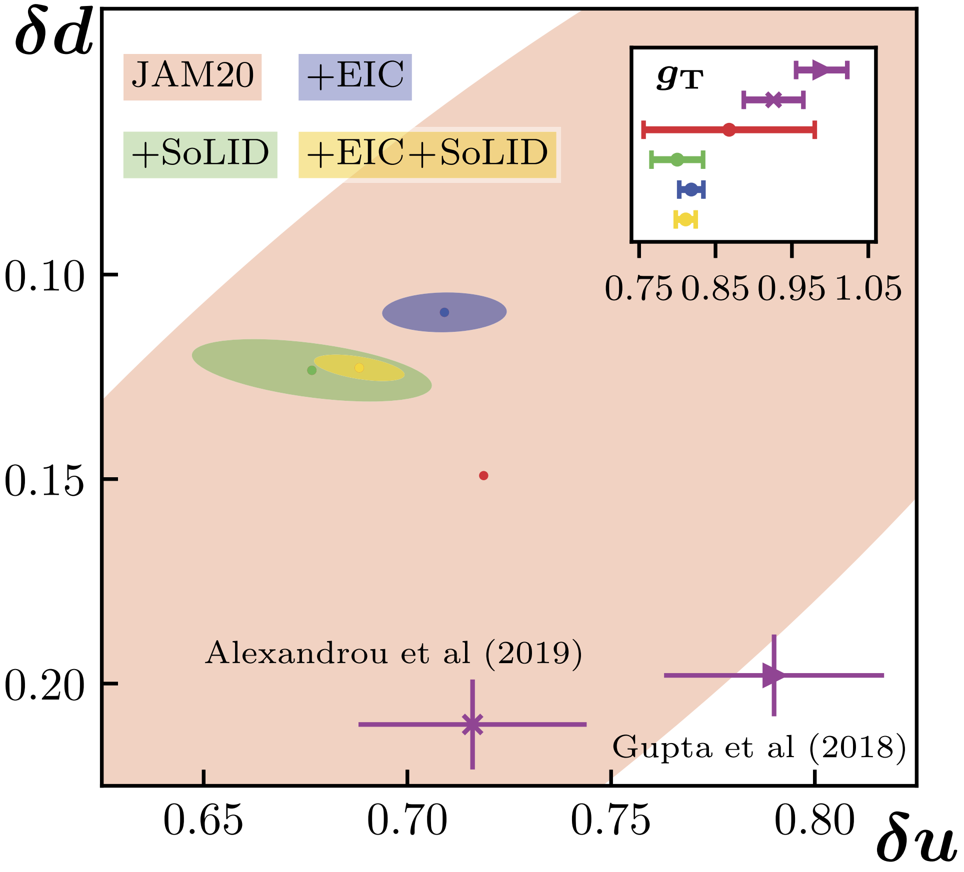


TENSOR CHARGE AT THE EIC AND JLAB

L. Gamberg, Z. Kang, D. Pitonyak, A. Prokudin, N. Sato Phys.Lett.B 816 (2021)

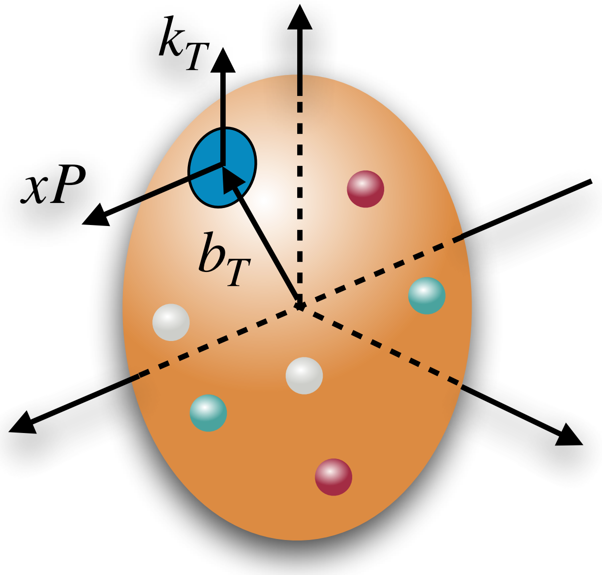


JAM20: Cammarota, Gamberg, Kang, Miller, Pitonyak, Prokudin, Rogers, Sato, Phys.Rev.D 102 (2020)



- EIC data will allow to have g_T extraction at the precision at the level of lattice QCD calculations
- JLab 12 data will allow to have complementary information on tensor charge to test the consistency of the extraction and expand the kinematical region

WIGNER DISTRIBUTIONS AND THE SPIN



- Wigner distributions have information on both position and motion

$$W(x,\vec{k},\vec{b}) = \int \frac{dz^- d\vec{z}}{2(2\pi)^3} \int \frac{d\vec{\Delta}}{(2\pi)^2}\, e^{ixP^+z^- - i\vec{k}\cdot\vec{z}} \langle P'S|\bar{\psi}(\vec{b}-\vec{z}/2)\gamma^+\psi(\vec{b}+\vec{z}/2)|PS\rangle$$

- The most intuitive definition of OAM involves Wigner functions

$$L_z = \int dx d^2\vec{b} d^2\vec{k} (\vec{b}\times\vec{k})_z W(x,\vec{b},\vec{k})$$

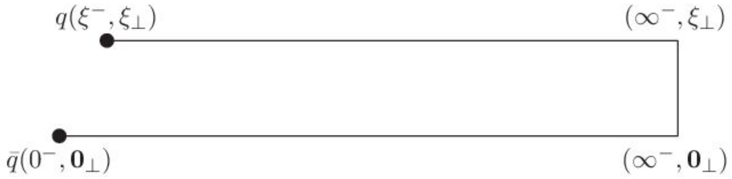
C. Lorcé, B. Pasquini, Phys. Rev. D 84, (2011)
C. Lorcé, B. Pasquini, X. Xiong and F. Yuan, Phys. Rev. D 85, (2012)

- The gauge link makes this definition gauge invariant and the two choices:

  Straight link - kinetic OAM, Ji: L_q X. Ji, X. Xiong and F. Yuan, Phys. Rev. D 88, no. 1, (2013)

  Y. Hatta, PLB 708 (2012) 186

  Stapple-like link - canonical OAM, Jaffe-Manohar: $\mathcal{L}_q$



- The difference between the two $\mathcal{L}_q - L_q$ is related to the torque force experienced by the struck quark and generated by the final state interactions M. Burkardt, Phys. Rev. D 88 (2013)
- How to fully access Wigner distributions in experiments is still to be explored

DATA DRIVEN SCIENCE

SLAC,USA

Fermilab, USA

BNL, USA
RHIC
EIC

HERA(DESY),
FAIR(GSI),
Germany

NICA(JINR),
Russia

JLab, USA
CEBAF

CERN
Switzerland
LHC
COMPASS

BESIII,China

BELLE II
Japan