

1. INTRODUCTION

Neutrino oscillation experiments show the existence of non-vanishing small **neutrino masses**, a very well established signal of **Physics Beyond the Standard Model** (SM).

The mechanism generating these masses is not yet known. The most widely accepted one is the **seesaw mechanism**, which involves **extra heavy neutral leptons** N_i (with i depending on the concrete model).

The Dirac or Majorana **nature** of both light and heavy neutrinos present in these models remains yet an **open question**.

Different experiments around the world look for different signals where their true nature is exposed.

2. THE LINEAR SEESAW MODEL

Besides the SM content, the minimal version of the **Linear seesaw model** (LSM) contains **two** different types of neutral SU(2) **singlet fermions** (N, S) per generation.

Working on the (ν_L^c, N, S) basis, the texture of the neutrino mass 9 x 9 matrix, given in a 3 x 3 block notation, reads:

$$M_\nu = \begin{pmatrix} 0 & m_D & M_\epsilon \\ m_D^T & 0 & M_R \\ M_\epsilon^T & M_R^T & 0 \end{pmatrix},$$

where $m_D = v_{SM} Y_D / \sqrt{2}$ and $M_\epsilon = v_{SM} Y_\epsilon / \sqrt{2}$.

Through a diagonalization-like procedure, considering $M_\epsilon \ll m_D < M_R$, the **light neutrino mass matrix** is linear in the extra Yukawas Y_ϵ and Y_D :

$$m_\nu = \frac{v_{SM}^2}{2} (Y_D M_R^{-1} Y_\epsilon^T + Y_\epsilon M_R^{-1} Y_D^T),$$

hence the name of the model. From the smallness of Y_ϵ , **large masses** M_R are **not required**, i.e. the heaviest neutrinos can live within the range of current experiments.

The analogous expressions for the two 3 x 3 mass matrices of the heavy neutrinos are

$$M_{N_a, N_b} \simeq \frac{M_R}{2} + \frac{m_D^2 M_R^{-1}}{4} \mp \frac{m_D M_R^{-1} M_\epsilon^T}{2} + \text{h.c.}$$

The SM flavour neutrinos are now a mixture of the light and heavy mass eigenstates

$$\nu_\ell = \sum_{k=1}^3 U_{\ell\nu_k} \nu_k + \sum_{k=1}^3 U_{\ell N_k} N_k + \sum_{k=1}^3 U_{\ell N'_k} N'_k,$$

implying that the PMNS matrix is not unitary anymore; instead the unitarity is only preserved for the more general (9×9) mixing matrix U .

An important **feature of the LSM** is that the mass splitting between the two heavy neutrinos within each generation is very small:

$$\Delta M_i \sim m_{\nu_i},$$

so that a **Quasi-Dirac (QD) behaviour** of the **heavy neutrinos** becomes a likely possibility and worth studying.

3. QUASI-DIRAC NEUTRINOS

The Dirac-Majorana dichotomy is somehow misleading: the **Dirac** case can be considered as a **limiting case** of a more **general Majorana scenario** with twice the neutrino content. When the Dirac limit is reached in a continuous way by gradually **switching off** the **lepton number violating (LNV) mass terms**, one crosses a narrow regime called **Quasi-Dirac**.

An **observable** commonly used at the **LHC** to look for **Majorana neutrinos** is the same-sign to opposite-sign dilepton ratio in $\ell\ell jj$ events with no missing p_T , the $R_{\ell\ell}$.

- Same-sign dilepton events occur through LNV processes mediated by Majorana neutrinos:

$$\bar{q}q \rightarrow W^\pm \rightarrow \ell_\alpha^\pm N^{(\prime)} \rightarrow \ell_\alpha^\pm \ell_\beta^\mp W'^\mp.$$

- Opposite-sign dilepton events occur through LNC processes mediated by either Majorana or Dirac neutrinos:

$$\bar{q}q \rightarrow W^\pm \rightarrow \ell_\alpha^\pm N^{(\prime)} \rightarrow \ell_\alpha^\pm \ell_\beta^\mp W'^\pm,$$

where the W'^\pm subsequently decays into quarks forming the jets.

Prompt searches of such signals are background dominated, while displaced vertex (DV) events are background free.

When the decay widths of the heavy neutrinos are approximately equal, i.e. $\Delta\Gamma = \Gamma_N - \Gamma_{N'} \ll \Gamma$, what in our case entails $Y_\epsilon \ll Y_D$, this ratio becomes [1]

$$R_{\ell\ell} = \frac{\Delta M^2}{2\Gamma^2 + \Delta M^2}.$$

$R_{\ell\ell} = 1$ corresponds to the Majorana case, while $R_{\ell\ell} = 0$ to the Dirac case. The **Quasi-Dirac** regime is characterized by $0 < R_{\ell\ell} < 1$.

In the LSM $\Delta M_i \sim m_{\nu_i}$, so that the window of $R_{\ell\ell}$ values compatible with **QD** neutrinos is **determined** by m_{ν_i} and Γ , namely $\Gamma(N) \sim m_\nu$.

We considered $M_{N_1} \ll M_{N_2} \lesssim M_{N_3}$ and computed the total decay width of the first heavy neutrino [2], focusing on the $M_{N_1} \lesssim 2.5$ GeV regime, where QCD is non-perturbative. For the **hadronization** of the **quark currents** in this regime, we made use of **Chiral Perturbation Theory** [3] and **Resonance Chiral Theory** [4].

4. PARAMETRIZATION

We intended to cover in the most general way the **parameter space** of the linear seesaw, paying special attention to the **regions** current and near-future **experiments** aim to **explore**.

For this purpose, we take the **master parametrization** [5], which allows to fit any Majorana neutrino mass model and automatically reproduce current experimental data.

For the LSM the Yukawas are parametrized as:

$$Y_D^T = c M_R^{1/2} W T \hat{m}_\nu^{1/2} U_{\ell\nu}^\dagger,$$

$$Y_\epsilon^T = c M_R^{1/2} W^* (T^T)^{-1} (I - K) \hat{m}_\nu^{1/2} U_{\ell\nu}^\dagger,$$

where W encloses all possible rotations in the Yukawa parameter space, while T and K contain the scaling of the different components of the Yukawa couplings.

We considered **two** main **scenarios** within this parametrization:

- Scenario a:** $W = U_{\ell\nu}$, $T = \frac{10^{-1}\alpha}{f'} \times (v_{SM}/\hat{m}_\nu[\text{GeV}])^{1/2}$ and $K = 0$; with $\alpha = (246)^{-1/2} \rightarrow Y_\epsilon \propto f'$; $Y_D \propto 1/f'$ and **diagonal**.
- Scenario b:** $W = I$, $T = gI$ and $K = 0 \rightarrow Y_D = g^2 Y_\epsilon$; For $g = 1 \rightarrow Y_D = Y_\epsilon$ the **traditional seesaw** scenario is recovered.

Any different choice in the parametrization structure either explores the same region or falls into non-testable or excluded regions.

5. RESULTS

Dilepton ration in the LSM

The **Quasi-Dirac** regime $0 < R_{\ell\ell} < 1$ occurs when $\Delta M \sim \Gamma$. Since $\Delta M_i \sim m_{\nu_i}$, and due to the M_{N_1} dependence of Γ , for **smaller** M_{N_1} , **smaller** m_{ν_1} are needed (see Fig. 1).

In contrast to the **inverse seesaw model**, where values of $R_{\ell\ell} < 1$ are still obtained for **larger** values of M_{N_1} , in the LSM $R_{\ell\ell} = 0$ for $M_{N_1} \gtrsim 100$ GeV.

In Fig. 2 we show some QD ranges in the $m_{\nu_1} - M_{N_1}$ plane for several f' . Note though that the **QD regime** is a **continuum**: the upper-left corner represents the Majorana case, while the lower-right corner approaches the Dirac limit.

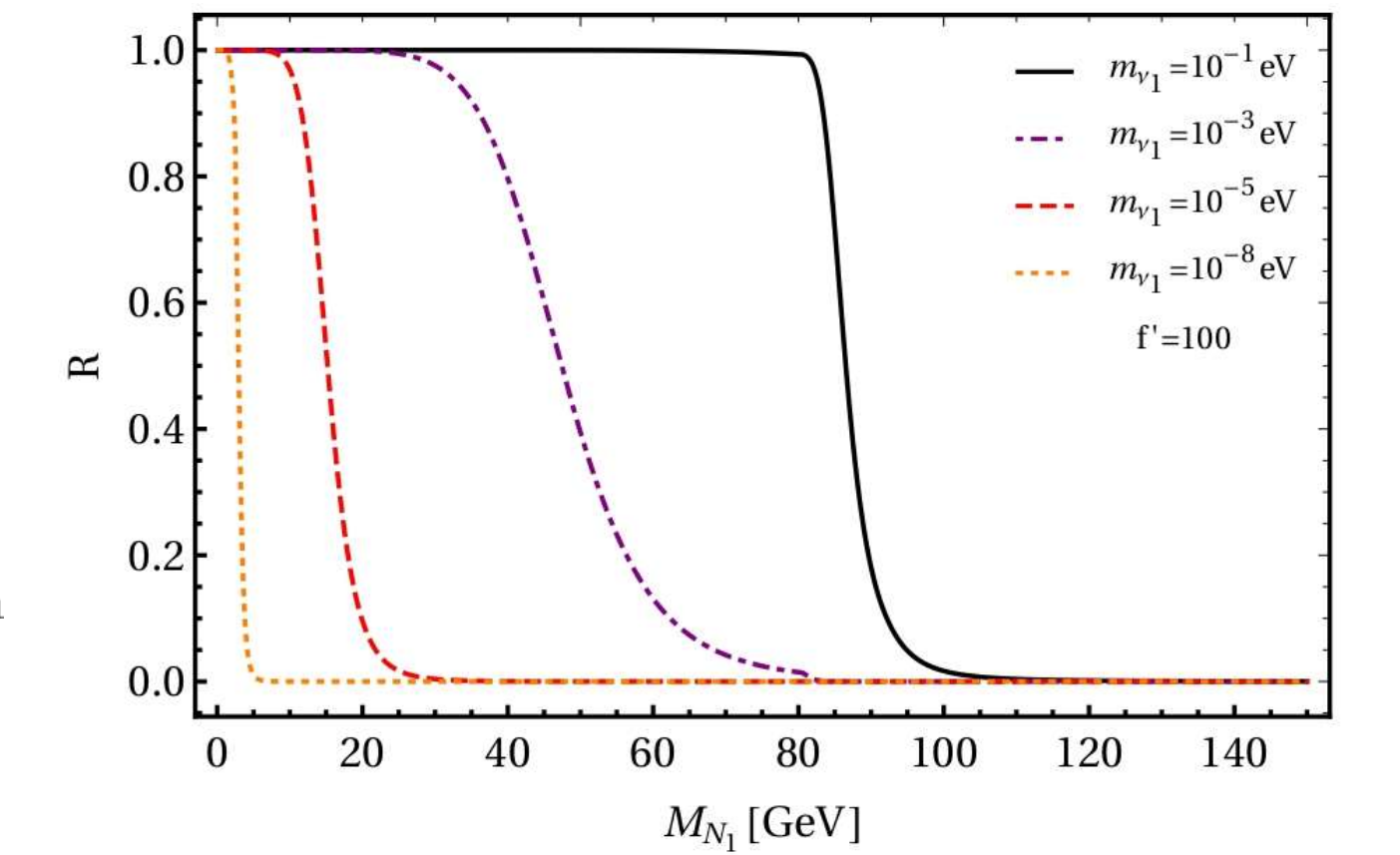


Figure 1. $R_{\ell\ell}$ vs M_{N_1} for different values of m_{ν_1} .

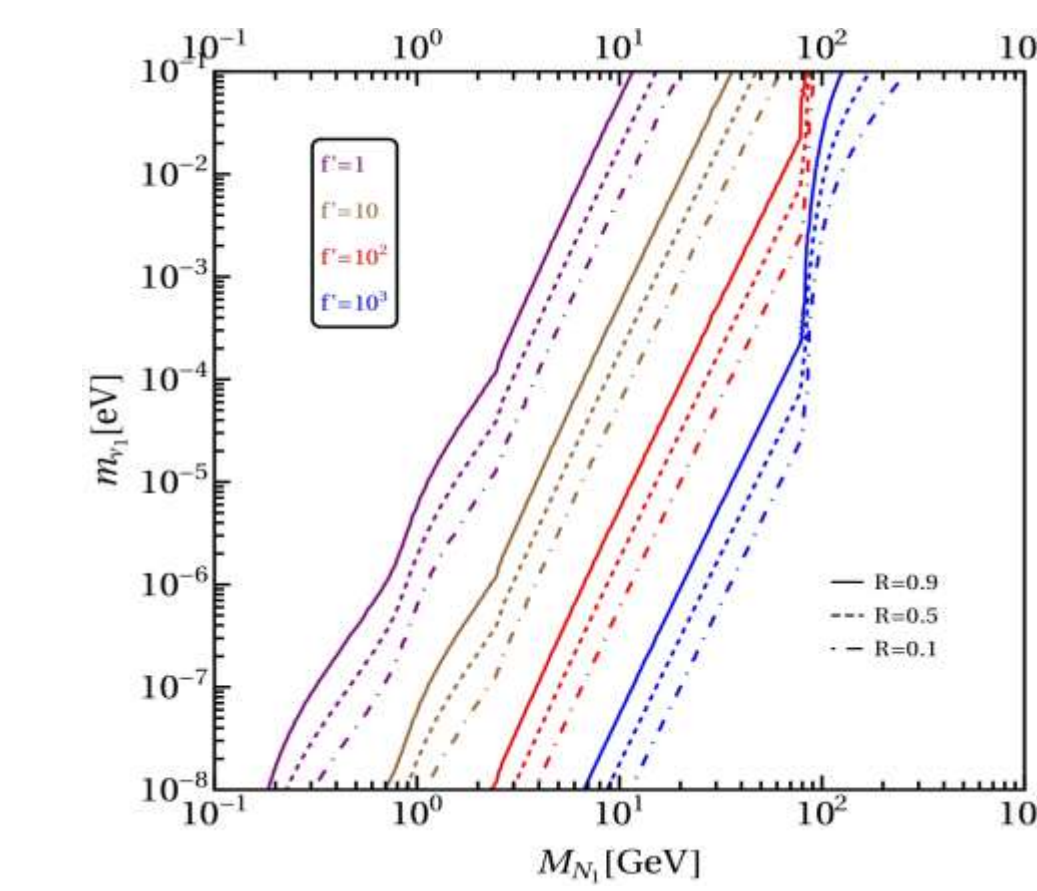


Figure 2. QD ranges in the $m_{\nu_1} - M_{N_1}$ plane for different values of f' .

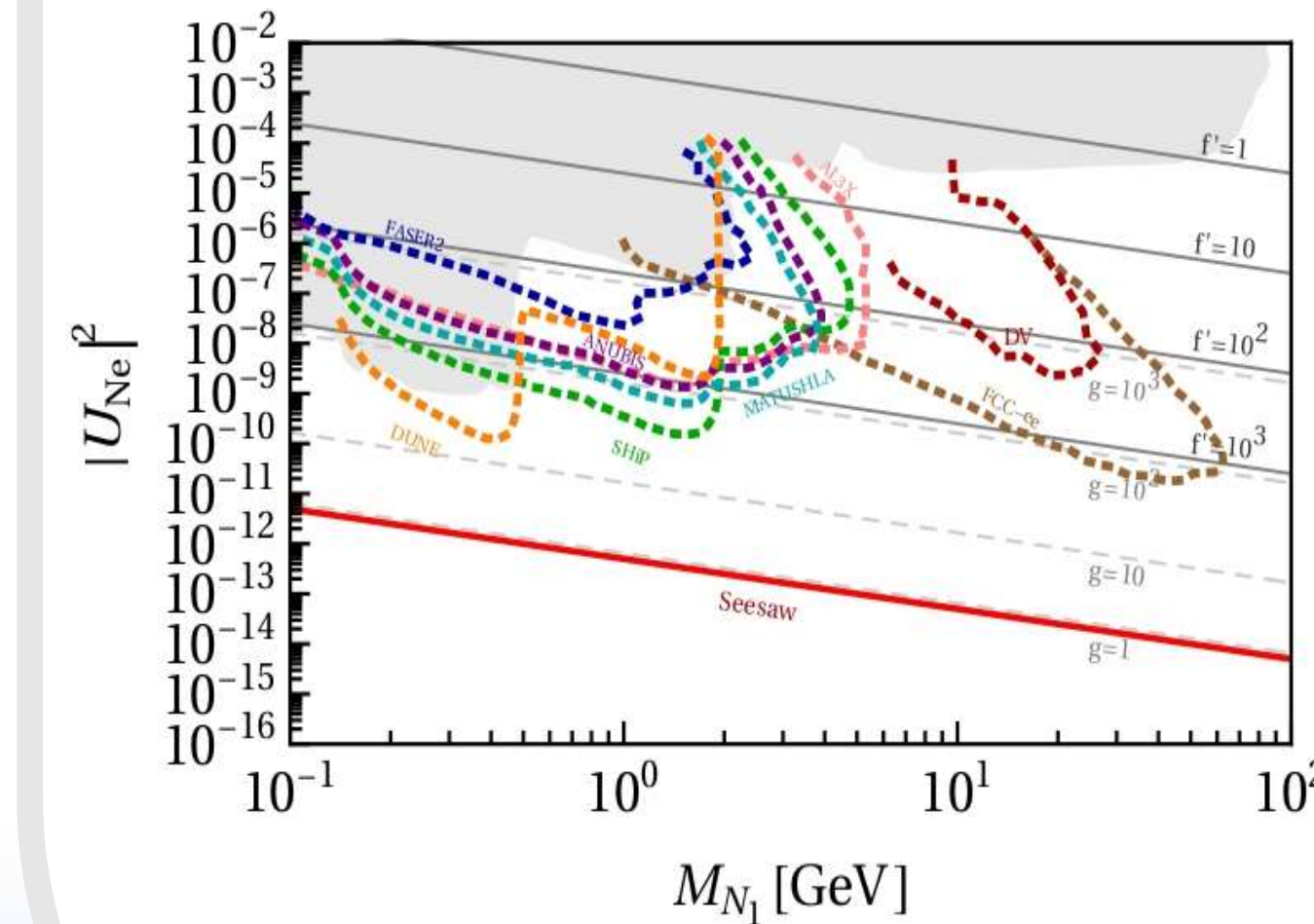


Figure 3. Constraints on $U_{N\ell}(M_{N_1})$.

Heavy to light neutrino mixing $U_{N\ell}$

We performed a **numerical analysis** based on the systematic **diagonalization** of the 9 x 9 mass matrix of the neutral states M_ν [6].

We found (see Fig. 3):

- Scenario a:** all neutrino masses m_{ν_i} enter all Y_ϵ entries, while Y_D is independent of the light neutrino masses. Then the **mixing** $U_{N_1 e}$ does **not depend** on m_{ν_1} .
- Scenario b:** light neutrino masses enter separately all Y_ϵ and Y_D entries, an explicit dependence on m_{ν_1} is then expected (**less constrained**).
- Scenarios a and b:** if there is some appreciable **hierarchy between the Yukawas** Y_ϵ and Y_D , the predicted **mixing** falls into the range **testable** by present and near-future experiments.

The **bounds** on the **mixing** themselves can place already stringent **constraints** into the **QD** regime, which can be found by studying the interplay between Figures 2 and 3.

We have also studied the bounds stemming from the lepton-flavour violating $\mu \rightarrow e\gamma$ and the LNV neutrinoless double beta decay processes, and concluded that these are not competitive as compared to the ones given in Fig. 3.

6. CONCLUSIONS

We showed that in the **LSM** the same-sign to opposite-sign dilepton ratio – characterizing the **QD behaviour** of the heavy neutrinos – is **controlled** by both the **masses** of the **light neutrinos** and the **decay widths** of their **heavy** partners.

Unlike other seesaw models with QD regimes, in the linear seesaw the pair of heavy neutrinos exhibits a **Quasi-Dirac** behaviour for relatively **low masses**.

Despite the difficulty of measuring $R_{\ell\ell}$ by low-energy experiments, we could translate the mixing constraints into bounds on the Quasi-Dirac nature of the heavy neutrinos by looking at the interplay between Figs. 2 and 3.

We concluded that current and near-future **experiments** are actually **probing hierarchical Yukawas**, with the equal-Yukawa case remaining unbounded.

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