Minimal Froggatt Nielsen Textures

ROMA TRE UNIVERSITÀ DEGLI STUDI

INFN Istituto Nazionale di Fisica Nucleare

A.Mastroddi[1,2*]

[1]Dipartimento di Matematica e Fisica, Università di Roma Tre
[2] INFN-Sezione di Roma Tre

*alessio.mastroddi@uniroma3.it

Motivation

The Standard Model (SM) lagrangian can be split in two:
$\mathscr{L}_{\text{SM}} = \mathscr{L}_{\text{gauge}}(A_a,\psi_i) + \mathscr{L}_{\text{Higgs}}(\phi, A_a, \psi_i)$.

- $\mathscr{L}_{\text{gauge}}$ is invariant under $\mathscr{G}_F = U(3)_Q \otimes U(3)_u \otimes U(3)_d \otimes U(3)_L \otimes U(3)_e$;
- $\mathscr{L}_{\text{Higgs}}$ breaks $\mathscr{G}_F$, but how is it dynamically broken? Can we explain the hierarchy in fermion masses?
- Simplest explanation: horizontal U(1) symmetry [1, 2], spontaneously broken by vev v_ϕ of one flavon field.
- We want to explore viable **minimal** models (low charges, for a cheaper UV completion) that reproduce exactly the quark masses and mixings and are not fine tuned.

EFT for Froggatt Nielsen

- Yukawa Lagrangian in *up-aligned* basis: $-\mathscr{L}^{Y^{u,d}}_{\text{SM}} = \hat{y}^u_{ij}\,\bar{Q}_i\,\tilde{H}\,u_j + \left(V_{\text{CKM}}\,\hat{y}^d\right)_{ij}\bar{Q}_i\,H\,d_j + \text{ h.c.}$, with
$$\hat{y}^u = \frac{\sqrt{2}}{v_H}\text{diag}\,(m_u\,,m_c\,,m_t), \qquad \hat{y}^d = \frac{\sqrt{2}}{v_H}\text{diag}\,(m_d\,,m_s\,,m_b)\ .$$
- Yukawa coefficients $y^{u,d}_{ij}$ are not all O(1): rotate quarks under flavor U(1), $\psi_j \to e^{i\theta X_{\psi_j}}$, then introduce charged scalar ($X_\phi = 1$ without loss of generality) for invariance.
- Assuming $v_\phi < \Lambda$ (Λ= scale of heavy charged messengers [3]), when $\phi \to v_\phi$, one can write
$$-\mathscr{L}_{\text{FN}-\text{EFT}} = c^u_{ii}\,\varepsilon^{n^u_{ij}}\,\bar{Q}_i\,\tilde{H}\,u_j\,+\,c^d_{ij}\,\varepsilon^{n^d_{ij}}\,\bar{Q}_i\,H\,d_j + \text{ h.c.}\ , \varepsilon = \frac{v_\phi}{\Lambda}, \quad n^u_{ij} = |X_{Q_i} - X_{u_j}|, \quad n^d_{ij} = |X_{Q_i} - X_{d_j}|$$
- Now the coefficients c_{ij} are of O(1), the hierarchy comes only from different powers of the perturbative parameter ε.
- Usual approach: find hierarchical patterns and select suitable charges [4]

$$y_d \sim \lambda^6, y_s \sim \lambda^4, y_b \sim \lambda^2, y_u \sim \lambda^7, y_c \sim \lambda^3, y_t \sim \lambda^0$$
$$\left|V_{ud}\right| \sim \left|V_{cs}\right| \sim \left|V_{tb}\right| \sim \lambda^0, \left|V_{us}\right| \sim \left|V_{cd}\right| \sim \lambda,$$
$$\left|V_{cb}\right| \sim \left|V_{ts}\right| \sim \lambda^2, \left|V_{ub}\right| \sim \left|V_{td}\right| \sim \lambda^3$$
$$\Rightarrow$$
$$X_{Q_{1,2,3}} = \{3,2,0\}$$
$$X_{u_{1,2,3}} = \{-4,-1,0\}$$
$$X_{d_{1,2,3}} = \{-3,-2,-2\}$$
$$\varepsilon \sim \lambda$$

New approach: $\mathrm{U}(3)^3$ rotations

- $-\mathscr{L}_{\text{FN}-\text{EFT}} = c^u_{ii}\,\varepsilon^{n^u_{ij}}\,\bar{Q}_i\,\tilde{H}\,u_j\,+\,c^d_{ij}\,\varepsilon^{n^d_{ij}}\,\bar{Q}_i\,H\,d_j + \text{ h.c.}$;
- $-\mathscr{L}^Y_{\text{SM}} = \hat{y}^u_{ij}\,\bar{Q}_i\,\tilde{H}\,u_j + \left(V_{\text{CKM}}\,\hat{y}^d\right)_{ij}\bar{Q}_i\,H\,d_j + \text{ h.c.}$;
- There exist 3 U(3) rotations such that
$$\left(V^\dagger_Q\,\hat{y}^u\,V_u\right)_{ij} = c^u_{ij}\,\epsilon^{n^u_{ij}} \quad , \quad \left(V^\dagger_Q\,V_{\text{CKM}}\,\hat{y}^d\,V_d\right)_{ij} = c^d_{ij}\,\epsilon^{n^d_{ij}}\ .$$
- Relating O(1) coefficients to rotational parameters means that no fits are required: the starting point are the SM values for quark masses and mixings, so they will be perfectly reproduced in every FN-EFT model.
- To favour O(1) coefficients, minimise cost function
$$\chi^2_{\text{FN}} = \sum_{i,j=1}^{3}\left(\left|c^u_{ij}\left(\epsilon,\theta^{Q,u,d}_{1,2,3},\delta^{Q,u,d}_{1,2,3}\right)\right| - 1\right)^2 + \left(\left|c^d_{ij}\left(\epsilon,\theta^{Q,u,d}_{1,2,3},\delta^{Q,u,d}_{1,2,3}\right)\right| - 1\right)^2 .$$

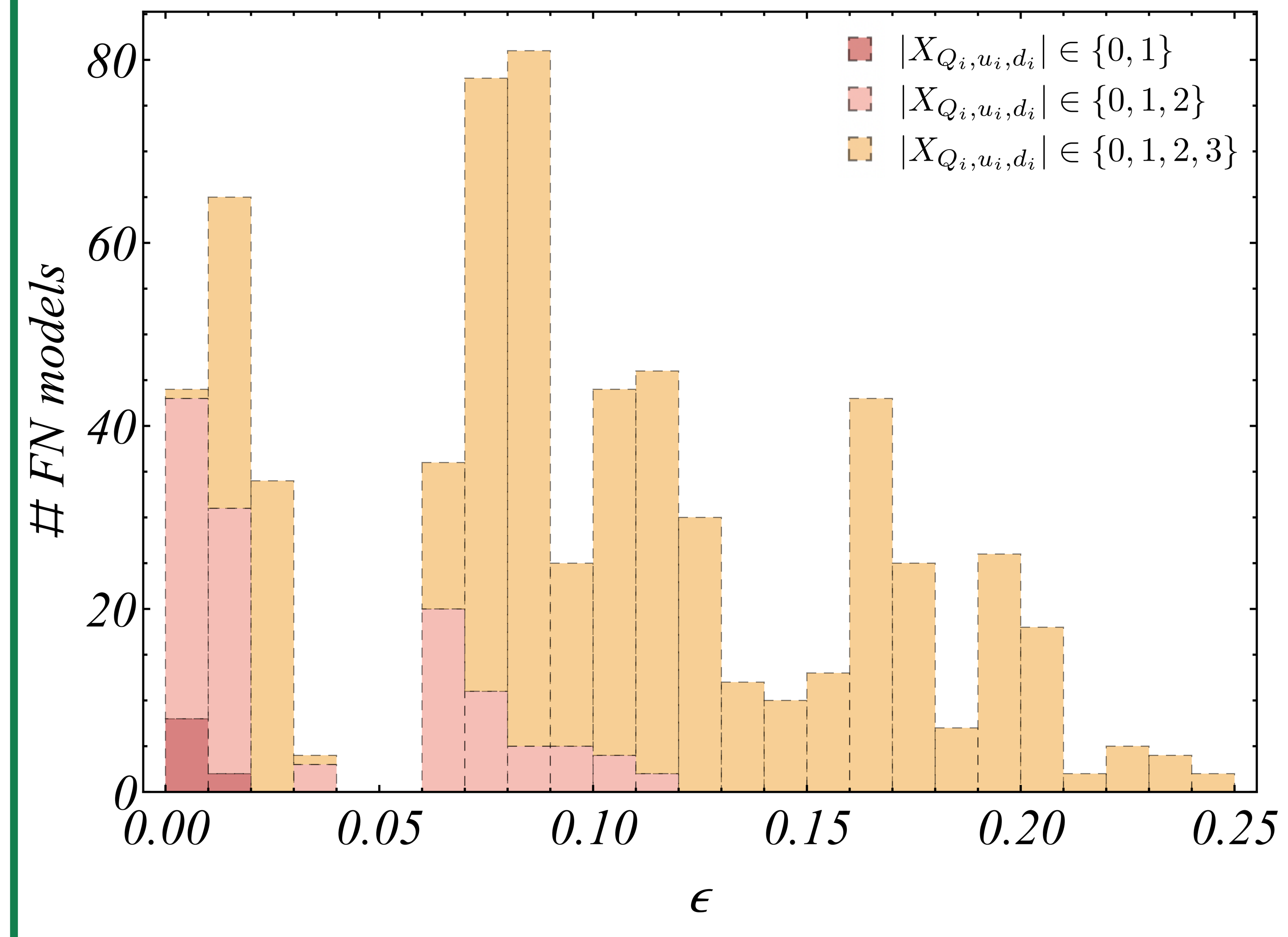
Tuning of the models

- O(1) coefficients are not enough: coefficients may cancel out and contribute to hierarchical pattern.
- Introduce **tuning score** (similar to Barbieri-Giudice [5]): $\Delta_{\text{FN}} \equiv \max_{K,i,j}\left|\delta_{K,ij}\right|$, $\delta_{K,ij} \equiv \frac{c^{u,d}_{ij}}{O_K}\frac{\delta O_K}{\delta c^{u,d}_{ij}}$
- If small change in c_{ij} results in large change of an observable O_K, $\Delta_{FN} \gg 1$ and the model is fine tuned.

Scanning the models

- Acceptance range of the models: $0.4 < |c^u_{ij}|, |c^d_{ij}| < 1.6\ , \Delta_{FN} \leq 100$.
- Consider all configurations with $X_{Q,u,d} \leq 3$: ~1.3k models can produce all coefficients in the accepted range; ~ 85% also have acceptable tuning [6].

Model population



- 2 dominant modes at $\varepsilon \sim 0.01, \varepsilon \sim 0.08$; Explored range: $0.005 \lesssim \varepsilon \lesssim 0.25$
- O(10) *minimal* models: all charges less than 1 in absolute value
- $\varepsilon \sim \lambda \approx 0.22$ (popular choice in literature) only captures tail of distribution when considering small charges

Explicit models

X_{Q_1}	X_{Q_2}	X_{Q_3}	X_{u_1}	X_{u_2}	X_{u_3}	X_{d_1}	X_{d_2}	X_{d_3}	ϵ
0	0	0	1	-1	0	-1	-1	-1	0.005
1	0	0	-1	-1	0	-1	-1	-1	0.006
1	0	0	0	-1	0	-1	-1	-1	0.006
1	1	0	0	-1	0	-1	-1	-1	0.012
0	0	0	1	1	0	-1	-1	-2	0.005
0	0	0	2	-1	0	-1	-1	-2	0.006
0	0	0	2	-1	0	-1	-1	-1	0.005
0	0	0	2	-1	0	2	1	-1	0.006
1	0	0	-1	-2	0	-1	-1	-2	0.008
1	0	0	-1	-1	0	-1	-1	-2	0.007
1	1	0	0	-2	0	-2	-2	-2	0.094
1	1	0	0	-1	0	-2	-2	-2	0.093
2	1	0	-2	-2	0	-2	-2	-2	0.109
2	1	0	-1	-2	0	-2	-2	-2	0.094
2	1	0	0	0	0	-2	-2	-2	0.094
2	2	0	-1	-1	0	-2	-2	-2	0.112
0	0	0	3	-3	0	-2	-2	-3	0.104
1	0	0	-2	-3	0	-2	-3	-3	0.098
1	1	0	-2	-3	0	-2	-2	-3	0.100
2	0	0	-2	-3	0	-2	-3	-3	0.104
2	1	0	-2	-3	0	-2	-2	-2	0.104

- Examples of minimal models
- Examples of $\varepsilon \lesssim 0.01$ models
- Examples of $\varepsilon \sim 0.1$ models with all charges ≤ 2 (top) and with no restrictions (bottom)

Future work

- Repeat same procedure for leptons: whatever the mechanism for neutrino masses, assume Weinberg operator in EFT to write lepton Yukawa lagrangian.
- Explore model with small charges for both Normal and Inverted Ordering, and for different values of Seesaw masses, then try unification with quark models.
- Preliminary results: similar percentage of non tuned models; ~5 completely minimal models when considering quarks and leptons together

References

[1] C. Froggatt and H. B. Nielsen, *Hierarchy of Quark Masses, Cabibbo Angles and CP Violation*, Nucl. Phys. B 147 (1979) 277–298.
[2] M. Leurer, Y. Nir, and N. Seiberg, *Mass matrix models*, Nucl. Phys. B 398 (1993) 319–342.
[3] L. Calibbi, Z. Lalak, S. Pokorski, and R. Ziegler, *The Messenger Sector of SUSY Flavour Models and Radiative Breaking of Flavour Universality*, JHEP 06 (2012) 018.
[4] F. Feruglio, *Pieces of the Flavour Puzzle*, Eur. Phys. J. C 75 (2015), no. 8 373.
[5] R. Barbieri and G. F. Giudice, *Upper Bounds on Supersymmetric Particle Masses*, Nucl. Phys. B306 (1988) 63–76.
[6] M. Fedele, A. Mastroddi, and M. Valli, *Minimal Froggatt-Nielsen textures*, JHEP 03, (2021) 135.