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Extension of Migdal-Watson formula and its application to binary breakup reaction

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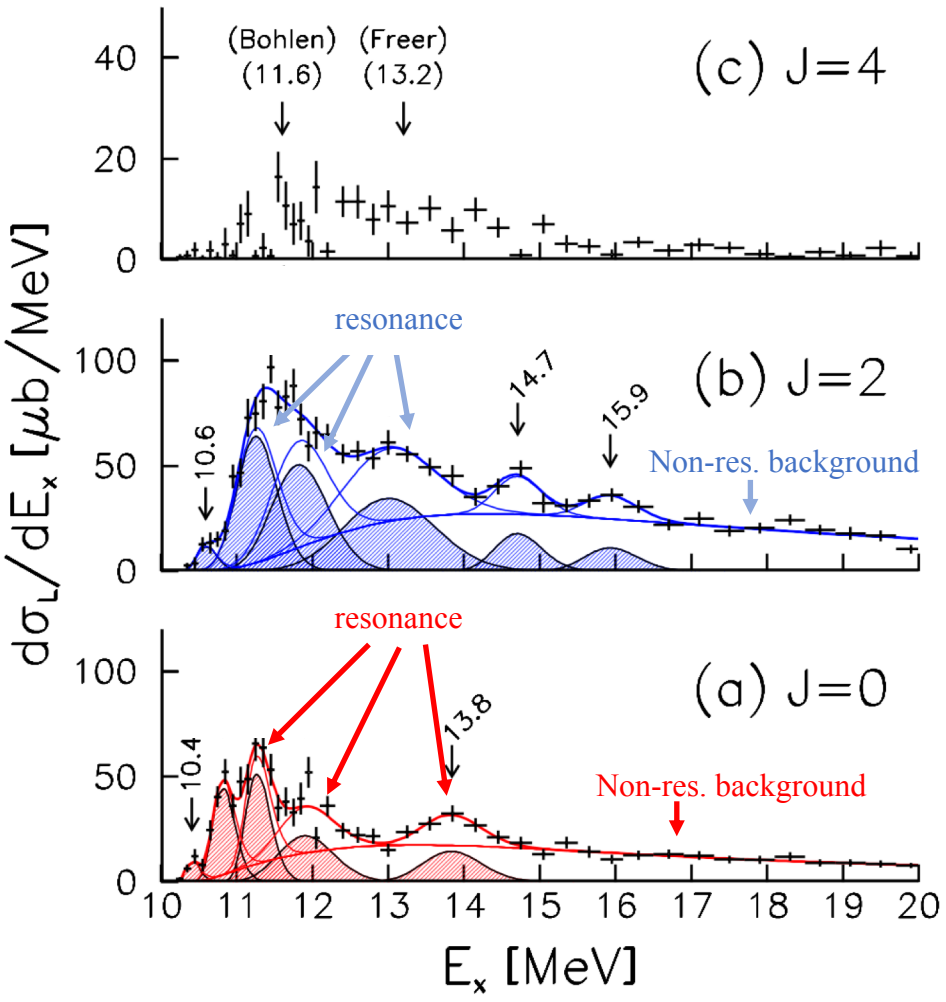
Introduction & Motivation

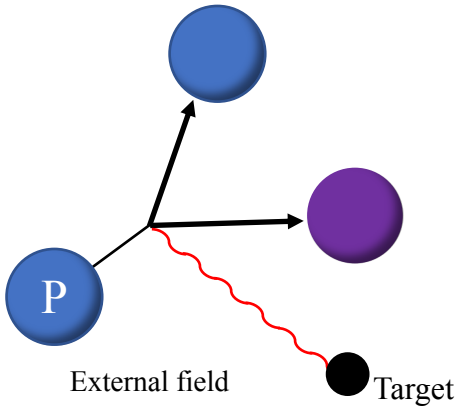
Example of $^{12}Be \rightarrow {}^{6}He + {}^{6}He$ experiment



A.Saito et al., Mod. Phys. Lett. A, Vol. 25, 21 (2010)

Breakup (BU) reaction is a useful tool to explore the unbound resonant states in nuclei

Simple image of breakup reaction (BU)



Exploration of resonances by BU is important for

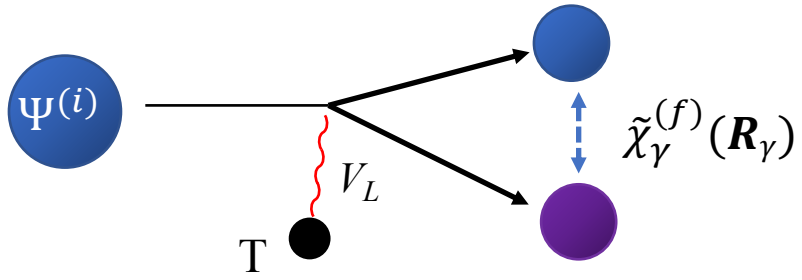
- Exotic molecular structure
- Nucleosynthesis

Evaluation of non-resonant background is important to derive resonance parameters, the resonant energy and the decay width

Non-resonant background strength in binary BU

The main origin of the non-resonant background is expected to be the one-step direct breakup

We define the direct breakup as an one-step transition from ground state (g.s.)→ binary continua



Exclusive measurement
→ Detection of a channel $\gamma\,(\phi_\gamma)$

Initial g.s. wave function

$$\Psi^{(i)} = \sum_\alpha \chi_\alpha^{(i)}(\boldsymbol{R}_\alpha)\phi_\alpha(\xi_\alpha)$$

relative internal

Expansion with various binary channels

V_L

Distorted wave of specific channel γ

$$\tilde{\chi}_\gamma^{(f)}(\boldsymbol{R}_\gamma)\phi_\gamma(\xi_\gamma)$$

Scattering boundary condition is imposed

Direct breakup → Transition from channel γ in the g.s. to the distorted wave in γ

$$S_L \propto \left|\left\langle \tilde{\chi}_\gamma^{(f)} \middle| V_L \middle| \chi_\gamma^{(i)} \right\rangle\right|^2$$

Distorted wave of channel γ

Component of channel γ in the initial state

Extension of Migdal-Watson formula

Our aim

We extend the Migdal-Watson (MW) formula so as to describe non-resonant background strength by direct breakup

MW formula

$$S_0(E) \propto \frac{\sqrt{E}}{AE^2 + BE + C}$$

Valid for binary breakup of

- s-wave → finite spin
- charge-less → finite charge
- short range initial w.f. → finite size

K. Watson, PRC88, 1163 (1952)
A. Migdal, Sov. Phys. JETP 1, 2 (1955)
S. Shimoura, EPJ Plus 133, 463 (2018)

Extended MW formula

$$S_L(E) \propto \frac{P_L(ka)e^{-\beta E}}{AE^2 + BE + C}$$

Fitting parameters : A, B, C, a, β

- Finite L & charge $\sqrt{E}$ → $P_L(ka)$
  P_L: penetration factor
  (finite spin and charge in final states)
- Effect of finite size → $e^{-\beta E}$
  (finite size of initial w.f.)

We simulate the non-resonant background strength by the extended MW formula

Evaluation of non-resonant background strength

We use the complex scaling method to simulate the strength of the non-resonant background

Complex Scaling Method (CSM)

Schrodinger equation: $H(r)\varphi_f(r) = E_f\varphi_f(r)$

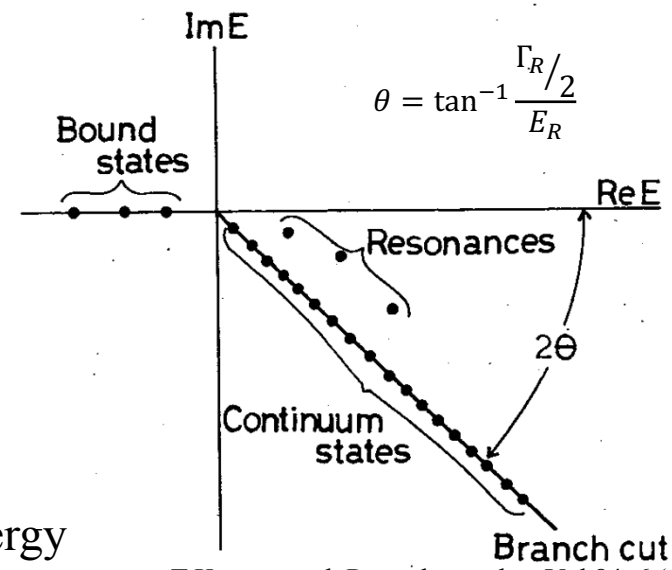
Complex scaling: $r \rightarrow re^{i\theta}$

Complex scaled equation: $H^\theta \varphi_f^\theta = E_f^\theta \varphi_f^\theta$

Complex eigenvalue: $E_R^\theta = E_R - i\frac{\Gamma_R}{2}$

E_R : res. energy

Γ_R : decay width



T.Kruppa et al. Prog, theor, phys,Vol.84, 6 (1990)

Calculation of strength function by CSM

T.Myo et al. Prog, theor, phys,Vol.99, 5 (1998)

$$S(E) = \sum_f |<\varphi_i|\hat{O}|\varphi_f>|^2 \delta(E-H)$$

Extended completeness relation

$$\sum_f |\varphi_f^\theta\rangle\langle\varphi_f^\theta| = 1$$

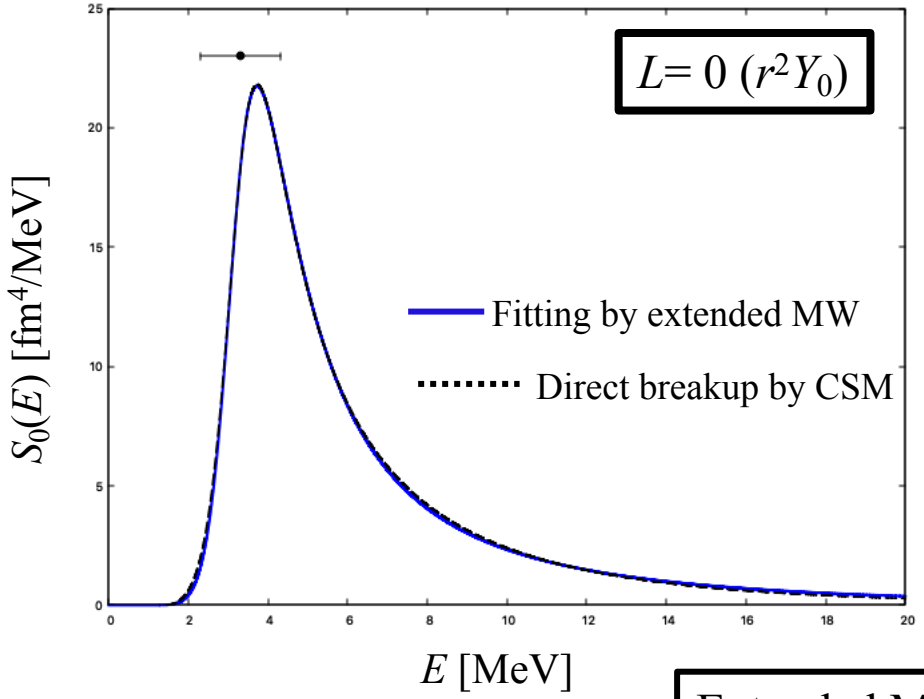
$$S(E) = -\frac{1}{\pi}\mathrm{Im}\left[\sum_f \frac{<\varphi_i^\theta|\hat{O}^\theta|\varphi_f^\theta>^2}{E - E_f^\theta}\right]$$

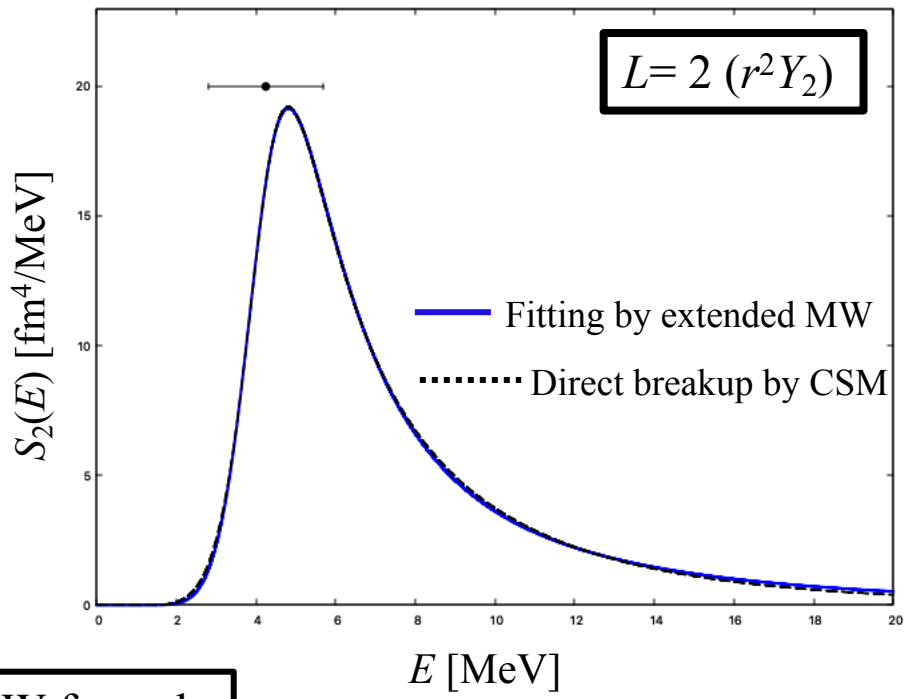
Original definition of strength function

Complex scaled strength function

Result (1) : Fitting of $^{20}Ne \rightarrow {}^{16}O + \alpha$ ($L = 0, 2$)

T.Myo et al. Prog, theor, phys,Vol.99, 5 (1998)





$A = 0.029$
$B = -0.187$
$C = 0.328$
$a = 5.0$, $\beta = 0.015$

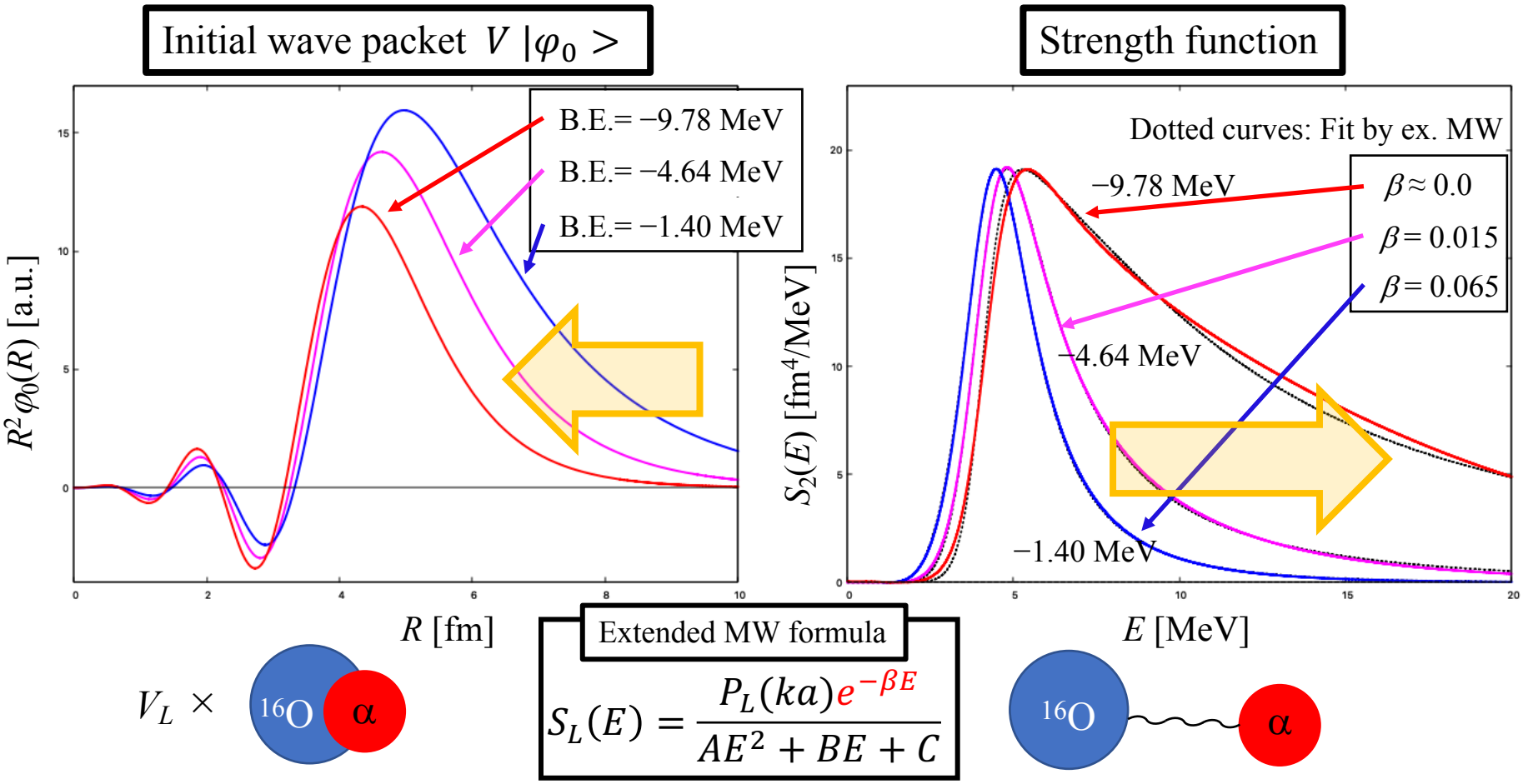
Extended MW formula

$$S_L(E) = \frac{P_L(ka)e^{-\beta E}}{AE^2 + BE + C}$$

$A = 0.025$
$B = -0.206$
$C = 0.479$
$a = 5.5$, $\beta = 0.015$

Extended MW formula nicely reproduces the results by CSM

Result (2) : Initial wave packet and strength function



1. Strength function is more extended as the initial wave packet is deeply bound

   ⇒ Strength func. corresponds to Fourier transformation by the final distorted wave

2. Spatial size of the initial wave packet is sensitive to the parameter β

   ⇒ $\beta \approx 0$ corresponds to the strongly localized wave packet inside of nuclear interaction

Summary & conclusion

R. Nakamoto et al, PRC 104, 034602 (2021)

Summary & Result

We have proposed a new formula to describe the non-resonant background strength by extending the Migdal-Watson (MW) formula, and it is checked by comparing with the CSM calculation

⇒ The new formula nicely reproduces the background strength induced by direct BU

Relation of the initial wave packet and the strength function is investigated

⇒ Energy dependence of direct BU is sensitive to the spatial size of initial wave packet

Extended MW formula

$$S_L(E) = \frac{P_L(ka)e^{-\beta E}}{AE^2 + BE + C}$$

Conclusion & Future work

In conclusion, the new formula is intuitively easy to understand, and it is expected to be useful to evaluate the background strength in the general binary breakup reaction

⇒ The application of new formula to the various binary breakup is strongly desired !

(We are now analyzing of the data of $^{15}\mathrm{C} \rightarrow \alpha + {}^{11}\mathrm{Be}$)

In future work, the similar analysis should be done for the three-body systems

⇒ Core + n + n system is especially important to consider