Unitarity, thermal corrections, and higher-order CP asymmetric reaction rates in leptogenesis (2102.05914, 2104.06395)

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Particle density evolution in the expanding universe

At classical level, number density n_{i_k} of any particle species included in $\{i\}$ is affected by $\{i\} \leftrightarrow \{f\}$ reactions,

$$\dot{n}_{i_k} + 3\mathcal{H}n_{i_k} = -\mathring{\gamma}_{fi} + \mathring{\gamma}_{if} + \dots \tag{1}$$

Reaction rates

$$\mathring{\gamma}_{fi} = \int\prod_{\{i\}}[d\mathbf{p}_i]\mathring{f}_i(p_i)\int\prod_{\{f\}}[d\mathbf{p}_f](2\pi)^4\delta^{(4)}(p_f-p_i)|\mathring{M}_{fi}|^2 \equiv \int_{i\to f}\mathring{f}_i|\mathring{M}_{fi}|^2 \tag{2}$$

- ← classical phase-space densities $\mathring{f}_i$ (Maxwell-Boltzmann densities in equilibrium),
- ← zero-temperature Feynman rules entering the calculation of $\mathring{M}_{fi}$.

CP asymmetries at zero-temperature

The S-matrix unitarity $S^\dagger = S^{-1}$, or $1 - \mathrm{i}T^\dagger = (1+\mathrm{i}T)^{-1}$, for the squared amplitude leads to

$$\mathrm{i}T^\dagger = \mathrm{i}T - (\mathrm{i}T)^2 + (\mathrm{i}T)^3 - \dots \quad\Rightarrow\quad |T_{fi}|^2 = -\mathrm{i}T_{if}\mathrm{i}T_{fi} + \sum_n \mathrm{i}T_{in}\mathrm{i}T_{nf}\mathrm{i}T_{fi} - \dots \tag{3}$$

Divide by $V_4 = (2\pi)^4\delta^{(4)}(0)$ to obtain $(2\pi)^4\delta^{(4)}(p_f-p_i)|\mathring{M}_{fi}|^2$. In CPT symmetric theory, the CP asymmetry $\Delta|T_{fi}|^2 = |T_{fi}|^2 - |T_{if}|^2$ can be expressed as [1]

$$\begin{aligned}\Delta|T_{fi}|^2 = &\sum_n(\mathrm{i}T_{in}\mathrm{i}T_{nf}\mathrm{i}T_{fi} - \mathrm{i}T_{if}\mathrm{i}T_{fn}\mathrm{i}T_{ni})\\ &-\sum_{n,m}(\mathrm{i}T_{in}\mathrm{i}T_{nm}\mathrm{i}T_{mf}\mathrm{i}T_{fi} - \mathrm{i}T_{if}\mathrm{i}T_{fm}\mathrm{i}T_{mn}\mathrm{i}T_{ni})\\ &+\dots\end{aligned} \tag{4}$$

From Eq. (4) the CPT and unitarity relations [2–4]

$$\sum_f \Delta|T_{fi}|^2 = 0 \tag{5}$$

are manifest at any perturbative order [1].

Thermal effects at the lowest order

As an example we consider $N_i \to lH$ decay within the seesaw type-I model. Right-handed neutrinos N_i, standard model lepton l and Higgs doublets H interact via Yukawa interactions

$$\mathcal{L} \supset -\frac{1}{2}M_i\bar{N}_iN_i - \left(\mathcal{Y}_{\alpha i}\bar{N}_iP_Ll_\alpha H + \text{H.c.}\right). \tag{6}$$

To the lowest $\mathcal{O}(\mathcal{Y}^2)$ order,

$$\mathring{\gamma}_{N_i\to lH} = \int_{N_i\to lH}\mathring{f}_{N_i}|\mathring{M}|^2, \quad |\mathring{M}|^2 = 4p_{N_i}{\cdot}p_l\sum_\alpha|\mathcal{Y}_{\alpha i}|^2 \tag{7}$$

that can be represented by the cut of the forward diagram Fig. 1a. Diagram in Fig. 1b leads to spurious contribution, equivalent to $\mathring{\gamma}_{N_i\to lH}$ times the total number of Higgs particles in the universe. Clearly, such contributions have to be omitted. Diagram in Fig. 1c corresponds to

$$\mathring{\gamma}_{N_iH\to lHH} = \int_{N_i\to lH}\mathring{f}_{N_i}\mathring{f}_H|\mathring{M}|^2 \tag{8}$$

with the same $|\mathring{M}|^2$ as in Eq. (7). To sum up the contributions such as in Fig. 1c avoiding those analogous to Fig. 1b, we shall draw the forward diagrams on a cylindrical surface [5]. In thermal equilibrium $\mathring{f}_H = e^{-E/T}$ and the summation leads to

$$\sum_{w=0}^{\infty}\mathring{f}_H^w = \frac{e^{E/T}}{e^{E/T}-1} \equiv 1 + f_H^{\text{eq}}. \tag{9}$$

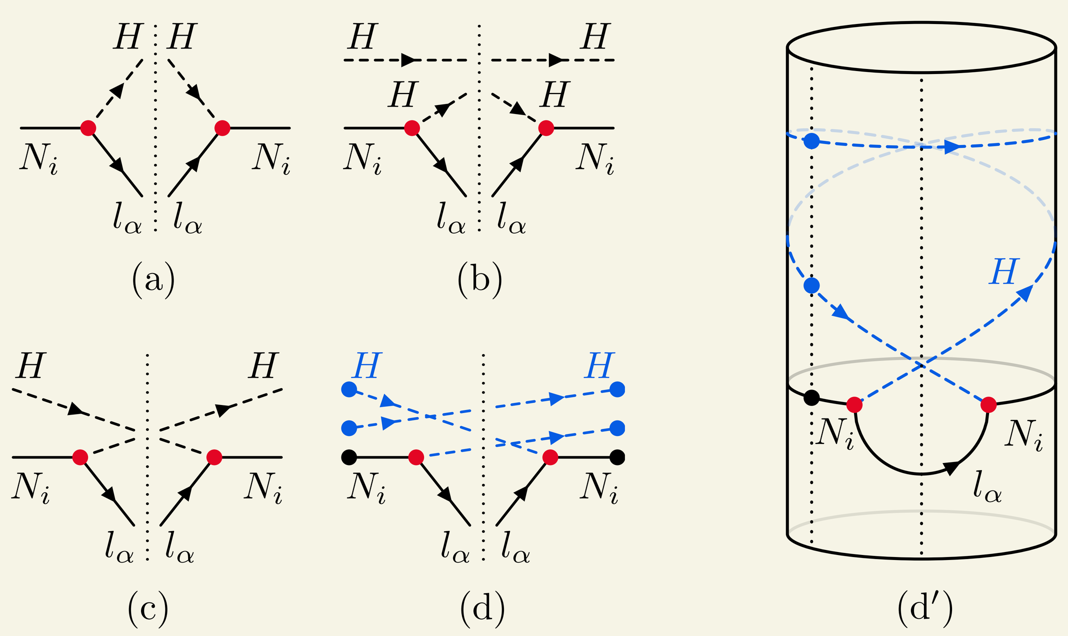


Figure 1: Planar and cylindrical diagrams contributing to the $N_i \to lH$ zero-temperature reaction rate and its thermal corrections.

Similarly, with alternating signs due to the crossing of fermionic legs, the Fermi-Dirac distributions for N_i and l are obtained. Thus, we introduce

$$\gamma_{N_i\to lH} = \int_{N_i\to lH} f_{N_i}(1+f_H^{\text{eq}})(1-f_l^{\text{eq}})|\mathring{M}|^2. \tag{10}$$

Thermal corrections to the lepton number source term

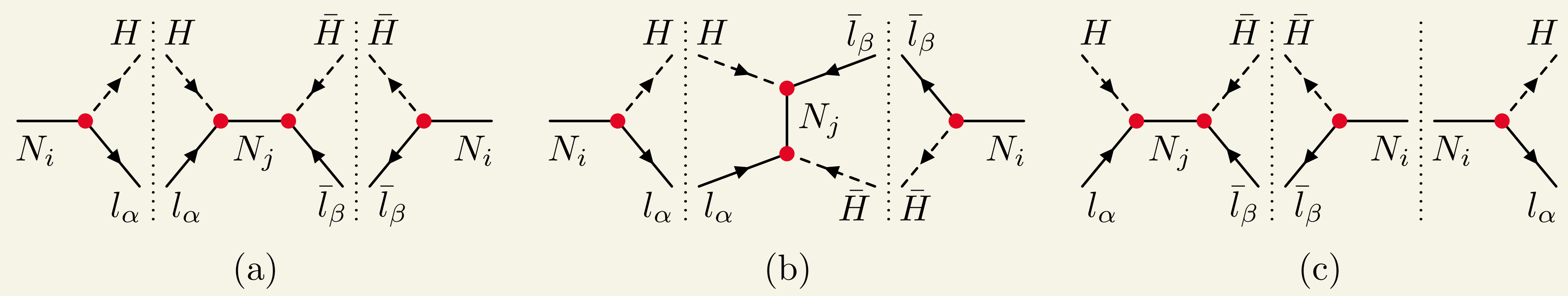
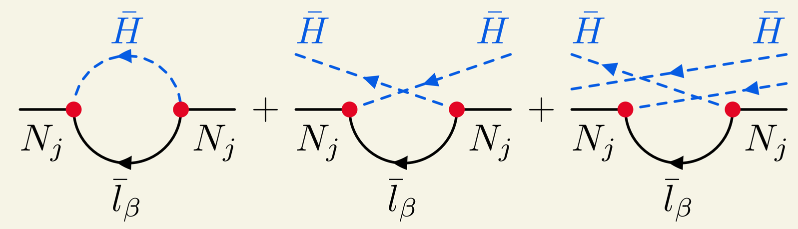


Figure 2: Lepton number violating contributions to the CP asymmetry in $N_i \to lH$ decay (Figs. 2a and 2b) and the s-channel part of the real-intermediate-state-subtracted $lH \to \bar{l}\bar{H}$ scattering (Fig. 2c). The latter can be obtained by a cyclic permutation of the diagrams in Fig. 2a using the approach of Refs. [1,6].

In leptogenesis, the matter asymmetry is generated from the $N_i \to lH$ decays and $lH \to \bar{l}\bar{H}$ scatterings (real-intermediate-state-subtracted scatterings). The zero-temperature asymmetry of the squared amplitude for the N_i decay, before the momentum integration is performed, equals

$$\Delta|\mathring{M}|^2 = 4p_l{\cdot}p_{\bar{l}}\sum_{j\neq i}2\text{Im}\left[\left(\mathcal{Y}\mathcal{Y}^\dagger\right)_{ij}^{2}\right]\left(\frac{2M_iM_j}{M_i^2-M_j^2} + \frac{M_iM_j}{(p_l-p_{\bar{H}})^2-M_j^2}\right). \tag{11}$$

At finite temperature, the on-shell parts of the propagators are modified, affecting the reaction rates asymmetries. If, for example, the first term in



$$+\ \dots, \tag{12}$$

occurs as a subdiagram, depending on the overall kinematics, the positive frequency on-shell part of the $\bar{H}$ propagator may contribute or not. If it does, all the other terms do as well. Thus, the $\bar{H}$ line can be replaced by

$$\frac{\mathrm{i}}{p_{\bar{H}}^2+\mathrm{i}\epsilon} + 2\pi\sum_{w=1}^{\infty}\mathring{f}_{\bar{H}}^w\theta(p_{\bar{H}}^0)\delta(p_{\bar{H}}^2) = \frac{\mathrm{i}}{p_{\bar{H}}^2+\mathrm{i}\epsilon} + 2\pi f_{\bar{H}}^{\text{eq}}\theta(p_{\bar{H}}^0)\delta(p_{\bar{H}}^2) \tag{13}$$

corresponding to the usual form of the positive frequency part of a thermal propagator. The negative frequencies contribute through the diagrams with opened $\bar{H}$ legs flipped to the opposite side.

Therefore, to take the thermal corrections to the reaction rate asymmetries into account, we should consider all the windings of the propagators involved, leading to

$$\Delta\gamma_{N_i\to lH} = \int_{N_i\to lH\to\bar{l}\bar{H}} f_{N_i}\left(1-f_l^{\text{eq}}\right)\left(1+f_H^{\text{eq}}\right)\left(1-f_{\bar{l}}^{\text{eq}}\right)\left(1+f_{\bar{H}}^{\text{eq}}\right)\Delta|\mathring{M}|^2, \tag{14a}$$

$$\Delta\gamma_{\bar{l}\bar{H}\to N_i} = \int_{N_i\to lH\to\bar{l}\bar{H}} f_{\bar{l}}^{\text{eq}}f_{\bar{H}}^{\text{eq}}(1-f_{N_i})\left(1-f_l^{\text{eq}}\right)\left(1+f_H^{\text{eq}}\right)\Delta|\mathring{M}|^2, \tag{14b}$$

$$\Delta\gamma_{lH\to\bar{l}\bar{H}} = \int_{N_i\to lH\to\bar{l}\bar{H}} f_l^{\text{eq}}f_H^{\text{eq}}\left(1-f_{\bar{l}}^{\text{eq}}\right)\left(1+f_{\bar{H}}^{\text{eq}}\right)(1-f_{N_i})\Delta|\mathring{M}|^2. \tag{14c}$$

Using the detailed balance conditions, such as

$$f_{N_i}^{\text{eq}}(1-f_l^{\text{eq}})(1+f_H^{\text{eq}}) = (1-f_{N_i}^{\text{eq}})f_l^{\text{eq}}f_H^{\text{eq}}, \tag{15}$$

and unitarity relations from Eq. (5) for asymmetries in Eq. (14), we obtain for the lepton number evolution

$$\dot{n}_L + 3\mathcal{H}n_L = \int_{N_i\to lH\to\bar{l}\bar{H}}\delta f_{N_i}(1-f_l^{\text{eq}})(1+f_H^{\text{eq}})(1-f_{\bar{l}}^{\text{eq}}+f_{\bar{H}}^{\text{eq}})\Delta|\mathring{M}|^2 + \text{wash-out terms}, \tag{16}$$

where $n_L \equiv n_l - n_{\bar{l}}$. Remarkably, the result is in agreement with non-equilibrium QFT calculations [7–9]. The source term vanishes in equilibrium, when $\delta f_{N_i} = f_{N_i} - f_{N_i}^{\text{eq}} = 0$.

Conclusions

- ← **A diagrammatic concept connecting the classical Boltzmann equation and quantum kinetic theory has been introduced.**
- ← **Statistical factors due to on-shell intermediate states have been formally represented by the cuts of forward diagrams with multiple spectator lines.**
- ← **Results are in agreement with the direct closed-time-path derivation of non-equilibrium quantum field theory [7–9].**
- ← **Unitarity and CPT relations between the reaction rate asymmetries can be derived in our approach to kinetic theory in the same way as in $T = 0$ calculations.**

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