

Light Exotic Λ -hypernuclei

S.V. SIDOROV^{1,2,3}, D.E. LANSKOY¹, T.YU. TRETAKOVA^{1,2,3}

¹ *Faculty of Physics, Moscow State University, Moscow, Russia*

² *Skobel'tsyn Institute of Nuclear Physics, Moscow State University, Moscow, Russia*

³ *Joint Institute for Nuclear Research, Dubna, Russia*

Skyrme-Hartree-Fock approach for hypernuclei

- Nucleon-nucleon Skyrme potential (Vautherin and Brink, 1972):

$$V_{NN}(\mathbf{r}_1, \mathbf{r}_2) = t_0(1 + x_0 P_\sigma)\delta(\mathbf{r}_{12}) + \frac{1}{2}t_1(1 + x_1 P_\sigma)(\mathbf{k}'^2\delta(\mathbf{r}_{12}) + \delta(\mathbf{r}_{12})\mathbf{k}^2) \\ + t_2(1 + x_2 P_\sigma)\mathbf{k}'\delta(\mathbf{r}_{12})\mathbf{k} + \frac{1}{6}t_3\rho^\alpha(\mathbf{R})(1 + x_3 P_\sigma)\delta(\mathbf{r}_{12}) + iW(\sigma_1 + \sigma_2)[\mathbf{k}' \times \delta(\mathbf{r})\mathbf{k}]$$

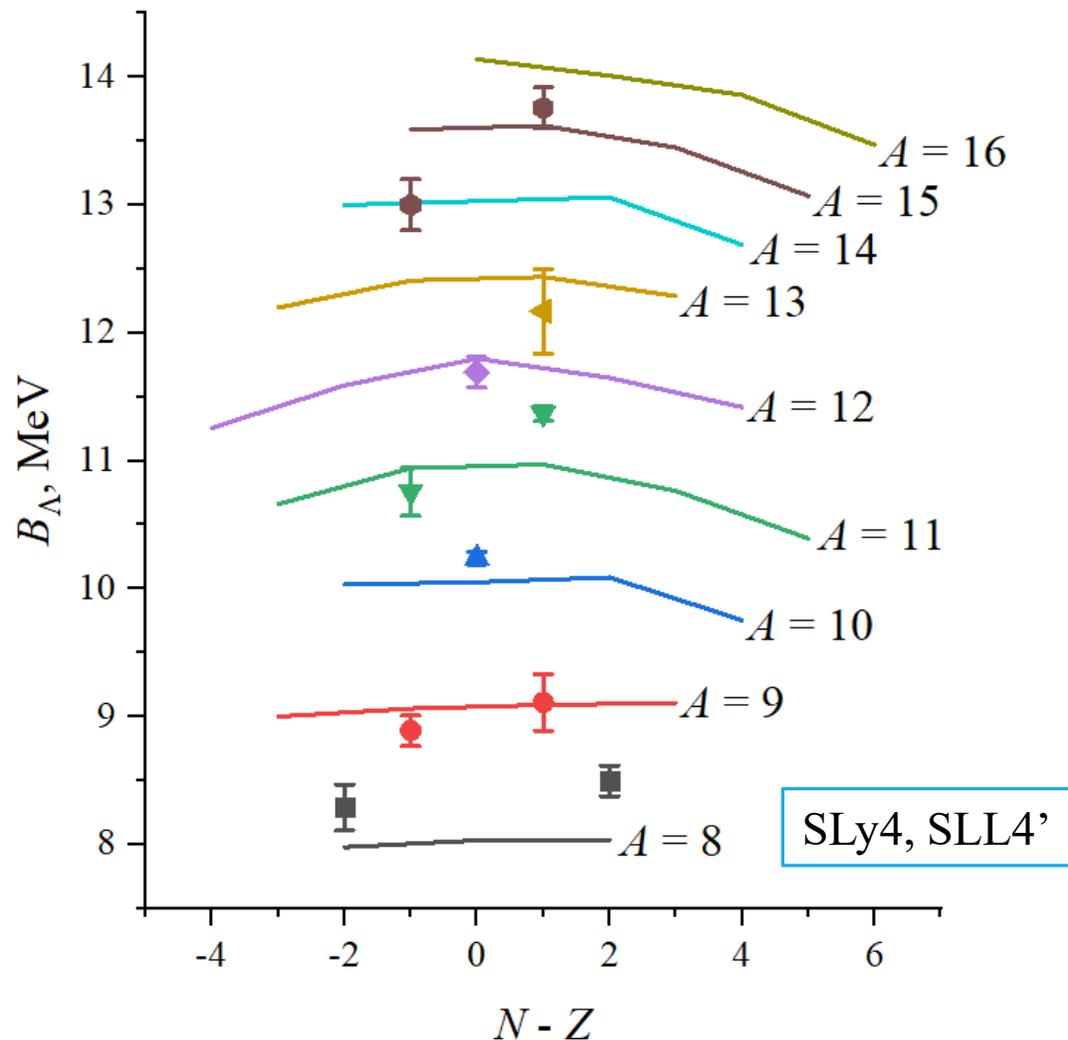
NN: SLy4, SkM*, SkIII

- Hyperon-nucleon Skyrme potential (Rayet, 1981):

$$V_{\Lambda N}(\mathbf{r}_\Lambda, \mathbf{r}_q) = t_0^\Lambda(1 + x_0^\Lambda P_\sigma)\delta(\mathbf{r}_{\Lambda q}) + \frac{1}{2}t_1^\Lambda(\mathbf{k}^2\delta(\mathbf{r}_{\Lambda q}) + \delta(\mathbf{r}_{\Lambda q})\mathbf{k}'^2) \\ + t_2^\Lambda\mathbf{k}'\delta(\mathbf{r}_{\Lambda q})\mathbf{k} + \frac{1}{6}t_3^\Lambda\rho^\alpha(\mathbf{R})\delta(\mathbf{r}_{\Lambda q})$$

ΛN : [SLL4](#), [SLL4'](#) (Schulze and Hiyama, 2014), [YBZ5](#) (Yamamoto et al, 1988),
[LY1](#) (Lanskoy and Yamamoto, 1997), [LY5r](#) (Zhang et al, 2021), [SkSH1](#)
(Fernandez et al, 1989)

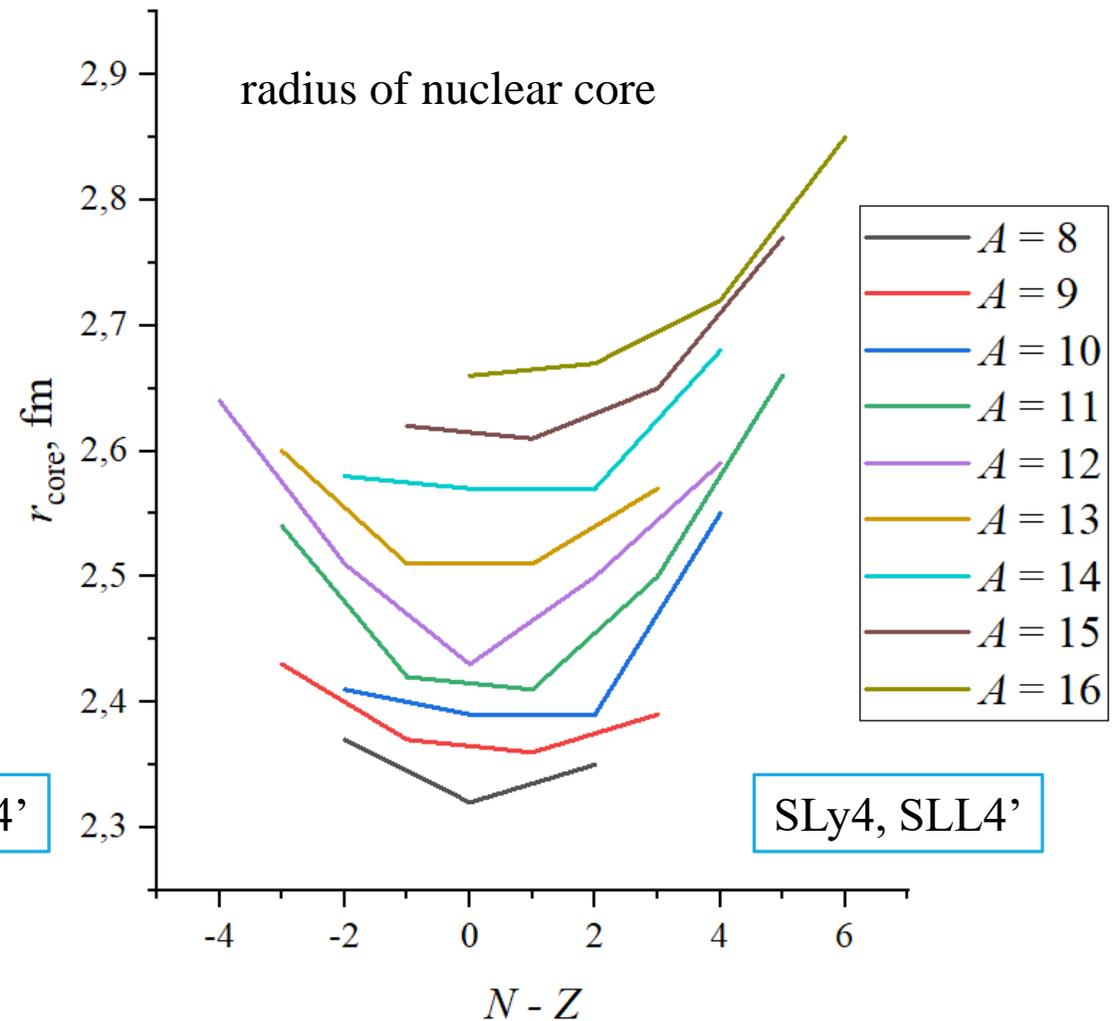
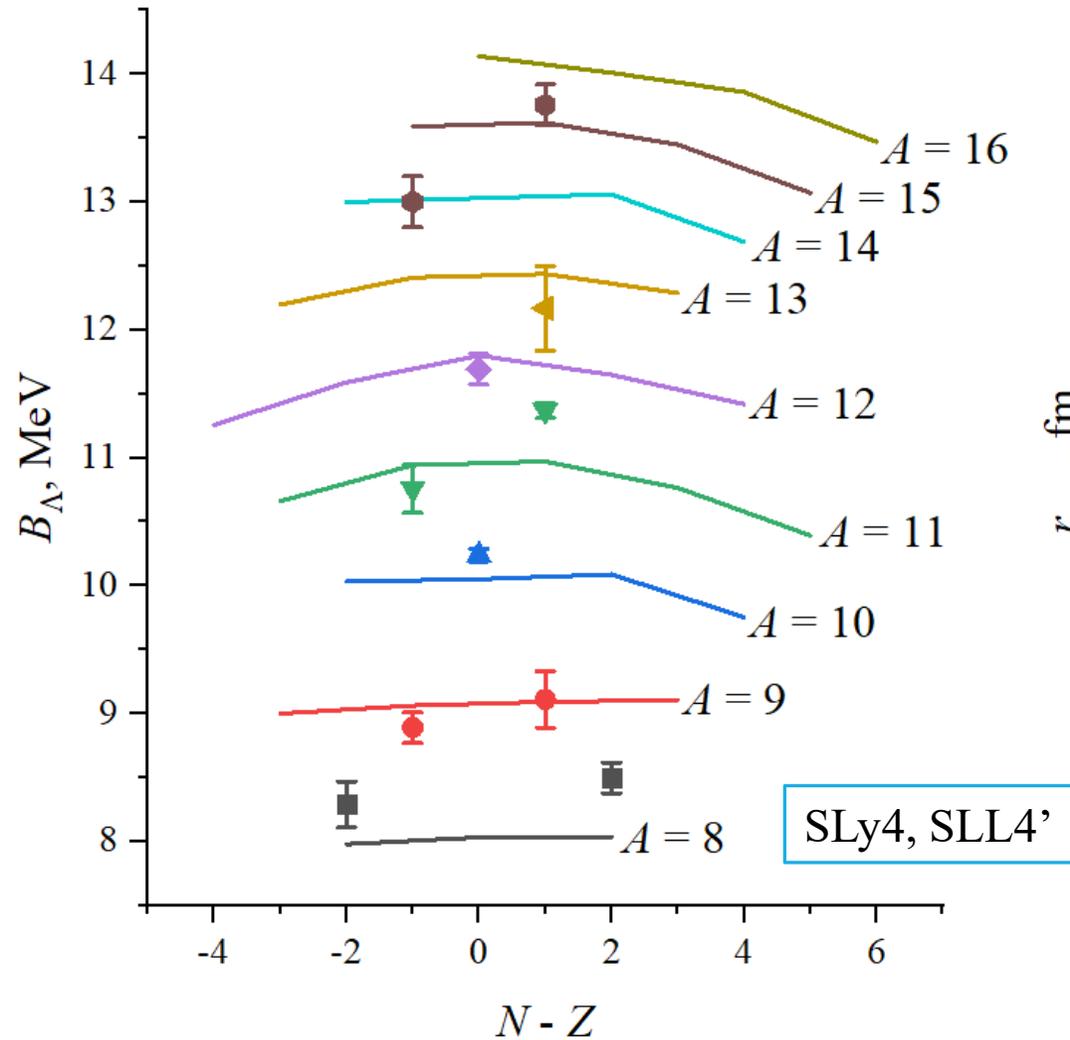
Hyperon binding energies in $A+1_{\Lambda}Z$ hypernuclei



$$B_{\Lambda}(^{A+1}_{\Lambda}Z) = B_{tot}(^{A+1}_{\Lambda}Z) - B_{tot}(^AZ)$$

- The difference in neighboring isobar chains is around 1 MeV for lighter hypernuclei, smaller as A increases
- Symmetric character of B_{Λ} with respect to isospin $N - Z$

Hyperon binding energies and radii of nuclear cores in $^{A+1}_{\Lambda}Z$ hypernuclei



Hypernuclei near the proton dripline

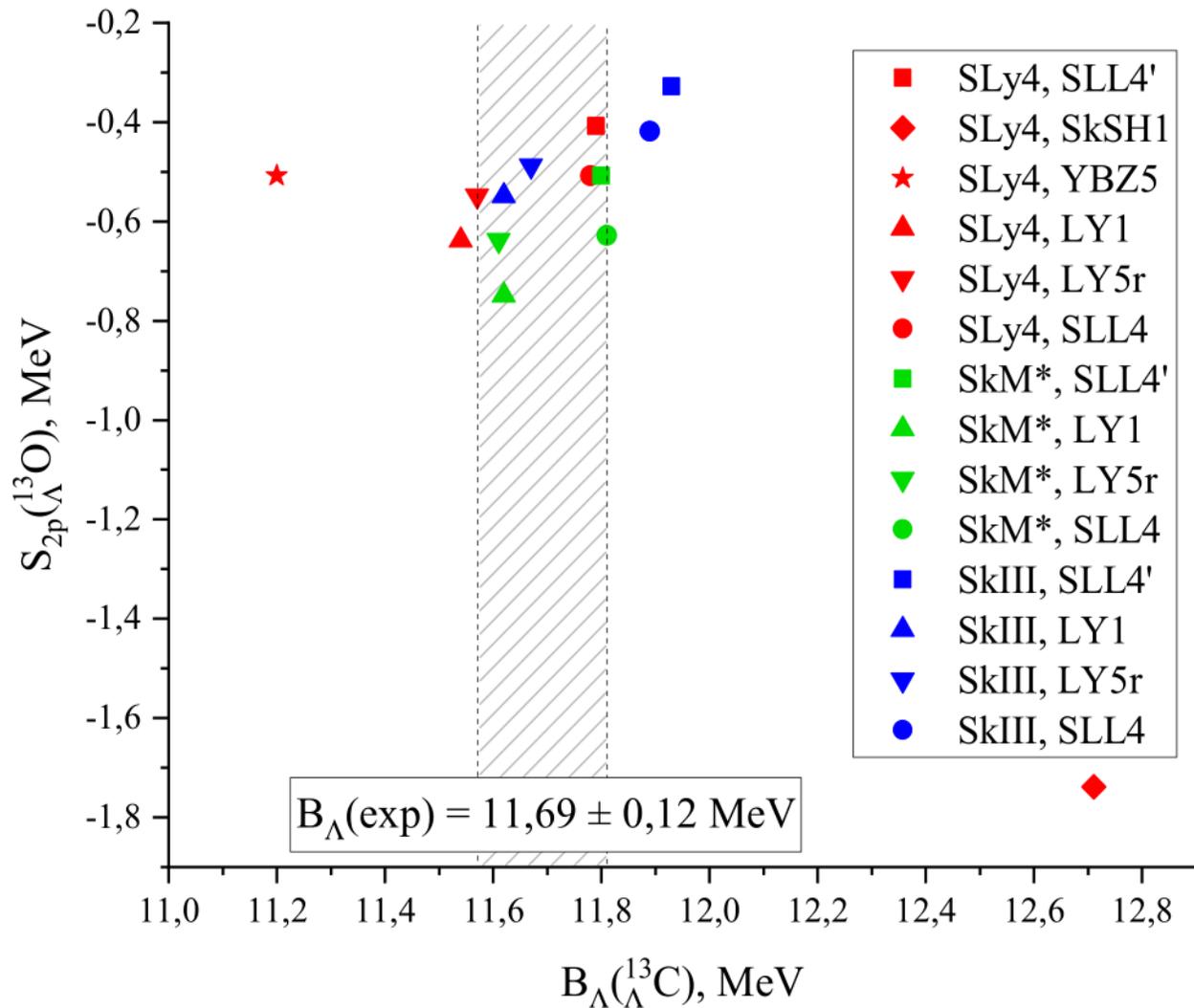
While we do not claim outright that hypernuclear HF approach gives precise predictions for proton (or two-proton) separation energies of hypernuclei, we set the following idea as the cornerstone of our method: hypernuclear HF allows to reproduce the hyperon binding energy B_Λ in light Λ -hypernuclei. Proton (two proton) separation energy S_p (S_{2p}) can then be found using the relation:

$$\begin{aligned} S_p({}^A_\Lambda Z) &= S_p({}^{A-1}Z) + B_\Lambda({}^A_\Lambda Z) - B_\Lambda({}^{A-1}_\Lambda(Z-1)), \\ S_{2p}({}^A_\Lambda Z) &= S_{2p}({}^{A-1}Z) + B_\Lambda({}^A_\Lambda Z) - B_\Lambda({}^{A-2}_\Lambda(Z-2)). \end{aligned}$$

and experimental data for $S_p({}^A_\Lambda Z)$ and $S_{2p}({}^A_\Lambda Z)$.

Here, $S_p({}^{A-1}Z)$ or $S_{2p}({}^{A-1}Z)$ is always taken from experiment, while B_Λ is calculated within HF approach when there are no experimental data available. Comparison between calculated values and experimental data on B_Λ in neighbouring nuclei can be used to verify the accuracy of our estimates for S_p (or S_{2p}) in proton-rich hypernuclei.

2p separation energy in $^{13}_{\Lambda}\text{O}$



^{12}O is unstable with respect to 2 proton decay ($S_{2p}(^{12}\text{O}) = -1,638 \text{ MeV}$), and $^{13}_{\Lambda}\text{O}$ is expected to decay similarly. On the left hand side we show

$$S_{2p}(^{13}_{\Lambda}\text{O}) = S_{2p}(^{12}\text{O}) + B_{\Lambda}(^{13}_{\Lambda}\text{O}) - B_{\Lambda}(^{11}_{\Lambda}\text{C})$$

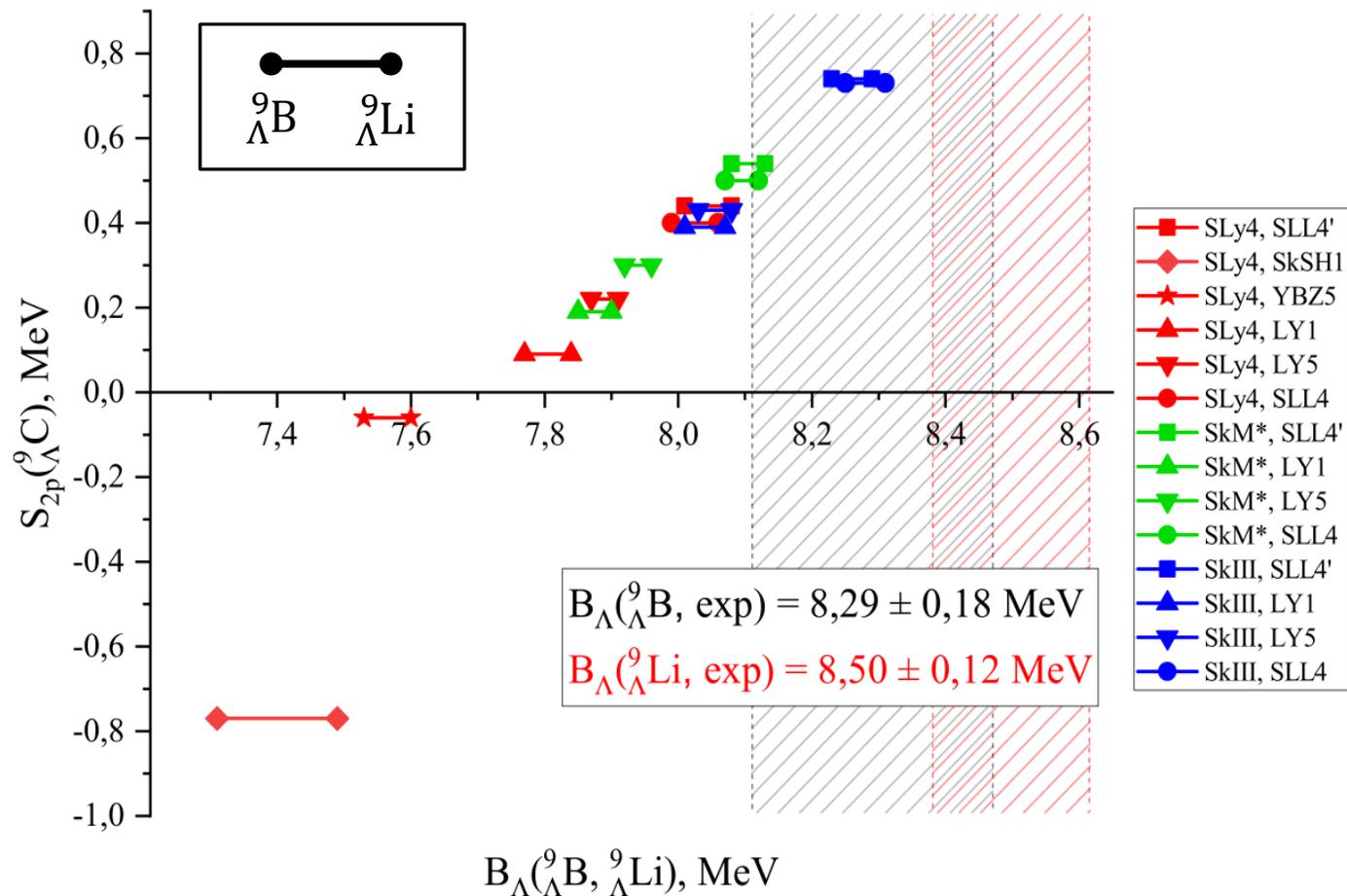
exp \curvearrowright
calc \curvearrowright

as a function of $B_{\Lambda}(^{13}_{\Lambda}\text{C})$ for different NN and ΛN Skyrme interactions. Most of the results are in good agreement with the experimental hyperon binding energy in $^{13}_{\Lambda}\text{C}$.

$^{13}_{\Lambda}\text{O}$ is unbound

$^8_{\Lambda}\text{B}$, $^{12}_{\Lambda}\text{N}$ are also unbound

2p separation energy in ${}^9_{\Lambda}\text{C}$



While ${}^8\text{C}$ decays primarily by emitting 4 protons, ${}^9_{\Lambda}\text{C}$ has 2 proton decay as critical decay mode, with bound ${}^7_{\Lambda}\text{Be}$ as a daughter hypernucleus. On the left hand side we show

$$S_{2p}({}^9_{\Lambda}\text{C}) = S_{2p}({}^8\text{C}) + B_{\Lambda}({}^9_{\Lambda}\text{C}) - B_{\Lambda}({}^7_{\Lambda}\text{Be})$$

calc \rightarrow
 exp (-2.14 MeV) \uparrow exp (5.16 MeV) \uparrow

as a function of B_{Λ} in ${}^9_{\Lambda}\text{B}$ and ${}^9_{\Lambda}\text{Li}$ for different NN and ΛN Skyrme interactions. The better $B_{\Lambda}({}^9_{\Lambda}\text{B})$ and $B_{\Lambda}({}^9_{\Lambda}\text{Li})$ are described, the stronger is the two-proton binding in ${}^9_{\Lambda}\text{C}$, indicating that $S_{2p}({}^9_{\Lambda}\text{C}) > 0$.

${}^9_{\Lambda}\text{C}$ is bound!

Skyrme $\Lambda\Lambda$ -interaction for double- Λ hypernuclei

Due to glue-like role of Λ -hyperon, there is a chance to stabilize the hypernuclei ${}^8_{\Lambda}\text{B}$, ${}^{12}_{\Lambda}\text{N}$ and ${}^{13}_{\Lambda}\text{O}$ by adding yet another Λ -hyperon. $\Lambda\Lambda$ -interaction then needs to be taken into account

- Hyperon-hyperon Skyrme potential:

$$V_{\Lambda\Lambda}(\mathbf{r}_1, \mathbf{r}_2) = \lambda_0 \delta(\mathbf{r}_{12}) + \frac{1}{2} \lambda_1 (\mathbf{k}'^2 \delta(\mathbf{r}_{12}) + \delta(\mathbf{r}_{12}) \mathbf{k}^2) + \lambda_2 \mathbf{k}' \delta(\mathbf{r}_{12}) \mathbf{k}$$

$\Lambda\Lambda$: S $\Lambda\Lambda$ 1', S $\Lambda\Lambda$ 3' (Lanskoy, 1998, Minato and Hagino, 2011)

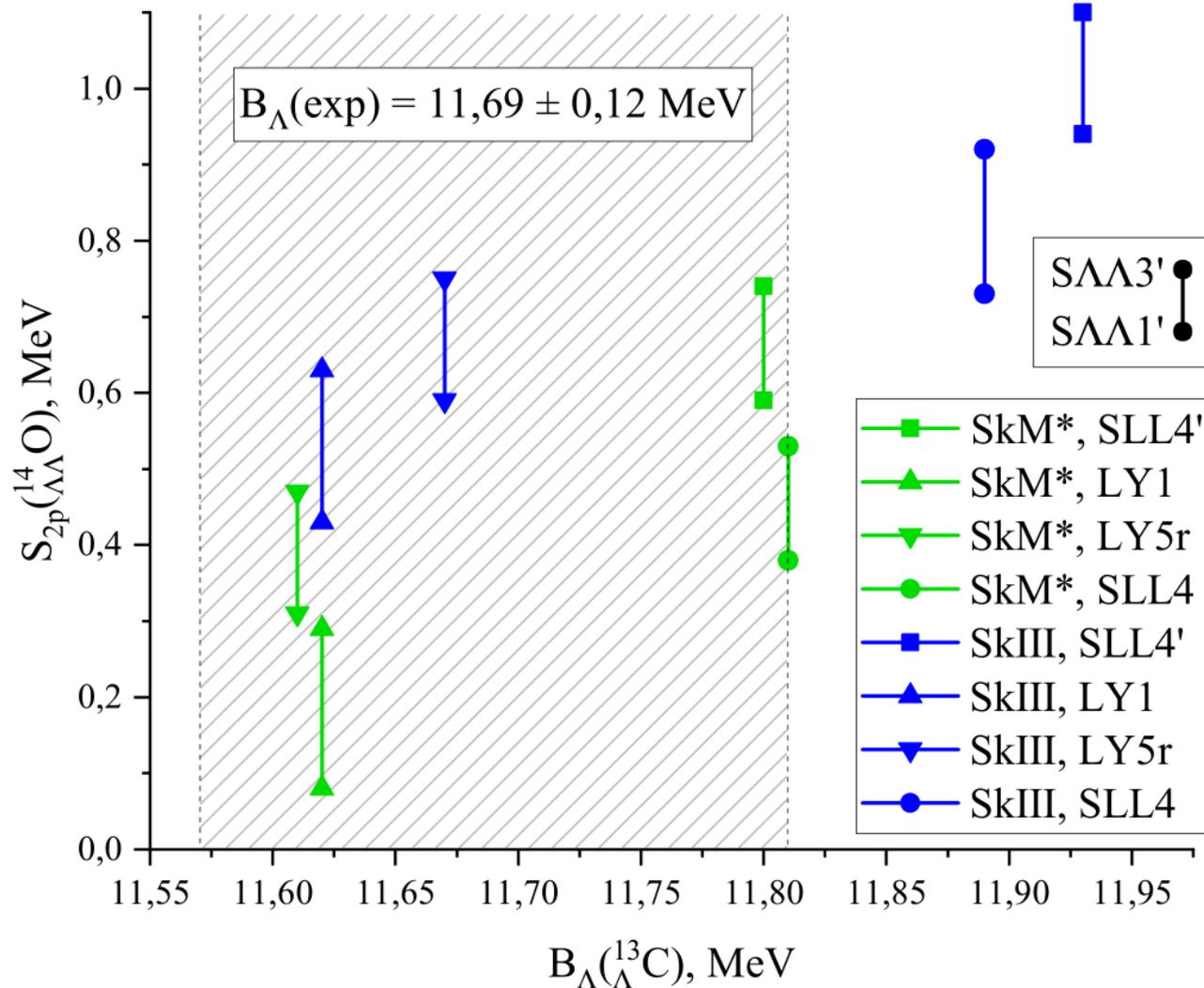
Proton (two proton) separation energy S_p (S_{2p}) can then be found using the relation:

$$\begin{aligned} S_p({}_{\Lambda\Lambda}^AZ) &= S_p({}^{A-2}Z) + B_{\Lambda\Lambda}({}_{\Lambda\Lambda}^AZ) - B_{\Lambda\Lambda}({}_{\Lambda\Lambda}^{A-1}(Z-1)), \\ S_{2p}({}_{\Lambda\Lambda}^AZ) &= S_{2p}({}^{A-2}Z) + B_{\Lambda\Lambda}({}_{\Lambda\Lambda}^AZ) - B_{\Lambda\Lambda}({}_{\Lambda\Lambda}^{A-2}(Z-2)); \end{aligned}$$

Here, 2 hyperon binding energy is:

$$B_{\Lambda\Lambda}({}_{\Lambda\Lambda}^AZ) = B_{tot}({}_{\Lambda\Lambda}^AZ) - B_{tot}({}^{A-2}Z)$$

2p separation energy in ${}_{\Lambda\Lambda}^{14}\text{O}$



$$S_{2p}({}_{\Lambda\Lambda}^{14}\text{O}) = S_{2p}({}^{12}\text{O}) + B_{\Lambda\Lambda}({}_{\Lambda\Lambda}^{14}\text{O}) - B_{\Lambda\Lambda}({}_{\Lambda\Lambda}^{12}\text{C})$$

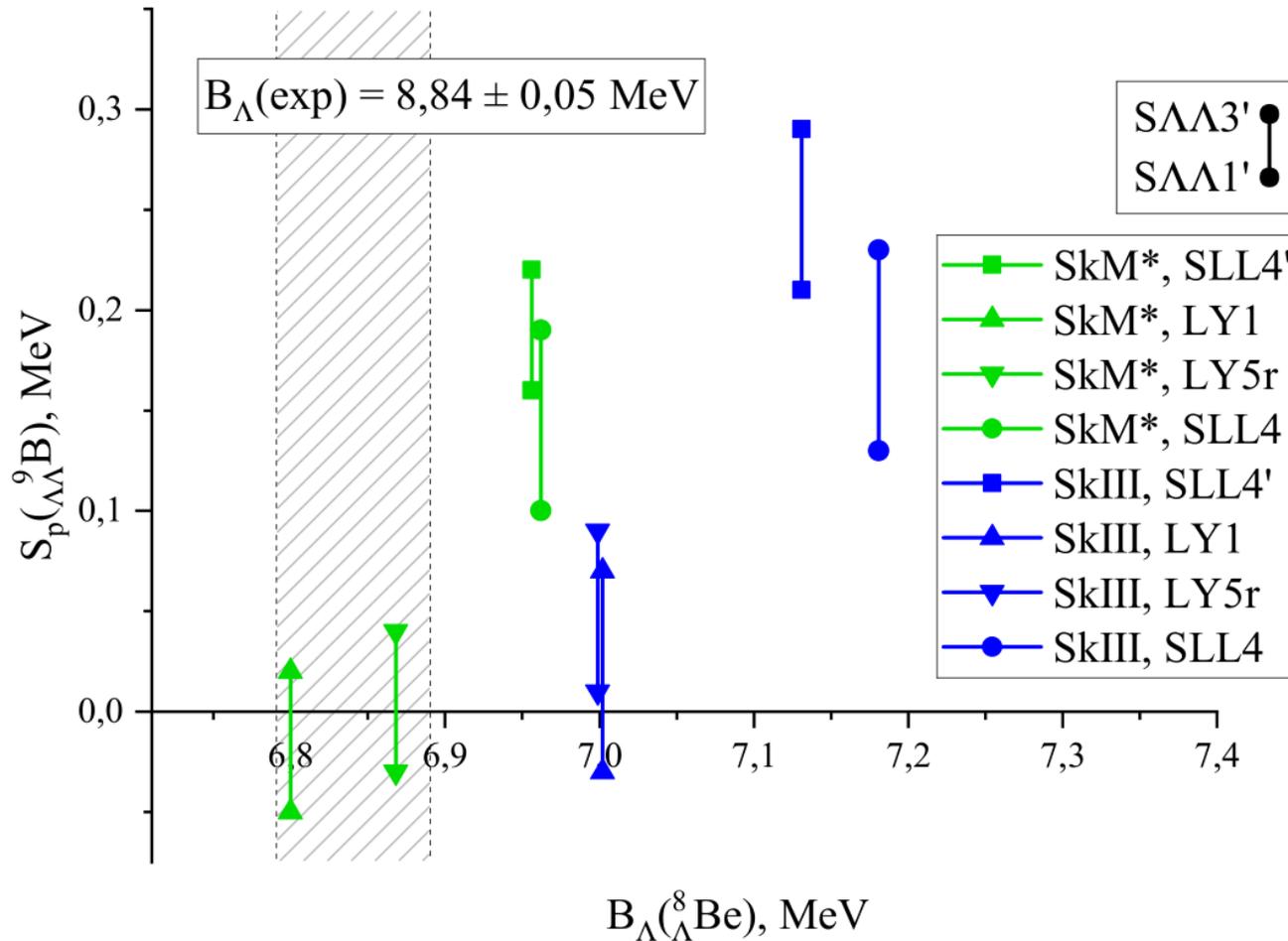
exp

calc

as a function of $B_{\Lambda}({}_{\Lambda}^{13}\text{C})$ for different NN and ΛN Skyrme interactions. Earlier we concluded ${}_{\Lambda}^{13}\text{O}$ is unbound; addition of another hyperon stabilizes the hypernucleus.

${}_{\Lambda\Lambda}^{14}\text{O}$ is bound!

Proton separation energy in ${}_{\Lambda\Lambda}{}^9\text{B}$



$$S_p({}_{\Lambda\Lambda}{}^9\text{B}) = S_p({}^7\text{B}) + B_{\Lambda\Lambda}({}_{\Lambda\Lambda}{}^9\text{B}) - B_{\Lambda\Lambda}({}_{\Lambda\Lambda}{}^8\text{Be})$$

exp (-2.01 MeV)

calc

as a function of $B_{\Lambda}({}_{\Lambda}{}^8\text{Be})$ for different NN and ΛN Skyrme interactions. While we concluded ${}_{\Lambda}{}^8\text{B}$ is unbound,

${}_{\Lambda\Lambda}{}^9\text{B}$ is possibly bound.

Hypernucleus ${}_{\Lambda\Lambda}{}^{13}\text{N}$ is found to be unbound.

Conclusions

Hypernuclear Hartree-Fock approach was utilized to study the properties of light proton-rich Λ -hypernuclei.

- Hyperon binding energy vs isospin dependence is more pronounced for the cases with more dramatic density changes, and the corresponding nuclei should be chosen as sources of information on Λ N interaction
- Among the studied hypernuclei:
 - $Z = 5$: ${}^8_{\Lambda}\text{B}$ is unbound, ${}^9_{\Lambda\Lambda}\text{B}$ could possibly be bound,
 - $Z = 6$: ${}^9_{\Lambda}\text{C}$ is bound, therefore ${}^{10}_{\Lambda\Lambda}\text{C}$ is also evidently bound,
 - $Z = 7$: ${}^{12}_{\Lambda}\text{N}$ and ${}^{13}_{\Lambda\Lambda}\text{N}$ are unbound,
 - $Z = 8$: ${}^{13}_{\Lambda}\text{O}$ is unbound, while ${}^{14}_{\Lambda\Lambda}\text{O}$ is bound.

Thank you for your attention