Femtoscopic study of coupled-channel baryon-baryon interactions with S=-2

Akira Ohnishi (YITP, Kyoto U.), Y. Kamiya, K. Sasaki, T. Fukui, T. Hyodo, K. Morita, K. Ogata, T. Hatsuda

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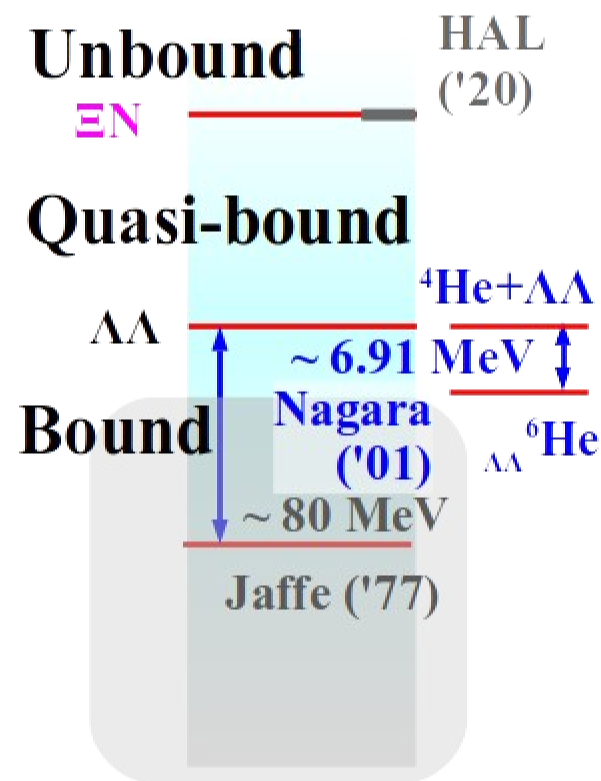
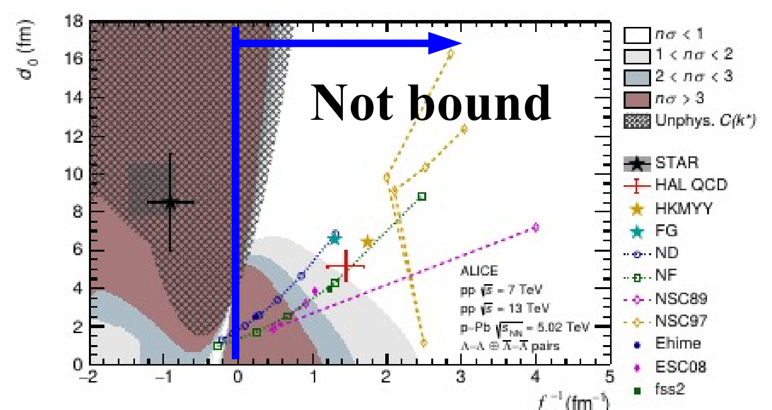
- **Introduction**
- **Coupled-channel NΞ-ΛΛ potential and correlation functions**
- **Comparison with $p\Xi^-$ and ΛΛ correlation function data**
- **Unbound nature of NΞ confirmed ?**
- **Summary**

Y. Kamiya, K. Sasaki, T. Fukui, T. Hyodo, K. Morita, K. Ogata, AO, T. Hatsuda, arXiv:2108.09644 [hep-ph]

Impact of S=−2 Baryon-Baryon Interactions (1)

- Is “H(uuddss)” bound, unbound, or quasi-bound ?
- It is plausible not to be bound below ΛΛ.
  - Bound H in the $SU(3)_f$ limit.
    Bag model: Jaffe, PRL38(1977)195.
    LQCD: HALQCD(‘11), NPLQCD('11,’13), Mainz (‘19).
  - But no discovery of bound H.
    No M(Λpπ⁻) peak; ΛΛ hypernucl.:Takahashi+ ('01);
    Femtoscopy:STAR(‘15); ALICE(‘19); Morita+(‘15).
- Quasi-bound state below NΞ or Unbound ?
  - Resonance “H” from (K^-, K^+) ?
    KEK-E522 ('07)
  - LQCD at almost physical m_q → Unbound
    HAL QCD(‘20).



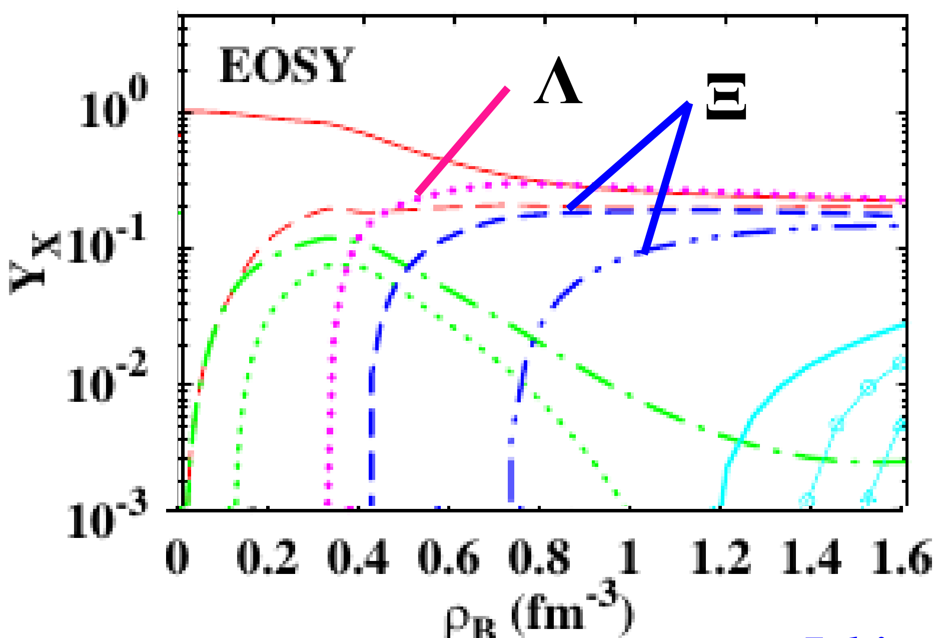


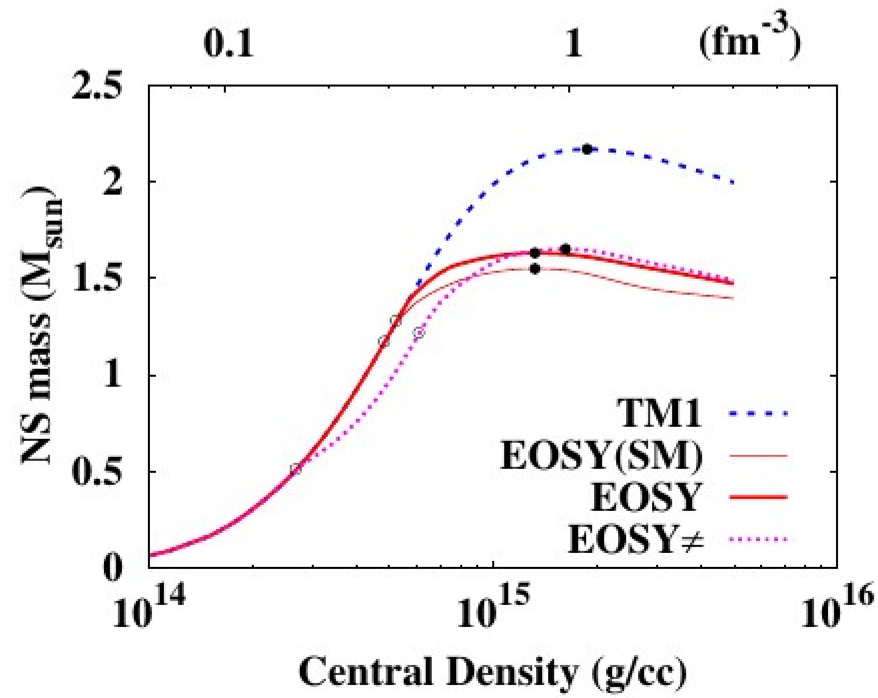
ALICE(‘19)

Impact of S= −2 Baryon-Baryon Interactions (2)

- **ΛΛ and NΞ interactions are relevant to "Hyperon Puzzle"**
  - **Λ and Ξ are predicted to appear at (2-4)ρ_0, and softened EOS cannot support 2 $M_\odot$ neutron stars.**
    → Repulsive YNN interactions, Quark Matter, Modified Gravity ?
  - **Precise ΛN, ΛΛ, NΞ, and ΛNN interactions need to be known.**
    - **Repulsive ΞN interaction (I=1) may help support 2 $M_\odot$ NS**

Weissborn et al., NPA881 ('12) 62.





Ishizuka, AO, Tsubakihara, Sumiyoshi, Yamada ('08)

S=−2 Baryon-Baryon Interactions

- **Theoretical Approaches**
  - **Phenomenological (Nijmegen, Jülich, Ehime, Quark model, …)**
  - **Chiral EFT** *[Haidenbauer, Meissner, Petschauer ('16); Li, Hyodo, Geng ('18)]*
  - **Lattice QCD** *[Sasaki+ [HAL QCD] ('20)]*
- **Experimental Information**
  - **Double Λ and Ξ hypernuclei**
    Takahashi+('01); Nakazawa+('15); Hayakawa+[E07]('21); Yoshimoto+[E07]('21).
  - **Femtoscopic study of hadron-hadron interactions**
    [See also Valentina Mantovani Sarti (Wed), Laura Šerkšnytė (Sun)]
    Adamczyk+[STAR]('15, ΛΛ); Acharya+[ALICE]('19(ΛΛ), '19(NΞ), '20(NΞ));
    Morita, Furumoto, AO ('15, ΛΛ); Hatsuda, Morita, AO, Sasaki ('17, NΞ);
    Haidenbauer ('19, ΛΛ-NΞ); Haidenbauer+ ('20).

We study $p\Xi^-$ and ΛΛ correlation functions in the coupled-channel framework (KPLLL formula) using S=−2 lattice baryon-baryon interaction from HAL QCD. [Kamiya+ (2108.09644)]

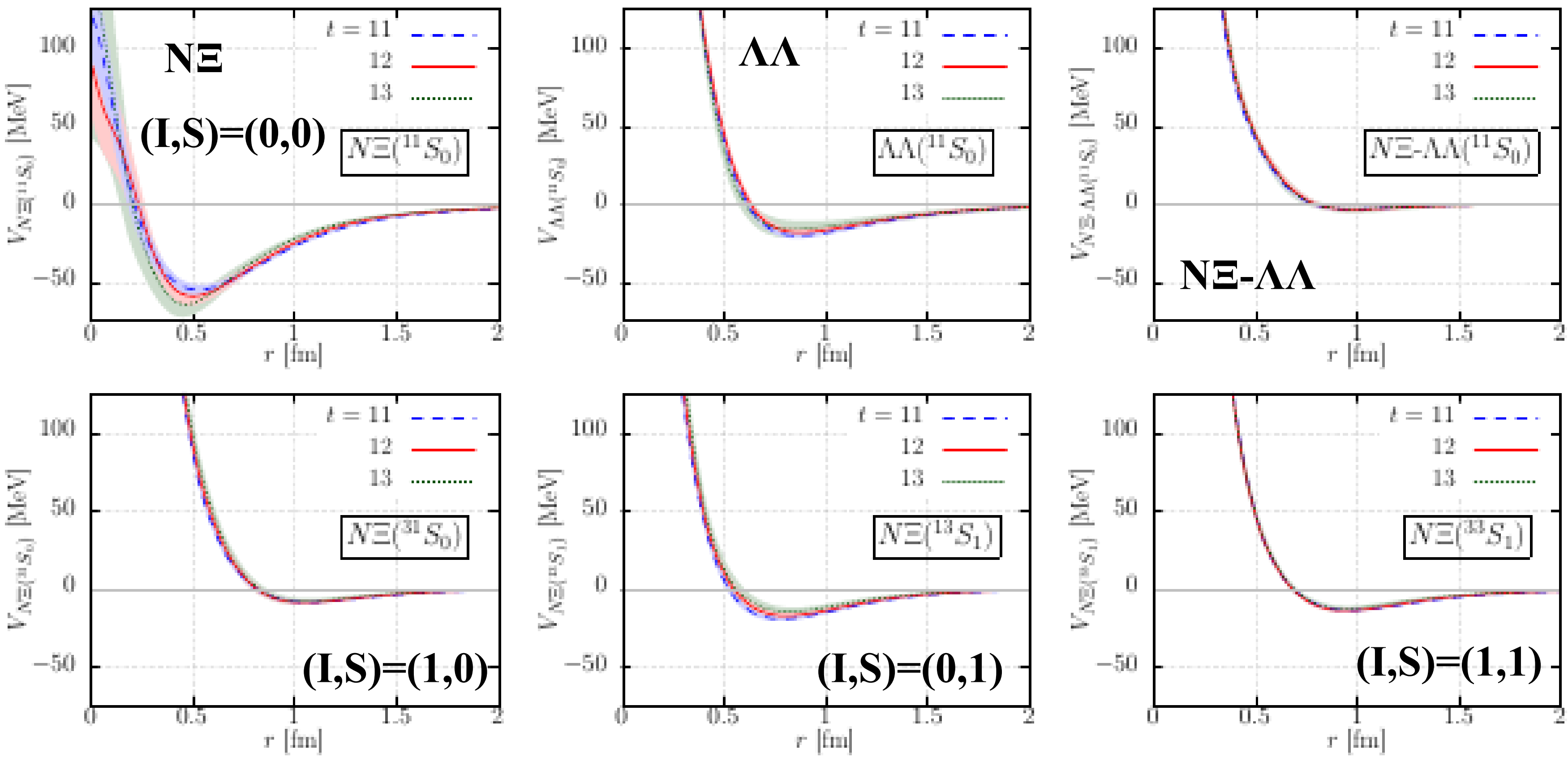
Coupled-channel NΞ-ΛΛ potential and correlation functions

NΞ-ΛΛ Potential from Lattice QCD

- **NΞ-ΛΛ potential at almost physical quark masses (m_π=146 MeV) by HAL QCD Collaboration**

  K. Sasaki et al. [HAL QCD], NPA 998 ('20) 121737 (1912.08630)

  - **Significant attraction in (I,S)=(0,0) of NΞ.**
  - **Weak attraction in ΛΛ (Coupling with NΞ causes ΛΛ attraction).**



$N\Xi$-$\Lambda\Lambda$ Potential from Lattice QCD

- **Low-energy scattering parameters**
  - **Nuclear physics convention** $$k\cot\delta = -\frac{1}{a_0} + \frac{1}{2} r_{\rm eff} k^2 + \mathcal{O}(k^2)$$

total spin	baryon pair	a_0 [fm]	$r_{\rm eff}$ [fm]
$J=0$	$p\Xi^-$	$-1.22(0.13)(^{+0.08}_{-0.00}) - i1.57(0.35)(^{+0.18}_{-0.23})$	$3.7(0.3)(^{+0.1}_{-0.1}) - i2.7(0.2)(^{+0.1}_{-0.3})$
	$n\Xi^0$	$-2.07(0.39)(^{+0.28}_{-0.35}) - i0.14(0.08)(^{+0.00}_{-0.01})$	$1.5(0.3)(^{+0.0}_{-0.0}) - i0.2(0.0)(^{+0.0}_{-0.1})$
	$\Lambda\Lambda$	$-0.78(0.22)(^{+0.00}_{-0.13})$	$5.4(0.8)(^{+0.1}_{-0.5})$
$J=1$	$p\Xi^-$	$-0.35(0.06)(^{+0.09}_{-0.07}) - i0.00$	$8.3(1.0)(^{+2.8}_{-1.2}) + i0.0(0.1)(^{+0.1}_{-0.0})$
	$n\Xi^0$	$-0.35(0.06)(^{+0.09}_{-0.07})$	$8.4(1.0)(^{+2.7}_{-1.2})$

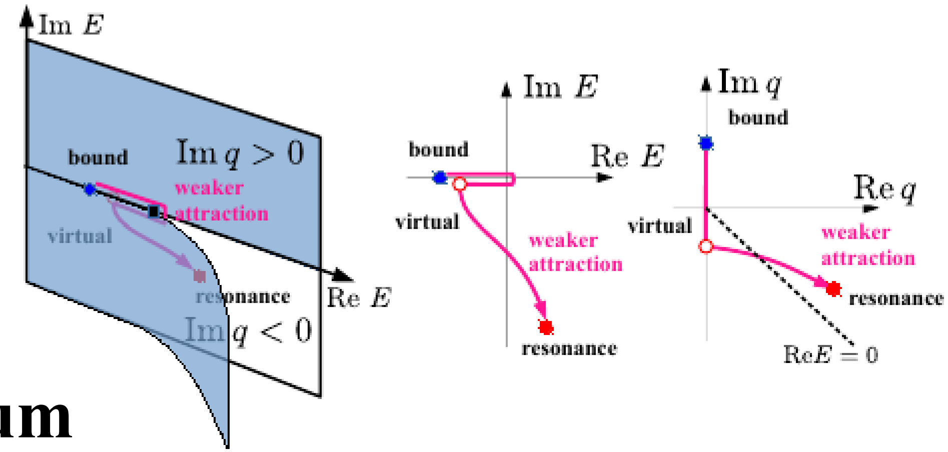
  - **Re (a_0) < 0 → No bound state in ΛΛ-NΞ systems.**
    (except for Ξ^- atom)
- **There is a virtual pole around the NΞ threshold**
  (3.93 MeV below $n\Xi^0$ threshold)
  on the irrelevant Riemann sheet, (+, –, +) [quasi-bound → (–,+,+)]

$$E_{\rm pole} = 2250.5 - i0.3 \text{ MeV}$$

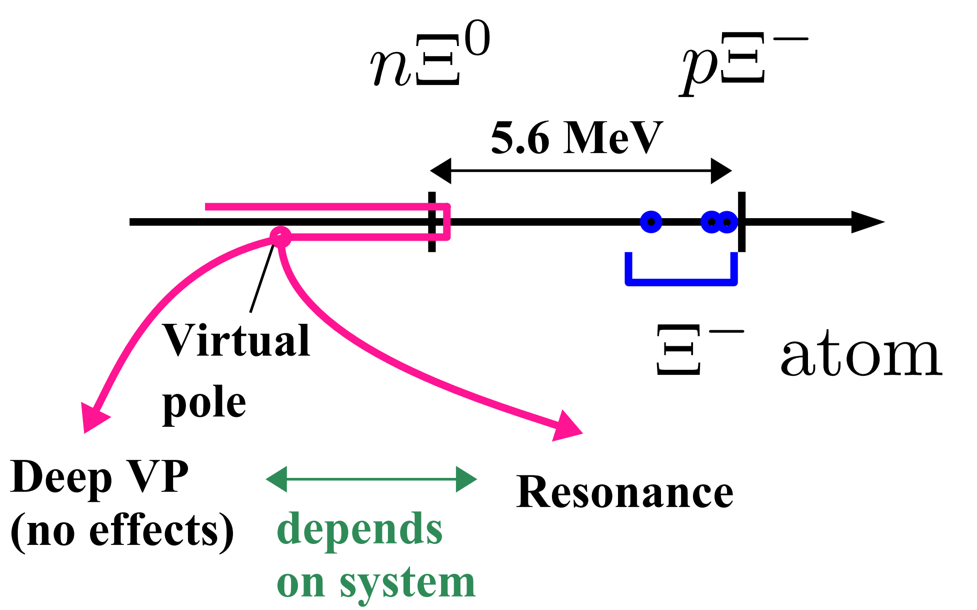
sign of Im(eignen momentum)

Virtual Pole

- Virtual pole (single channel case)
  = Eigen energy of the pole is below the threshold, but the wave function diverges at $r \to \infty$.
  (Imaginary part of eigen momentum is negative, $\exp(iqr)/r \to \infty$.)

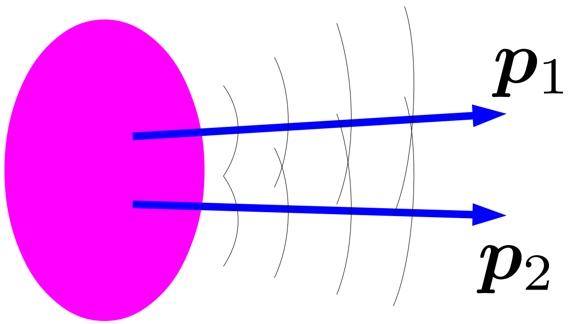


- Lattice BB potential at almost physical quark masses (HAL QCD)
  - With Coulomb potential and threshold mass difference, virtual pole appears on (+,-,+) Riemann sheet (w.f. of $n\Xi^0$ channel diverges).
  - Atomic states are well separated from VP. ($\mu\alpha^2/2n^2$=14.6 keV/n^2)



Femtoscopic Study of Hadron-Hadron Interaction

- **Correlation function (CF)**
  - **Koonin-Pratt formula**
    Koonin('77), Pratt+('86), Lednicky+('82)



$$C(\boldsymbol{p}_1,\boldsymbol{p}_2)=\frac{N_{12}(\boldsymbol{p}_1,\boldsymbol{p}_2)}{N_1(\boldsymbol{p}_1)N_2(\boldsymbol{p}_2)}\simeq\int d\boldsymbol{r}\,\underline{S_{12}(\boldsymbol{r})}\,\underline{|\varphi_{\boldsymbol{q}}(\boldsymbol{r})|^2}$$

source fn. **relative w.f.**

- **Source size from quantum stat. + CF (Femtoscopy)**
  Hanbury Brown & Twiss ('56); Goldhaber, Goldhaber, Lee, Pais ('60)

- ***Hadron-hadron interaction from source size + CF***
  - **CF of non-identical pair from Gaussian source**
    R. Lednicky, V. L. Lyuboshits ('82); K. Morita, T. Furumoto, AO ('15)

$$C(\boldsymbol{q})=1+\int d\boldsymbol{r}\,S(r)\left\{|\varphi_0(r)|^2-|j_0(qr)|^2\right\}\quad(\varphi_0=\text{s-wave w.f.})$$

CF shows how much $|\varphi|^2$ is enhanced → V_{hh} effects !

Coupled-Channel Correlation Function

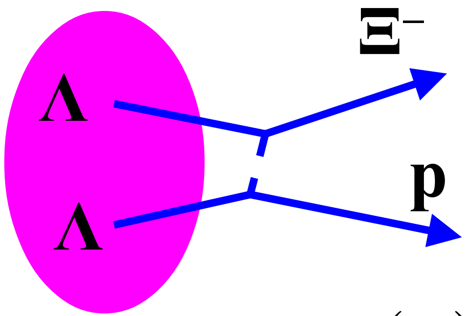
- **Correlation function with CC effects (KPLLL formula)**
  –> sum of j-th channel contributions leading to j=1
  with outgoing momentum *q*
  Lednicky, Lyuboshits, Lyuboshits ('98);
  Haudenbauer ('19)

$$C(\boldsymbol{q}) = \sum_j \omega_j \int d\boldsymbol{r} S_j(\boldsymbol{r}) |\Psi_j^{(-)}(\boldsymbol{r})|^2$$

$$\Psi_j^{(-)}(\boldsymbol{r}) = [e^{i\boldsymbol{q}\cdot\boldsymbol{r}} - j_0(qr)]\delta_{1j} + \psi_j^{(-)}(r)$$

$$\psi_j^{(-)}(q) \propto e^{-iqr}/r \text{ or } e^{-\kappa r}/r \ (r \to \infty)$$

(No Coulomb case)



Outgoing Mom. *q* in channel j=1

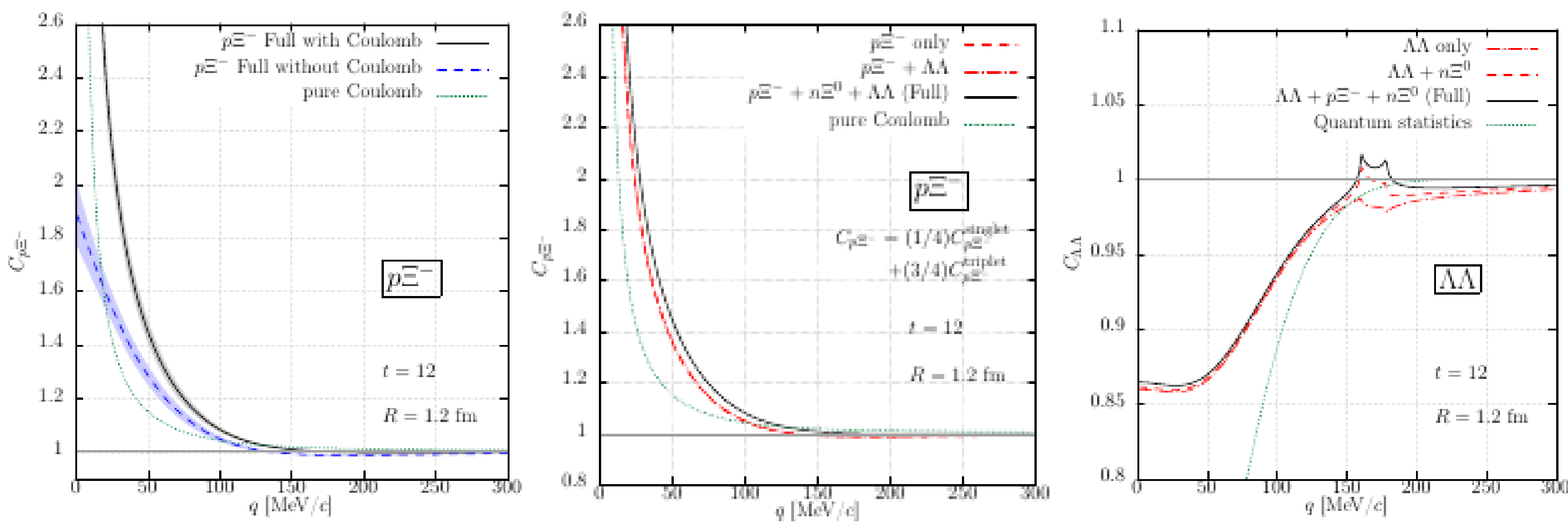
$$\omega_j S_j(\boldsymbol{r}) |\Psi_j^{(-)}(\boldsymbol{r})|^2$$

Source weight **Normalized Source fn.**

- **Effects of coupled-channel, strong & Coulomb pot., and threshold difference are taken into account in the charge base, $p\Xi^-$, $n\Xi^0$, ΛΛ.**
  Y. Kamiya+, PRL('20, K^-p)
- **Source size (R) and source weight (ω_j) need to be determined.**

Theoretical pΞ⁻ and ΛΛ Correlation Function

- **$p\Xi^-$ correlation function**
  - **Strongly enhanced at low q by the strong interaction, and further enhanced by the Coulomb potential at q < 50 MeV/c**
  - **ΛΛ source effect is small.**
- **ΛΛ correlation function**
  - **Suppressed by quantum statistics, but enhanced by the strong interaction at low q.**
  - **NΞ source effect is visible only around the thresholds.**



Kamiya+ (2108.09644)

Comparison with $p\Xi^-$ and $\Lambda\Lambda$ correlation function data

Parameters in Correlation Function Data

- **Actual data contains non-femtoscopic effects → Pair purity < 1. (jets, misidentified particles)**

$$C_{\rm exp}(q; R, \lambda, N, \omega) = N(q)\,[1 + \lambda(C_{\rm theory}(q; R, \omega) - 1)]$$

  - **We adopt Pair purity (λ) from MC analysis results by ALICE.**
  - **Source Weight (ω_j) is given by a simple statistical model. (Sensitivity is small.)**
  - **Normalization with jet effects (N(q)=a+bq) is determined by the fit to the data.**
  - **Source size (R) is determined by the fit to the data for pp 13 TeV collisions,**

$$R_{p\Xi^-}(pp) \simeq 1.05~{\rm fm} \quad [R^{\rm ALICE}_{p\Xi^-}(pp) = 1.02 \pm 0.05~{\rm fm}]$$

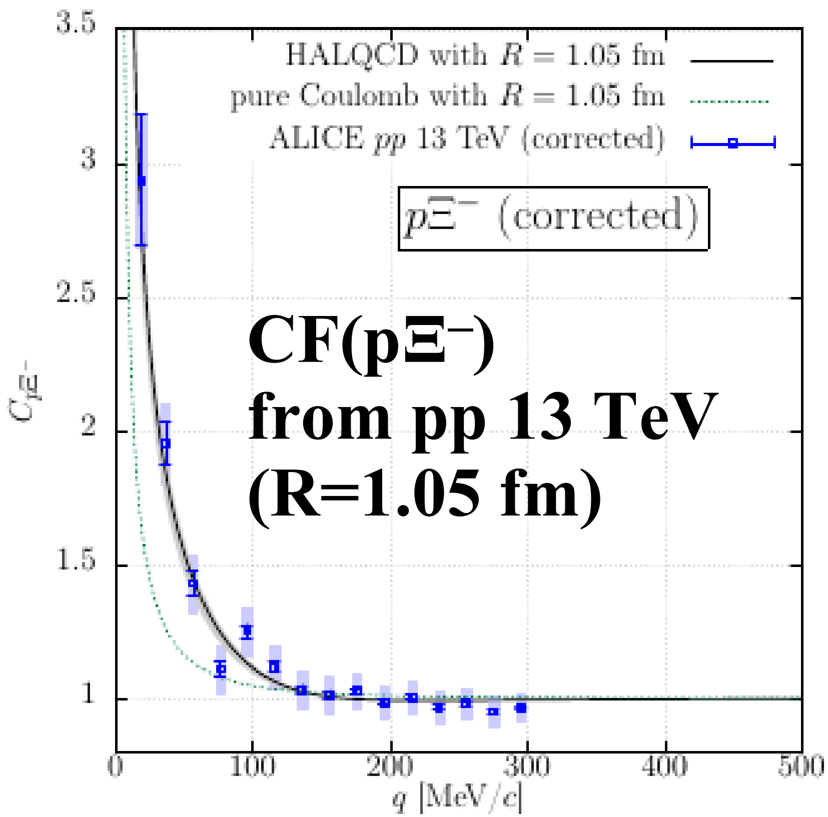
and based on the scaling relation for p Pb 5.02 TeV collisions.

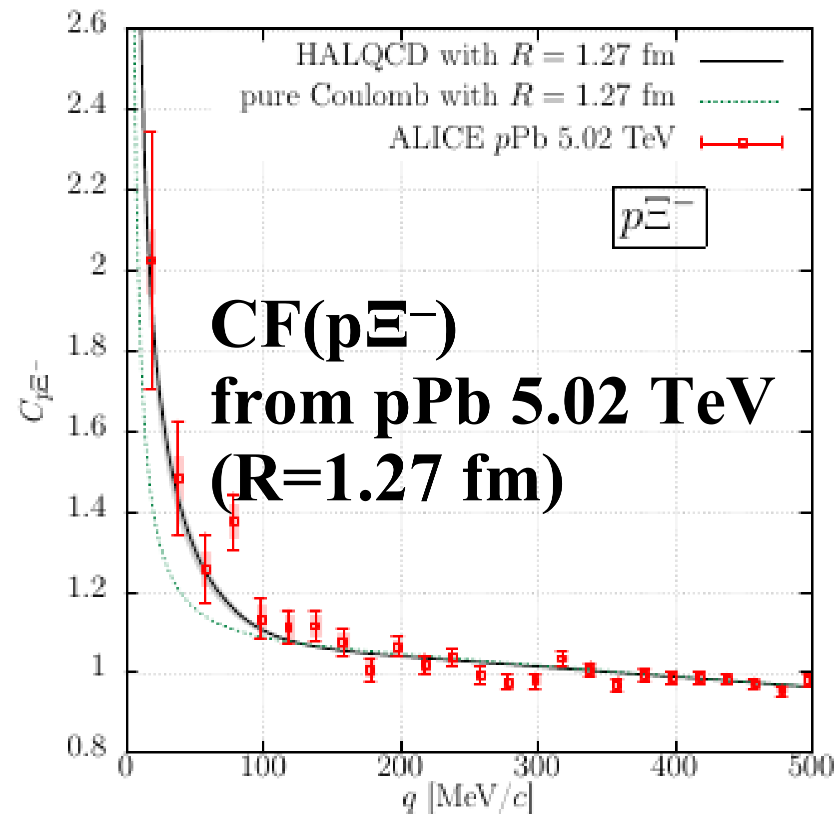
$$R_{p\Xi^-}(p{\rm Pb})/R_{p\Xi^-}(pp) \simeq R^{\rm ALICE}_{pp}(p{\rm Pb})/R^{\rm ALICE}_{pp}(pp)\ [R_{p\Xi^-}(p{\rm Pb}) = 1.27~{\rm fm}]$$

(ΛΛ and pΞ^- source sizes are assumed to be the same.)

$p\Xi^-$ Correlation Function

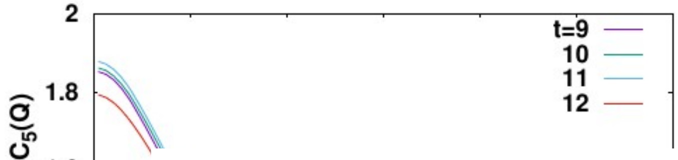
- $p\Xi^-$ correlation function data implies attractive $N\Xi$ interaction.
  - Strong enhancement from pure Coulomb CF
  - $\Lambda\Lambda$ source effect is negligible. $n\Xi^0$ source effect is visible.
  - Calculated CF agrees with ALICE data.



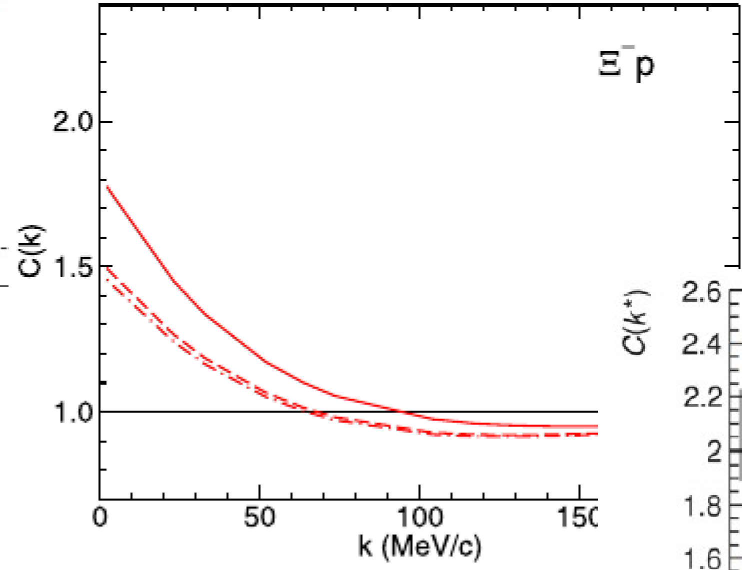


Kamiya+ (2108.09644); Acharya+(ALICE), PRL('19), Nature ('20)

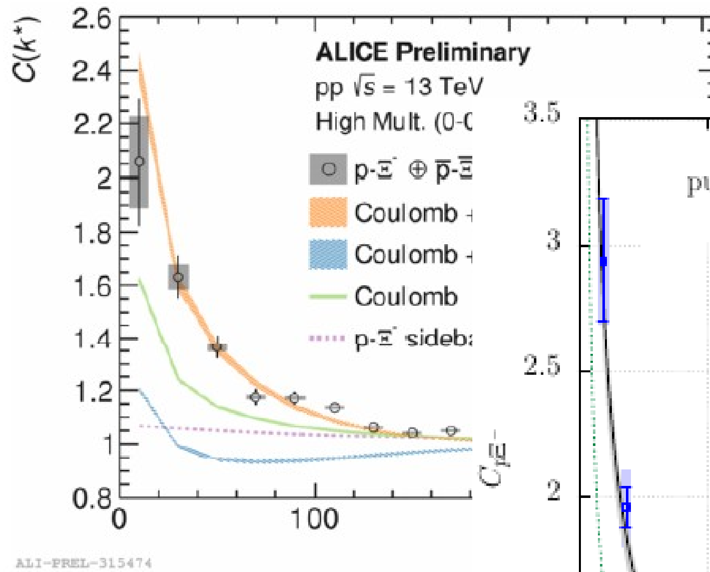
Comparison with other results



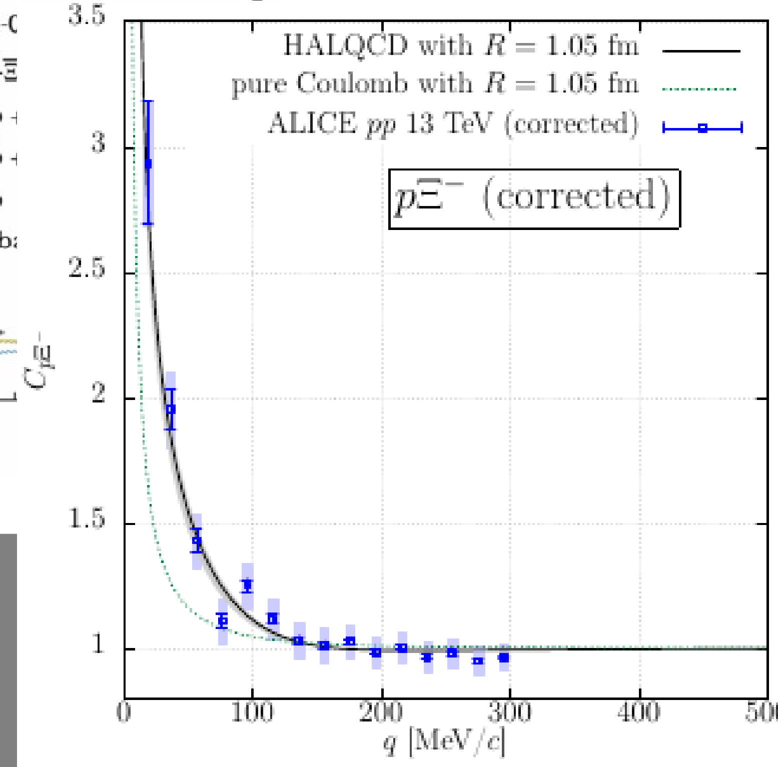
T. Hatsuda, K. Morita, AO,
K. Sasaki, NPA967('17)856.
(heavier quark mass)



J. Haidenbauer, NPA981('19)1.
(NLO(600), w/ CC effects, w/o Coulomb)
(w/ Coulomb, it will be comparable with data.)



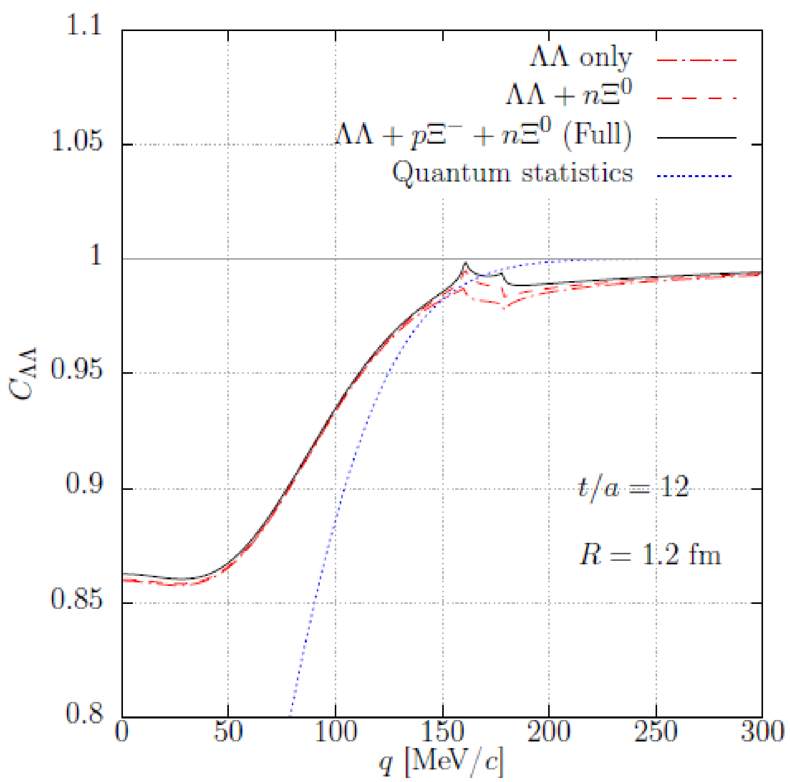
D. L. Mihairov+[ALICE],
NPA1005('21)121760 (QM2019).
(Nijmegen potential does not
explain the data.)

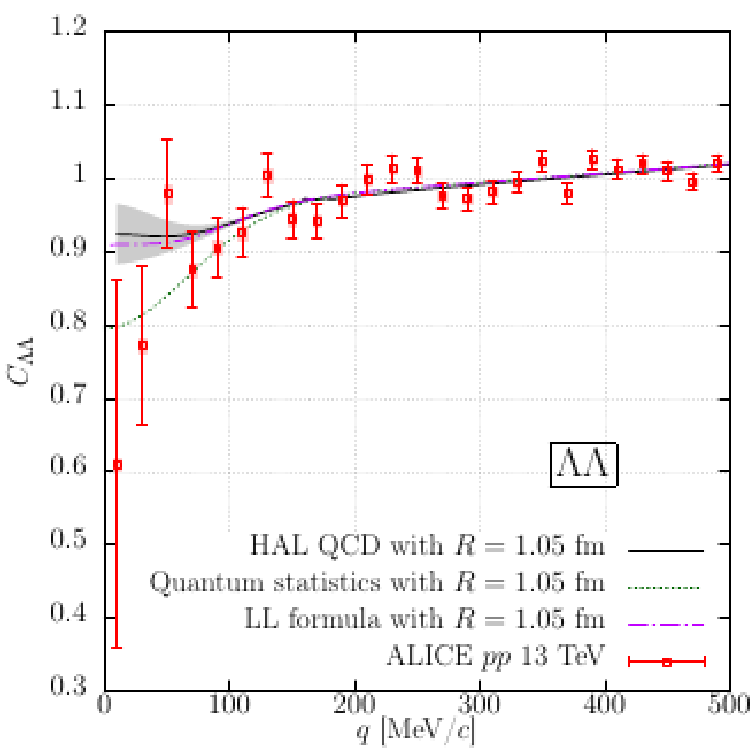


Kamiya+(2108.09644).
(w/ Lattice BB pot. at phys. m_q
and CC effects with ΛΛ)

ΛΛ correlation function

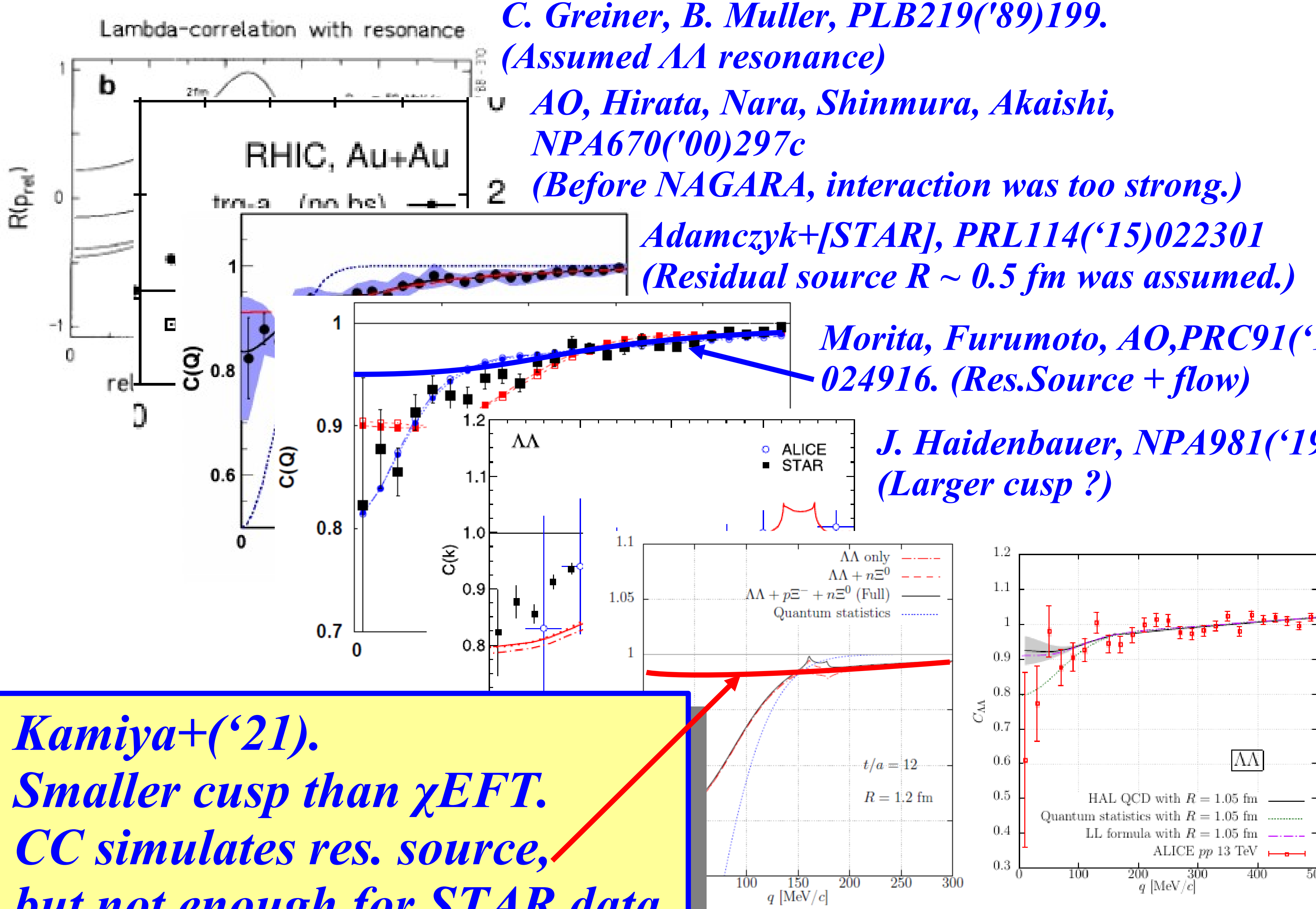
- **ΛΛ correlation function**
  - **Enhancement from pure quantum statistic CF**
  - **NΞ source effect is visible only around thresholds.**
  - **Calculated CF agrees with ALICE data.**
    Analytic model (Lednicky-Lyuboshits formula) works well.





Kamiya+ (2108.09644); Acharya+[ALICE] ('19)

Comparison with other results



C. Greiner, B. Muller, PLB219('89)199. (Assumed ΛΛ resonance)

AO, Hirata, Nara, Shinmura, Akaishi, NPA670('00)297c (Before NAGARA, interaction was too strong.)

Adamczyk+[STAR], PRL114('15)022301 (Residual source R ~ 0.5 fm was assumed.)

Morita, Furumoto, AO,PRC91('15) 024916. (Res.Source + flow)

J. Haidenbauer, NPA981('19)1. (Larger cusp ?)

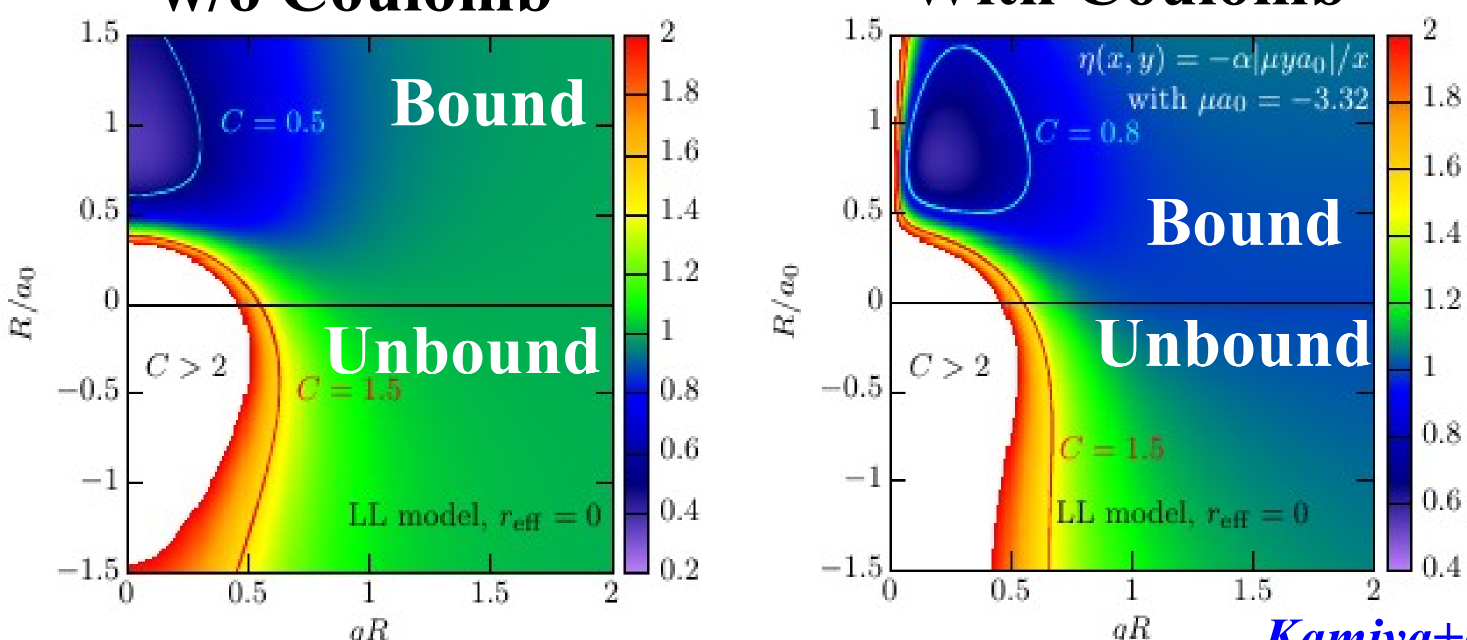
Kamiya+('21).
Smaller cusp than χEFT.
CC simulates res. source,
but not enough for STAR data

Unbound nature of NΞ confirmed ?

R Dependence of Correlation Function

- **Source size (R) dependence of C(q) is helpful to deduce the existence of a bound state.**
  Morita+('16, '20), Kamiya+('20), Kamiya+(2108.09644)
  - **With a bound state, C(q) is suppressed at small q when R ~ $|a_0|$. (w.f. has a node at r ~ $|a_0|$ with a bound state.)**
  - **Qualitative understanding by the analytic model (LL formula)** *[Lednickey, Lyuboshits ('82)]* **with the zero range approx. (r_{eff}=0)**

w/o Coulomb **With Coulomb**

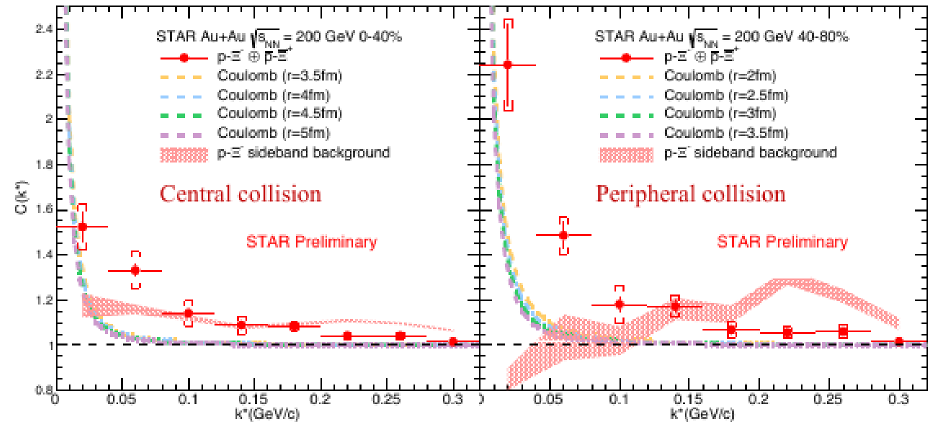


Kamiya+(2108.09644)

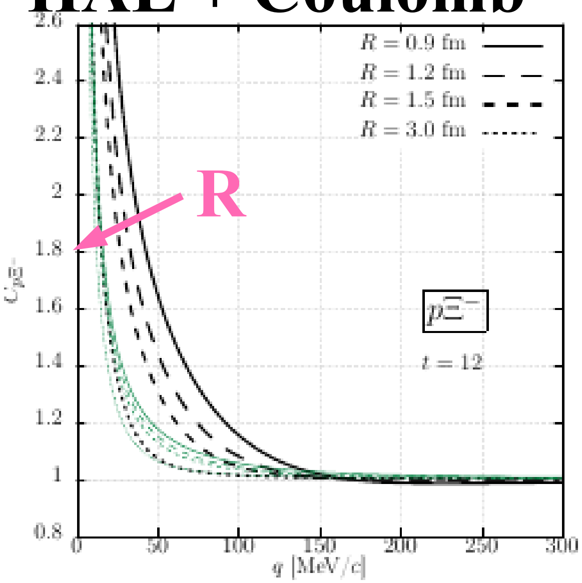
R dependence of pΞ⁻ correlation function

- R dep. of calculated results
  → Enhanced region shrinks
  with larger R. No Dip.
- Larger R data from Au+Au seem to show similar behavior.

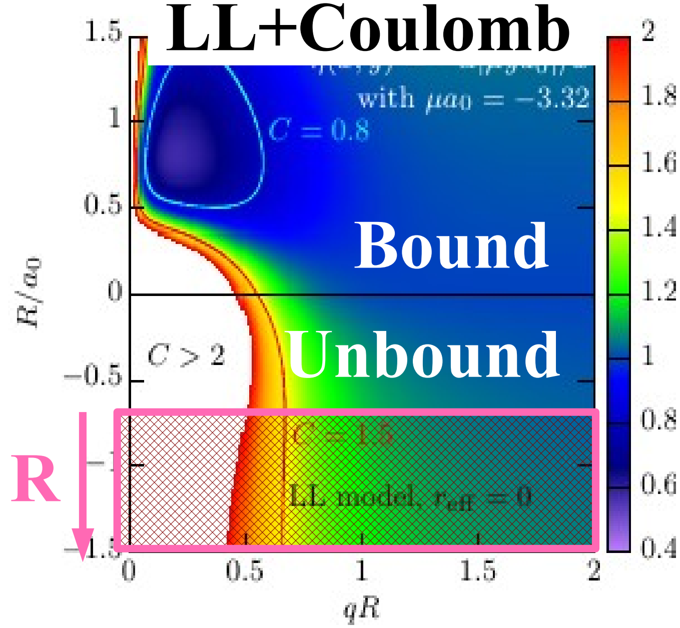
K. Mi+(STAR, preliminary), Au+Au 200 AGeV, APS2021.
(No Dip at larger R)

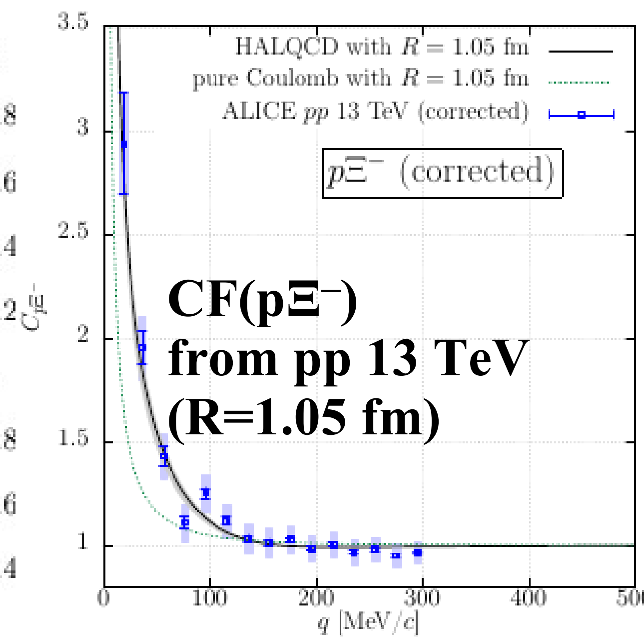


HAL + Coulomb



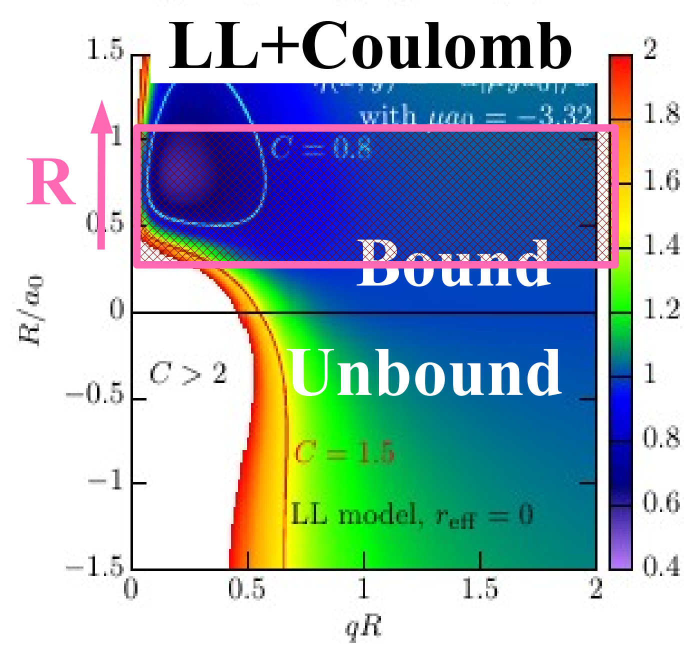
LL+Coulomb

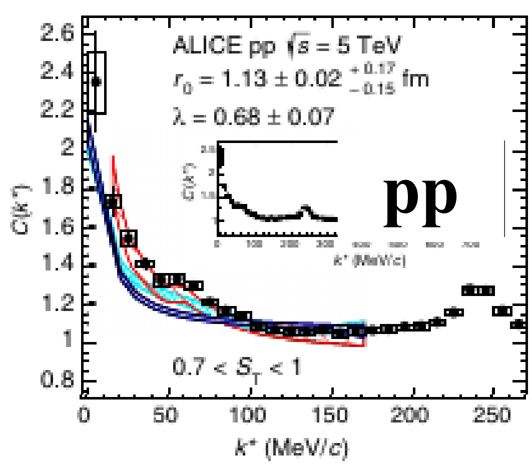




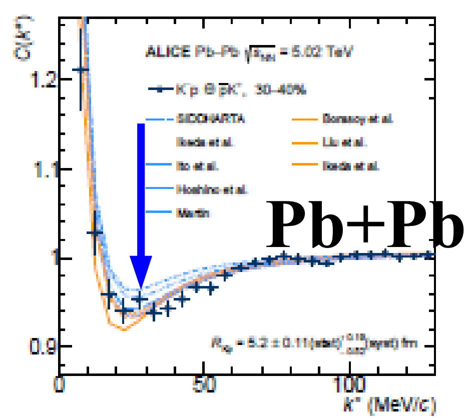
c.f. R dependence of pK⁻ correlation function

- **Enhanced C(q) from pp collisions, and dip in heavy-ion collisions.**
  = Typical behavior expected from LL formula + Coulomb with a bound state.
  Kamiya+(PRL, '20)
- **These R dependence of C(q) supports again the $\bar{K}N$ bound state nature of Λ(1405).**

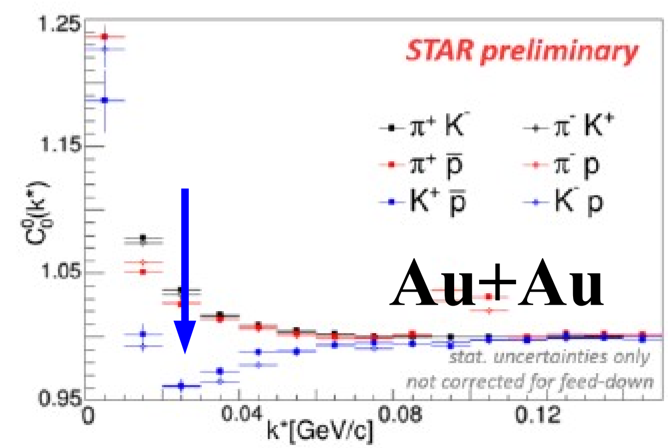




S. Acharya+[ALICE], PRL124('20)092301



S. Acharya+[ALICE], 2105.05683



Siejka+[STAR, preliminary], NPA982 ('19)359.

Summary

- **Correlation functions are helpful to constrain / examine hadron-hadron interactions as well as to deduce the existence of a bound state.**
- **We have calculated pΞ^- and ΛΛ correlation functions by using lattice NΞ-ΛΛ coupled-channel (CC) potential.**
  - **w/ effects of CC, Coulomb, threshold difference.**
  - **ALICE pΞ^- and ΛΛ correlation function data are consistent with the HAL QCD potential.**
  - **Source weight effect from conversion channel is not big, except for the cusps at NΞ thresholds in ΛΛ corr. fn. (Solving CC equation is still important.)**
- **Unbound nature of NΞ will be supported by studying the source size dependence of the pΞ^- correlation function. (Any way to confirm the virtual pole nature ?)**

Thank you for attention !

Coauthors of Y. Kamiya et al. (pΞ^-), arXiv:2108.09644.

Y. Kamiya

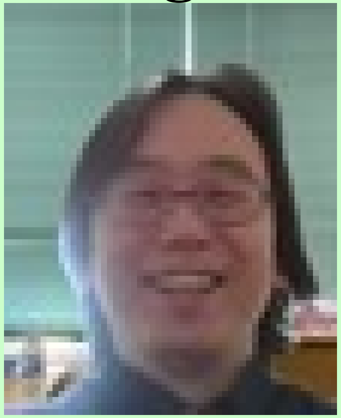
K. Sasaki

T. Fukui

T. Hyodo

K. Morita

K. Ogata



AO

T. Hatsuda

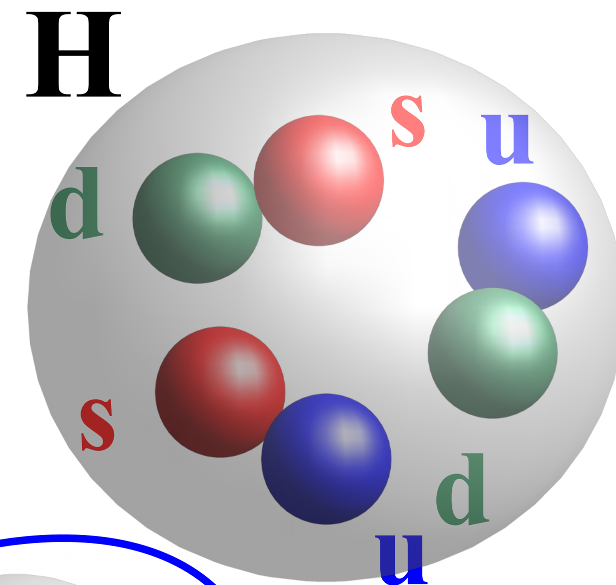
Y. Kamiya, K. Sasaki, T. Fukui, T. Hyodo, K. Morita, K. Ogata, AO, T. Hatsuda, arXiv:2108.09644 [hep-ph].

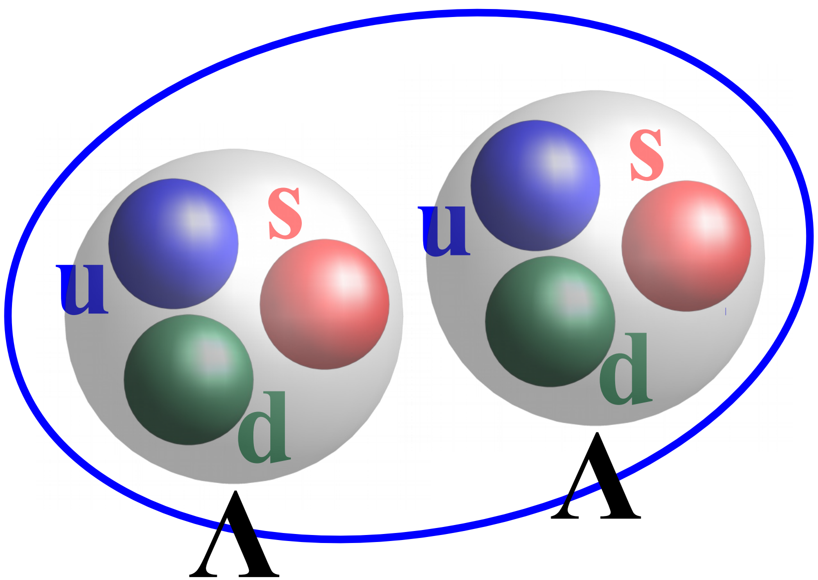
To be, or not to be, that is the question.

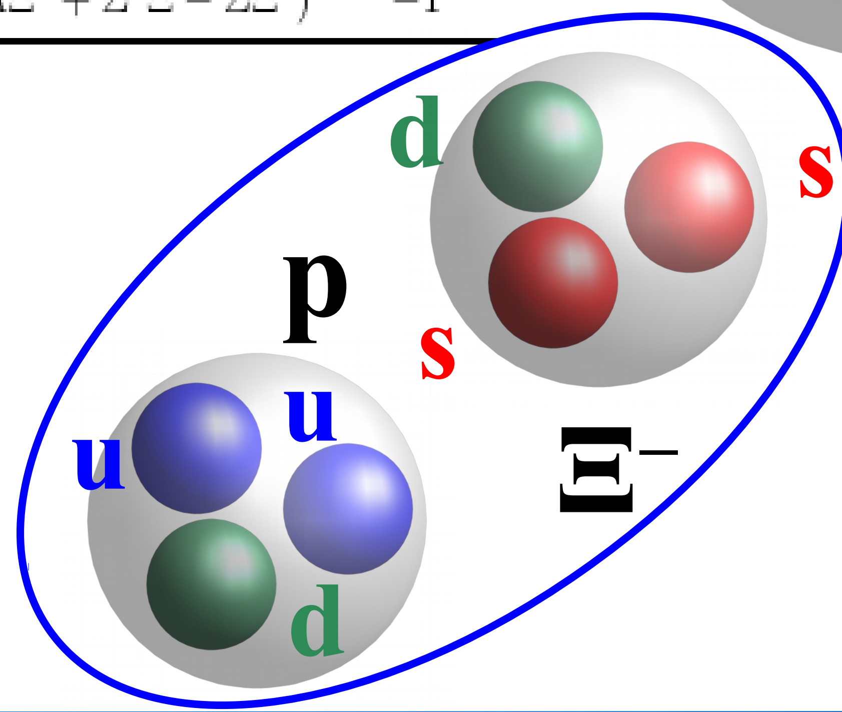
A. Gal ('16); M. Oka ('88)

Table 1. Leading 6q $L = 0$ dibaryon candidates [12], their BB' structure and the CM interaction gain with respect of the lowest BB' threshold calculated by means of Eq. (2). Asterisks are used for the $\mathbf{10}_f$ baryons $\Sigma^* \equiv \Sigma(1385)$ and $\Xi^* \equiv \Xi(1530)$. The symbol [i,j,k] stands for the Young tablaux of the $SU(3)_f$ representation, with i arrays in the first row, j arrays in the second row and k arrays in the third row, from which $\mathcal{P}_f$ is evaluated. The $\overline{\mathbf{10}}$ $SU(3)_f$ representation is denoted here $\mathbf{10}^*$.

$-S$	$SU(3)_f$	I	J^π	BB' structure	$\frac{\Delta\langle V_{CM}\rangle}{M_0}$
0	[3,3,0] $\mathbf{10}^*$	0	3^+	$\Delta\Delta$	0
1	[3,2,1] $\mathbf{8}$	1/2	2^+	$\frac{1}{\sqrt{5}}(N\Sigma^* + 2\Delta\Sigma)$	-1
2	[2,2,2] $\mathbf{1}$	0	0^+	$\frac{1}{\sqrt{8}}(\Lambda\Lambda + 2N\Xi - \sqrt{3}\Sigma\Sigma)$	-2
3	[3,2,1] $\mathbf{8}$	1/2	2^+	$\frac{1}{\sqrt{5}}(\sqrt{2}N\Omega - \Lambda\Xi^* + \Sigma^*\Xi - \Sigma\Xi^*)$	-1







Potentially measurable hh pairs

- **Correlation function is useful to access hadron-hadron interactions as well as to deduce the existence of a bound state.**

Scatt.+Nuclei

Scatt.+Mesic atom

Scatt. +Hyper Nuclei

	n	p	K^-	K^+	π^-	π^+	Λ	Σ	Ξ^-	Ω^-	D^-	D^+	Ks	d	pp	φ	+α
n																	
p		O	O	O	△	△	O	O	O	O	O	O		O	O	O	
K^-		O	O	O	O	O							O				
K^+		O	O	O	O	O							O				
π^-		△	O	O	O	O											
π^+		△	O	O	O	O											
Λ		O					O		O						O		
Σ		O						O									
Ξ^-		O															
Ω^-		O															
D^-		O															
D^+		O															
Ks			O	O													
d		O															
pp		O					O										
φ		O															
+α																	

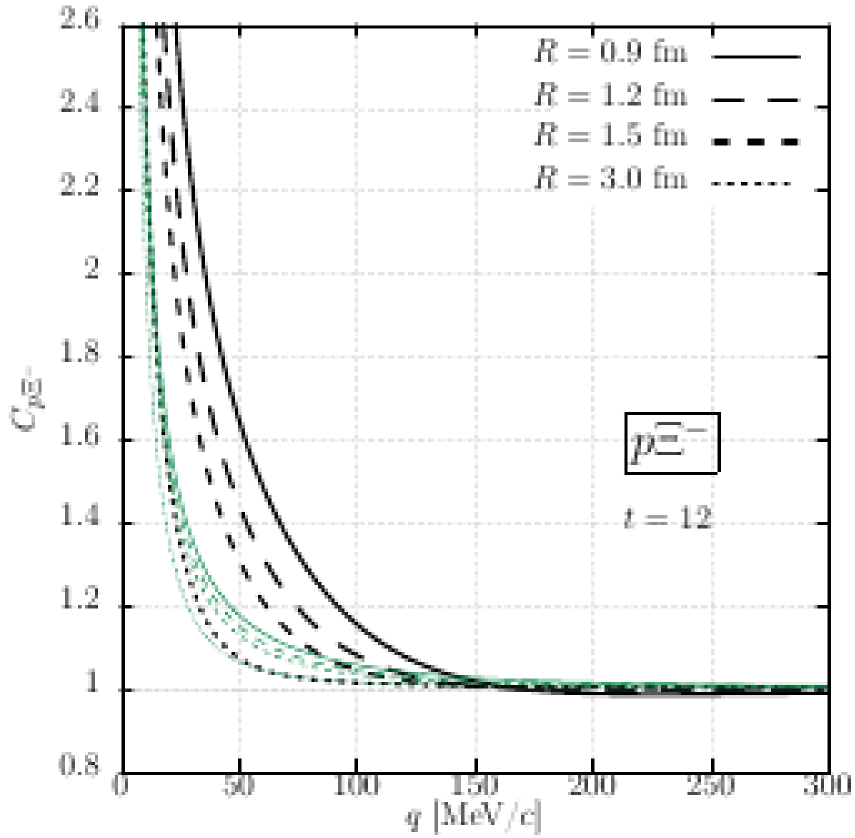
ΛΛ hypernuclei

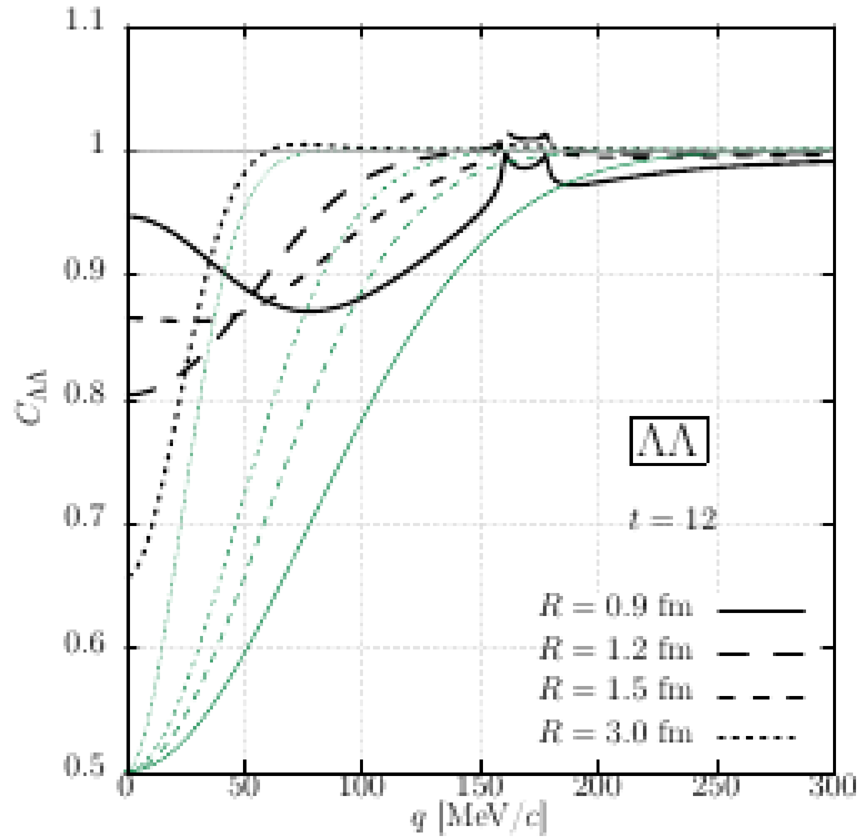
Femtoscopy

Blue: Pairs we have studied, O: Experimentally measured

Source size dependence of correlation functions

- pΞ–
  - Smooth dependence on R. (No bound state, Non-identical particles)
- ΛΛ
  - Complicated R dependence (Quantum statistics)
  - No long-tail (q > 200 MeV/c) with R > 1.5 fm.





Kamiya+(2108.09644)

Non-Femtoscopic Parameters

- **Relevant parameters=R, λ, N=a+bq**
  (ω's are almost irrelevant for pΞ^- and ΛΛ correlation functions.)

$$C_{\rm exp}(q;R,\lambda,N,\omega) = N(q)\left[1+\lambda(C_{\rm theory}(q;R,\omega)-1)\right]$$

collision	pair	λ	a	b [(MeV/c)$^{-1}$]	R [fm]
pp (13 TeV)	$p\Xi^-$	1 [15]	1 [15]	0 [15]	1.05
	$\Lambda\Lambda$	0.338 [9]	0.95	1.28×10^{-4}	
pPb (5.02 TeV)	$p\Xi^-$	0.513 [14]	1.09	-2.56×10^{-4}	$1.27^{(*)}$
	$\Lambda\Lambda$	0.239 [9]	0.99	0.29×10^{-4}	

TABLE II. The pair purity λ, non-femtoscopic parameters a and b, and the effective source size R in the fitting function $C_{\rm th}(q)$. The parameters a and b in pp ($\Lambda\Lambda$ pairs) and pPb ($p\Xi^-$ and $\Lambda\Lambda$ pairs) collisions and R in pp collisions are the actual fitting parameters. Numbers with references are taken from Refs. [9, 14, 15], and the number with $(*)$ is estimated from other other parameters. See the text for details.

[9] S. Acharya et al. [ALICE], Phys. Lett. B **797** (2019), 134822 [arXiv 1905.07209].

[14] S. Acharya et al. [ALICE], Phys. Rev. Lett. **123** (2019), 112002 [arXiv 1904.12198].

[15] S. Acharya et al. [ALICE], Nature **588** (2020), 232-238 [arXiv 2005.11495].

Kamiya+(2108.09644)

Correlation function from T-matrix

- **s-wave w.f. using the half-off-shell T-matrix (T_0)**

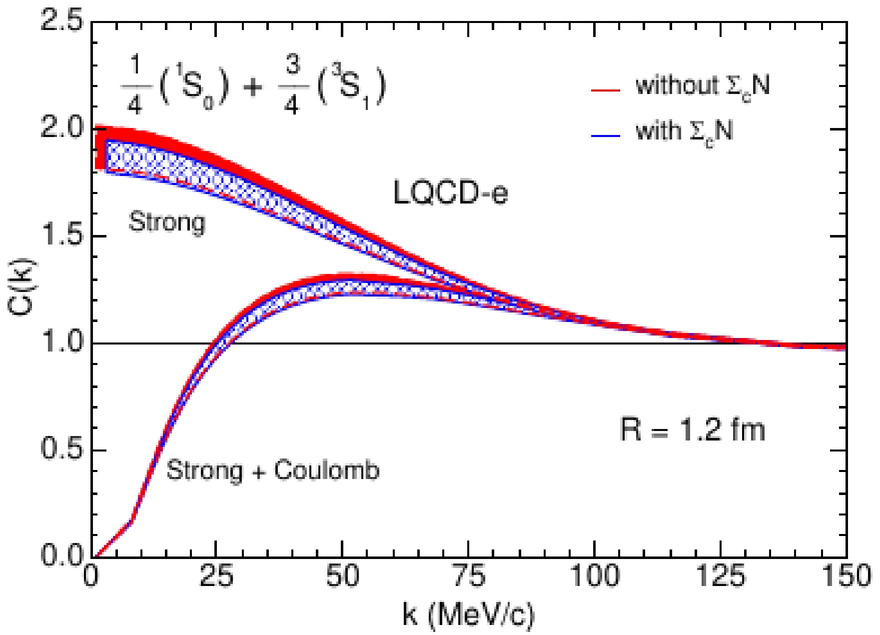
J. Haidenbauer, NPA 981('19)1.

$$\widetilde{\psi}_0(k,r) = j_0(kr) + \frac{1}{\pi}\int dq\, q^2 j_0(qr)\, \frac{1}{E - E_1(q) - E_2(q) + i\varepsilon} T_0(q,k;E)$$

$$\psi_0^{(-)}(k,r) = e^{-2i\delta_0}\widetilde{\psi}_0(k,r) \to \frac{e^{-i\delta_0}}{kr}\sin(kr+\delta_0) = \frac{1}{2ikr}\left(e^{ikr} - e^{-2i\delta_0}e^{-ikr}\right)$$

- **Strong T-matrix + Coulomb potential**

J. Haidenbauer, G. Krein, and T. C. Peixoto, EPJA 56 ('20)184;
using the Vincent-Phatak method
[C.M. Vincent and S.C. Phatak, PRC10('74)391;
B. Holzenkamp, K. Holinde and J. Speth,
NPA 500('89)485 (1989)]



Analytic model of correlation function

- **Asymptotic w.f. is described by the scattering amplitude *f(q)***
  (non-identical particle pair, short range int. (only s-wave is modified),
  single channel, no Coulomb pot.)

$$\Phi^{(+)}(\boldsymbol{r}) = e^{i\boldsymbol{q}\cdot\boldsymbol{r}} - j_0(qr) + \varphi_0^{(+)}(r;q)$$

$$\varphi_0^{(+)}(r;q) \to \frac{e^{i\delta}\sin(qr+\delta)}{qr} = \frac{1}{2iqr}(Se^{iqr} - e^{-iqr}) = \frac{\sin qr}{qr} + f(q)\frac{e^{iqr}}{r}$$

$$\varphi_0^{(-)}(r;q) = S^{-1}\varphi^{(+)}(r;q) \quad \left[S = \exp(2i\delta),\, f = (S-1)/2iq = [q\cot\delta - iq]^{-1}\right]$$

- **Correlation function in Lednicky-Lyuboshits (LL) formula**
  (with static Gaussian source, real δ) (Lednickey, Lyuboshits ('82))

$$C(q) = \int d\boldsymbol{r} S(r) \left|\Phi^{(-)}(\boldsymbol{r})\right|^2 = 1 + \int d\boldsymbol{r} S(r) \left[\left|\varphi_0^{(-)}(\boldsymbol{r})\right|^2 - (j_0(qr))^2\right]$$

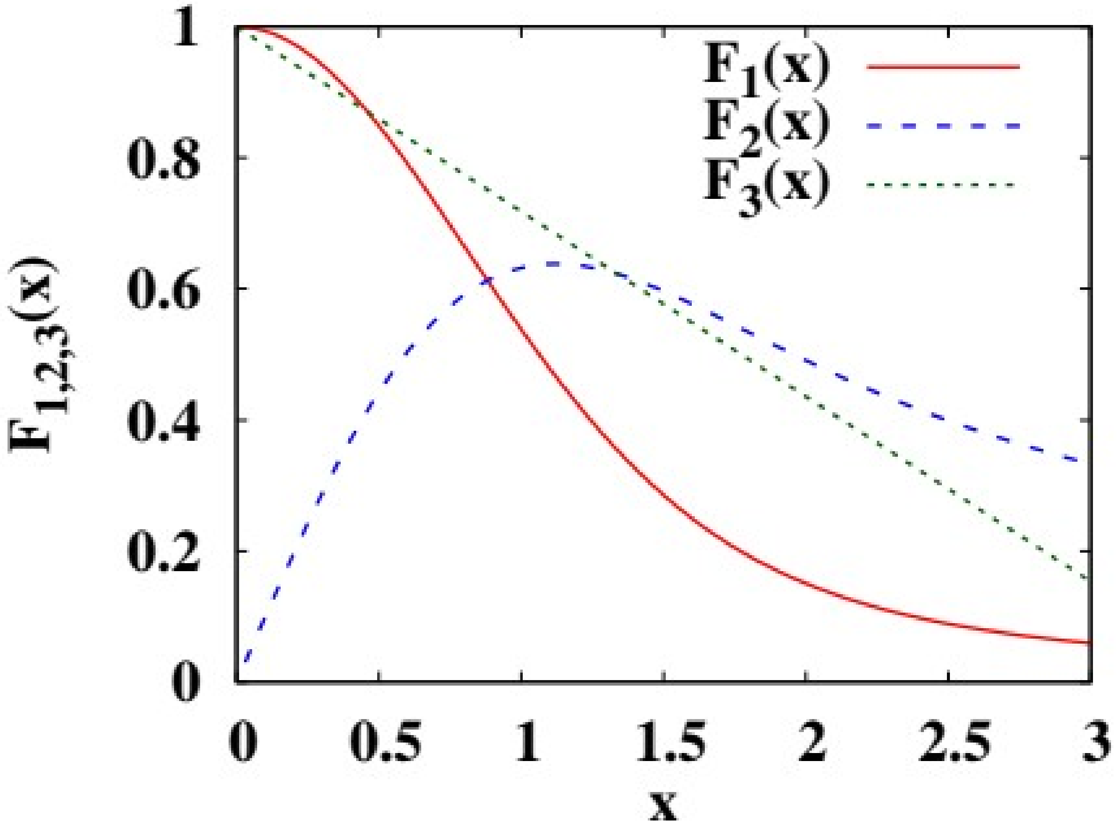
$$\simeq 1 + \int 4\pi dr\, S(r) \left[|f(q)|^2 + \frac{\sin qr}{q}\left\{f(q)e^{iqr} + f^*(q)e^{-iqr}\right\}\right]$$

$$C_{\rm LL}(q) = 1 + \frac{|f(q)|^2}{2R^2} F_3\left(\frac{r_{\rm eff}}{R}\right) + \frac{2{\rm Re}\, f(q)}{\sqrt{\pi}R} F_1(2qR) - \frac{{\rm Im}\, f(q)}{R} F_2(2qR)$$

$$\left[f(q) = (q\cot\delta - iq)^{-1},\ F_1(x) = \frac{1}{x}\int_0^x dt e^{t^2-x^2},\ F_2(x) = (1-e^{-x^2})/x,\ F_3(x) = 1 - \frac{x}{2\sqrt{\pi}}\right]$$

Lednicky-Lyuboshits functions

$$F_1(x) = \frac{1}{x}\int_0^x dt e^{t^2-x^2},\ F_2(x) = (1-e^{-x^2})/x,\ F_3(x) = 1 - \frac{x}{2\sqrt{\pi}}$$



$$F_1(x) \simeq \frac{1 + c_1 x^2 + c_2 x^4 + c_3 x^6}{1 + (c_1 + 2/3)x^2 + c_4 x^4 + c_5 x^6 + c_3 x^8}\ (0 \le x < 20)$$

$$(c_1, c_2, c_3, c_4, c_5) = (0.123, 0.0376, 0.0107, 0.304, 0.0617)$$

AO,Morita,Mihayara,Hyodo, NPA 954 ('16)294.

Bird's-eye view of C(q)

- **Zero eff. range pot. → C(q)=F(R/a_0, qR)**

$$r_{\rm eff}=0 \to q\cot\delta = -1/a_0 \to f(q)=(q\cot\delta - iq)^{-1} = -\frac{R}{R/a_0+iqR}$$

$$C(x,y)=1+\frac{1}{x^2+y^2}\left[\frac{1}{2}-\frac{2y}{\sqrt{\pi}}F_1(2x)-xF_2(2x)\right] \quad (x=qR, y=R/a_0)$$

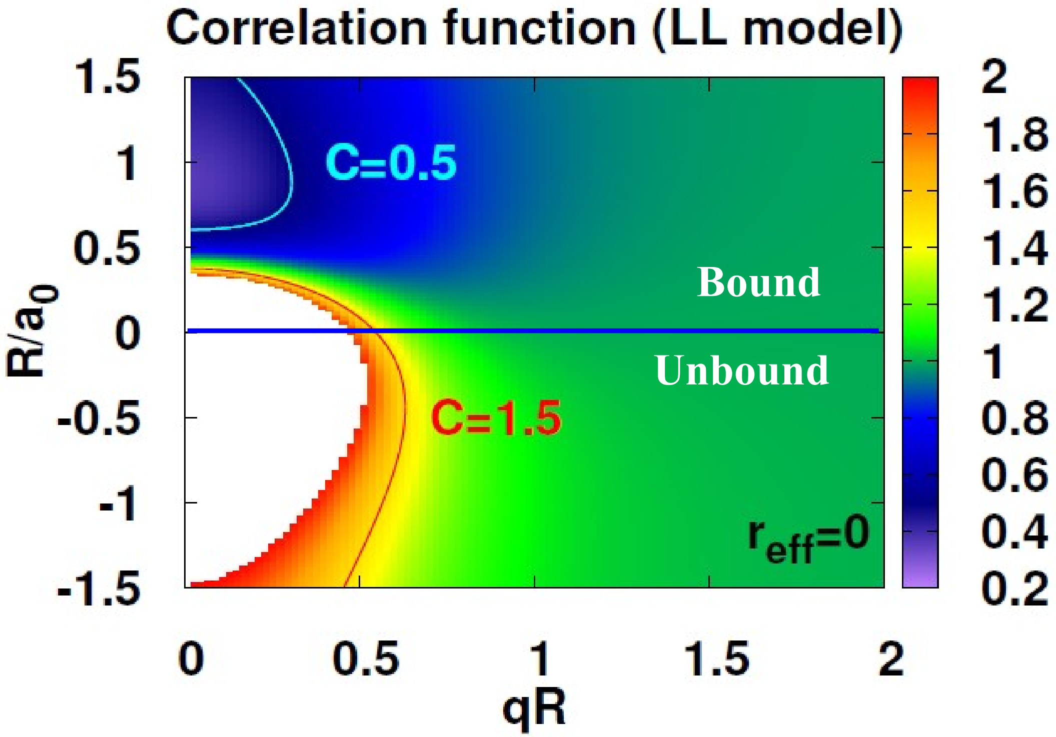
- **Low momentum limit**

$$C(x,y)\to\frac{1}{2}\left(\frac{1}{y}-\frac{2}{\sqrt{\pi}}\right)^2+1-\frac{2}{\pi} \quad (F_1\to 1, F_2\to 0 \text{ at } x\to 0)$$

  - **Enhanced C(q) at small q with a_0<0**

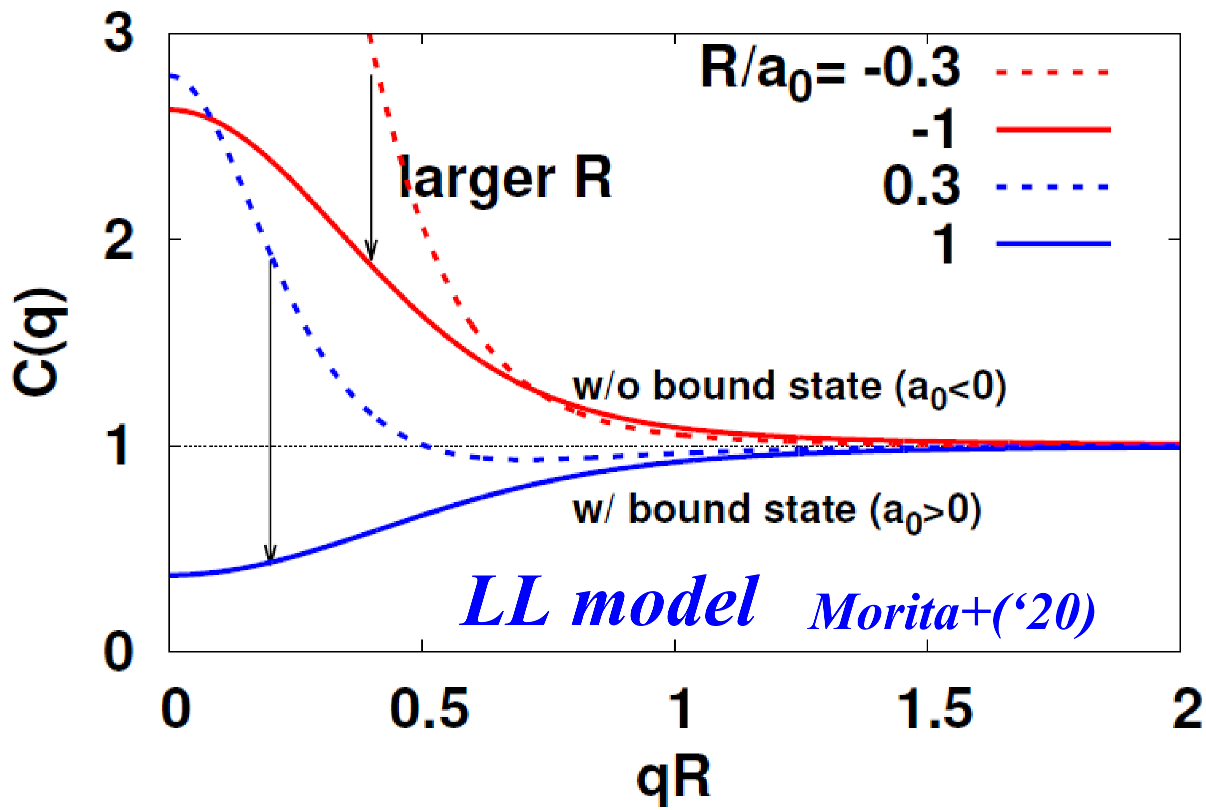
$$C_{\rm LL}(0)=1-\frac{2}{\sqrt{\pi}}\left(\frac{a_0}{R}\right)+\frac{1}{2}\left(\frac{a_0}{R}\right)^2$$

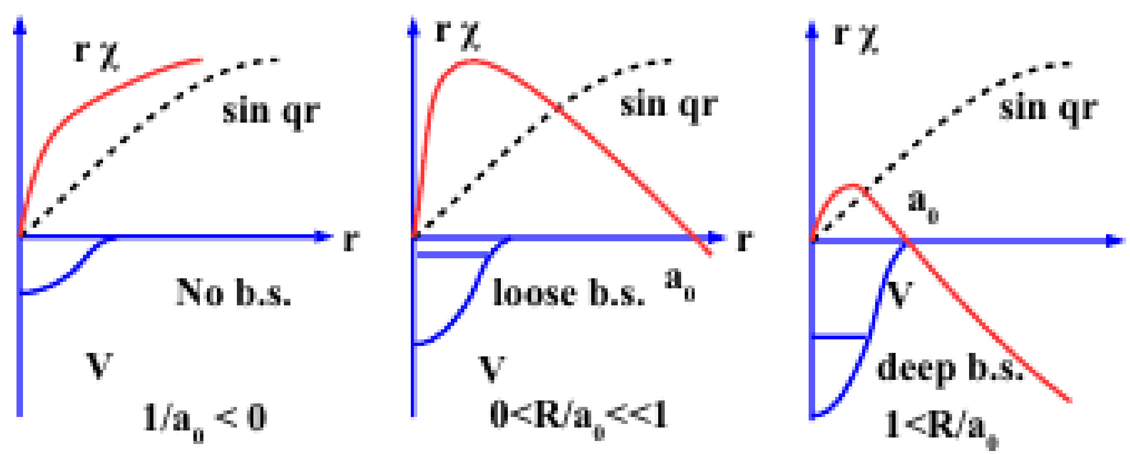
  - **a_0>0 → Size dependent C(q)**
    - **C(q) > 1 at small R**
    - **C(q) < 1 at R ~ a_0 (w.f. node at r ~ a_0)**



Bound state diagnosis by femtoscopy

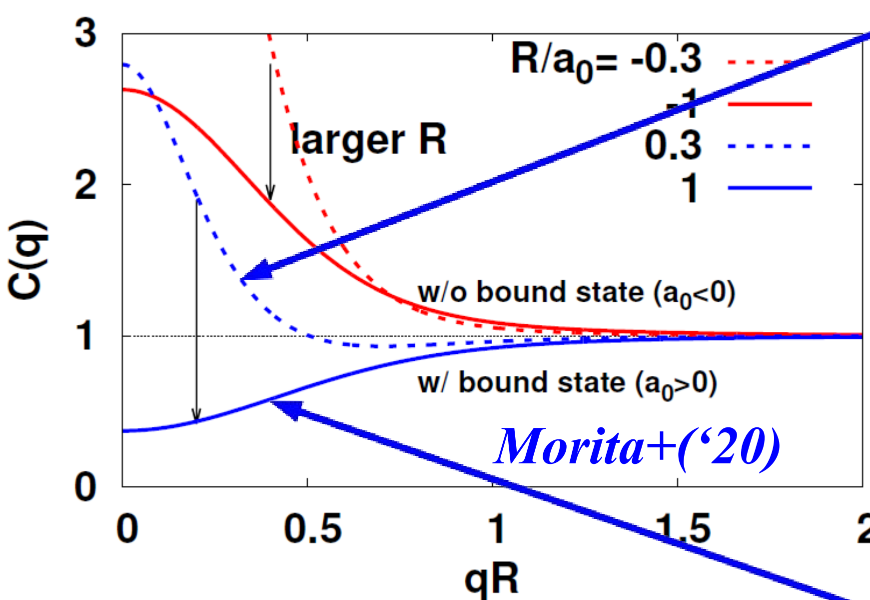
- Source size dep. of CF tells the sign of the scattering length (a_0).
  - With attraction, Large CF at small R.
  - With a bound state ($a_0>0$), CF is suppressed at $R \sim a_0$



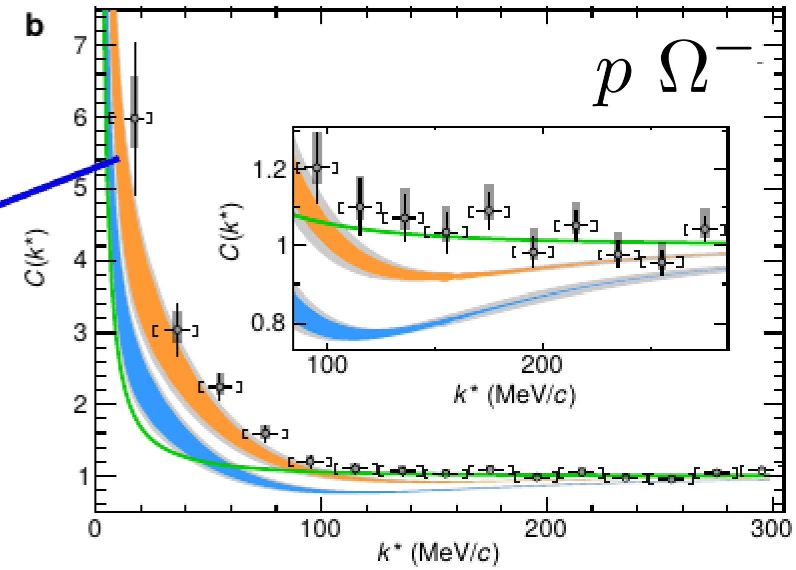


Source size dep. of CF
→ To be bound,
or not to be bound.

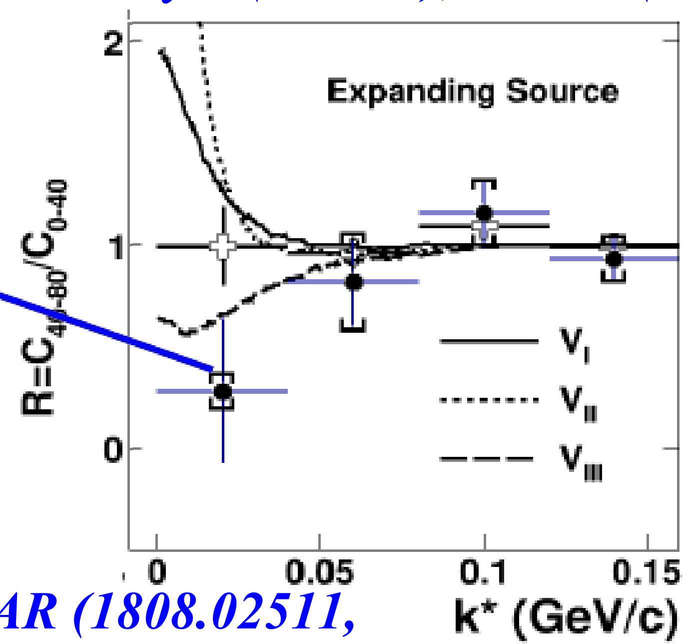
ALICE+STAR = NΩ Dibaryon



Morita+('20)

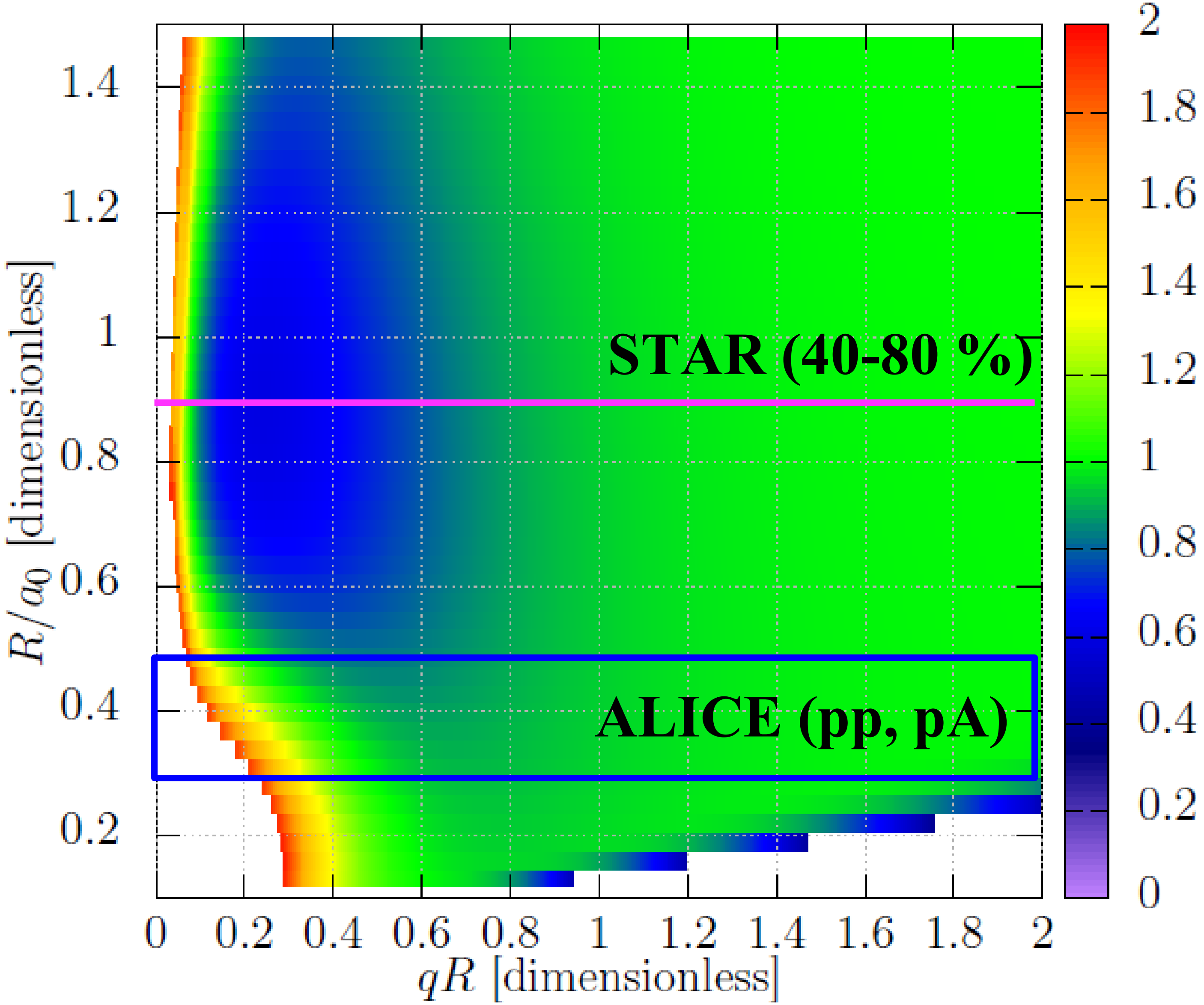


Acharya+(ALICE), Nature ('20)



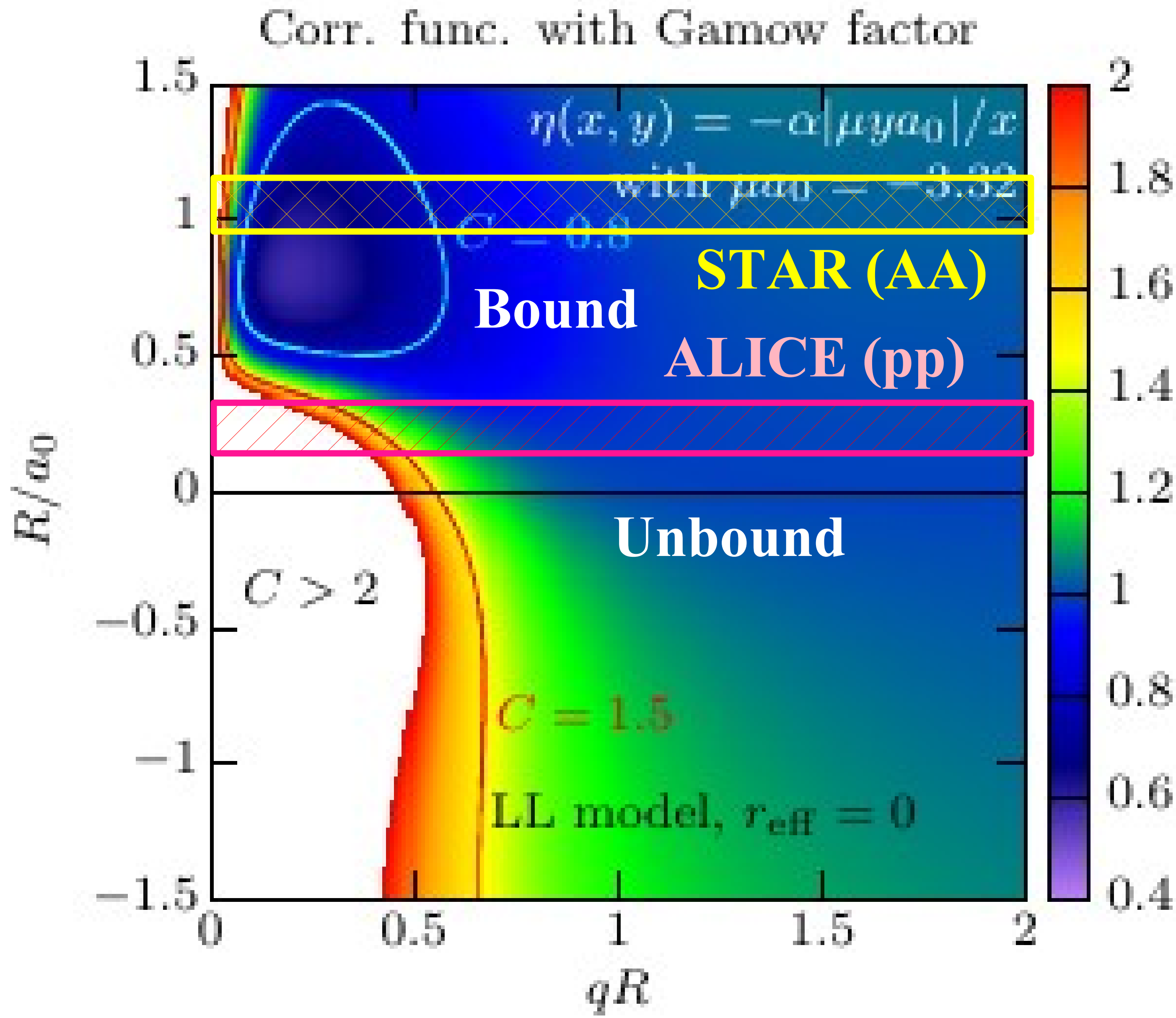
STAR (1808.02511, PLB790 ('19) 490)

Correlation Function with Gaussian source



NΩ potential (J=2, HAL QCD, a_0=3.4 fm) + Coulomb

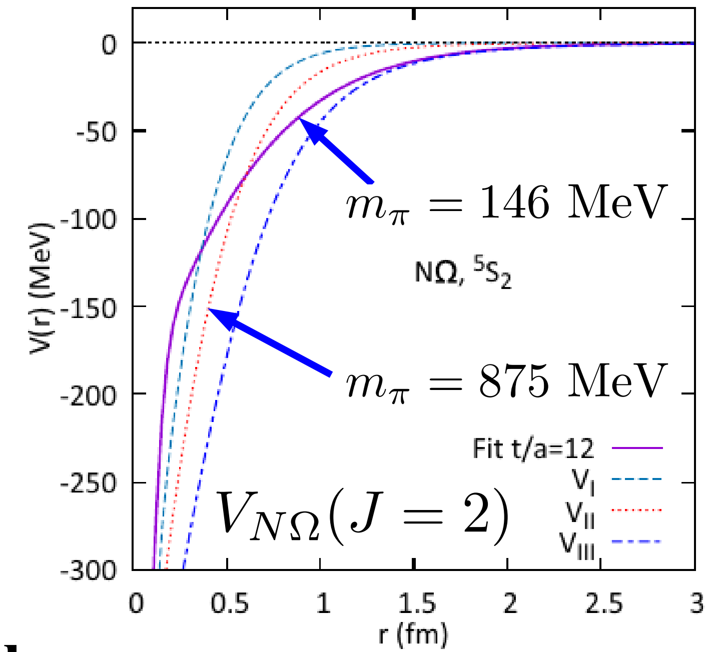
Source Size Dep. of CF w/ Coulomb potential



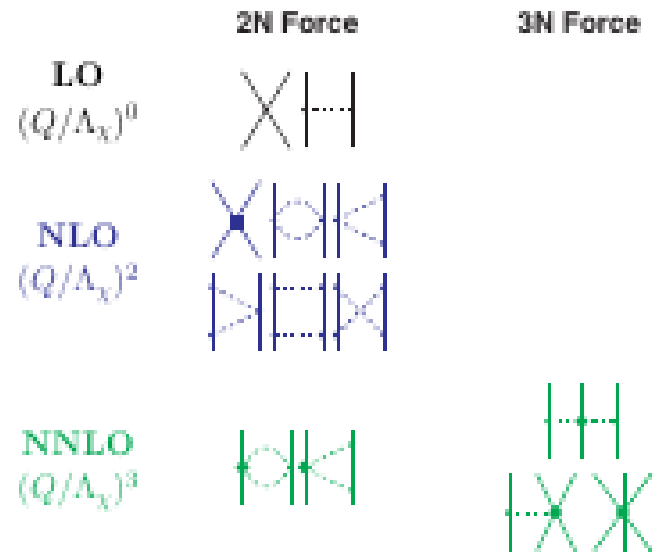
Y. Kamiya+(2108.09644) (LL × Gamow factor)

Modern Hadron-Hadron Interactions

- **Lattice QCD *hh* potential**
  - V_{hh} is obtained from the Schrödinger eq. for the Nambu-Bethe-Salpeter (NBS) amplitude.
    N. Ishii, S. Aoki, T. Hatsuda, PRL99('07)022001.
    → ΩΩ, NΩ, ΛΛ-NΞ potentials
    at phys. quark mass are published



- **Chiral EFT / Chiral SU(3) dynamics**
  - V_{hh} at low E. can be expanded systematically in powers of Q/Λ.
    S. Weinberg ('79);R. Machleidt, F. Sammarruca ('16); Y. Ikeda, T. Hyodo, W. Weise ('12).
    → NN, NY, YY, $\bar{K}$N-πΣ-πΛ, ...

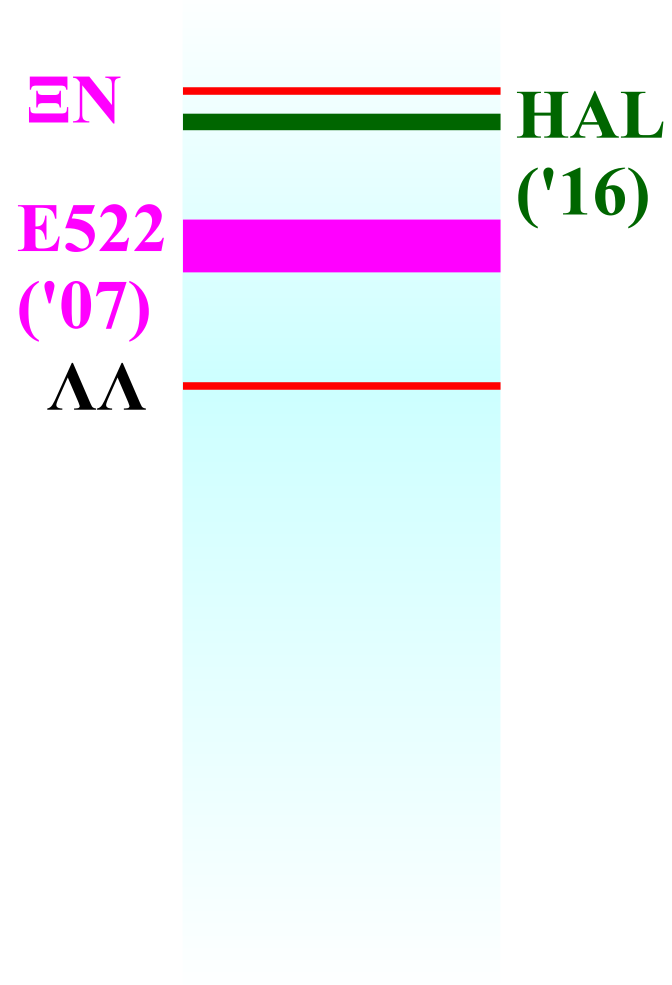


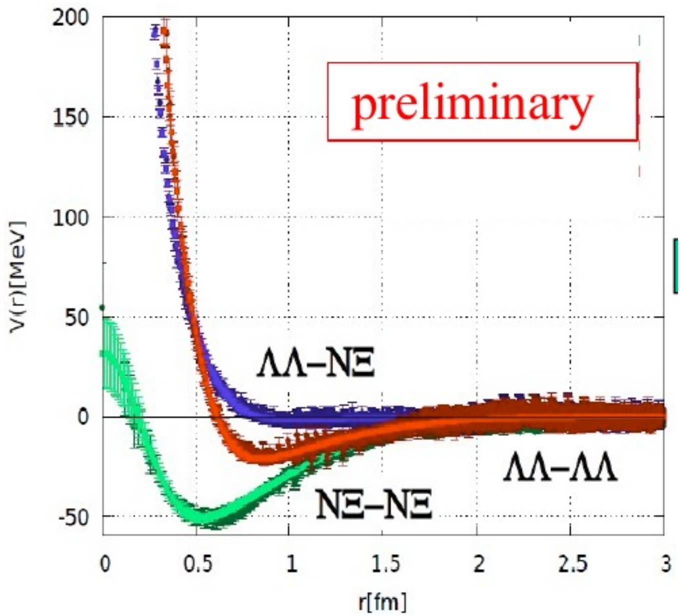
- **Quark cluster models,**
  Meson exchange models,
  More phenomenological models, …

Let us examine modern hh interactions !

Relevance of ΞN interaction to physics

- **H-particle: 6-quark state (uuddss) may be realized as a loosely bound state of ΞN (I=0)**
  K. Sasaki et al. (HAL QCD, '16,'17)
- **Repulsive ΞN interaction (I=1) may help to support 2 $M_\odot$ Neutron Star**
  Weissborn et al., NPA881 ('12) 62.

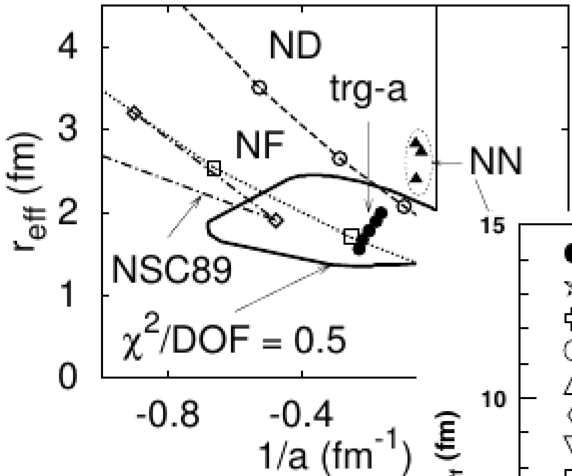




K. Sasaki et al. (HAL QCD Collab.), EPJ Web Conf. 175 ('18) 05010.

ΛΛ correlation and ΛΛ interaction

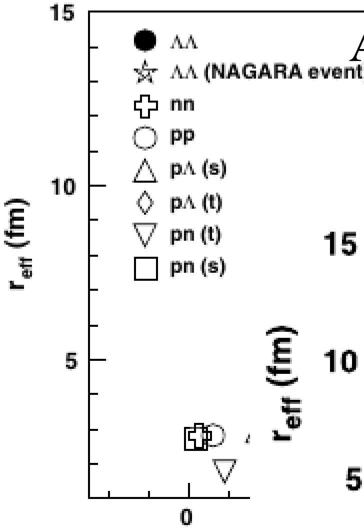
It is unlikely that ΛΛ bound state exists.



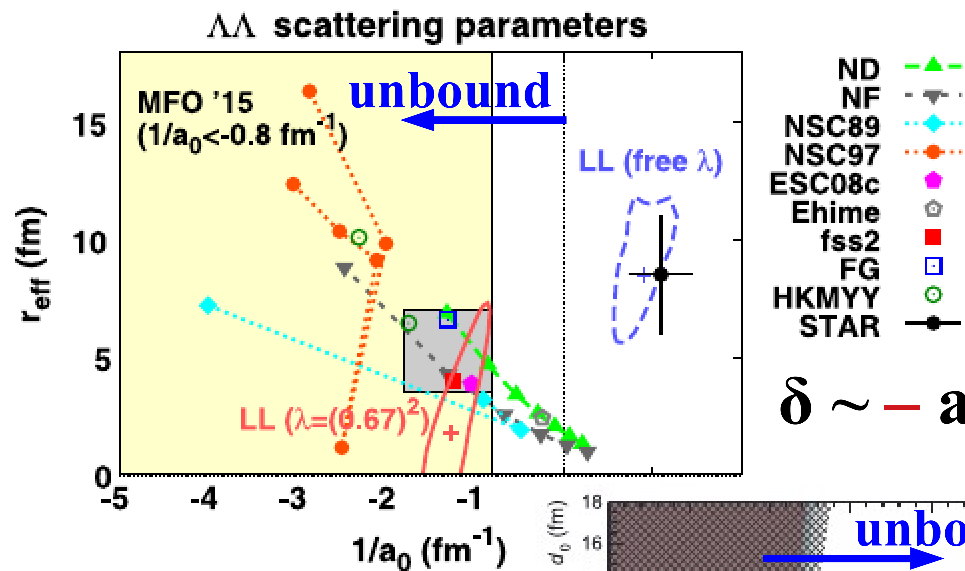
AO+(‘00)

L. Adamczyk+[STAR], PRL114(‘15)022301

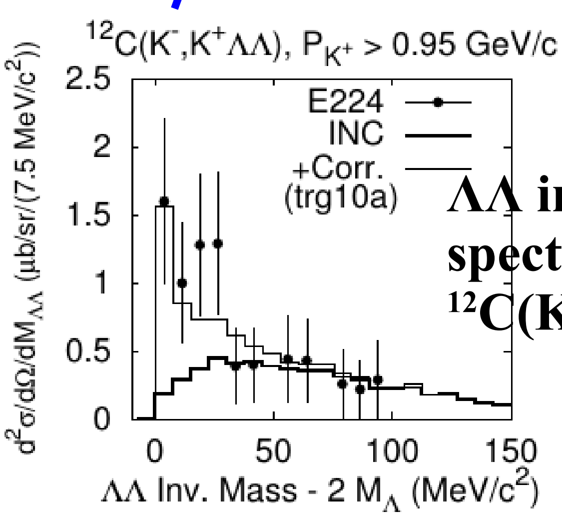
Au+Au → ΛΛ



AO, K. Morita, K. Miyahara, T. Hyodo, NPA954(‘16)294

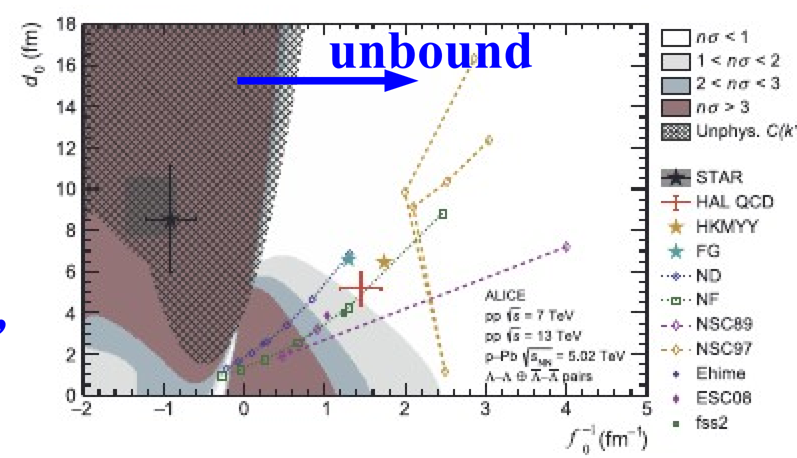


$\delta \sim - a_0 q$



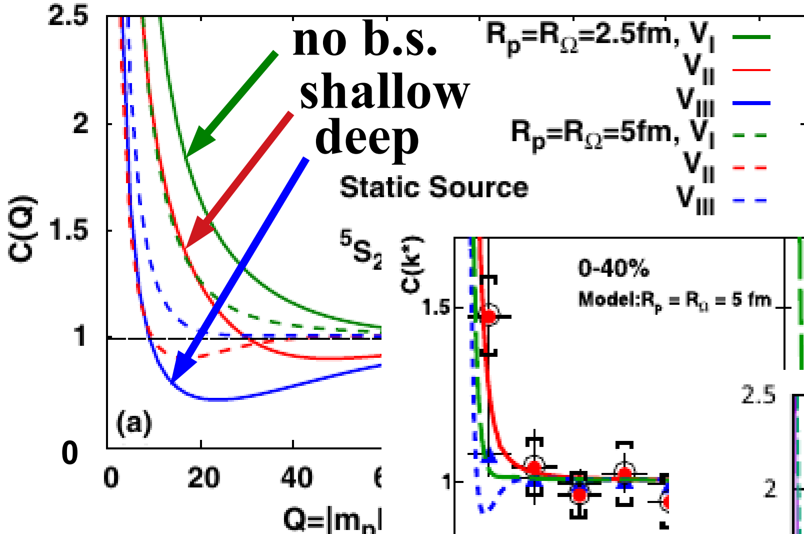
ΛΛ inv. mass spectrum from ^{12}C(K^-, K^+ΛΛ)

S. Acharya+[ALICE], PLB797(‘19)134822

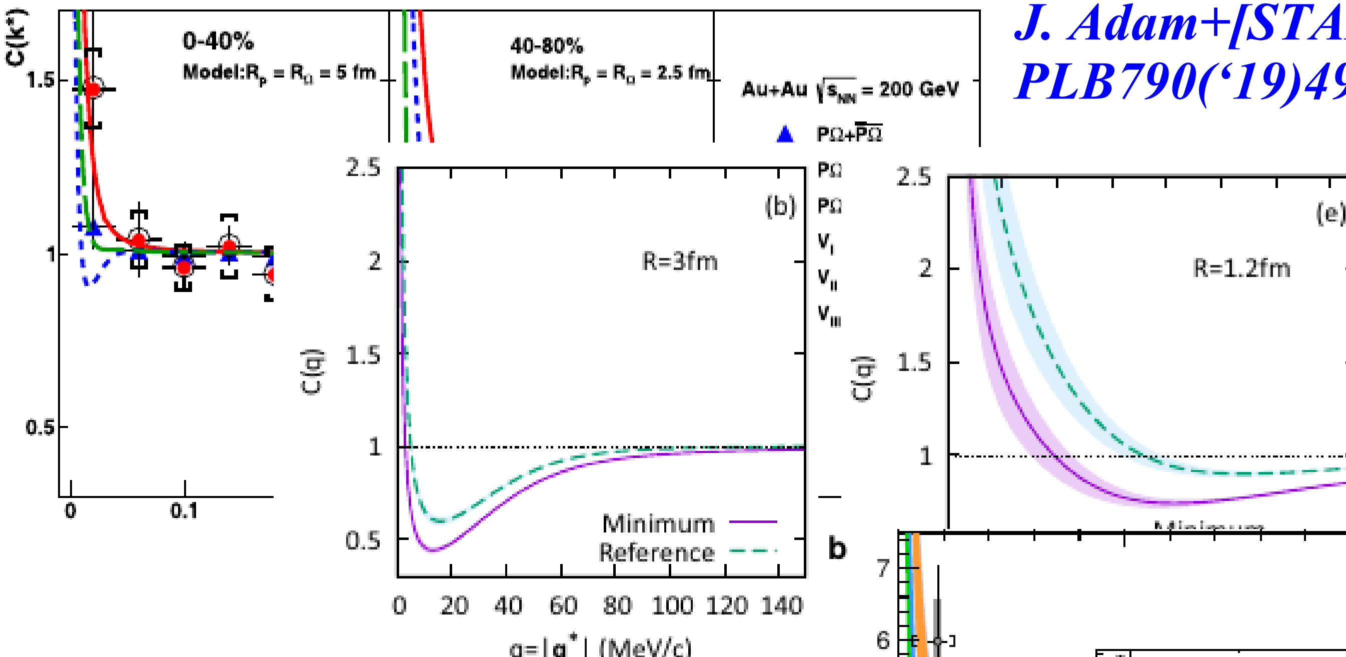


$\delta \sim + a_0 q$

$p\Omega^-$ correlation

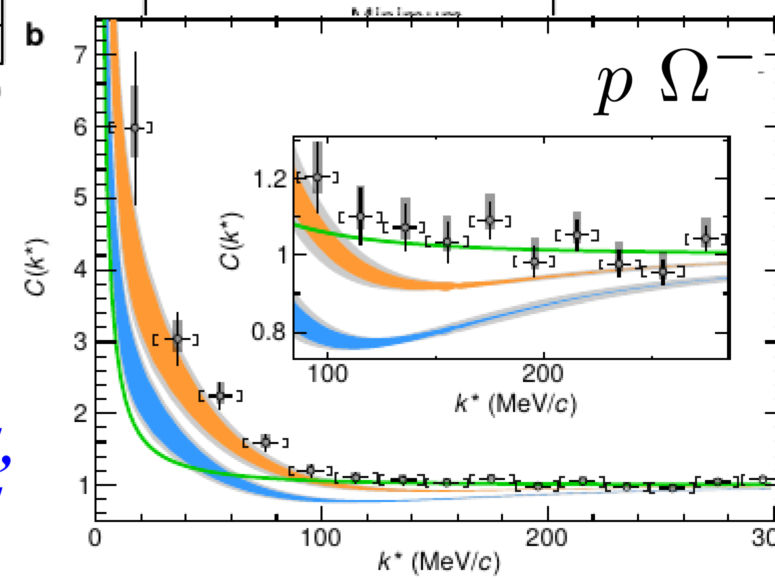


K. Morita, AO, F. Etminan, T. Hatsuda, PRC94('16)031901(R) (w/ Lattice potential with heavier quark mass)



J. Adam+[STAR], PLB790('19)490.

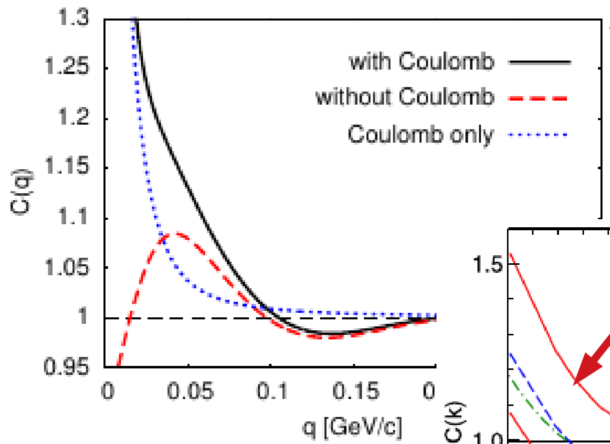
K. Morita, S. Gongyo, T. Hatsuda, T. Hyodo, Y. Kamiya, AO, PRC 101('20)015201. (w/ Lattice potential at physical quark mass)



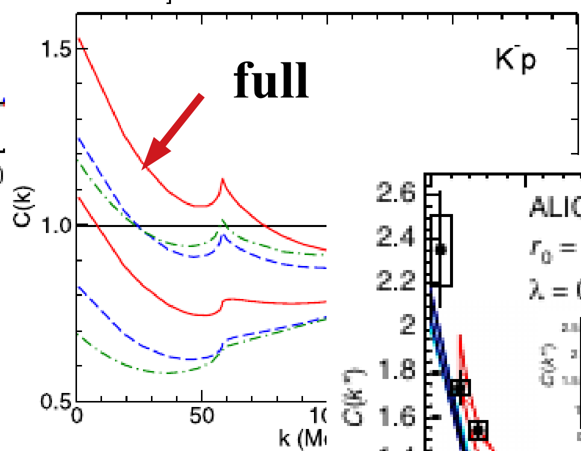
S. Acharya+[ALICE], 2005.11495 [nucl-ex] (pp 13 TeV)

Bound state ?

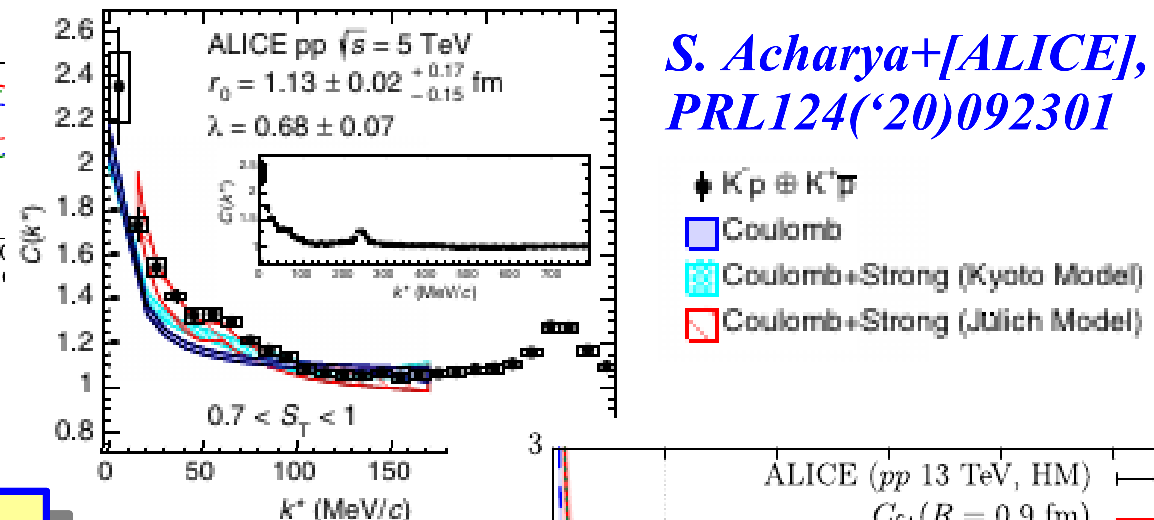
pK^- correlation



S. Cho+ [ExHIC], PPNP95('17)279.
(Insufficient coupled-channel effects)

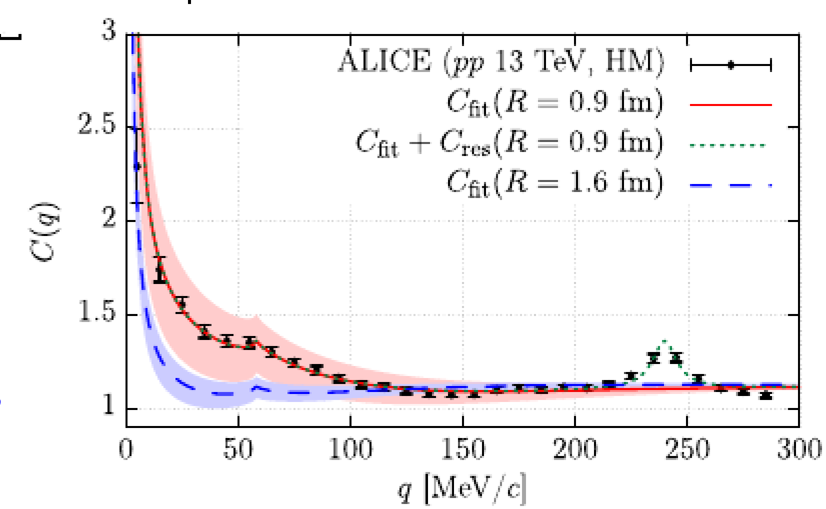


J. Haidenbauer, NPA981('19)1.
(w/ CC effects, w/o Coulomb)



S. Acharya+[ALICE], PRL124('20)092301

Source size dep. shows interesting feature.



Y. Kamiya, T. Hyodo, K. Morita, AO, W. Weise, PRL124('20)132501.
(Chiral SU(3) dynamics)

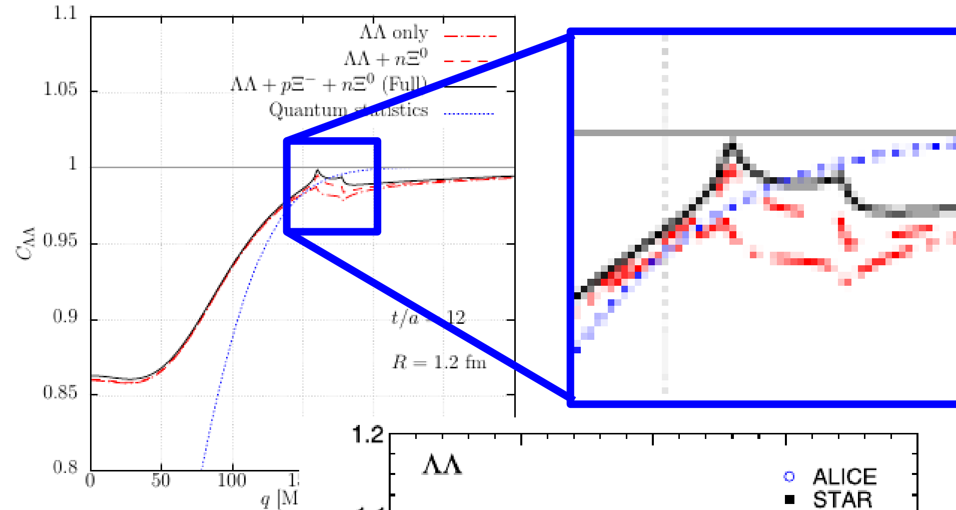
Other bound states ?

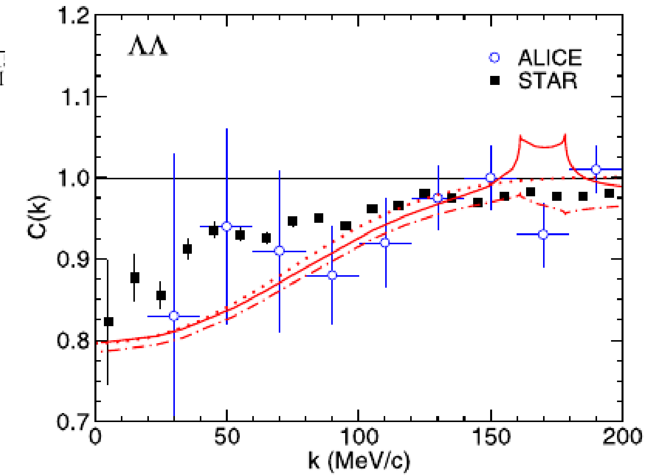
- **ΛΛ-NΞ**
  - **$C_{\Lambda\Lambda}(q)$ in AA(RHIC) and pp(LHC) are similar (No b.s. below ΛΛ).**
  - **LQCD predicts a virtual pole near NΞ threshold, which can be detected as the cusp in $C_{\Lambda\Lambda}(q)$.**

    NLO(600) potential predicts the same.

    (The fate of H particle)

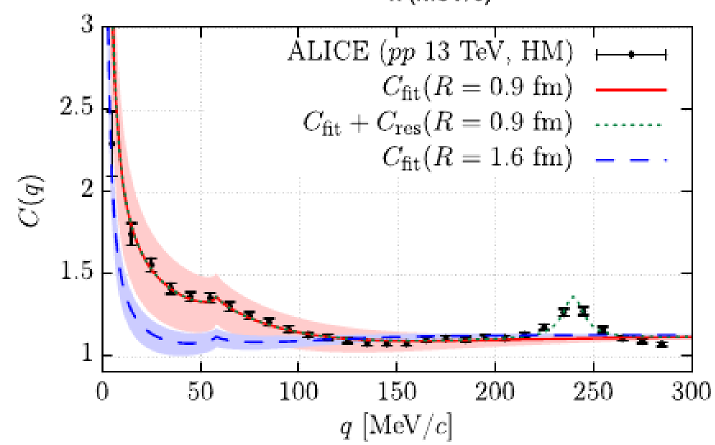
    K. Sasaki+[HAL QCD], NPA998('20)121737; Y. Kamiya+, in prep.; Haidenbauer('19).





- **$\overline{\text{K}}$N**
  - **Λ(1405) is believed to be the bound state of $\overline{\text{K}}$N, and "dip" is expected at $R \sim a_0$.**
  - **However, Coulomb and coupled-channel effects modify the dip-like behavior.**

    Kamiya+ ('20).



Correlation Function with Coupled-Channels Effects

J. Haidenbauer, NPA 981('19)1; R. Lednicky, V. V. Lyuboshits, V. L. Lyuboshits, Phys. At. Nucl. 61('98)2950.

- **Single channel, w/o Coulomb (non-identical pair)**

$$C(\boldsymbol{q}) = \underline{1} + \int d\boldsymbol{r} S(\boldsymbol{r}) \left[\underline{|\chi^{(-)}(r,q)|^2} - \underline{|j_0(qr)|^2} \right]$$

- **Single channel, w/ Coulomb**

$$C(\boldsymbol{q}) = \int d\boldsymbol{r} S(\boldsymbol{r}) \left[\underline{|\varphi^{C,\mathrm{full}}(\boldsymbol{q},\boldsymbol{r})|^2} + \underline{|\chi^{C,(-)}(r,q)|^2} - \underline{|j_0^C(qr)|^2} \right]$$

Full free Coulomb w.f. — **s-wave w.f. with Coul.** — **s-wave Coul. w.f.**

- **Coupled channel, w/ Coulomb**

$$C_i(\boldsymbol{q}) = \int d\boldsymbol{r} S_i(\boldsymbol{r}) \left[\underline{|\varphi^{C,\mathrm{full}}(\boldsymbol{q},\boldsymbol{r})|^2} + \underline{|\chi_i^{C,(-)}(r,q)|^2} - \underline{|j_0^C(qr)|^2} \right]$$

$$+ \sum_{j \neq i} \omega_j \int d\boldsymbol{r} S_j(\boldsymbol{r}) \underline{|\chi_j^{C,(-)}(r,q)|^2}$$

s-wave w.f. in j-th channel

Outgoing B.C. in the i-th channel, ω_j = Source weight (ω_j=1)