

# Four-body Faddeev-type calculation of $\bar{K}NNN$ system

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Interest to antikaon-nucleon systems: quasi-bound state in the  $K^-pp$  system

[Y.Akaishi, T.Yamazaki, *Phys.Rev.C65, 044005 (2002)*]

→ experimental and theoretical efforts with different results

**Our contribution:** series of dynamically exact calculations of three-body antikaon-nucleon systems

- Quasi-bound states in  $\bar{K}NN$ , spin 0 ( $K^-pp$ ) and  $\bar{K}\bar{K}N$  systems - binding energy and width
- No quasi-bound state in  $\bar{K}NN$ , spin 1 ( $K^-np$ ) - proven
- Near-threshold scattering in  $\bar{K}NN$ , spin 1 ( $K^-d$ ) (Faddeev-type AGS equations with coupled  $\bar{K}NN - \pi\Sigma N$  channels)
- 1s level shift and width of kaoninc deuterium (Faddeev-type equations with directly included Coulomb interaction)

[N.V. S., *Three-Body Antikaon-Nucleon Systems. Few Body Syst. 58, 6 (2017)*]

Next step – **four-body calculations** of the  $\bar{K}NNN$  using Faddeev-type equations.

The four-body Faddeev-type AGS equations, written for separable potentials  
 [P. Grassberger, W. Sandhas, Nucl. Phys. B 2, 181-206 (1967)]

$$\bar{U}_{\alpha\beta}^{\sigma\rho}(z) = (1 - \delta_{\sigma\rho})(\bar{G}_0^{-1})_{\alpha\beta}(z) + \sum_{\tau,\gamma,\delta} (1 - \delta_{\sigma\tau})\bar{T}_{\alpha\gamma}^{\tau}(z)(\bar{G}_0)_{\gamma\delta}(z)\bar{U}_{\delta\beta}^{\tau\rho}(z),$$

$$\bar{U}_{\alpha\beta}^{\sigma\rho}(z) = \langle g_{\alpha} | G_0(z) U_{\alpha\beta}^{\sigma\rho}(z) G_0(z) | g_{\beta} \rangle,$$

$$\bar{T}_{\alpha\beta}^{\tau}(z) = \langle g_{\alpha} | G_0(z) U_{\alpha\beta}^{\tau}(z) G_0(z) | g_{\beta} \rangle,$$

$$(\bar{G}_0)_{\alpha\beta}(z) = \delta_{\alpha\beta} \tau_{\alpha}(z).$$

Operators  $\bar{U}_{\alpha\beta}^{\sigma\rho}$  and  $\bar{T}_{\alpha\beta}^{\tau}$  contain four-body  $U_{\alpha\beta}^{\sigma\rho}(z)$  and three-body  $U_{\alpha\beta}^{\tau}(z)$  transition operators of the general form, correspondingly.

$\bar{K}NNN$  system:

- two partitions of 3 + 1 type:  $|\bar{K} + (NNN)\rangle$ ,  $|N + (\bar{K}NN)\rangle$ ,
- one of the 2 + 2 type:  $|(\bar{K}N) + (NN)\rangle$

## Four-body equations

Separable form of the "effective three-body potentials" :

$$\bar{T}_{\alpha\beta}^{\tau}(z) = |\bar{g}_{\alpha}^{\tau}\rangle \bar{\tau}_{\alpha\beta}^{\tau}(z) \langle \bar{g}_{\beta}^{\tau}|$$

→ the four-body equations can be rewritten as

[A. Casel, H. Haberzettl, W. Sandhas, *Phys. Rev. C* 25, 1738 (1982)]

$$\bar{X}_{\alpha\beta}^{\sigma\rho}(z) = \bar{Z}_{\alpha\beta}^{\sigma\rho}(z) + \sum_{\tau,\gamma,\delta} \bar{Z}_{\alpha\gamma}^{\sigma\tau}(z) \bar{\tau}_{\gamma\delta}^{\tau}(z) \bar{X}_{\delta\beta}^{\tau\rho}(z)$$

with new four-body transition  $\bar{X}^{\sigma\rho}$  and kernel  $\bar{Z}^{\sigma\rho}$  operators

$$\begin{aligned}\bar{X}_{\alpha\beta}^{\sigma\rho}(z) &= \langle \bar{g}_{\alpha}^{\sigma} | \bar{G}_0(z)_{\alpha\alpha} \bar{U}_{\alpha\beta}^{\sigma\rho}(z) \bar{G}_0(z)_{\beta\beta} | \bar{g}_{\beta}^{\rho} \rangle, \\ \bar{Z}_{\alpha\beta}^{\sigma\rho}(z) &= (1 - \delta_{\sigma\rho}) \langle \bar{g}_{\alpha}^{\sigma} | \bar{G}_0(z)_{\alpha\beta} | \bar{g}_{\beta}^{\rho} \rangle.\end{aligned}$$

Full system of equations with:

- 1-term separable  $\bar{K}N$  potential, 2-term separable  $NN$  potential (input)
- 1-term separabilized 3-body  $NNN$ ,  $\bar{K}NN$  "T-matrices" and "3-body"  $\bar{K}N + NN$  "T-matrices"

→ system of  $18 \times 18$  coupled equations

$\bar{K}N$  and  $NN$  potentials: separable by construction, separabelization of 3 + 1 and 2+2: **Energy Dependent Pole Expansion/Approximation (EDPE/ EDPA)**  
 [S. Sofianos, N.J. McGurk, H. Fiedeldeldey, *Nucl. Phys. A* 318, 295 (1979)]

Three-body Faddeev-type AGS equations written in momentum basis:

$$X_{\alpha\beta}(p, p'; z) = Z_{\alpha\beta}(p, p'; z) + \sum_{\gamma=1}^3 \int_0^{\infty} Z_{\alpha\gamma}(p, p''; z) \tau_{\gamma}(p''; z) X_{\gamma\beta}(p'', p'; z) p''^2 dp'',$$

eigenvalues  $\lambda_n$  and eigenfunctions  $g_{n\alpha}(p; z)$  of the system – from

$$g_{n\alpha}(p; z) = \frac{1}{\lambda_n} \sum_{\gamma=1}^3 \int_0^{\infty} Z_{\alpha\gamma}(p, p'; z) \tau_{\gamma}(p'; z) g_{n\gamma}(p'; z) p'^2 dp'$$

with normalization condition

$$\sum_{\gamma=1}^3 \int_0^{\infty} g_{n\gamma}(p'; z) \tau_{\gamma}(p'; z) g_{n'\gamma}(p'; z) p'^2 dp' = -\delta_{nn'}.$$

**EDPE/EDPA method:** solution of the eigenequations for a fixed energy  $z$ , usually  $z = E_B$ . After that energy dependent form-factors

$$g_{n\alpha}(p; z) = \frac{1}{\lambda_n} \sum_{\gamma=1}^3 \int_0^{\infty} Z_{\alpha\gamma}(p, p'; z) \tau_{\gamma}(p'; E_B) g_{n\gamma}(p'; E_B) p'^2 dp'$$

and propagators

$$\begin{aligned} (\Theta(z))_{mn}^{-1} &= \sum_{\gamma=1}^3 \int_0^{\infty} g_{m\gamma}(p'; z) \tau_{\gamma}(p'; E_B) g_{n\gamma}(p'; E_B) p'^2 dp' \\ &\quad - \sum_{\gamma=1}^3 \int_0^{\infty} g_{m\gamma}(p'; z) \tau_{\gamma}(p'; z) g_{n\gamma}(p'; z) p'^2 dp' \end{aligned}$$

calculated. The separable version of a three-body amplitude:

$$X_{\alpha\beta}(p, p'; z) = \sum_{m,n=1}^{\infty} g_{m\alpha}(p; z) \Theta_{mn}(z) g_{n\beta}(p'; z).$$

In contrast to Hilbert-Schmidt expansion, EDPE method needs only one solution of the eigenvalue equations and calculations of the integrals after that. The method is accurate already with one term (i.e. EDPA), and it converges faster than Hilbert-Schmidt expansion.

Two-term Separable New potential (TSN) of nucleon-nucleon interaction

$$V_{NN}^{\text{TSN}}(k, k') = \sum_{m=1}^2 g_m(k) \lambda_m g_m(k'),$$

$$g_m(k) = \sum_{n=1}^3 \frac{\gamma_{mn}}{(\beta_{mn})^2 + k^2}, \quad \text{for } m = 1, 2$$

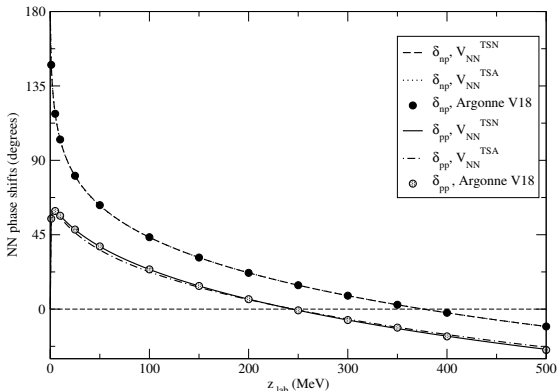
fitted to Argonne V18 potential [*R.B. Wiringa, V.G.J. Stoks, R. Schiavilla, Phys. Rev. C 51, 38 (1995)*] phase shifts

Triplet and singlet scattering lengths  $a$  and effective ranges  $r_{\text{eff}}$

$$a_{np}^{\text{TSN}} = -5.400 \text{ fm}, \quad r_{\text{eff},np}^{\text{TSN}} = 1.744 \text{ fm}$$

$$a_{pp}^{\text{TSN}} = 16.325 \text{ fm}, \quad r_{\text{eff},pp}^{\text{TSN}} = 2.792 \text{ fm},$$

deuteron binding energy  $E_{\text{deu}} = 2.2246 \text{ MeV}$ .



**Fig:** Phase shifts of  $np$  and  $pp$  scattering calculated using the new  $V_{NN}^{TSN}$  and  $V_{NN}^{TSA-B}$  potentials plus phase shifts of Argonne V18



## By-product result

$K^-np$  subsystem ( $\bar{K}NN, S = 1$ ) of the 4-body  $K^-ppn$  system:

$$I^{(4)} = 0, S^{(4)} = 1/2, \text{ orbital momentum } L^{(4)} = 0$$

- Previously used  $NN$  potential [*P.Doleschall*] – **no quasi-bound state** (proven)
- Newly constructed and used  $NN$  potential – **quasi-bound state** in the  $K^-np$  subsystem **caused by strong interactions** was found

## Antikaon-nucleon interaction

Three our potentials:

- phenomenological  $\bar{K}N - \pi\Sigma$  with **one-pole**  $\Lambda(1405)$  resonance
- phenomenological  $\bar{K}N - \pi\Sigma$  with **two-pole**  $\Lambda(1405)$  resonance
- chirally motivated  $\bar{K}N - \pi\Sigma - \pi\Lambda$  potential, **two-pole**  $\Lambda(1405)$

reproduce (with the same level of accuracy):

- 1s level shift and width of kaonic hydrogen (*SIDDHARTA*)  
**direct inclusion of Coulomb interaction, no Deser-type formula used**
- Cross-sections of  $K^-p \rightarrow K^-p$  and  $K^-p \rightarrow MB$  reactions
- Threshold branching ratios  $\gamma$ ,  $R_c$  and  $R_n$
- $\Lambda(1405)$  resonance (*one- or two-pole structure*)  
 $M_{\Lambda(1405)}^{PDG} = 1405.1_{-1.0}^{+1.3}$  MeV,  $\Gamma_{\Lambda(1405)}^{PDG} = 50.5 \pm 2.0$  MeV [*PDG (2016)*]

Subsystems of the  $\bar{K}NNN$  system

- $\bar{K}NN$  ( $I^{(3)} = 1/2, S^{(3)} = 0$  or  $1$ )

Binding energy  $B$  (MeV) and width  $\Gamma$  (MeV) of the quasi-bound state in the  $K^-pp$  and  $K^-np$  (sub)systems calculated with exact optical  $V_{\bar{K}N}^{\text{Opt}}$

|                                | $B_{K^-pp}$ | $\Gamma_{K^-pp}$ | $B_{K^-np}$ | $\Gamma_{K^-np}$ |
|--------------------------------|-------------|------------------|-------------|------------------|
| $V_{\bar{K}N}^{1,\text{SIDD}}$ | 53.3        | 63.3             | 0.8         | 68.3             |
| $V_{\bar{K}N}^{2,\text{SIDD}}$ | 46.7        | 47.4             | 3.8         | 63.2             |
| $V_{\bar{K}N}^{\text{Chiral}}$ | 30.0        | 46.6             | 0.9         | 43.6             |

- $NNN$  ( $I^{(3)} = 1/2, S^{(3)} = 1/2$ )       $B_{NNN} = 9.52$  MeV
- $\bar{K}N + NN$  ( $I^{(4)} = 0, S^{(4)} = 1/2$ ) – a special system with two non-interacting pairs of particles; 3-body system of equations to be solved

## Results: $K^-ppn$ quasi-bound state

$K^-ppn - \bar{K}^0nnp$  system:  $\bar{K}NNN$  with  $I^{(4)} = 0, S^{(4)} = 1/2, L^{(4)} = 0$

Binding energy  $B_{K^-ppn}$  (MeV) and width  $\Gamma_{K^-ppn}$  (MeV) of the quasi-bound state in the  $K^-ppn$  system

|                   | Present results         |                         |                         | Others |      |             |
|-------------------|-------------------------|-------------------------|-------------------------|--------|------|-------------|
|                   | $V_{\bar{K}N}^{1,SIDD}$ | $V_{\bar{K}N}^{2,SIDD}$ | $V_{\bar{K}N}^{Chiral}$ | AY     | BGL  | ME          |
| $B_{K^-ppn}$      | 35.5                    | 34.8                    | 16.0                    | 108    | 29.3 | 41.4 – 73.5 |
| $\Gamma_{K^-ppn}$ | 56.7                    | 60.3                    | 67.2                    | 20     | 32.9 | 22.0 – 31.5 |

[AY: Y. Akaishi, T. Yamazaki, *Phys. Rev. C* 65, 044005 (2002)]

[BGL: N. Barnea, A. Gal, E.Z. Liverts, *Phys. Lett. B* 712, 132 (2012)]

[ME: S. Marri, J. Esmaili, *Eur. Phys. J. A* 55, 43 (2019)]

- **quasi-bound  $K^-ppn$  state:** binding energy is smaller (16 – 36 MeV), while the width (57 – 67 MeV) is comparable the width of the  $K^-pp$  quasi-bound state
- binding energy and width of the  **$K^-ppn$  quasi-bound state** strongly depends on the model of  $\bar{K}N$  interaction; the largest difference is between phenomenological and chiral models, difference between one- and two-pole phenomenological models is small
- **by-product result:** a quasi-bound state caused by strong interactions found in  $K^-d(K^-np)$  (sub)system while the new  $NN$  potential is used
- $K^-d(K^-np)$  binding energy is small (1 – 2 MeV), the width (40 – 60 MeV) is comparable with the width of the  $K^-pp$  quasi-bound state