1. Running b-quark mass
   - Quarks can not be observed
   - "Single" quark masses are not observables, and they are observed as running parameters (running mass)
   - Running mass is described by RGE:
     \[ \frac{d\mu}{dy} = -y \left( a_0 \rho_0 + a_1 \rho_1 \right) \mu \]
     - \( a_0 \): renormalization scale
     - \( a_1 \): Perturbative function

2. Inferring of running b-quark mass
   - Quarks and gluons appear as jets
   - Running quark masses are obtained from hadronic observables
     - Exclusive observables (e.g. three jet rates) have better sensitivity (by a factor of 10 @250-GeV) of quark mass
       - Jet should be defined so that avoid infrared/soft/collinear divergence
     - Jet-Clustering algorithm (JADE, DURHAM, CAMBRIDGE) uses
     - Radiative return cut
       - \( R^q = \frac{\Gamma(\bar{q}q \to f \bar{f})}{\Gamma(\bar{q}q)} \quad \text{Total width for } e^+e^- \to \bar{q}q \)
       - The b-quark mass at Z-pole has been measured precisely at LEP/SLD
       - b-quark mass at high energies
         - Estimate dominant systematic errors at 250GeV ILC and Giga-Z ILC

3. Definition of the Observable
   - Consider double-ratio \( R^q \) as the observable
     - Cancel or reduce EW corrections and systematic uncertainties (hadronization effect)
     - \( R^q = \frac{\Gamma(\bar{q}q \to f \bar{f})/\Gamma(\bar{q}q)}{\Gamma(\bar{q}q \to f \bar{f})/\Gamma(\bar{q}q)} \)
       - (history correction)
       - \( \Delta m_q \) (e.g. CAMBRIDGE predictions are given below)

4. Sensitivity of b-quark mass at high energies
   - Sensitivity of b-quark mass for 250GeV is given by \( a_{\mu} \sim (1-3R^q) \)
     - If we want \( \Delta m_q = 0.4 \) GeV, we need to measure \( R^q \)
     - with a precision of 1% (for Z-pole) and 0.1% (for 250 GeV)
     - The sensitivity at 250 GeV is ~5 times deteriorated

5. Environment of 250GeV measurement
   - Signal event: \( e^+e^- \rightarrow q\bar{q} \quad (q = u, d, s, b) \)
   - BKG events:
     1. Radiative return (\( w < 50 \) GeV ISR)
     2. Di-boson events
   - Luminosity: 2ab^{-1} with two polarizations \( (P_+ P_-) \) = (-0.8, 0.3) and (+0.8, -0.3)
   - Used old DBD sample, event generated by LO for massless quarks in WHIZARD
   - Mass effects are only implemented in PYTHIA/PS+Hadronization
   - Situation is completely different from LEP’s Z-pole measurement

6. Event selection
   - Radiative return cut
     - Construct ISR energy from 2-jets kinematically and remove visible \( y_s \)
     - \( \phi \): angle of ISR jet
   - Di-boson events cut: use \( \text{Thrust} > 0.85 \)
   - Flavor-tagging Efficiency: 80% (for b), 58% (for uds)
   - Purity: 98% (for b), 96.1% (for uds)
   - Jet-reconstruction: CAMBRIDGE algorithm \( w/\chi^2 < 0.01 \)

7. Assessment of uncertainties
   - The mass effects are not implemented in the current MC, but corrections between different levels are worthwhile:
     - Hadronization model
       - LEP's time: 0.2% uncertainty on \( c_{\text{had}} \)
       - Both corrections are close to 1
   - Estimate systematic uncertainties from these corrections:
     - Detector correction
       - From trigger to signal, 100 times larger statistics
     - Propagated flavor-tagging efficiency (0.1-0.5%) and BKG contaminations
     - Assumed high efficiency
     - Updated b-quark mass evolution

8. Conclusion and Prospects
   - ILC 250GeV measurement has limited b-quark mass sensitivity, but it will add a new point at never proved energy
   - Giga-Z ILC will provide superior result at 2-pole than LEP and better QCD test