Neutrino mixing from flavour symmetry

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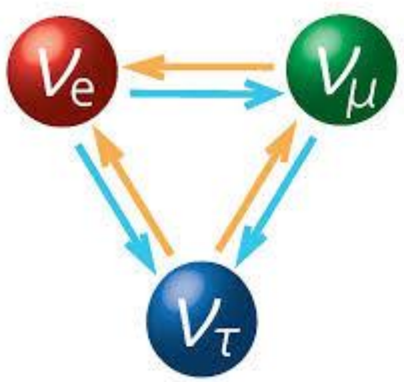
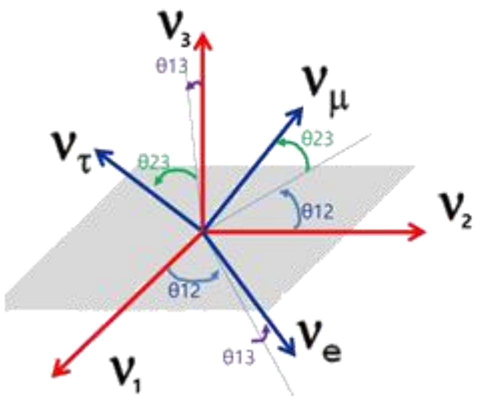


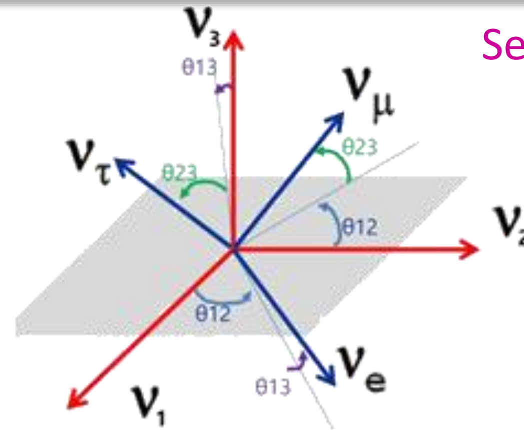


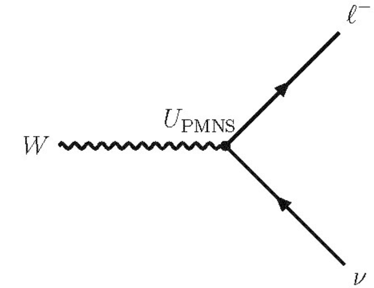
Current experimental results on lepton mixing

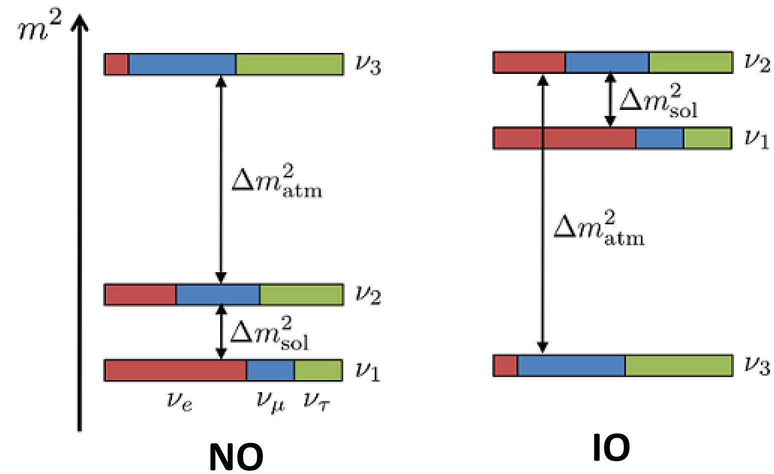
[P.F. de Salas et al, arXiv: 2006.11237]

See Tortola's talk

parameter	best fit $\pm 1\sigma$	2σ range	3σ range
$\Delta m^2_{21}[10^{-5}\mathrm{eV}^2]$	$7.50^{+0.22}_{-0.20}$	7.12–7.93	6.94–8.14
$\lvert\Delta m^2_{31}\rvert[10^{-3}\mathrm{eV}^2]$ (NO)	$2.55^{+0.02}_{-0.03}$	2.49–2.60	2.47–2.63
$\lvert\Delta m^2_{31}\rvert[10^{-3}\mathrm{eV}^2]$ (IO)	$2.45^{+0.02}_{-0.03}$	2.39–2.50	2.37–2.53
$\sin^2\theta_{12}/10^{-1}$	3.18 ± 0.16	2.86–3.52	2.71–3.69
$\theta_{12}/^\circ$	34.3 ± 1.0	32.3–36.4	31.4–37.4
$\sin^2\theta_{23}/10^{-1}$ (NO)	5.74 ± 0.14	5.41–5.99	4.34–6.10
$\theta_{23}/^\circ$ (NO)	49.26 ± 0.79	47.37–50.71	41.20–51.33
$\sin^2\theta_{23}/10^{-1}$ (IO)	$5.78^{+0.10}_{-0.17}$	5.41–5.98	4.33–6.08
$\theta_{23}/^\circ$ (IO)	$49.46^{+0.60}_{-0.97}$	47.35–50.67	41.16–51.25
$\sin^2\theta_{13}/10^{-2}$ (NO)	$2.200^{+0.069}_{-0.062}$	2.069–2.337	2.000–2.405
$\theta_{13}/^\circ$ (NO)	$8.53^{+0.13}_{-0.12}$	8.27–8.79	8.13–8.92
$\sin^2\theta_{13}/10^{-2}$ (IO)	$2.225^{+0.064}_{-0.070}$	2.086–2.356	2.018–2.424
$\theta_{13}/^\circ$ (IO)	$8.58^{+0.12}_{-0.14}$	8.30–8.83	8.17–8.96
δ/π (NO)	$1.08^{+0.13}_{-0.12}$	0.84–1.42	0.71–1.99
$\delta/^\circ$ (NO)	194^{+24}_{-22}	152–255	128–359
δ/π (IO)	$1.58^{+0.15}_{-0.16}$	1.26–1.85	1.11–1.96
$\delta/^\circ$ (IO)	284^{+26}_{-28}	226–332	200–353





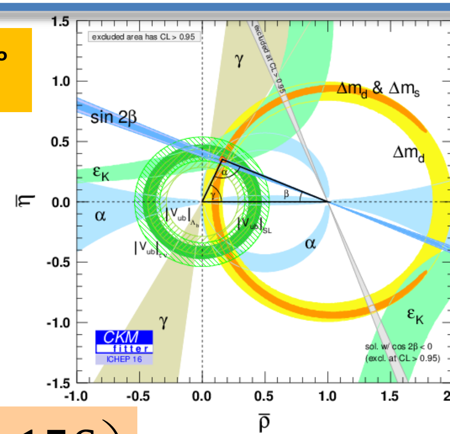


- Octant of θ_{23} : > or <45° ?
- What is the value of δ_{CP} ?
- Mass hierarchy: NO or IO?
- Absolute mass scale: $m_{lightest}$=?
- Majorana or Dirac neutrinos?
- Why m_ν so small?
- Connections to other new physics?

CKM vs PMNS

[Particle Data Group 2020]

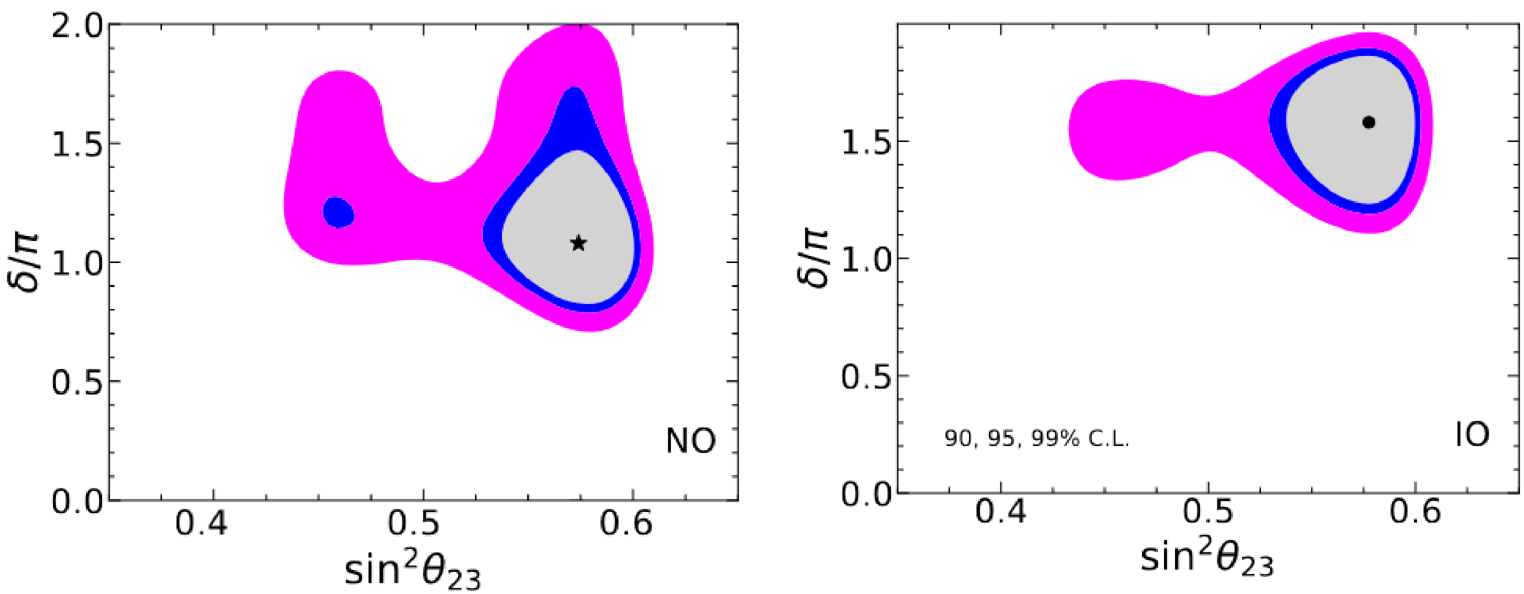
$\alpha=(88.8\pm2.3)^\circ$



Quarks: $|V_{CKM}|\approx\begin{pmatrix}0.97434 & 0.22506 & 0.00357\\ 0.22492 & 0.97351 & 0.0414\\ 0.00875 & 0.0403 & 0.99915\end{pmatrix}$

Leptons: $|U_{PMNS}|=\begin{pmatrix}0.799\sim0.844 & 0.516\sim0.582 & 0.141\sim0.156\\ 0.242\sim0.494 & 0.467\sim0.678 & 0.639\sim0.774\\ 0.284\sim0.521 & 0.490\sim0.695 & 0.615\sim0.754\end{pmatrix}$

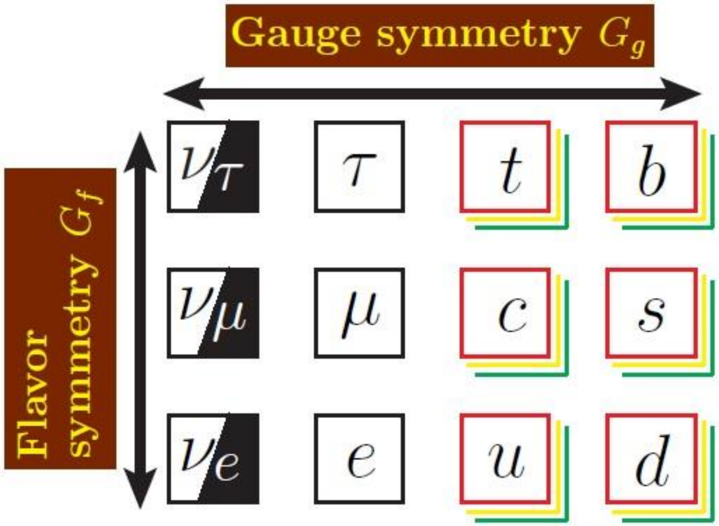
[P.F. de Salas et al, arXiv: 2006.11237]



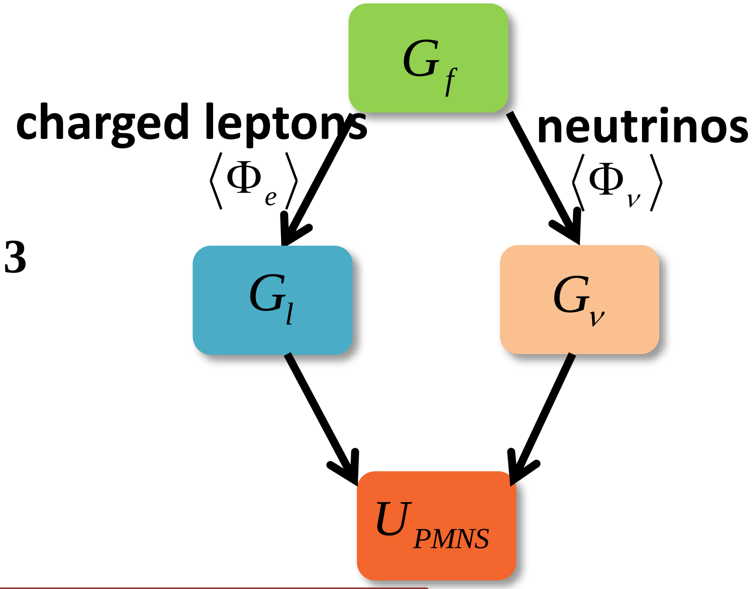
Why quark and lepton mixings are so different?

Symmetry as a guiding principle to flavor puzzle

Relate lepton mixing to how are G_f broken, lepton mixing matrix arises from mismatch of the different residual subgroups G_l and G_ν



$$\begin{pmatrix} L_e \\ L_\mu \\ L_\tau \end{pmatrix} = \begin{pmatrix} \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \\ \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L \\ \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L \end{pmatrix} \sim \mathbf{3}$$



$$\boxed{\mathcal{L}_m = -Y^e_{ij}(\langle \Phi_e \rangle)\overline{L_i} H e_{Rj} - \frac{1}{2} Y^\nu_{ij}(\langle \Phi_\nu \rangle)\overline{L^c_i} H H^T L_j + \ldots}$$

high dimensional operators:
Φ_e, Φ_ν cross terms

	Continuous	Discrete
Abelian	**U(1)**	**Z_n**
Non-Abelian	**U(2), SU(3),SO(3) …**	**A_4, S_4, A_5, $\Delta(6n^2)$, …**

[Altarelli, Feruglio,1002.0211;Tanimoto et al., 1003.3552; King and Luhn, 1301.1340; Feruglio,Romanino, arXiv:1912.06028]

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Direct approach of flavor symmetry

For Majorana neutrinos

[King and Luhn, 1301.1340]

G_f — Generators: **S, T, U**

T preserved → Charged lepton sector $G_l = \langle T \rangle$ → $T^\dagger m_l^\dagger m_l T = m_l^\dagger m_l$ → $U_l^\dagger T U_l = T^{\rm diag}$

S, U preserved → Neutrino sector $G_\nu = \langle S, U \rangle$ → $S^T m_\nu S = m_\nu$, $U^T m_\nu U = m_\nu$ → $U_\nu^\dagger S U_\nu = S^{\rm diag}$, $U_\nu^\dagger U U_\nu = U^{\rm diag}$

→ $U_{PMNS} = U_l^\dagger U_\nu$

- Klein symmetry **S,U** and **T** are **each** identified as subgroups of some flavor symmetry G_f
- Only **trimaximal** mixing can be compatible with data

$$U_{PMNS} = \frac{1}{\sqrt{3}} \begin{pmatrix} \sqrt{2}\cos\theta & 1 & -\sqrt{2}\sin\theta \\ -\sqrt{2}\cos(\theta - \pi/3) & 1 & \sqrt{2}\sin(\theta - \pi/3) \\ -\sqrt{2}\cos(\theta + \pi/3) & 1 & \sqrt{2}\sin(\theta + \pi/3) \end{pmatrix}$$

θ depends on groups G_f , G_l ,G_ν

- Dirac CP phase δ_{CP} is predicted to be **conserved**.

[Lindner et al., 1212.2411; King, Neder, Stuart,1305.3200;Fonseca and Grimus, 1405.3678;Yao, Ding, 1505.03798]

- If the lepton mixing matrix is partially determined by the flavor symmetry G_f , G_l and G_ν, e.g. G_ν=Z_2

[Ge, Dicus, Repko, 1104.0602; Hernandez and Smirnov, 1204.0445]

For example, two deformations of TBM

TM_1

$$U = U_{TBM}\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta e^{-i\delta} \\ 0 & -\sin\theta e^{i\delta} & \cos\theta \end{pmatrix},$$

TM_2

$$U = U_{TBM}\begin{pmatrix} \cos\theta & 0 & \sin\theta e^{-i\delta} \\ 0 & 1 & 0 \\ -\sin\theta e^{i\delta} & 0 & \cos\theta \end{pmatrix}$$

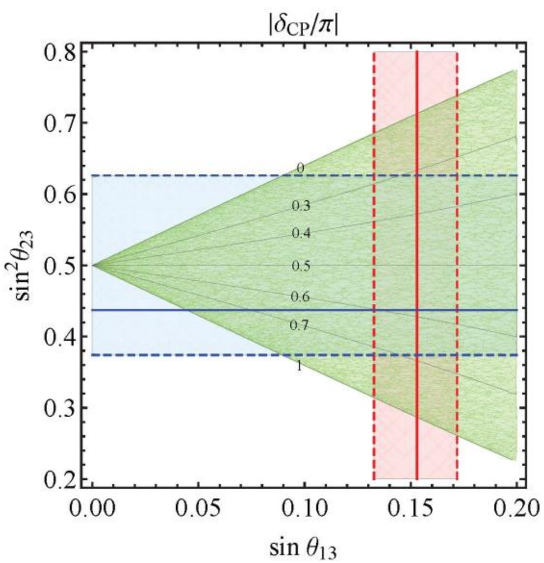
Two predictions in terms of sum rules

$$3\cos^2\theta_{12}\cos^2\theta_{13} = 2$$

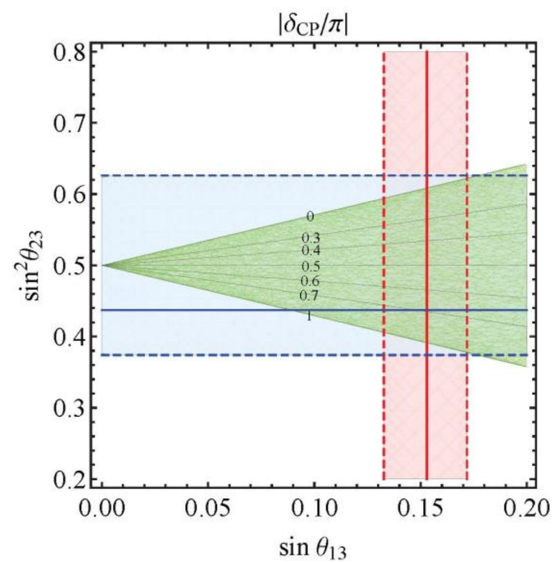
$$\sin^2\theta_{23} \simeq \frac{1}{2} - \sqrt{2}\sin\theta_{13}\cos\delta_{CP}$$

$$3\sin^2\theta_{12}\cos^2\theta_{13} = 1$$

$$\sin^2\theta_{23} \simeq \frac{1}{2} + \frac{1}{\sqrt{2}}\sin\theta_{13}\cos\delta_{CP}$$

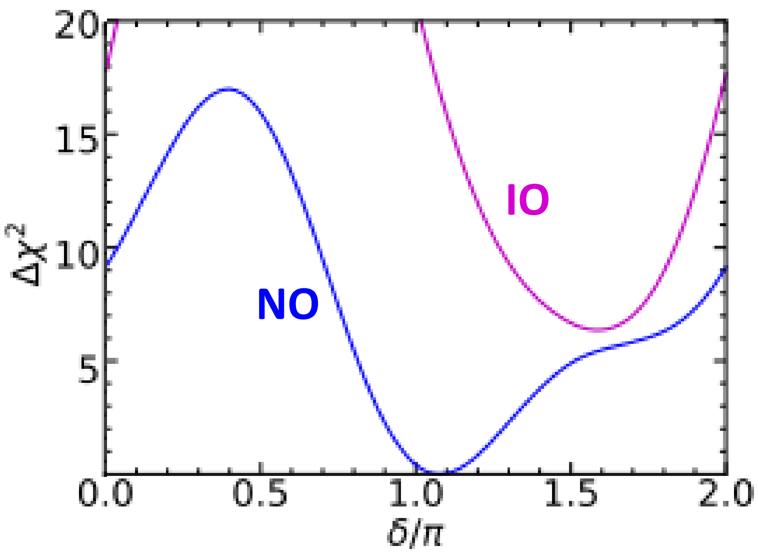


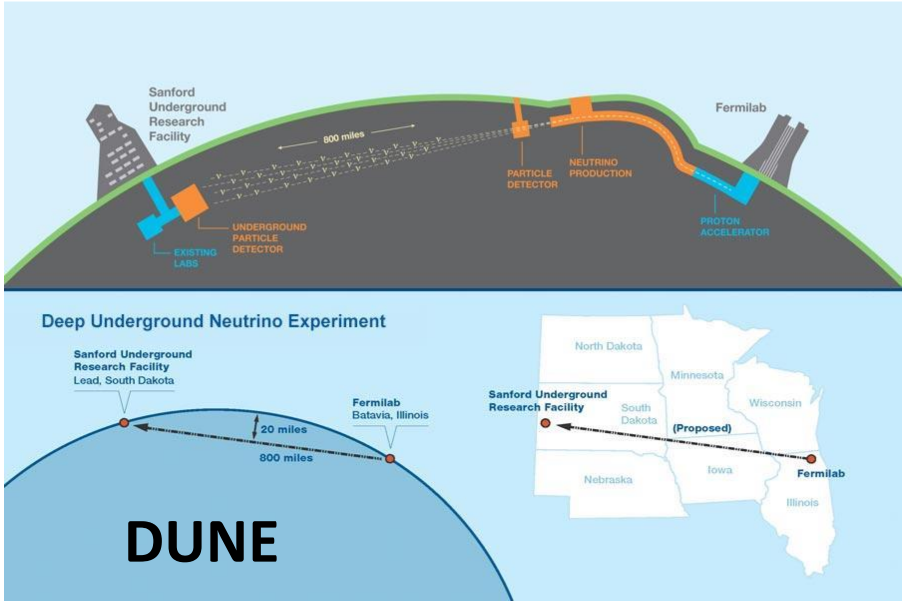
He, Zee, 07 and 11; Grimus, Lavoura, 08; Albright, Rodejohann, 09; King, Luhn 11; Chen, Vale et., 1812.04663, 1905.11997…



Leptonic CP violation

[P.F. de Salas et al, arXiv: 2006.11237]





➢Theoretical idea: **flavor symmetry → flavor+CP symmetries**

Flavor symmetry

⇕

CP symmetry

} **Mixing angles &CP phases**

"closure" relations have to hold!

$$
\begin{tikzcd}
 & X\varphi^*(\mathcal{P}x) \arrow[dr, "g"] & \\
\varphi(x) \arrow[ur, "\mathcal{CP}"] \arrow[dr, "g'"'] & \boxed{X\rho^*(g)X^{-1} = \rho(g')} & X\rho(g)^*\varphi^*(\mathcal{P}x) \arrow[dl, "\mathcal{CP}^{-1}"] \\
 & \rho(g')\varphi(x) = X\rho(g)^*X^{-1}\varphi(x) &
\end{tikzcd}
$$

[Grimus,Rebelo,hep-ph/9506272; Feruglio et al., 1211.5560; Lindner et al., 1211.6953]

a simple predictive CP symmetry: μτ reflection

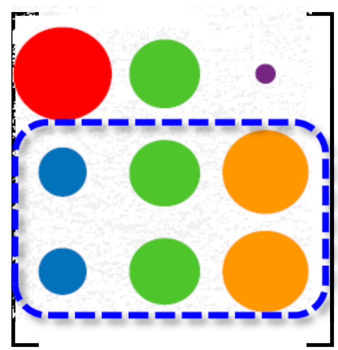
μτ reflection =μτ exchange+canonical CP

$$\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix} \rightarrow \begin{pmatrix}\nu_e^c\\ \nu_\tau^c\\ \nu_\mu^c\end{pmatrix} = \begin{pmatrix}1&0&0\\0&0&1\\0&1&0\end{pmatrix}\begin{pmatrix}\nu_e^c\\ \nu_\mu^c\\ \nu_\tau^c\end{pmatrix}$$

This CP transformation is **not** a unit matrix.

If the neutrino mass matrix is **invariant** under the μτ reflection

$$m_\nu = \begin{pmatrix} a & b & b^* \\ b & c & d \\ b^* & d & c^* \end{pmatrix} \Longrightarrow |U_{PMNS}| =$$



$$\Longrightarrow \boxed{\theta_{23} = \frac{\pi}{4}, \quad \delta_{CP} = \pm\frac{\pi}{2}}$$

$$\nu_e \qquad \nu_\mu \leftrightarrow \nu_\tau^c$$

The last two rows have equal magnitudes

[Harrison, Scott, hep-ph/0210197; Grimus, Lavoura,hep-ph/0305309]

Generalized μτ reflection

the generalized μτ reflection of neutrino

[Chen, Ding, Valle et al, 1512.01551 , 1802.04275]

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} \to \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\Theta & i\sin\Theta \\ 0 & i\sin\Theta & \cos\Theta \end{pmatrix} \begin{pmatrix} \nu_e^c \\ \nu_\mu^c \\ \nu_\tau^c \end{pmatrix},$$

μτ reflection: $\Theta = \pi/2$

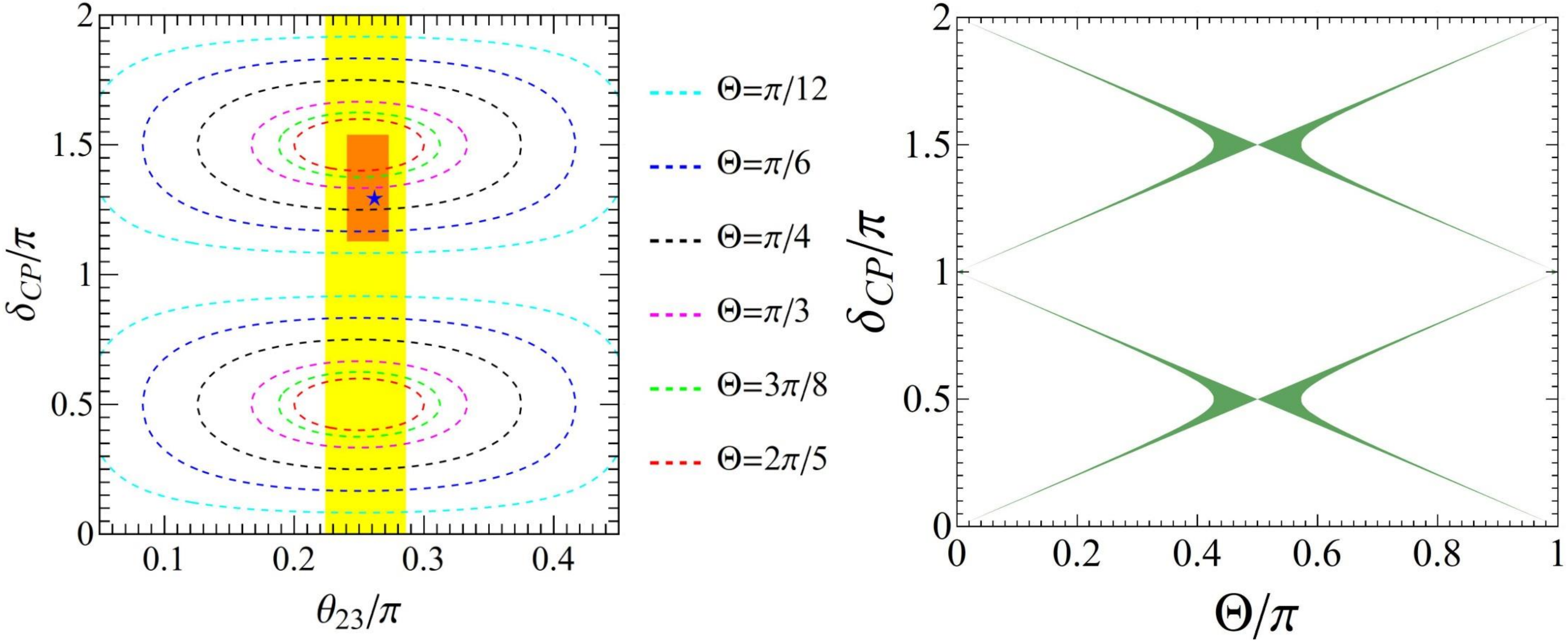
In the charged lepton diagonal basis, the lepton mixing matrix is

$$U = \Sigma_{\mu\tau} O_{3\times3} diag(1, i^{k_1}, i^{k_2}), \qquad \Sigma_{\mu\tau} \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\frac{\Theta}{2} & i\sin\frac{\Theta}{2} \\ 0 & i\sin\frac{\Theta}{2} & \cos\frac{\Theta}{2} \end{pmatrix}$$

Lepton mixing matrix is determined up to a real orthogonal matrix O_{3x3}

Predictions for lepton mixing parameters:

$$\sin^2\delta_{CP}\sin^2 2\theta_{23} = \sin^2\Theta, \qquad \sin\alpha_{21} = \sin\alpha'_{31} = 0$$



Numerical example:

$$\Theta = \frac{\pi}{3}, \quad \theta_1 = 0.274\pi, \quad \theta_2 = 0.0475\pi, \quad \theta_3 = 0.813\pi,$$

$$\sin^2\theta_{12} = 0.307, \quad \sin^2\theta_{13} = 0.0221, \quad \sin^2\theta_{23} = 0.538, \quad \delta_{CP} = 1.335\pi$$

Semi-direct approach to lepton mixing

[Feruglio et al., 1211.5560; Ding et al., 1303.6180,...., Hagedorn et al ,2107.07537]

$$G_f \rtimes H_{CP}$$

charged lepton

neutrino

$$Z_n^{g_l}, n \geq 3$$

$$Z_2^{g_\nu} \times X_\nu$$

$$X_\nu = \Sigma_\nu \Sigma_\nu^T,$$

$$\Sigma_\nu^\dagger \rho_3(g_\nu) \Sigma_\nu = \pm \mathrm{diag}(1,-1,-1)$$

$$U_l^\dagger \rho_3(g_l) U_l = \rho_3(g_l)_{diag}$$

$$U_l$$

$$U_\nu = \Sigma_\nu R_{23}(\theta) Q_\nu$$

$$U_l^\dagger m_l^\dagger m_l U_l \equiv \mathbf{diag}(m_e^2, m_\mu^2, m_\tau^2)$$

$$U_\nu^T m_\nu U_\nu \equiv \mathbf{diag}(m_1, m_2, m_3)$$

$$U_{PMNS} = U_l^\dagger \Sigma_\nu R_{23}(\theta) Q_\nu$$

- The mixing angles and CP violating phases are predicted in terms of a single real parameter $0 \leq \theta < \pi$. One column is fixed.
- quark mixing can not be explained in semi-direct approach.

Benchmark examples

For popular flavor symmetries **A_4, S_4, A_5:** [Li et al, 1503.03711, Iura et al., 1503.04140, Ballett et al.,1503.07543]

$\delta_{CP} = \pm\pi/2,\ \theta_{23} = \pi/4$ or $\delta_{CP} = 0, \pi,\ \theta_{23} \neq \pi/4$

- Flavor group $G_f = \Delta(96)$, residual symmetry $G_e = Z_3^{ac^3}, G_\nu = Z_2^{c^2}, X_\nu = c^2 d$

[Ding et al., 1409.8005; Hagedorn et al., 1408.7118]

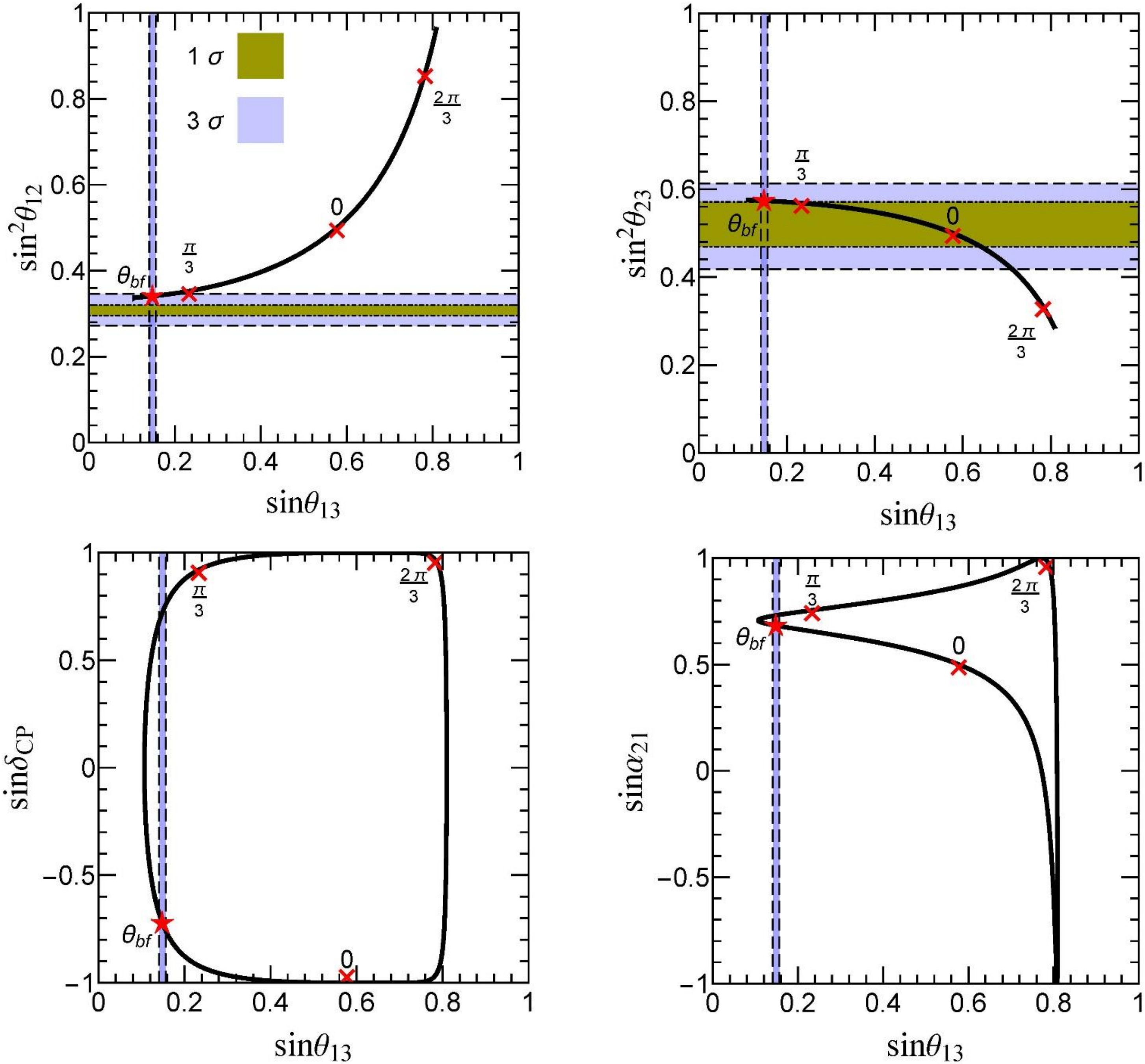
$$U_{PMNS} = \frac{1}{\sqrt{3}}\begin{pmatrix} \omega^2 e^{i\pi/4}\cos\theta - \omega\sin\theta & 1 & \omega^2 e^{i\pi/4}\sin\theta + \omega\cos\theta \\ e^{i\pi/4}\cos\theta - \sin\theta & 1 & e^{i\pi/4}\sin\theta + \cos\theta \\ \omega e^{i\pi/4}\cos\theta - \omega^2\sin\theta & 1 & \omega e^{i\pi/4}\sin\theta + \omega^2\cos\theta \end{pmatrix}, \quad \omega \equiv e^{2\pi i/3}$$

$$\Rightarrow \begin{cases} \sin^2\theta_{13} = \frac{1}{3} - \frac{\sqrt{3}+1}{6\sqrt{2}}\sin 2\theta, \quad \sin^2\theta_{12} = \frac{2\sqrt{2}}{4\sqrt{2} + \left(\sqrt{3}+1\right)\sin 2\theta}, \quad J_{CP} = -\frac{\cos 2\theta}{6\sqrt{3}}, \\ \sin^2\theta_{23} = \frac{1}{2} + \frac{\left(3-\sqrt{3}\right)\sin 2\theta}{8\sqrt{2} + 2\left(\sqrt{3}+1\right)\sin 2\theta}, \quad I_1 = \frac{1}{18}\left(1 + \left(\sqrt{3}-1\right)\sin^2\theta + \sqrt{2}\sin 2\theta\right), \quad I_2 = \frac{\cos 2\theta}{18} \end{cases}$$

Numerical results: **non-regular values of CP phases**

$$\theta_{bf} = 0.209\pi, \quad \chi^2_{min} = 7.960, \quad \sin^2\theta_{12} = 0.341, \quad \sin^2\theta_{13} = 0.0220,$$

$$\sin^2\theta_{23} = 0.574, \quad \sin\delta_{CP} = -0.722, \quad \sin\alpha_{21} = 0.683, \quad \sin\alpha_{31} = -0.091.$$



1 σ
3 σ
$\sin^2\theta_{12}$
θ_{bf}
$\frac{\pi}{3}$
0
$\frac{2\pi}{3}$
$\sin\theta_{13}$
$\sin^2\theta_{23}$
θ_{bf}
$\frac{\pi}{3}$
0
$\frac{2\pi}{3}$
$\sin\theta_{13}$
$\sin\delta_{CP}$
$\frac{\pi}{3}$
$\frac{2\pi}{3}$
θ_{bf}
0
$\sin\theta_{13}$
$\sin\alpha_{21}$
$\frac{\pi}{3}$
$\frac{2\pi}{3}$
θ_{bf}
0
$\sin\theta_{13}$

Universal flavor symmetry for quark and lepton mixing

[Lu, Ding, 1610.05682, 1806.02301; Li, Lu, Ding, 1706.04576]

$$G_f \rtimes H_{CP}$$

charged lepton

neutrino

$$Z_2^{g_l} \times X_l$$

$$Z_2^{g_\nu} \times X_\nu$$

$$X_l = \Sigma_l \Sigma_l^T, \quad \Sigma_l^\dagger \rho_3(g_l) \Sigma_l = \pm \mathrm{diag}(1,-1,-1)$$

$$X_\nu = \Sigma_\nu \Sigma_\nu^T, \quad \Sigma_\nu^\dagger \rho_3(g_\nu) \Sigma_\nu = \pm \mathrm{diag}(1,-1,-1)$$

$$U_l = \Sigma_l R_{23}(\theta_l) P_l \qquad U_\nu = \Sigma_\nu R_{23}(\theta_\nu) P_\nu Q_\nu$$

$$U_l^\dagger m_l^\dagger m_l U_l \equiv \mathbf{diag}(m_e^2, m_\mu^2, m_\tau^2)$$

$$U_\nu^T m_\nu U_\nu \equiv \mathbf{diag}(m_1, m_2, m_3)$$

$$U_{PMNS}(\theta_l, \theta_\nu) = R_{23}^T(\theta_l) \Sigma_l^\dagger \Sigma_\nu R_{23}(\theta_\nu) P_\nu Q_\nu$$

- All mixing angles and CP phases are expressed in terms of two free angles $\theta_{l,\nu} \in [0,\pi)$
- This scheme can be extended to quark sector, and the CKM mixing matrix is of similar form

Quark and lepton mixing from Dihedral group D_n and CP

Quark sector:

[Lu, Ding, 1901.07414]

Assignment: 2+1

$$\left(\begin{pmatrix} u_L \\ d_L \end{pmatrix} \begin{pmatrix} c_L \\ s_L \end{pmatrix}\right) \sim \mathbf{2}_1, \quad \begin{pmatrix} t_L \\ b_L \end{pmatrix} \sim \mathbf{1}_1$$

Residual symmetry: $Z_2^{g_u} = Z_2^{SR^x}$, $X_u = SR^{n/2}$, $Z_2^{g_d} = Z_2^{SR^y}$, $X_d = S$, $x, y = 0, \ldots, n-1$

- The CKM matrix is determined to be $c_{u,d} \equiv \cos\theta_{u,d}$, $s_{u,d} \equiv \sin\theta_{u,d}$

$$V_{CKM} = \begin{pmatrix} -c_d \sin\varphi_1 & \boxed{\cos\varphi_1} & s_d \sin\varphi_1 \\ c_u c_d \cos\varphi_1 + i s_u s_d & c_u \sin\varphi_1 & -c_u s_d \cos\varphi_1 + i c_d s_u \\ -c_d s_u \cos\varphi_1 + i c_u s_d & -s_u \sin\varphi_1 & s_u s_d \cos\varphi_1 + i c_u c_d \end{pmatrix}$$

with $\varphi_1 = \dfrac{y-x}{n}\pi \longleftarrow$ **fixed by residual symmetry**

- Sum rules

$$\begin{cases} \cos^2\theta_{13}^q \sin^2\theta_{12}^q = \cos^2\varphi_1, \\ J_{CP}^q \approx \dfrac{1}{2}\sin 2\varphi_1 \sin\varphi_2 \sin\theta_{13}^q \sin\theta_{23}^q \end{cases}$$

- Benchmark value: $\varphi_1 = 3\pi/7$ which can be achieved in **D_{14}** group

	$\theta_1^{\rm bf}/\pi$	$\theta_2^{\rm bf}/\pi$	$\sin\theta_{12}^q$	$\sin\theta_{23}^q$	$\sin\theta_{13}^q$	J_{CP}^q
Our	0.01326	0.00117	0.22252	0.04166	0.00357	3.224×10^{-5}
Data	—	—	0.22500 ± 0.00100	0.04200 ± 0.00059	0.003675 ± 0.000095	$(3.120\pm0.090)\times10^{-5}$

The measured quark mixing angles and CP violation phases can be accommodated.

- Lepton sector : $\varphi_1 = 2\pi/7$

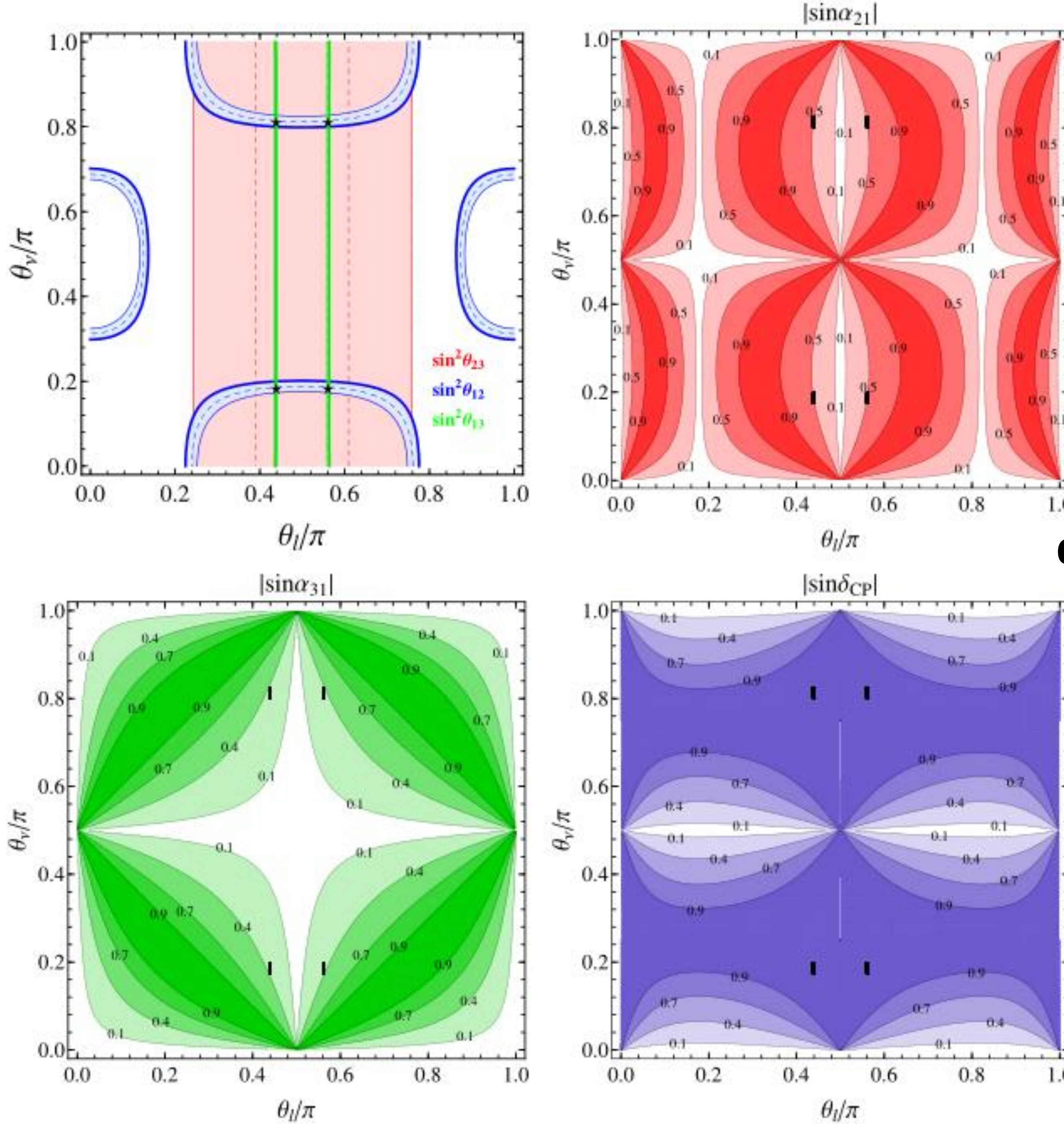
$$U_{PMNS}=R_{12}(\theta_l)\begin{pmatrix}0 & \cos\frac{2\pi}{7} & \sin\frac{2\pi}{7}\\ i & 0 & 0\\ 0 & -\sin\frac{2\pi}{7} & \cos\frac{2\pi}{7}\end{pmatrix}R_{13}(\theta_\nu)Q_\nu \Longrightarrow \cos^2\theta_{23}\cos^2\theta_{13}=\cos^2\frac{2\pi}{7}$$

Numerical benchmark:

$$\theta_l=0.562\pi,\quad \theta_\nu=0.186\pi,\quad \chi^2_{\min}=1.824,$$

$$\sin^2\theta_{13}=0.0226,\quad \sin^2\theta_{12}=0.310,\quad \sin^2\theta_{23}=0.602,$$

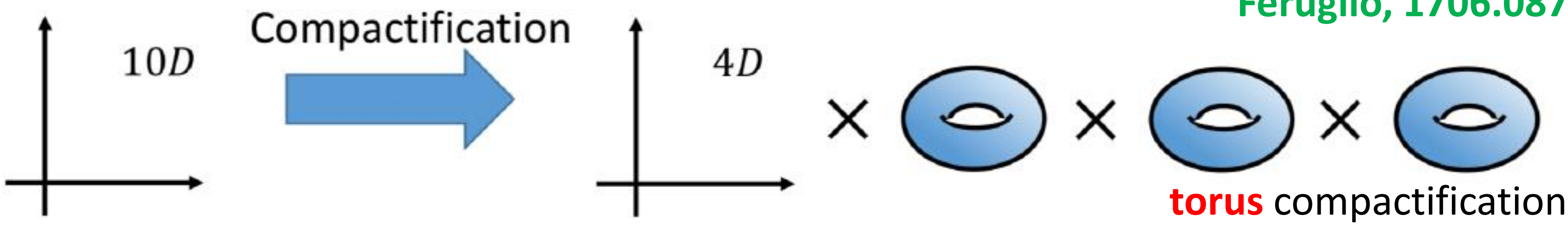
$$\delta_{CP}=1.467\pi,\quad |\sin\alpha_{21}|=0.503,\quad |\sin\alpha_{31}|=0.358$$



quite predictive!

Recent progress: modular symmetry

String theory in d=10 need 6 compact dimensions [Ferrara et al, 1989; Feruglio, 1706.08749]



integrating over 6D: $S = \int d^4x d^6y \, \mathcal{L}_{10D} \Rightarrow \int d^4x \, \mathcal{L}_{\text{eff}}(\varphi, \tau_i)$

The shape of the torus is parametrized by the modulus τ, and it is invariant under the modular transformation

$$\boxed{\tau \to \gamma\tau = \frac{a\tau + b}{c\tau + d} \Leftrightarrow \begin{pmatrix} a & b \\ c & d \end{pmatrix}} \qquad \begin{array}{c} ad - bc = 1 \\ a, b, c, d \;\; \text{integers} \end{array}$$

Modular group

$\Gamma \cong SL(2, Z)$

- Generators S and T: $S^4 = (ST)^3 = 1, \;\; S^2T = TS^2$

$$S = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad T = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad \tau \xrightarrow{S} -\frac{1}{\tau}, \;\; \tau \xrightarrow{T} \tau + 1$$

- Finite modular groups: the quotient over the principal congruence subgroups Γ(N)

Γ_2	Γ_3	Γ_4	Γ_5
S_3	A_4	S_4	A_5
Γ'_2	Γ'_3	Γ'_4	Γ'_5
S_3	T'	S'_4	A'_5

$$\Gamma_N \equiv SL(2, Z) / \pm\Gamma(N), \quad \Gamma'_N \equiv SL(2, Z) / \Gamma(N)$$

Γ'_N is the "double covering" of Γ_N

Modular invariance as flavor symmetry

- Modular transformation of chiral superfield in MSSM

$$\gamma\tau = \frac{a\tau+b}{c\tau+d}, \quad \chi^{(I)} \to (c\tau+d)^{-k_I} \rho^{(I)}(\gamma)\chi^{(I)}$$

Modular weight

unitary representation of finite modular groups Γ'_N or Γ_N

- Modular invariant superpotential

$$W = \sum_n Y_{I_1\cdots I_n}(\tau)\chi^{(I_1)}\cdots\chi^{(I_n)}$$

Yukawa couplings only depend on τ

Modular invariance requires a holomorphic $Y_{I_1 I_2\cdots I_n}(\tau)$ satisfying

$$\boxed{Y_{I_1\cdots I_n}(\gamma\tau) = (c\tau+d)^{k_Y(n)}\rho^{(Y)}(\gamma)Y_{I_1\cdots I_n}(\tau)}$$

$Y_{I_1 I_2\cdots I_n}(\tau)$ are modular forms of level N and weight k_Y

obeying $\begin{cases} k_Y(n)+k_{I_1}+\ldots+k_{I_n} = 0 \\ \rho^{(Y)}\otimes\rho^{(I_1)}\otimes\cdots\otimes\rho^{(I_n)} \supset 1 \end{cases}$

- Lowest weight **2** modular forms of level 3 transforms as a **triplet of A_4**

$$Y_3^{(2)} = \begin{pmatrix} \varepsilon^2(\tau) \\ \sqrt{2}\,\vartheta(\tau)\varepsilon(\tau) \\ -\vartheta^2(\tau) \end{pmatrix} = \begin{pmatrix} 1+12q+36q^2+\cdots \\ -6q^{1/3}(1+7q+8q^2+\cdots) \\ -18q^{2/3}(1+2q+5q^2+\cdots) \end{pmatrix}$$

$$\vartheta(\tau) = 3\sqrt{2}\frac{\eta^3(3\tau)}{\eta(\tau)}, \quad q = e^{2\pi i\tau},$$

$$\varepsilon(\tau) = -\frac{3\eta^3(3\tau)+\eta^3(\tau/3)}{\eta(\tau)}$$

tesor product generate high weight modular forms

Example: a minimal model based on Γ_3=A_4

➢ Field content

Including gCP symmetry to constrain all couplings real

	L	(e^c, μ^c, τ^c)	N^c	H_u	H_d
$SU(2)_L \times U(1)_Y$	$(2, -1/2)$	$(1,1)$	$(1,0)$	$(2,1/2)$	$(2,-1/2)$
A_4	$\mathbf{3}$	$(\mathbf{1}, \mathbf{1}, \mathbf{1}')$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}$
k_I	-1	$(3,5,9)$	1	0	0

Charged lepton mass terms

[Yao, Lu, Ding, 2012.13390]

$$W_e = \alpha e^c (LY_3^{(2)})_1 H_d + \beta \mu^c (LY_3^{(4)})_1 H_d + \gamma \tau^c (LY_{3I}^{(8)})_{1''} H_d + \gamma' \tau^c (LY_{3II}^{(8)})_{1''} H_d$$

$$M_e = \begin{pmatrix} \alpha Y_{3,1}^{(2)} & \alpha Y_{3,3}^{(2)} & \alpha Y_{3,2}^{(2)} \\ \beta Y_{3,1}^{(4)} & \beta Y_{3,3}^{(4)} & \beta Y_{3,2}^{(4)} \\ \gamma Y_{3I,3}^{(8)} + \gamma' Y_{3II,3}^{(8)} & \gamma Y_{3I,2}^{(8)} + \gamma' Y_{3II,2}^{(8)} & \gamma Y_{3I,1}^{(8)} + \gamma' Y_{3II,1}^{(8)} \end{pmatrix} v_d$$

NO flavons

Neutrino mass terms: $W_\nu = g(N^c L)_1 H_u + \Lambda (N^c N^c Y_3^{(2)})_1$

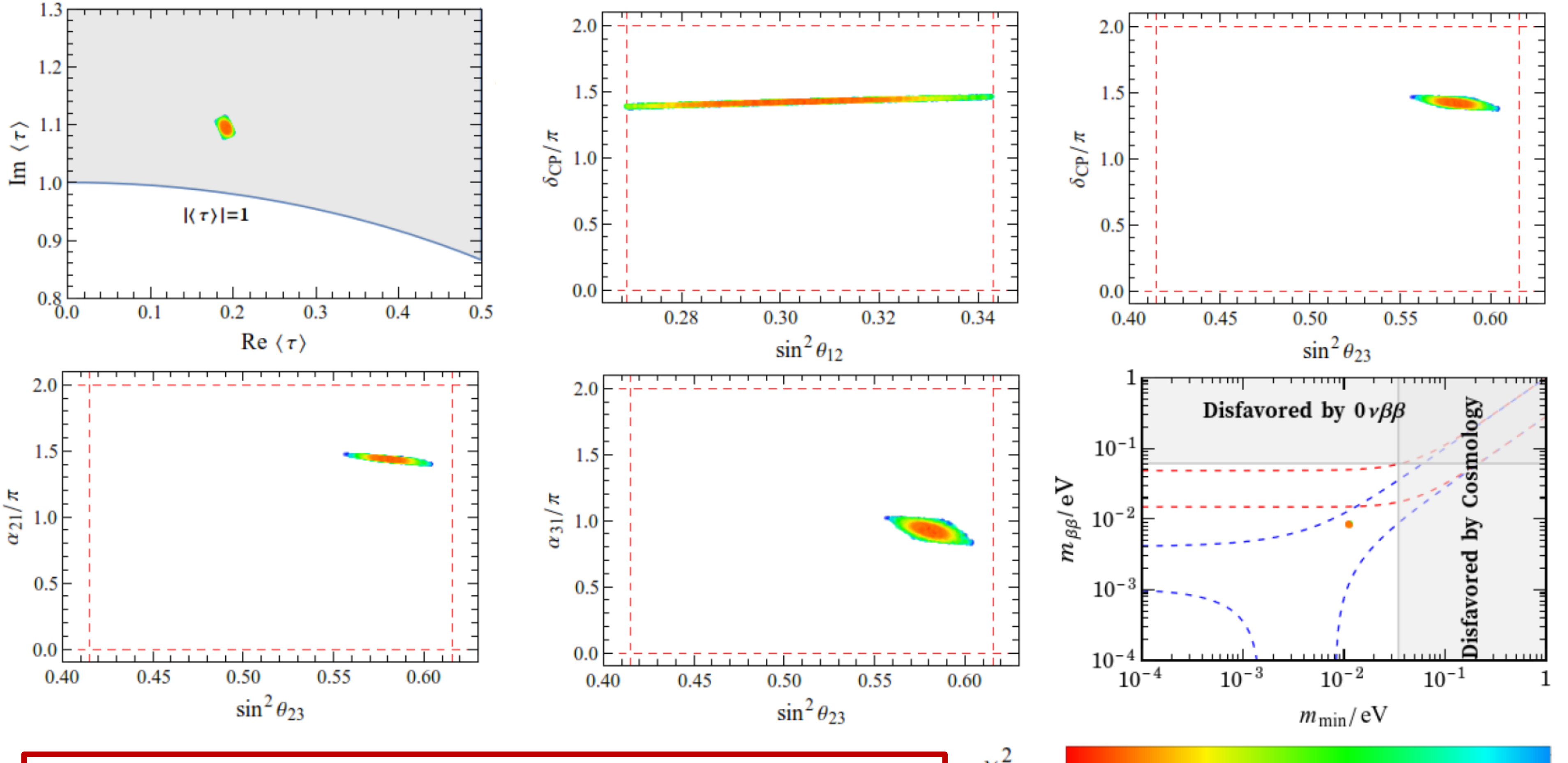
$$M_D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}_3 g v_u, \quad M_N = \begin{pmatrix} 2Y_{3,1}^{(2)} & -Y_{3,3}^{(2)} & -Y_{3,2}^{(2)} \\ -Y_{3,3}^{(2)} & 2Y_{3,2}^{(2)} & -Y_{3,1}^{(2)} \\ -Y_{3,2}^{(2)} & -Y_{3,1}^{(2)} & 2Y_{3,3}^{(2)} \end{pmatrix} \Lambda$$

The light neutrino mass matrix only depends on τ up to an overall scale

7 real input parameters describe 12 obervables: $m_{e,\mu,\tau}$, $m_{1,2,3}$, θ_{12}, θ_{13}, θ_{23}, δ, α_{21}, α_{31}

✓ **The complex modulus τ is the only source of both CP violation and flavor symmetry breaking**

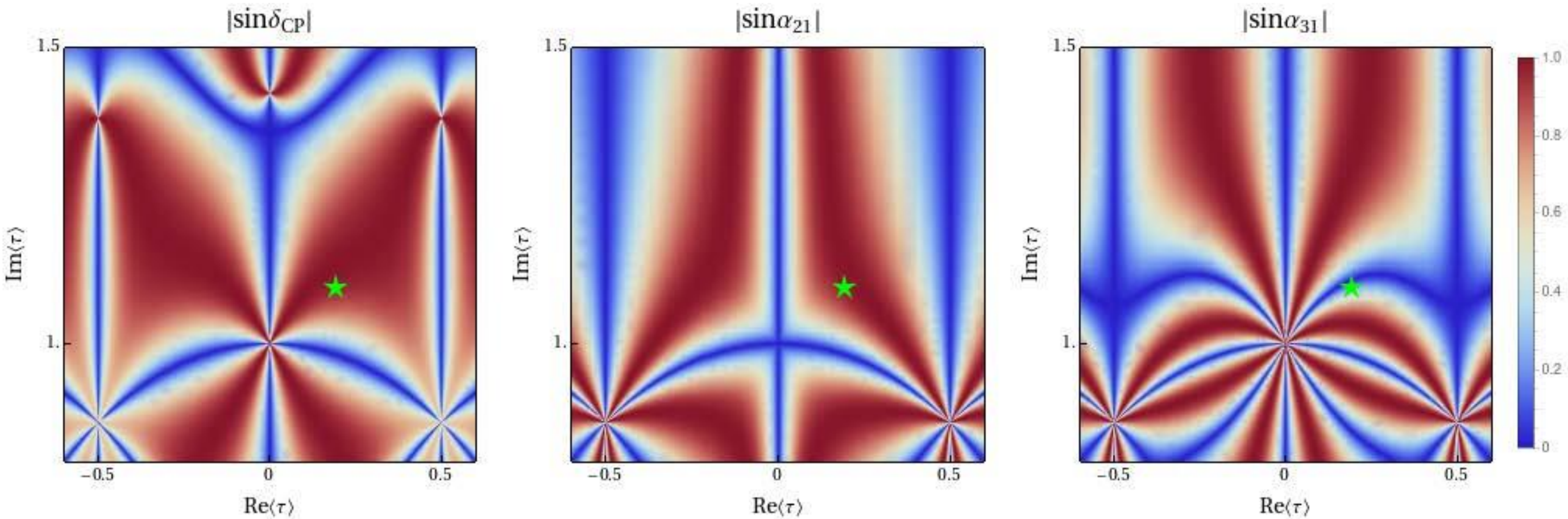
✓ The best fit value $\tau \simeq 0.19+1.09i$ is close to the self-dual point $\tau=i$



$$\sin^2\theta_{23} \sim 0.58,\quad \delta \sim 1.42\pi,\quad \alpha_{21} \sim 1.43\pi,\quad \alpha_{31} \sim 0.92\pi$$
$$\sum_i m_i \sim 76.18\,\text{meV},\quad m_{\beta\beta} \sim 8.57\text{meV},\quad \chi^2 \sim 4.87$$

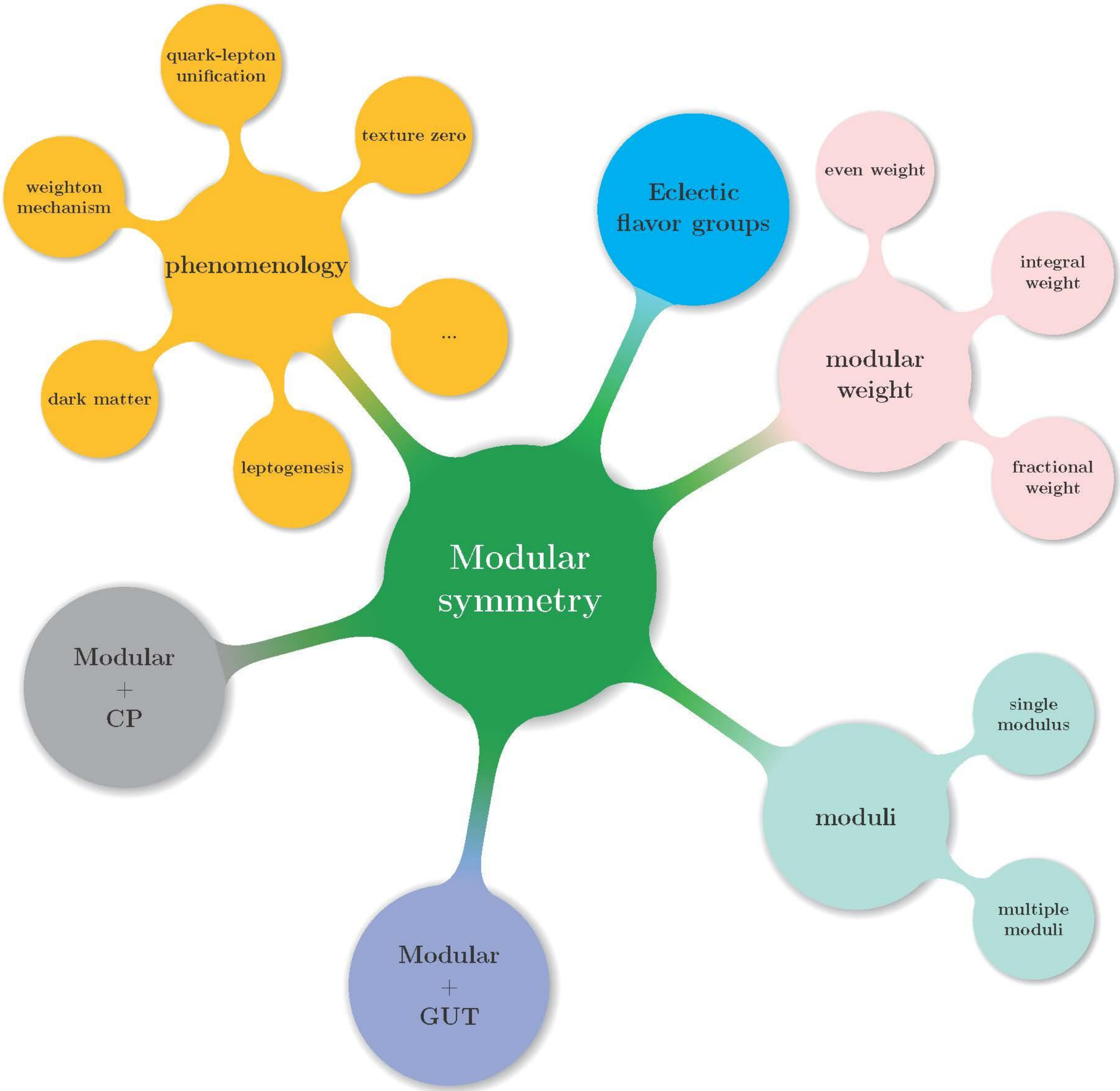
[Yao, Lu, Ding, 2012.13390]

Numerical results for CP violation phases:



CP is conserved for modulus τ imaginary or at the border of the fundamental domain, a small deviation can generate large CP violation.

Further development of modular symmetry



Baur, Chen, Criado, Ding,Feruglio, King, Kobayashi, Li, Liu, Lu,Nilles, Nomura , Novichkov, Okada, Penedo, Petcov, Ramos-Sanchez, Ratz, Y.Shimizu, Tanimoto, Tatsuishi, Titov, Trautner, Uemura, Varzielas , Vaudrevange, Wang,Yao, Yu, Zhang, Zhou...

Summary

- flavour symmetries are a useful tool to understand the origin flavor mixing and CP violation, but no compelling and unique picture have emerged so far.

- More precise neutrino data calls for convincing model of neutrino masses and mixings, with testable and confirmed predictions. Modular symmetry is a new promising approach to the flavor puzzles. Modular symmetry is still at the early stage of its development, many aspects still need to be understood.

- Future experimental data on θ_{23}, $\delta_{\rm CP}$, the effective Majorana mass $m_{\beta\beta}$ can exclude many models, will provide important hints for the underlying principle.