Effect of finite system size on the thermodynamics of hot and magnetized hadron resonance gas

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Abstract

The thermodynamic properties of a non-interacting ideal Hadron Resonance Gas (HRG) of finite volume have been studied in the presence of an external magnetic field. The inclusion of background magnetic field in the calculation of thermodynamic potential is done by the modification of the dispersion relations of charged hadrons in terms of Landau quantization. The generalized Matsubara prescription has been employed to take into account the finite size effects in which a periodic (anti-periodic) boundary conditions is considered for the mesons (baryons). We find significant effects of the magnetic field as well as system size on the temperature dependence of energy density, longitudinal and transverse pressure especially in low temperature regions.

Introduction

Since few decades, considerable amount of research interest has been grown on the study of hot and/or dense 'strongly' interacting matter produced in the heavy ion collision (HIC) experiments at RHIC and LHC. On top of that, recently, another contemporary research topic is the investigation of the effect of a strong background magnetic field on various properties of QCD matter at extreme condition of high temperature and/or baryon density. Interestingly, a non-central or asymmetric HIC at RHIC and LHC energies has the potential to create strong magnetic field of the order of $\sim 10^{18}$ Gauss or more. As the magnitude of the magnetic field is comparable to QCD energy scale, various novel phenomena owing to the rich vacuum structure of QCD could take place [1] such as chiral magnetic effect, magnetic catalysis, inverse magnetic catalysis etc.

Through the HIC experiments, it is possible to probe the bulk thermodynamic properties or the phase structure of QCD. The non-perturbative aspects of QCD restrict a first principle analytic calculation of the QCD thermodynamics especially in the low temperature region. The numerical lattice QCD (LQCD) based calculations [2] is one of the best alternatives to study the QCD thermodynamics, but is limited to the low baryon density region of the QCD phase diagram due to its 'sign' problem. On the other hand, the hadron resonance gas (HRG) model [3] is a statistical thermal model for studying the QCD thermodynamics at finite temperature, baryon density as well as external magnetic field [4, 5]. Interestingly, at low temperature and small baryon density, the results from HRG model agrees well with the LQCD.

In the calculation of thermodynamic quantities, one generally assumes the system size to be infinite. However, in the HIC experiments, the created fireball has finite volume ($\sim$ few fm^3). So, it is justified to consider the boundary effects in the calculation of thermodynamical quantities pertaining to the HIC [6]. In this presentation, we will be showing the calculation of various thermodynamic quantities like energy density, longitudinal and transverse pressure and magnetization of an ideal HRG of finite size in presence of external magnetic field. The formalism of generalized Matsubara prescription [7] will be used to incorporate the finite size effect whereas the effect of external magnetic field will enter through the Landau quantization of the dispersion relations of charged hadrons.

HRG at $B = 0$ and $L = \infty$

Let us start with the standard expression of the thermodynamic potential (density) Ω of an ideal HRG at zero-magnetic field in infinite volume as [6]

$$\Omega = -T \sum_{i\in\{\text{hadrons}\}} g_i a_i \int \frac{d^3k}{(2\pi)^3} \ln\left(1 + a_i e^{-\beta\omega^i_k}\right) \tag{1}$$

where $g_i = (2s_i + 1)$ is the spin degeneracy of hadron i having spin s_i,

$$a_i = -(-1)^{2s_i} = \begin{cases} +1 & \text{if } i \in \{\text{baryons}\} \text{ (half-integer spin)} \\ -1 & \text{if } i \in \{\text{mesons}\} \text{ (integer spin),} \end{cases} \tag{2}$$

$\beta = 1/T$ is the inverse temperature and $\omega^i_k = \sqrt{\vec{k}^2 + m_i^2}$ is the single particle energy of hadron i having mass m_i.

From Eq. (1), all the other thermodynamic quantities can be calculated. For example, the iostropic pressure is $P = -\Omega$, energy density (ε) is

$$\varepsilon = -T^2 \frac{\partial}{\partial T}\left(\frac{\Omega}{T}\right) = \sum_{i\in\{\text{hadrons}\}} g_i \int \frac{d^3k}{(2\pi)^3} \frac{\omega^i_k}{e^{\beta\omega^i_k} + a_i}, \tag{3}$$

and the entropy density is $s = (\varepsilon + P)/T$ etc.

HRG at $B \neq 0$ and $L = \infty$

Let us now consider an external magnetic field $\vec{B} = B\hat{z}$ in the positive-$\hat{z}$ direction. The single particle energies of charged hadrons will now be Landau quantized as

$$\omega^i_{kls} = \sqrt{k_z^2 + \{2l + 1 - 2s\,\text{sign}(e_i)\}|e_i|B + m_i^2}$$
$$\text{with } s = -s_i, -s_i + 1, \cdots, s_i \text{ and } l = 0, 1, 2, \cdots \tag{4}$$

where, e_i is the electronic charge of hadron i. It is to be noted that, in Eq. (4), l is related to the orbital angular momentum quantum number and not the Landau level index in the usual sense though they are inter-connected. To obtain the expressions of the thermodynamic quantities at $B \neq 0$, we make the following standard replacement of the phase-space integral as

$$\int \frac{d^3k}{(2\pi)^3} f(\omega^i_k) \to \sum_{l=0}^{\infty} \sum_{s=-s_i}^{s_i} \frac{|e_i|B}{2\pi} \int_{-\infty}^{\infty} \frac{dk_z}{2\pi} f(\omega^i_{kls}). \tag{5}$$

With the above replacement, the thermodynamic potential of ideal HRG of Eq. (1) in presence of external magnetic field modifies to

$$\Omega_B = \Omega_{\substack{\text{neutral}\\\text{hadrons}}} - T \sum_{i\in\left\{\substack{\text{charged}\\\text{hadrons}}\right\}} a_i \sum_{l=0}^{\infty} \sum_{s=-s_i}^{s_i} \frac{|e_i|B}{2\pi} \int_{-\infty}^{\infty} \frac{dk_z}{2\pi} \ln\left(1 + a_i e^{-\beta\omega^i_{kls}}\right) \tag{6}$$

where, the first term on the right hand side (RHS) corresponds to the contribution from the neutral hadrons which are not affected by the external magnetic field in the leading order.

In the presence of external magnetic field, the pressure becomes anisotropic and have different values along the longitudinal and transverse direction with respect to the direction of the magnetic field [8]. The longitudinal pressure is obtained from $P_\parallel = -\Omega_B$ whereas the transverse pressure is $P_\perp = P_\parallel - MB$ where $M = \left(\frac{\partial P_\parallel}{\partial B}\right)$ is the magnetization of the system. Thus, the explicit expression of transverse pressure comes out to be

$$P_\perp = P_{\substack{\text{neutral}\\\text{hadrons}}} + \sum_{i\in\left\{\substack{\text{charged}\\\text{hadrons}}\right\}} \sum_{l=0}^{\infty} \sum_{s=-s_i}^{s_i} \frac{|e_i|B}{2\pi} \int_{-\infty}^{\infty} \frac{dk_z}{2\pi} \frac{1}{2\omega^i_{kls}} \{2l + 1 - 2s\,\text{sign}(e_i)\}|e_i|B \times \frac{1}{e^{\omega^i_{kls}} + a_i} \tag{7}$$

where the first term on the RHS corresponds to the contribution of isotropic pressure from the neutral hadrons. All the other thermodynamic quantities of interest can be calculated from Eq. (6). The energy density in this case is given by

$$\varepsilon = \varepsilon_{\substack{\text{neutral}\\\text{hadrons}}} + \sum_{i\in\left\{\substack{\text{charged}\\\text{hadrons}}\right\}} \sum_{l=0}^{\infty} \sum_{s=-s_i}^{s_i} \frac{|e_i|B}{2\pi} \int_{-\infty}^{\infty} \frac{dk_z}{2\pi} \frac{\omega^i_{kls}}{e^{\omega^i_{kls}} + a_i} \tag{8}$$

where the first term on the RHS again corresponds to the contribution to energy density from the neutral hadrons.

HRG with Finite Size

Till now, we have considered the system size to be infinite. To take into account the finite-volume effect in HRG thermodynamics, we employ the formalism of generalized Matsubara prescription as discussed in Ref. [7]. For this, we consider our system to be a cube of length L so that, the spatial coordinates lie in the interval $x^i \in [0, L]$. As a consequence of the generalized Matsubara prescription [9, 10], the momentum integral at $B = 0$ will have to be replaced with sum over discrete Matsubara modes as

$$\int \frac{d^3k}{(2\pi)^3} f(\vec{k}) \to \frac{1}{L^3} \sum_{n_x=-\infty}^{\infty} \sum_{n_y=-\infty}^{\infty} \sum_{n_z=-\infty}^{\infty} f(\vec{k}_{n_x n_y n_z}) \tag{9}$$

where,

$$\vec{k}_{n_x n_y n_z} = \frac{2\pi}{L}\left[(n_x + b)\hat{x} + (n_y + b)\hat{y} + (n_z + b)\hat{z}\right] \tag{10}$$

in which the parameter b can be chosen appropriately to consider periodic or anti-periodic boundary condition in the compactified spatial coordinates. It is well known from the Kubo-Martin-Schwinger (KMS) relation [9] in the context of finite temperature field theory, that the imaginary time coordinate ($\tau = it$) becomes periodic (anti-periodic) for Bosonic (Fermionic) system.

However, no restrictions are applied for the compactified spatial coordinates. Though, the Lattice QCD calculations [11] generally employ the periodic boundary condition on the spatial coordinates of Fermions, yet other work on QCD effective models [12, 13] takes the identical boundary conditions in the temporal as well as spatial coordinates. Hence, following Ref. [7], in this work we consider periodic boundary condition for the mesons and anti-periodic boundary condition for the baryons. Hence, in our calculation, we choose the parameter b in Eq. (10) as

$$b = \begin{cases} 0 & \text{for Mesons,} \\ 1/2 & \text{for Baryons.} \end{cases} \tag{11}$$

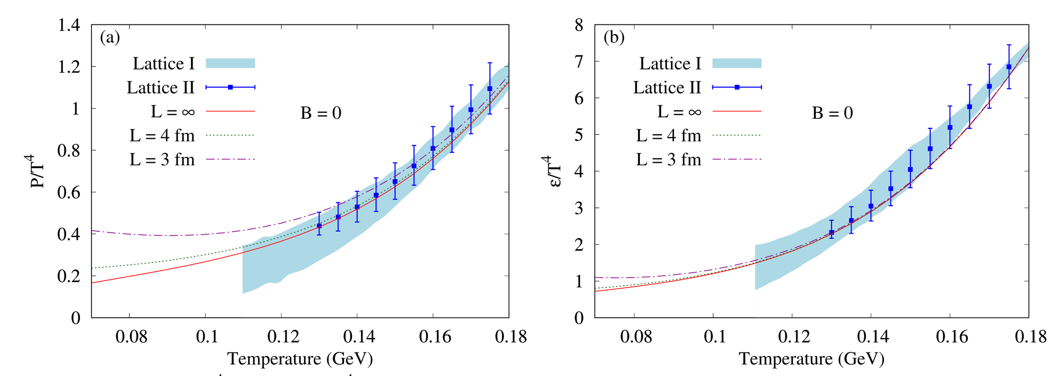
For, non-zero external magnetic field, due to dimensional reduction, the transverse momentum is already Landau quantized. Thus, the following Matsubara prescription has been used for a magnetized HRG with finite system

$$\int_{-\infty}^{\infty} \frac{dk_z}{2\pi} f(k_z) \to \frac{1}{L} \sum_{n_z=-\infty}^{\infty} f(k_{z;n_z}) \tag{12}$$

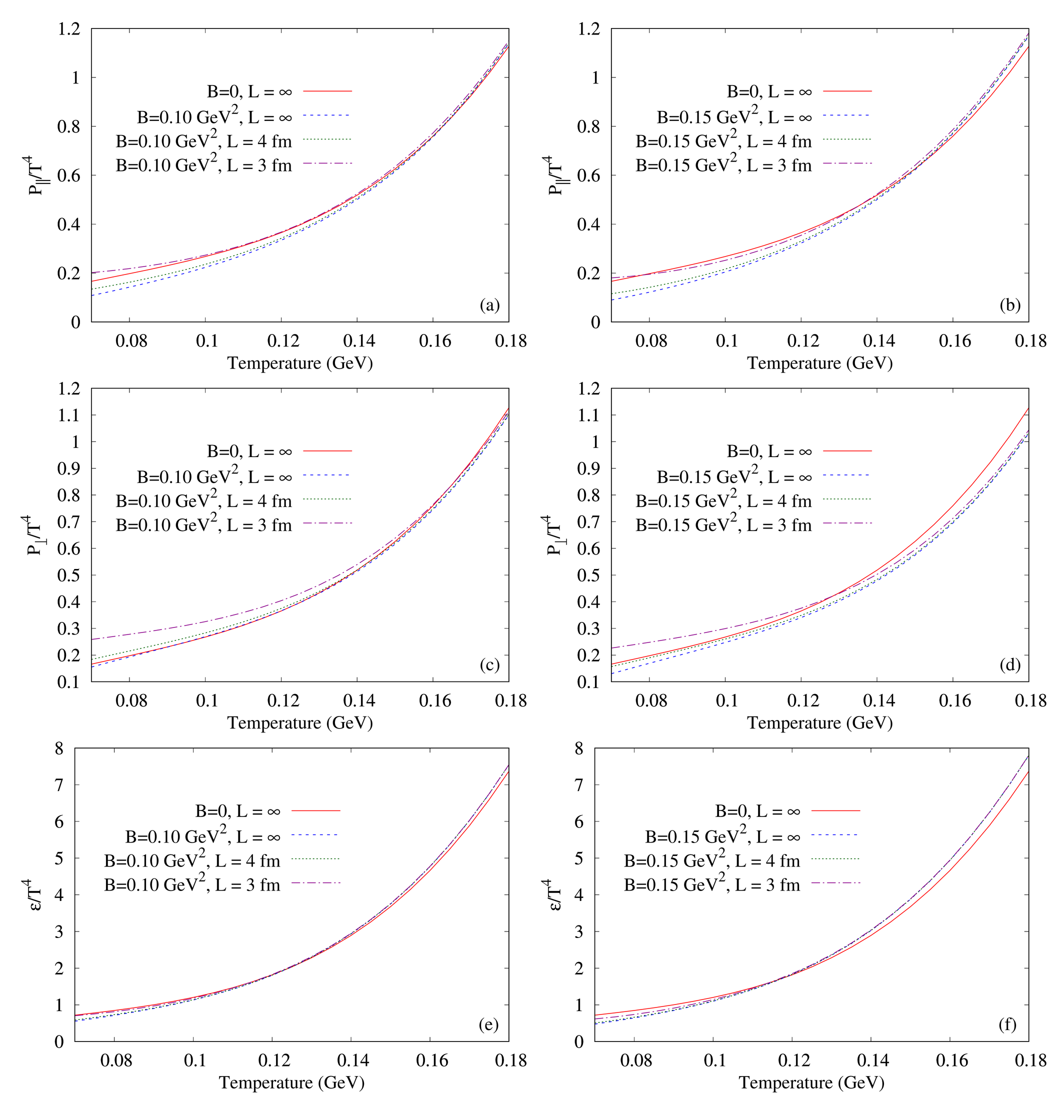
where,

$$k_{z;n_z} = \frac{2\pi}{L}(n_z + b). \tag{13}$$

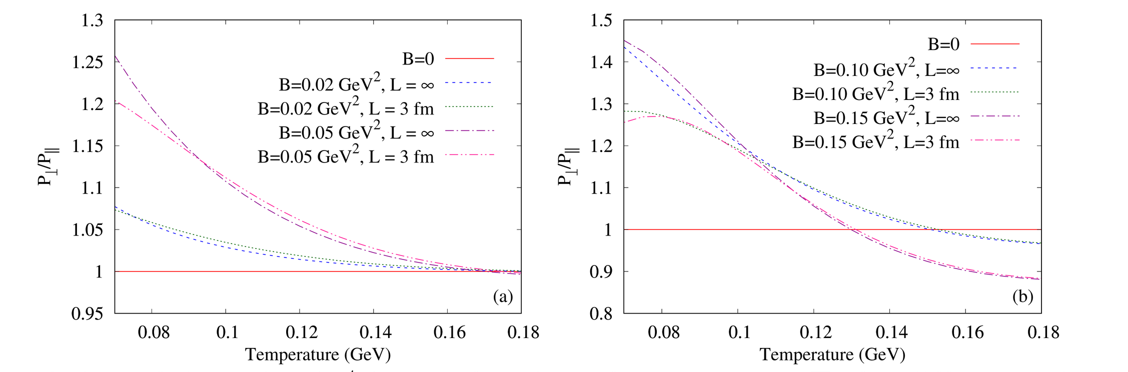
Numerical Results



The variation of P/T^4 and ε/T^4 as a function of temperature (T) for different values of system size (L) at $B = 0$. For comparison, the results of Lattice QCD calculations from Ref. [14] and Ref. [15] are also shown as Lattice I and Lattice II respectively.



The variation of $P_\parallel/T^4$, $P_\perp$ and ε/T^4 as a function of temperature (T) for different values of system size (L) and magnetic field B.



The variation of the ratio $P_\perp/P_\parallel$ as a function of temperature (T) for different values of system size (L) and magnetic field B.

Summary & Discussions

In summary, we have calculated the energy density, pressure and magnetization of a non-interacting ideal HRG having finite volume in the presence of a magnetic background. The background magnetic field modifies the energy dispersion relation of the charged hadrons owing to Landau quantization. Whereas, the restrictions in spatial dimension quantizes the three momenta of all the hadrons in terms of generalized Matsubara frequencies. We have used a periodic (anti-periodic) boundary conditions for the mesonic (baryonic) Matsubara modes. The current study is important in the context of HIC experiments, where the created fireball has finite volume ($\sim$ few fm^3) and is exposed to a very strong external magnetic field. We find, the magnetic field makes the pressure of the system anisotropic, and the pressure has different values in the longitudinal and transverse direction. Moreover, significant effects of the magnetic field as well as system size on the temperature dependence of energy density, longitudinal and transverse pressure are observed. These effects are more in low temperature regions.

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