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Flow fluctuation studies using a multiharmonic cumulant analysis

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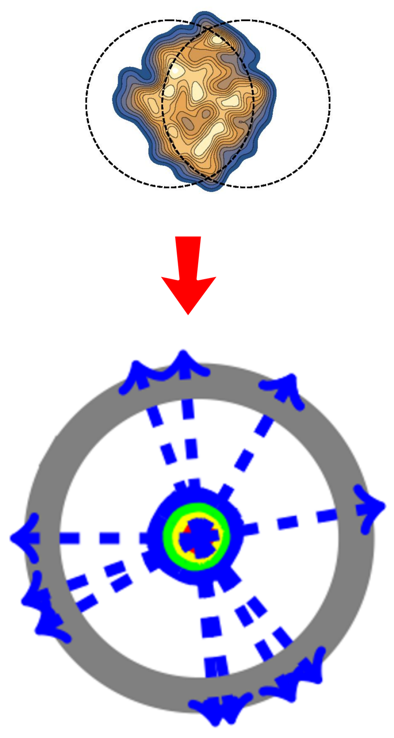
Introduction

- Significant progress has happened in modeling heavy-ion collisions.
- A typical model contains 10 to 20 parameters, approximately. Two most interesting ones are η/s and ζ/s.

- It is important to introduce new experimental observables.

Flow harmonics in a nutshell!



- $\frac{dN}{d\varphi} \propto 1 + \sum_{n=1}^{\infty} 2\, v_n \cos[n(\varphi - \psi_n)]$
- Flow harmonic fluctuation distribution:
- $p_f(v_1, v_2, v_3, \ldots, \psi_1 - \psi_2, \psi_2 - \psi_3, \ldots)$
- Some examples for the distribution's cumulant: [1]
- $c_n\{2k\}$, $\mathrm{SC}(n,m)$, $\mathrm{SC}(n,m,\ell)$

One package for all cumulants

- Mathematica package **MultiharmonicCumulants_v2_1.m**

https://github.com/FaridTaghavi/MultiharmonicCumulants.git

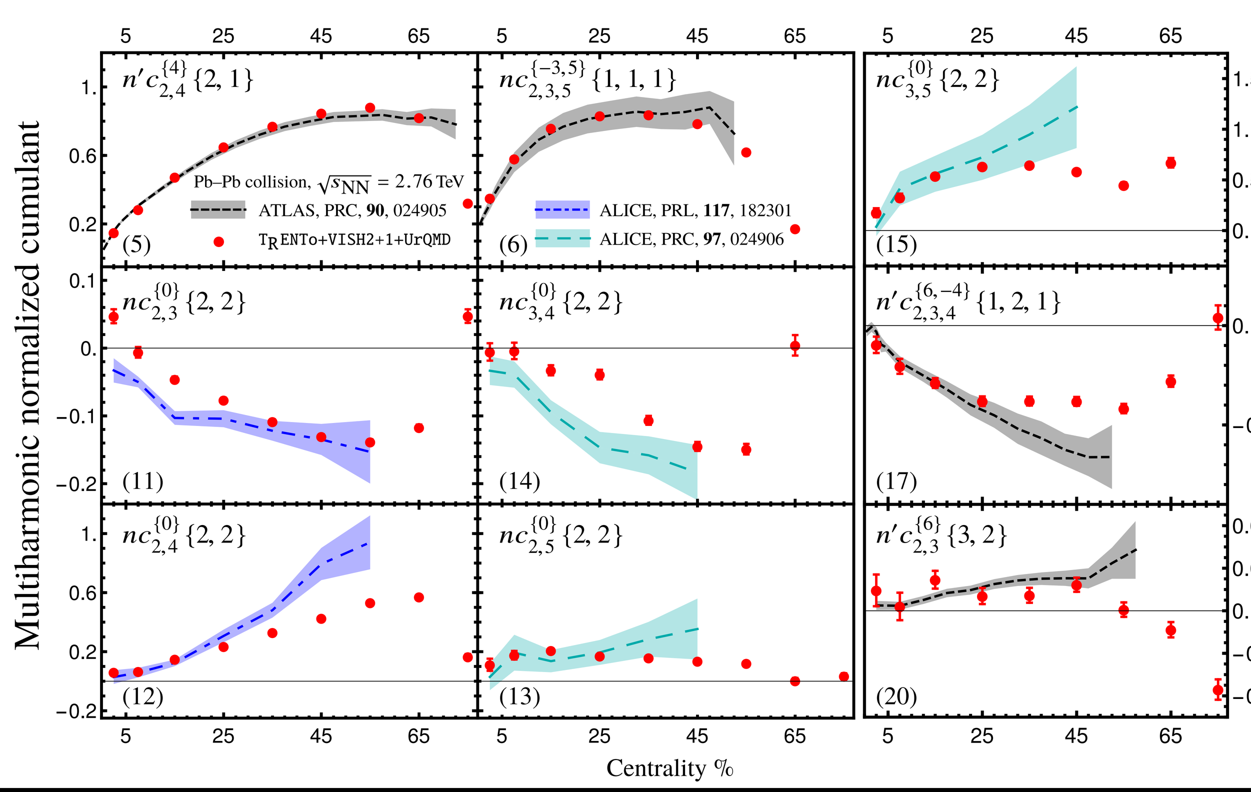
- Returns the cumulants in terms of correlation functions, Q-vectors, and their statistical uncertainty relations.
- Example: $\mathrm{SC}(2,3)$, statistical error of $c_2\{2\}$

$\text{In}[1] := \mathrm{c}[\{2,2\},\{0\},\{2,3\},v,\psi]$, $\text{In}[2] := \mathrm{Nsigma2}[\mathrm{cCorr}[\{2\},\{\},\{2\},\mathrm{corr}]]$

$\text{Out}[1] := \langle v_2^2 v_3^2\rangle - \langle v_2^2\rangle\langle v_3^2\rangle$, $\text{Out}[2] := \langle\langle 2\rangle^2_{-2,2}\rangle - \langle\langle 2\rangle_{-2,2}\rangle^2$

Normalized cumulant at the LHC

- From 29 normalized cumulants, nine of them have been measured.

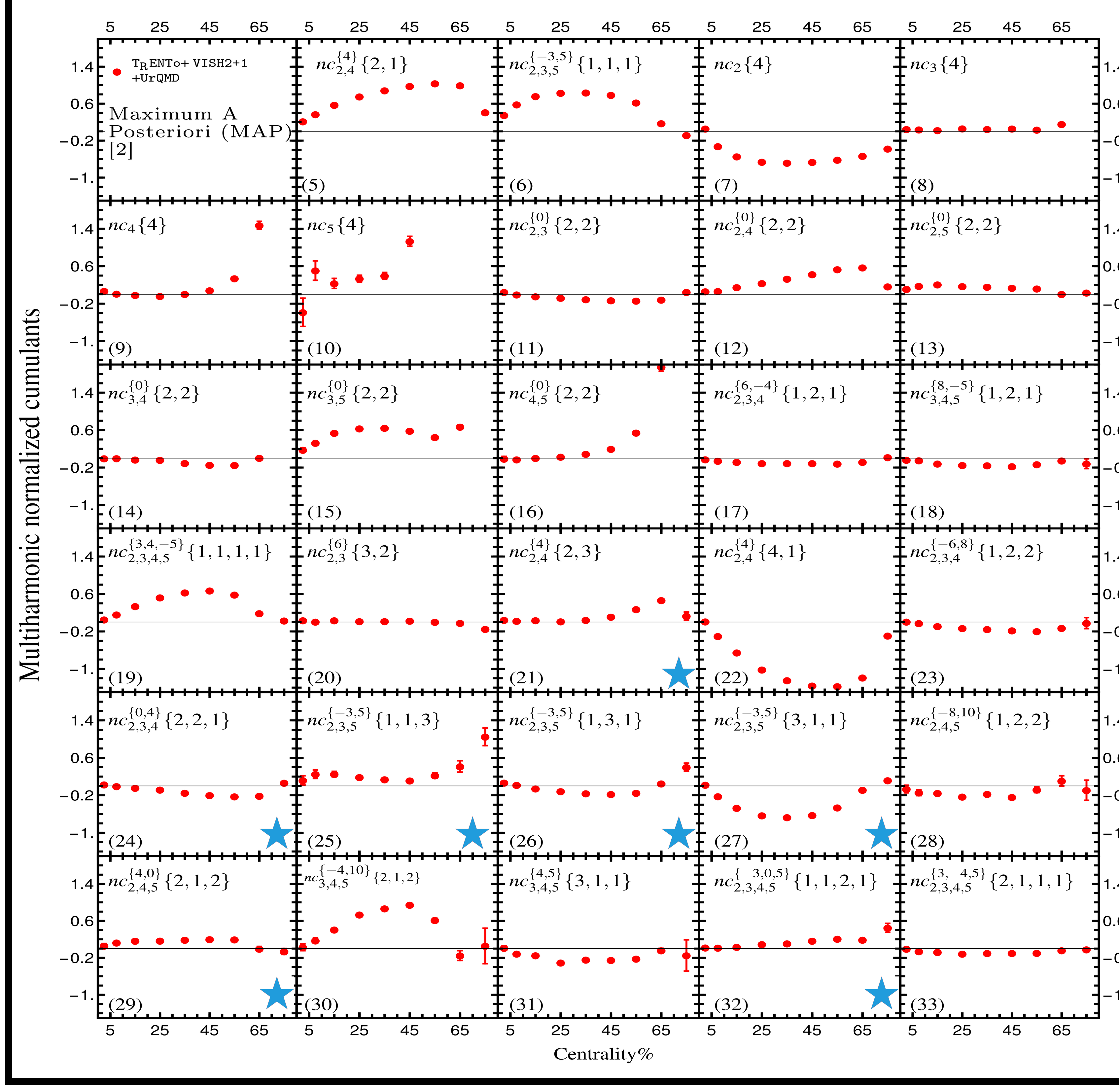


All cumulants: harmonics 2,3,4,5 and orders 2,3,4,5

- There are 33 distinct cumulants; seven of them have been missed in previous theoretical and experimental studies (marked with **blue arrows**).

	cumulant	order	cumulant expression
1	$c_2\{2\}$	2	$\langle v_2^2\rangle$
2	$c_3\{2\}$	2	$\langle v_3^2\rangle$
3	$c_4\{2\}$	2	$\langle v_4^2\rangle$
4	$c_5\{2\}$	2	$\langle v_5^2\rangle$
5	$c_{2,4}^{\{4\}}\{2,1\}$	3	$\langle v_2^2 v_4 \cos(4(\psi_2-\psi_4))\rangle$
6	$c_{2,3,5}^{\{-3,5\}}\{1,1,1\}$	3	$\langle v_2 v_3 v_5 \cos(2\psi_2+3\psi_3-5\psi_5)\rangle$
7	$c_2\{4\}$	4	$\langle v_2^4\rangle - 2\langle v_2^2\rangle^2$
8	$c_3\{4\}$	4	$\langle v_3^4\rangle - 2\langle v_3^2\rangle^2$
9	$c_4\{4\}$	4	$\langle v_4^4\rangle - 2\langle v_4^2\rangle^2$
10	$c_5\{4\}$	4	$\langle v_5^4\rangle - 2\langle v_5^2\rangle^2$
11	$c_{2,3}^{\{0\}}\{2,2\}$	4	$\langle v_2^2 v_3^2\rangle - \langle v_2^2\rangle\langle v_3^2\rangle$
12	$c_{2,4}^{\{0\}}\{2,2\}$	4	$\langle v_2^2 v_4^2\rangle - \langle v_2^2\rangle\langle v_4^2\rangle$
13	$c_{2,5}^{\{0\}}\{2,2\}$	4	$\langle v_2^2 v_5^2\rangle - \langle v_2^2\rangle\langle v_5^2\rangle$
14	$c_{3,4}^{\{0\}}\{2,2\}$	4	$\langle v_3^2 v_4^2\rangle - \langle v_3^2\rangle\langle v_4^2\rangle$
15	$c_{3,5}^{\{0\}}\{2,2\}$	4	$\langle v_3^2 v_5^2\rangle - \langle v_3^2\rangle\langle v_5^2\rangle$
16	$c_{4,5}^{\{0\}}\{2,2\}$	4	$\langle v_4^2 v_5^2\rangle - \langle v_4^2\rangle\langle v_5^2\rangle$
17	$c_{2,3,4}^{\{6,-4\}}\{1,2,1\}$	4	$\langle v_3^2 v_2 v_4 \cos(2(\psi_2-3\psi_3+2\psi_4))\rangle$
18	$c_{3,4,5}^{\{8,-5\}}\{1,2,1\}$	4	$\langle v_4^2 v_3 v_5 \cos(3\psi_3-8\psi_4+5\psi_5)\rangle$
19	$c_{2,3,4,5}^{\{3,4,-5\}}\{1,1,1,1\}$	4	$\langle v_2 v_3 v_4 v_5 \cos(2\psi_2-3\psi_3-4\psi_4+5\psi_5)\rangle$
20	$c_{2,3}^{\{6\}}\{3,2\}$	5	$\langle v_2^3 v_3^2 \cos(6(\psi_2-\psi_3))\rangle$
21	$c_{2,4}^{\{4\}}\{2,3\}$	5	$\langle v_2^2 v_4^3 \cos(4(\psi_2-\psi_4))\rangle - 2\langle v_4^2\rangle\langle v_2^2 v_4 \cos(4(\psi_2-\psi_4))\rangle$
22	$c_{2,4}^{\{4\}}\{4,1\}$	5	$\langle v_2^4 v_4 \cos(4(\psi_2-\psi_4))\rangle - 3\langle v_2^2\rangle\langle v_2^2 v_4 \cos(4(\psi_2-\psi_4))\rangle$
23	$c_{2,3,4}^{\{-6,8\}}\{1,2,2\}$	5	$\langle v_4^2 v_3^2 v_2 \cos(2(\psi_2+3\psi_3-4\psi_4))\rangle$
24	$c_{2,3,4}^{\{0,4\}}\{2,2,1\}$	5	$\langle v_3^2 v_2^2 v_4 \cos(4(\psi_2-\psi_4))\rangle - \langle v_3^2\rangle\langle v_2^2 v_4 \cos(4(\psi_2-\psi_4))\rangle$
25	$c_{2,3,5}^{\{-3,5\}}\{1,1,3\}$	5	$\langle v_5^3 v_2 v_3 \cos(2\psi_2+3\psi_3-5\psi_5)\rangle - 2\langle v_5^2\rangle\langle v_2 v_3 v_5 \cos(2\psi_2+3\psi_3-5\psi_5)\rangle$
26	$c_{2,3,5}^{\{-3,5\}}\{1,3,1\}$	5	$\langle v_3^3 v_2 v_5 \cos(2\psi_2+3\psi_3-5\psi_5)\rangle - 2\langle v_3^2\rangle\langle v_2 v_3 v_5 \cos(2\psi_2+3\psi_3-5\psi_5)\rangle$
27	$c_{2,3,5}^{\{-3,5\}}\{3,1,1\}$	5	$\langle v_2^3 v_3 v_5 \cos(2\psi_2+3\psi_3-5\psi_5)\rangle - 2\langle v_2^2\rangle\langle v_2 v_3 v_5 \cos(2\psi_2+3\psi_3-5\psi_5)\rangle$
28	$c_{2,4,5}^{\{-8,10\}}\{1,2,2\}$	5	$\langle v_5^2 v_4^2 v_2 \cos(2(\psi_2+4\psi_4-5\psi_5))\rangle$
29	$c_{2,4,5}^{\{4,0\}}\{2,1,2\}$	5	$\langle v_5^2 v_2^2 v_4 \cos(4(\psi_2-\psi_4))\rangle - \langle v_2^2 v_4 \cos(4(\psi_2-\psi_4))\rangle\langle v_5^2\rangle$
30	$c_{3,4,5}^{\{-4,10\}}\{2,1,2\}$	5	$\langle v_5^2 v_3^2 v_4 \cos(6\psi_3+4\psi_4-10\psi_5)\rangle$
31	$c_{3,4,5}^{\{4,5\}}\{3,1,1\}$	5	$\langle v_3^3 v_4 v_5 \cos(9\psi_3-4\psi_4-5\psi_5)\rangle$
32	$c_{2,3,4,5}^{\{-3,0,5\}}\{1,1,2,1\}$	5	$\langle v_4^2 v_2 v_5 v_3 \cos(2\psi_2+3\psi_3-5\psi_5)\rangle - \langle v_4^2\rangle\langle v_2 v_3 v_5 \cos(2\psi_2+3\psi_3-5\psi_5)\rangle$
33	$c_{2,3,4,5}^{\{3,-4,5\}}\{2,1,1,1\}$	5	$\langle v_2^2 v_3 v_4 v_5 \cos(4\psi_2-3\psi_3+4\psi_4-5\psi_5)\rangle$

- normalized cumulant $=$ cumulant$/\sqrt{c_{n_1}^{m_1}\{2\}\cdots c_{n_k}^{m_k}\{2\}}$
  n_i : the involving harmonics in the cumulant.
  m_i : the power of the flow amplitude v_{n_i} in the cumulant.
- We have 29 distinct normalized cumulants; new cumulants are marked with **blue asterisks**.



References

[1] N. Borghini, P. M. Dinh and J. Y. Ollitrault, PRC **64**, 054901 (2001); A. Bilandzic, C. H. Christensen, K. Gulbrandsen, A. Hansen and Y. Zhou, PRC **89**, no.6, 064904 (2014); C. Mordasini, A. Bilandzic, D. Karakoç and S. F. Taghavi, PRC **102**, no.2, 024907 (2020); S. Acharya *et al.* [ALICE], [arXiv:2101.02579 [nucl-ex]].

[2] J. E. Bernhard, J. S. Moreland and S. A. Bass, Nature Phys. **15**, no.11, 1113-1117 (2019)

Summary and outlook

- We introduced a systematic method to extract flow harmonic cumulants and its statistical uncertainties up to a given order.
- What are the experimental values for unmeasured normalized cumulants? Can the new cumulants help the Bayesian analysis?