Chiral Symmetry Restoration, thermal resonances and the $U(1)_A$ symmetry

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Ward Identities (WI) provide useful conclusions regarding chiral symmetry restoration, including $U(1)_A$ restoration, a relevant open problem for the QCD phase diagram [1, 2, 3, 6]:

- ⋆ $U(1)_A$ **restored** for **exact chiral** restoration of scalar/pseudoscalar lightest nonet [2, 3]. **OK with** $N_f = 2$ **lattice.**
- ⋆ **Sizable gap for** $N_f = 2 + 1$ **driven by strangeness.**
- ⋆ K/κ **degeneration suitable alternative sign for** $O(4) \times U(1)_A$**. Predicts** χ_S^κ **peak and explains role of strangeness through** $\langle \bar{s}s \rangle$ [6].

$$\chi_P^{ls}(T) = -2\frac{m_l}{m_s}\chi_{5,disc}(T) = -\frac{2}{m_l m_s}\chi_{top}(T)$$

$$\pi^a = i\bar{\psi}_l\gamma_5\tau^a\psi_l \overset{SU(2)_A}{\longleftrightarrow} \sigma_l = \bar{\psi}_l\psi_l$$
$$\updownarrow U(1)_A \qquad \updownarrow U(1)_A$$
$$\delta^a = \bar{\psi}_l\tau^a\psi_l \overset{SU(2)_A}{\longleftrightarrow} \eta_l = i\bar{\psi}_l\gamma_5\psi_l$$

$\chi_P^{ls} = \int_T dx \langle \mathcal{T}\eta_l(x)\eta_s(0)\rangle$

$\chi_{5,disc} = \frac{1}{4}(\chi_P^\pi - \chi_P^\eta)$ **measures** $O(4) \times U(1)_A$ **restoration**

$\chi_{top} = -\frac{1}{36}\int_T dx \langle \mathcal{T}A(x)A(0)\rangle$ **topological susceptibility,** $A(x) = \frac{3\alpha_s}{4\pi}Tr_c G_{\mu\nu}\tilde{G}^{\mu\nu}$

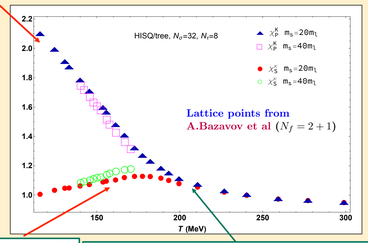
$SU(2)_A$ **transforms** $P_{ls} \longrightarrow \langle \delta\eta_s \rangle = 0$ **by parity**

$$\eta_l \overset{O(4)}{\sim} \delta \Rightarrow \chi_P^{ls} \overset{O(4)}{\sim} 0 \Rightarrow \chi_{5,disc} \overset{O(4)}{\sim} 0,\ \chi_{top} \overset{O(4)}{\sim} 0$$

$\Longrightarrow$ $O(4) \times U(1)_A$ **pattern for** ***exact*** **chiral restoration**
(as for $N_f = 2$**,** $m_l \to 0$**, consistent with lattice)**

Reconstructed K/κ **susceptibilities from WI and lattice condensate data**

$$\chi_P^K(T) = -\frac{\langle \bar{q}q \rangle_l(T) + 2\langle \bar{s}s \rangle(T)}{m_l + m_s}$$

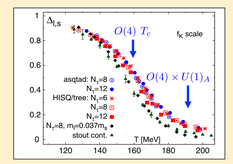


$$\chi_S^\kappa(T) = \frac{\langle \bar{q}q \rangle_l(T) - 2\langle \bar{s}s \rangle(T)}{m_s - m_l}$$

- $\chi_S^\kappa \to \chi_P^K$ **in** $O(4) \times U(1)_A$ **region, above** χ_S^κ **peak**
- **Peak behaviour driven by** m_l/m_s

$$\chi_S^\kappa(T) - \chi_P^K(T) = \frac{2m_s}{m_s^2 - m_l^2}\left[\langle \bar{q}q \rangle_l(T) - 2\frac{m_l}{m_s}\langle \bar{s}s \rangle(T)\right]$$

subtracted condensate $\Delta_{l,s}$
(removes lattice finite-size divergences)



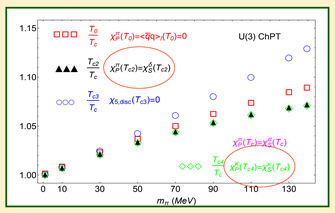
- **Consistent with** $O(4) \times U(1)_A$ **restoration for** $m_l/m_s, \langle \bar{q}q \rangle_l \to 0^+$ $(N_f = 2)$
- **In physical limit provides well-determined** $O(4) \times U(1)_A$ **breaking above** T_c **via** $\Delta_{l,s}$ **tail modulated by** $\langle \bar{s}s \rangle$

Effective Theories allow to check $O(4) \times U(1)_A$ pattern and generate **thermal resonances** dominating most relevant observables [3, 4, 5, 6]:

- ⋆ $U(3)$ **ChPT analysis confirms WI and lattice results** [3, 5, 6].
- ⋆ **Scalar susceptibilities saturated by thermal** $f_0(500)(\sigma)$ **and** $K_0^*(700)(\kappa)$ **reproduce expected peaks consistently with lattice** [4, 6].

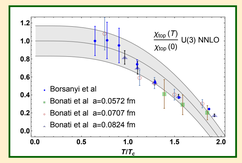
Partners in $U(3)$ **ChPT**

$1/N_c \sim m_q \sim T^2 \sim p^2$ **counting**



$\to O(4) \times U_A(1)$ **in chiral limit**
(note $K - \kappa$ **and** $\pi - \delta$ **coincidence)**

Topological Susceptibility in $U(3)$ **ChPT**



- $T = 0$ **and thermal dependence captured by ChPT within uncertainties**
- **Dominated by light quarks.** K, η, η' **loops and** $\eta - \eta'$ **mixing included.**
- **From WI** $\chi_{top} = -\frac{1}{4}\left[m_{ud}\langle \bar{q}q \rangle_l + m_{ud}^2\chi^{\eta_l}\right]$, T **scaling dominated by** $\langle \bar{q}q \rangle_l$ **at low** T**, but second term relevant near** T_c
  → **consistent with finite** $O(4)$**-**$U(1)_A$ **gap in physical case**

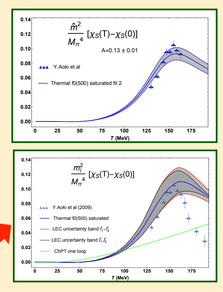
Scalar susceptibility and the (thermal) $\sigma/f_0(500)$ **state**

⇒ χ_S **saturated by lightest** $p = 0$ **state** $f_0(500)$ (σ)

$$\chi_S(T) \simeq \chi_S(0)\frac{M_S^2(0)}{M_S^2(T)}$$

$M_S^2(T) = \mathbf{Re}(s_{pole}(T)) \sim \mathbf{Re}\Sigma_{f_0}(T)$

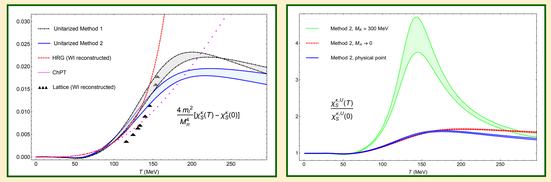
Pole from UChPT $\pi\pi$ **scattering at finite** T
(AGN et al 2002)



⇒ **Reproduces expected peak, OK with lattice**
⇒ **Thermal interactions crucial**
⇒ **Uncertainties within LEC range**

Saturated κ **scalar susceptibility in UChPT**

⇒ χ_S^κ **saturated now by** $K_0^*(700)$ **thermal pole in** πK **scattering**



thermal interactions crucial to reproduce peak

- $m_l/m_s \to 0^+$**:** $\chi_S^\kappa \to \chi_P^K$ **at lower** T
- $m_l/m_s \to 1$**:** $\chi_S^\kappa \to \chi_S^\sigma$ $SU(3)$ **deg,**

References

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[6] A. Gómez Nicola, J. Ruiz de Elvira, A. Vioque-Rodríguez and D. Álvarez-Herrero, "The role of strangeness in chiral and $U(1)_A$ restoration," Eur. Phys. J. C 81, 637 (2021).