Causal Equation of State of Hadron Resonance Gas with Relativistic Excluded Volumes and Its Relation to Morphological Thermodynamics

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Overview

We present a solution to the long-standing problem of constructing the causal equation of state of hadron resonance gas model (HRGM) with Lorentz contracted eigenvolumes of particles with the hard-core repulsion. It is based on the concept of Induced Surface and Curvature Tension (ISCT) [1] to treat the excluded volumes of hard spheres in the high-pressure region. Its mathematically sound and extensive derivation was obtained according to principles of morphological thermodynamics [2]. Following the Hadwiger theorem, the concept of morphological thermodynamics [3] assumes that the change of free energy of a convex rigid body $\mathcal{B}$ immersed into the fluid whose state is away both from the critical point and from wetting and drying transitions can be completely described by four thermodynamic characteristics only: the system pressure p, the mean surface tension coefficient Σ, the mean curvature tension coefficient K and the bending rigidity coefficient ψ, i.e. $-\Delta\Omega = pV_{\mathcal{B}} + \Sigma S_{\mathcal{B}} + KC_{\mathcal{B}} + \psi X_{\mathcal{B}}$. Here the quantities $V_{\mathcal{B}}$, $S_{\mathcal{B}}$, $C_{\mathcal{B}}$ and $X_{\mathcal{B}}$ are, respectively, the volume of the rigid body $\mathcal{B}$, its surface, mean curvature integrated over the surface of $\mathcal{B}$ and the mean Gaussian curvature integrated over the surface of $\mathcal{B}$

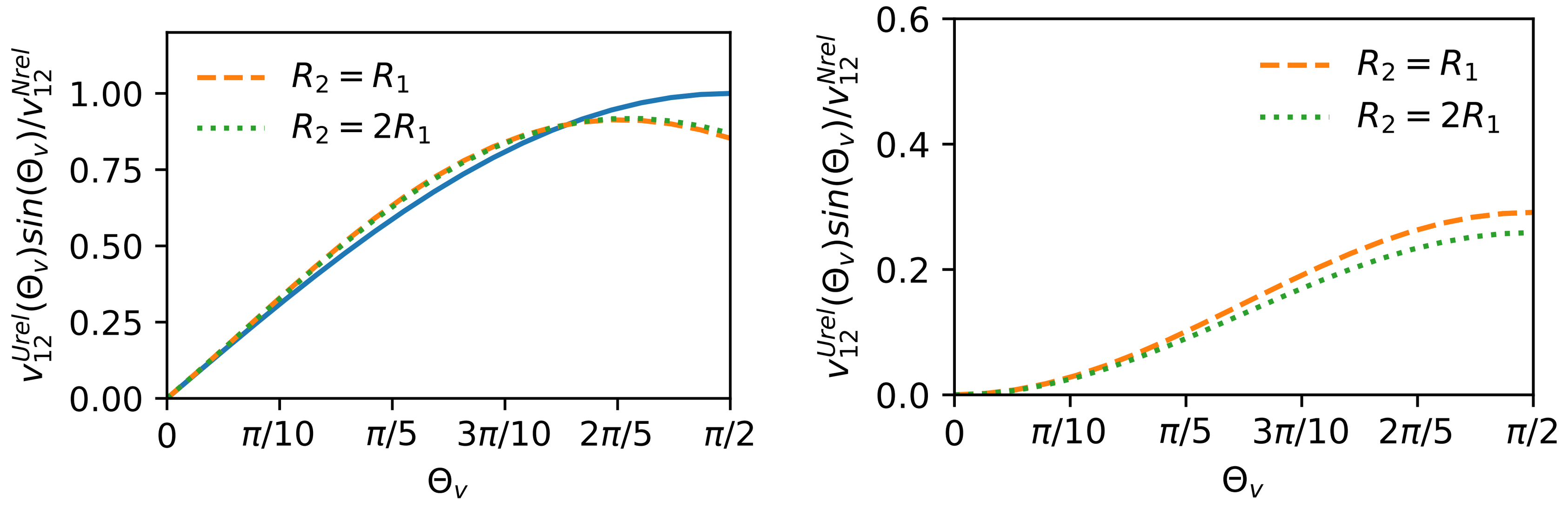

Figure 1: The excluded volume $v_{12}^{Urel}(\Theta_v)\sin(\Theta_v)$ of two Lorentz contracted hard spheres in units of the excluded volume of two nonrelativistic hard spheres v_{12}^{Nrel} for the radii $R_2 = R_1$ fm (dashed curve) and for the radii $R_2 = 2R_1$ fm (dotted curve) as a function of the angle Θ_v between the 3-momentum vectors of the particles. *Left panel* shows the nonrelativistic limit $\gamma_1 = \gamma_2 = 1$ (for two spheres). The solid curve is the exact result for $v_{12}^{Nrel}\sin(\Theta_v)$, while the dashed and dotted curves are obtained approximate formula [4]. *Right panel* shows the ultra-relativistic limit $\gamma_1 = \gamma_2 = 1000$ (for two thin disks).

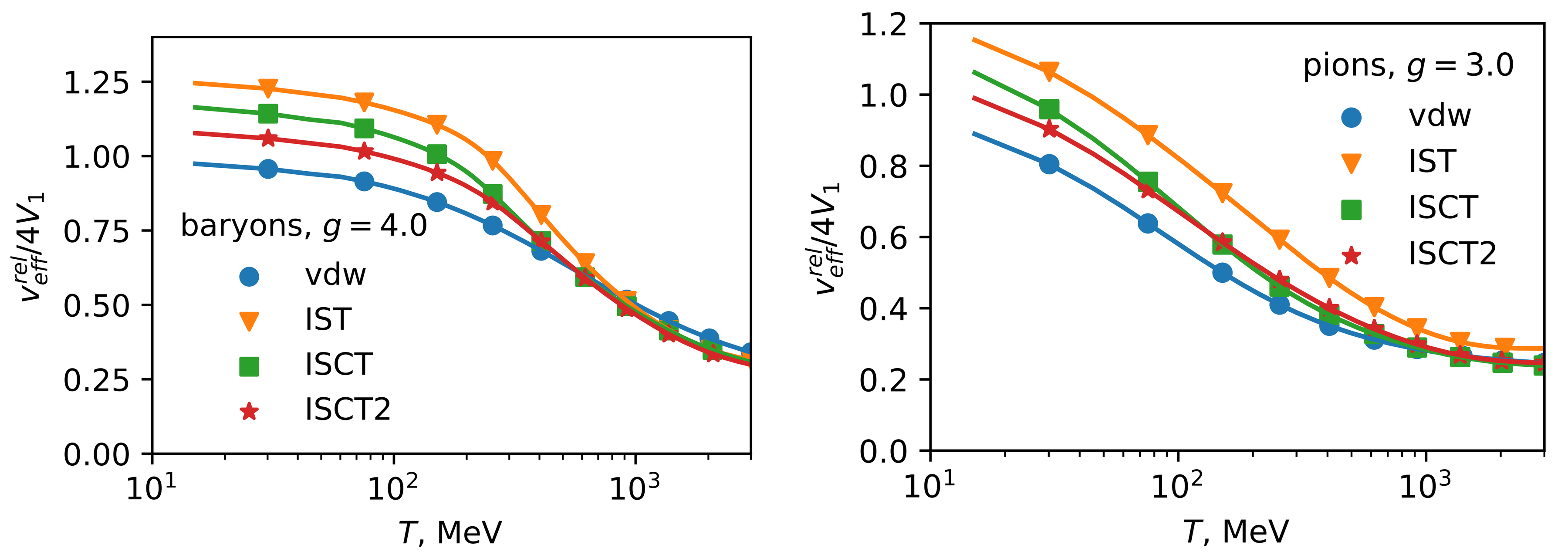

Figure 2: Averaged excluded volume of gas of Lorentz contracted rigid spheres in units of the excluded volume of two non-relativistic hard spheres of radius $R_1 = 0.39$ fm as a function of the temperature of the system T. Various markers correspond to different sets of ISCT EoS parameters. *Left panel:* a gas of baryons (nucleons and anti-nucleons) with masses $m_1 = 940$ MeV and degeneracy factor $g_1 = 4$. *Right panel:* a gas of pions with masses $m_1 = 140$ MeV and degeneracy factor $g_1 = 3$.

which can be defined as $C_{\mathcal{B}} = \int_{\partial\mathcal{B}} d^2r \frac{1}{2}\left[\frac{1}{R_{c1}} + \frac{1}{R_{c2}}\right]$ and $X_{\mathcal{B}} = \int_{\partial\mathcal{B}} d^2r \frac{1}{R_{c1}R_{c2}}$ using local principal curvature radii R_{c1} and R_{c1}. This concept was extended to the grand canonical ensemble of systems with not conserved number of particles. Practically an exact formula for the relativistic second virial coefficient (excluded volume) (see Fig 1) was obtained and investigated for various equations of state and a wide range of temperatures T and was shown that it reproduces a close packing of equal spheres limit $\eta = 1 - \frac{\pi}{3\sqrt{2}} \approx 0.26$ in case of high temperatures with sufficient accuracy without any prior knowledge about such system configuration (see Fig 2). We as well propose an ansatz to take into account the effect of Lorentz contraction for higher-order virial coefficients of Boltzmann particles with hard-core repulsion. Such an ansatz allows us to obtain the expected vanishing limit for the effective relativistic excluded volume for high temperatures $T \gg m$.

Conclusions

The present work is a first and important step to the development of the novel hadron resonance gas model with multicomponent hard-core repulsion which is causal inside the whole hadronic phase of the QCD matter. In contrast to all

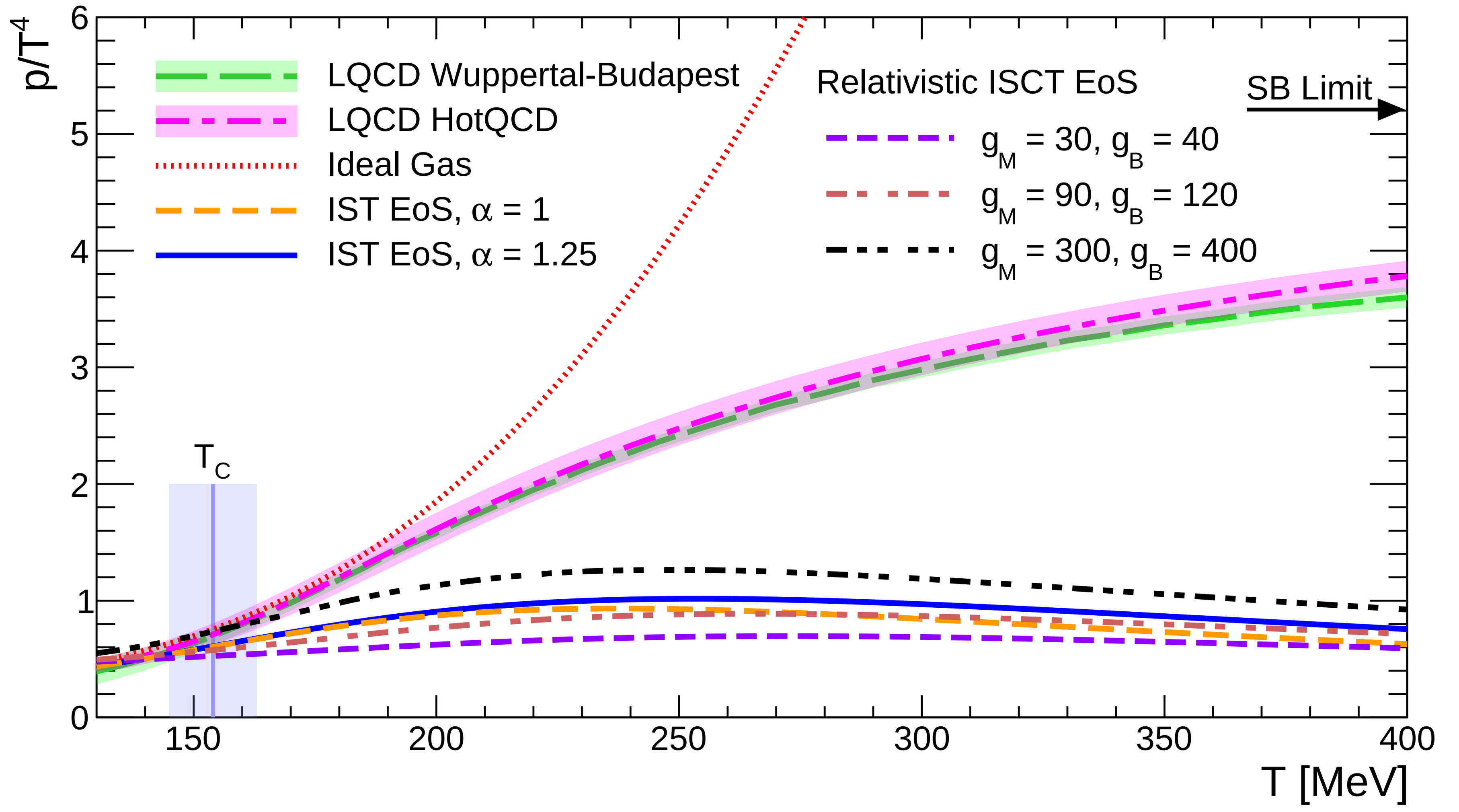

Figure 3: Comparison of the pressure of ISCT EoS for Lorentz contracted hard spheres which is able to mimic the hadrons resonance gas model (solid and dashed curves) with the lattice QCD thermodynamics data (long dashed curve for [5] and the dashed-dotted curve for [6] as the function of temperature at vanishing baryonic chemical potential.

known formulations of the hadronic matter EoS which in some way are taking into account the Lorentz contraction of particle's eigenvolumes, the present formulation correctly reproduces the relativistic excluded volume of two hard-core particles with arbitrary velocities and masses. This automatically provides the correct value of their second virial coefficient, which in the relativistic case differs from the relativistic excluded volume and comparison with lattice QCD thermodynamics is shown in Fig 3.

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