A Large N Expansion for Minimum Bias

Tom Melia (Kavli IPMU)

PANIC 2021, Hot and Dense Matter - QGP and Heavy Ion Collisions

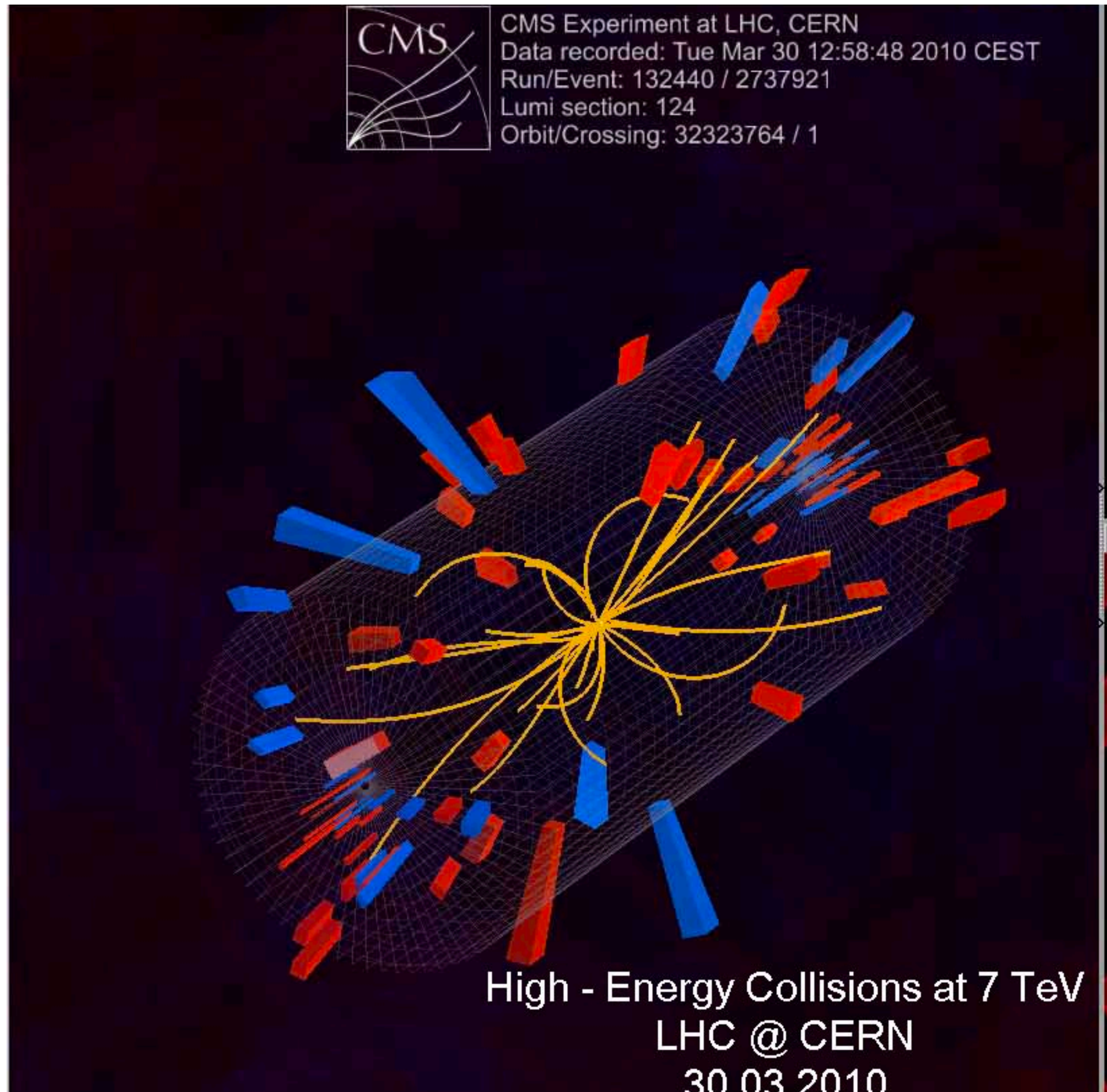
8th September 2021

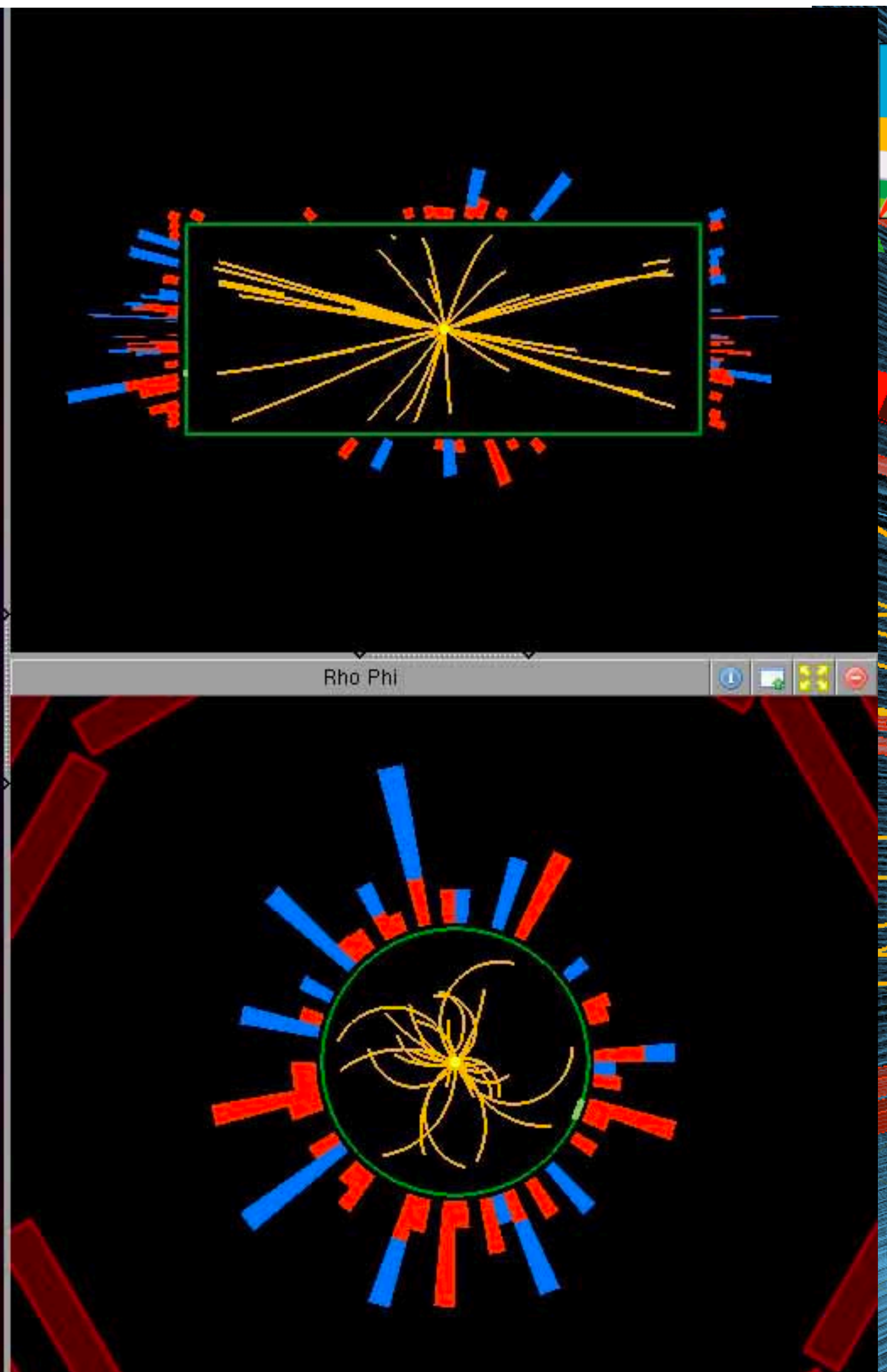
Based on: Andrew Larkoski, TM, arXiv:2107.04041

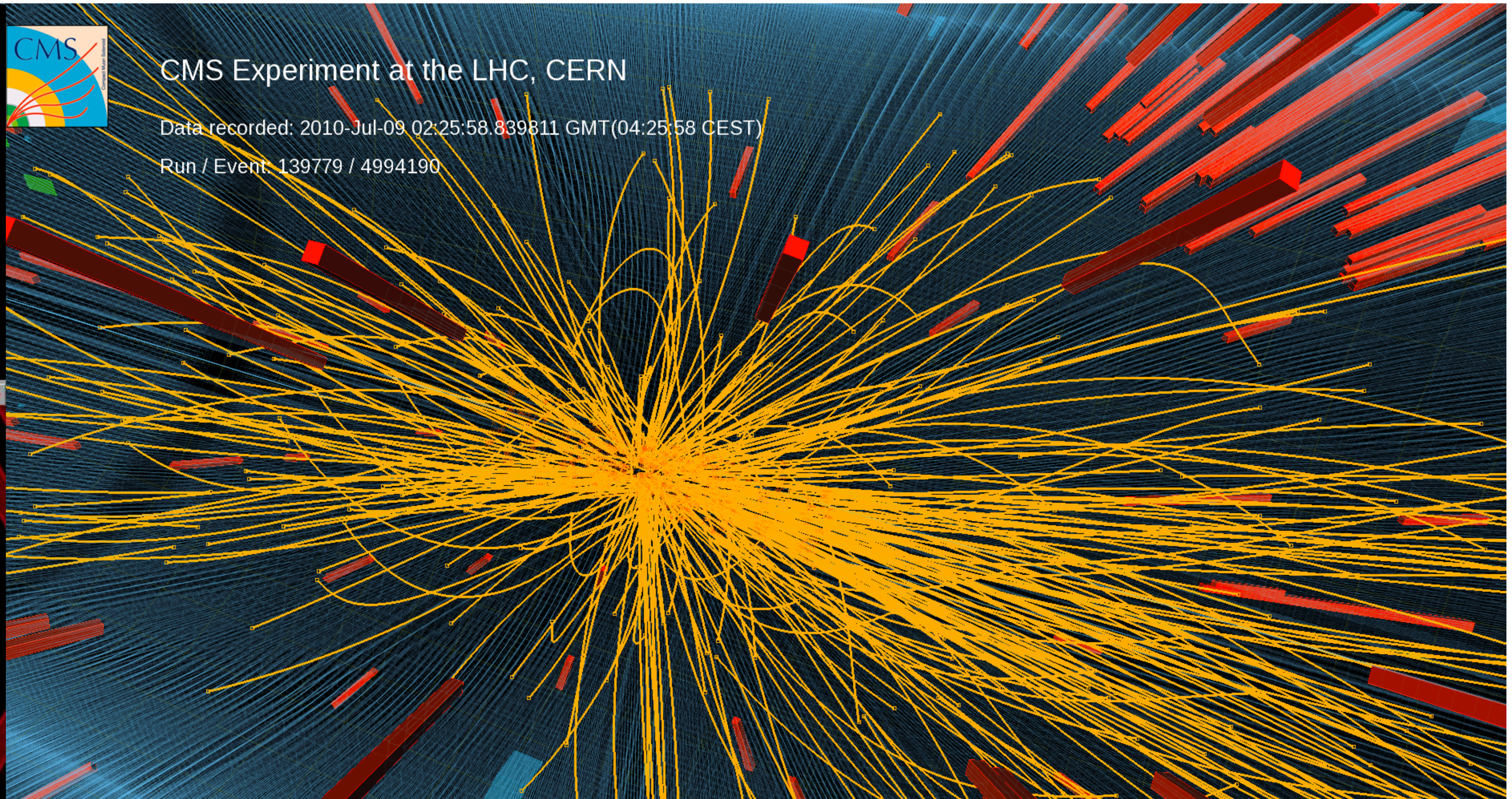
Propose and discuss a framework that can provide a first principles effective description of minimum bias events

Minimum bias: experimentally, some minimal trigger, typically some forward calorimeter activity

Soft QCD, where strong nature of interactions dominate. *Ergodic*







Reasons to seek first principles approach

EFT is a powerful symmetry based approach

pp or AA to N hadrons has *some* S-matrix element, that has to obey certain symmetries

Connections with bootstrap

Understanding of strongly coupled theories from a bootstrap approach, recently been applied to QFT, i.e. to the S-matrix

Starting with: M. F. Paulos, J. Penedones, J. Toledo, B. C. van Rees, and P. Vieira, ’16, ’17. More recently e.g. L. Cordova and P. Vieira, ’18; D. Mazac and M. Paulos ’18,’19; Cordova, He, Kruczenski, Vieira, ’19; Karateev, Kuhn, Penedones ’19; Correia, Sever, Zhiboedov, ’20; Homrich, Penedones, Toledo, van Rees, Vieira, ’20 …

Those are 2 to 2. This is 2 to N>>1; shortly see how “large N” helps

(This is a novel large N=multiplicity; it is not number of colours a la ’t Hooft)

Equal footing

Treat both small and large systems, at both low and high energy, all within the same framework.

Potential to aid in elucidation of nature of small scale (p p collision) collective phenomena in QCD; jet quenching

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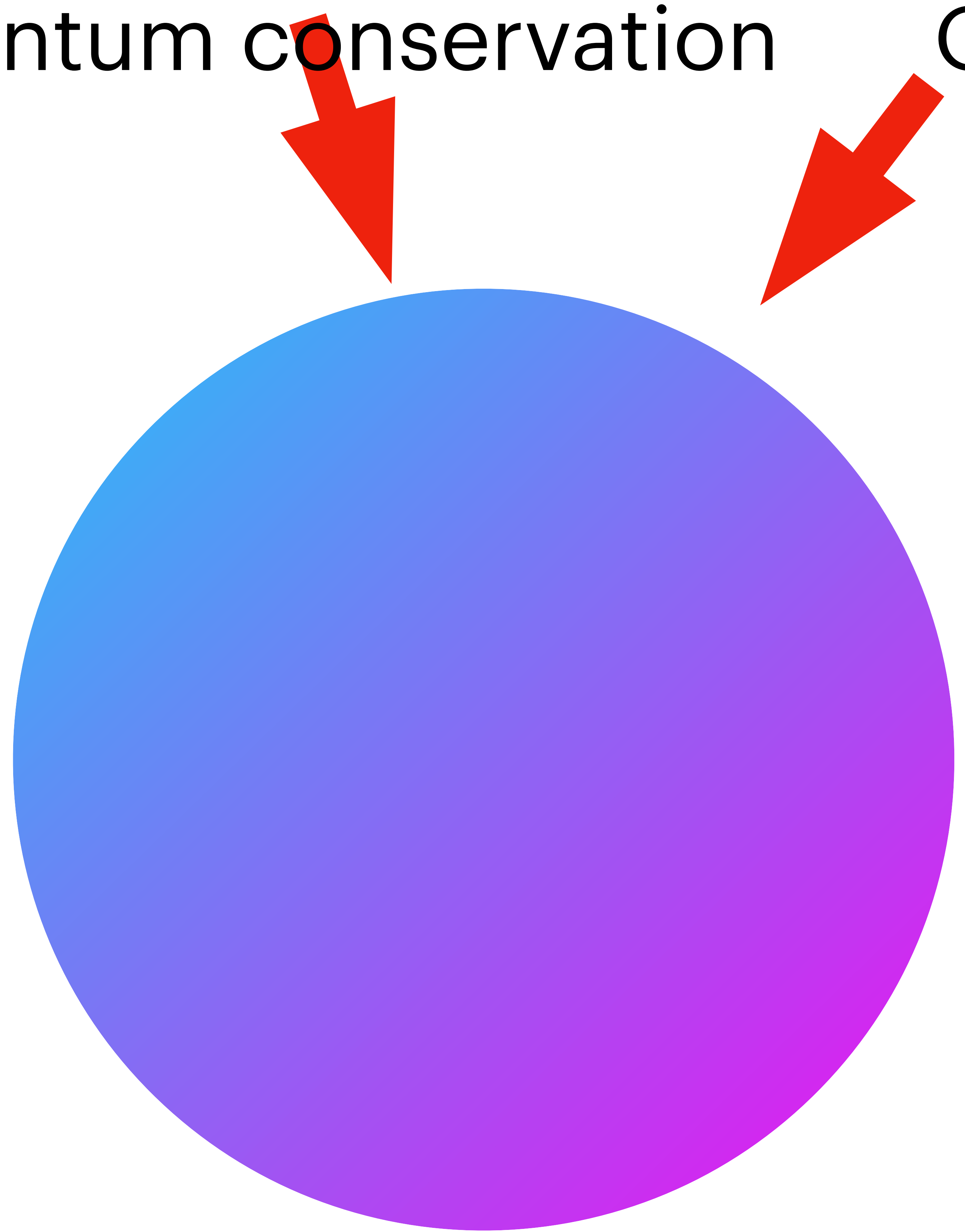
A broad picture

$$d\sigma(p_a, p_b, p_1, \ldots, p_N) = \sigma_N(p_a, p_b, p_1, \ldots, p_N)\, \delta^{(4)}(p_a + p_b - p_1 - \ldots - p_N) \prod_{i=1}^{N} \delta(p_i^2 - m_i^2)$$

Momentum conservation On-shell



Potential to aid in elucidation of nature of small scale collective phenomena in QCD; jet quenching

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Momentum conservation

On-shell

We would like to have

$$= 1 + \dots$$

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On-shell

We would like to have

$$= 1 + \sum_{l=1} c_l\, Y_l(\{p_i\})$$

Harmonics

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A broad picture

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Momentum conservation On-shell

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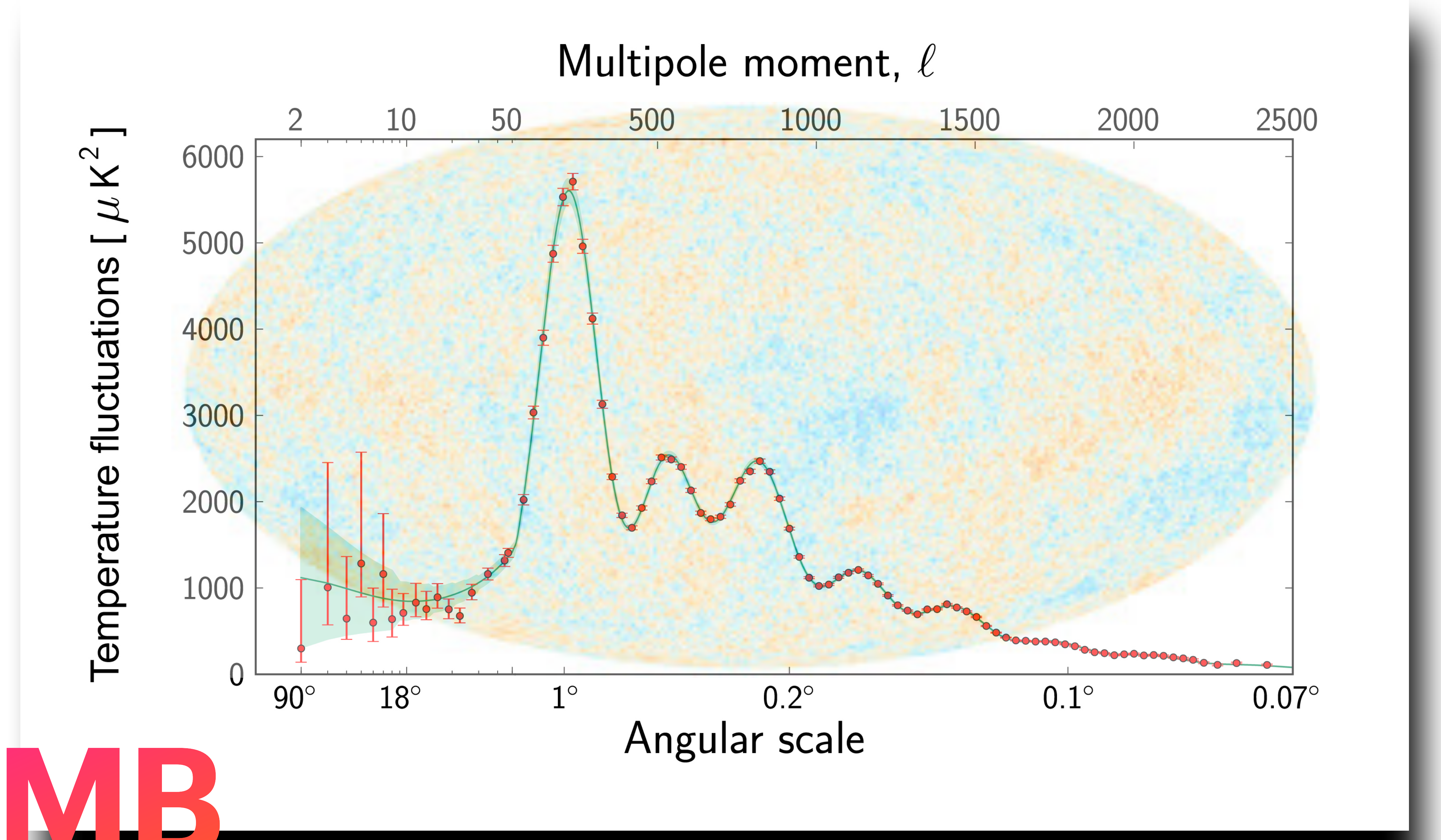
Harmonics

Power spectrum

C.f. the CMB



Treat both small and large systems, at both low and high energy, all within the same framework.

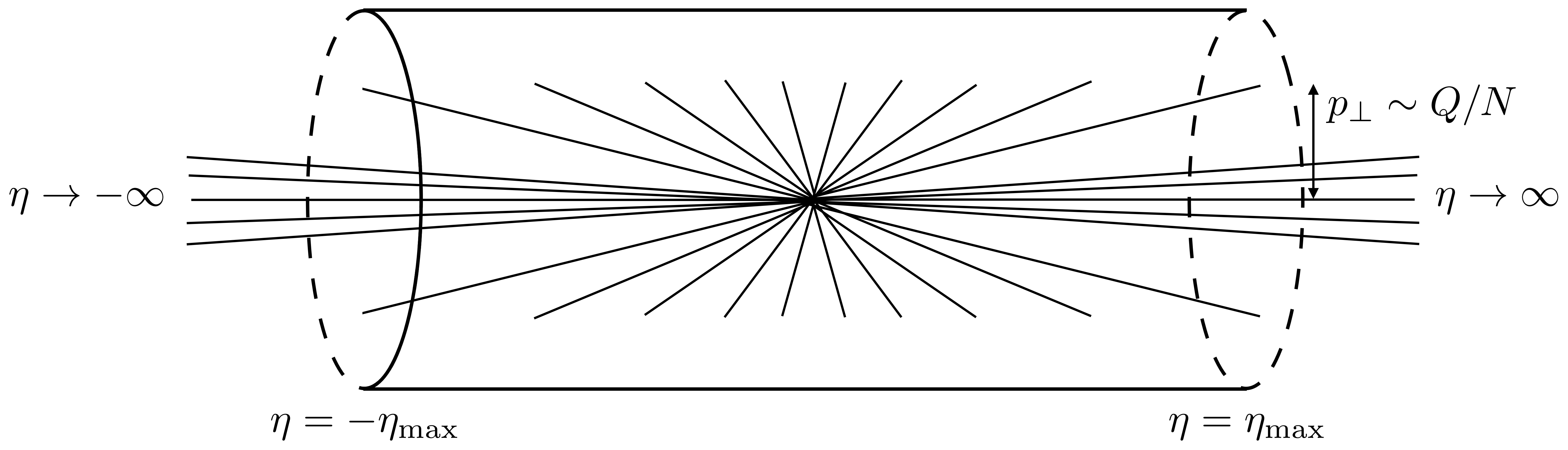
What will be addressed; what will not

Assume that events are binned in multiplicity, N

i.e. **Not** attempt a description of fluctuations in multiplicity

Therefore, can capture how *normalized* distributions (or, better yet, power spectra), binned in N, change as a function of N, and as a function of Q

We work at fixed Q, and take the large N limit, meaning we do not consider a scaling of Q and N such that Q/N (c.f. ‘t Hooft coupling) remains finite. (Although this could be interesting)



Power counting and symmetries for pp/AA min bias

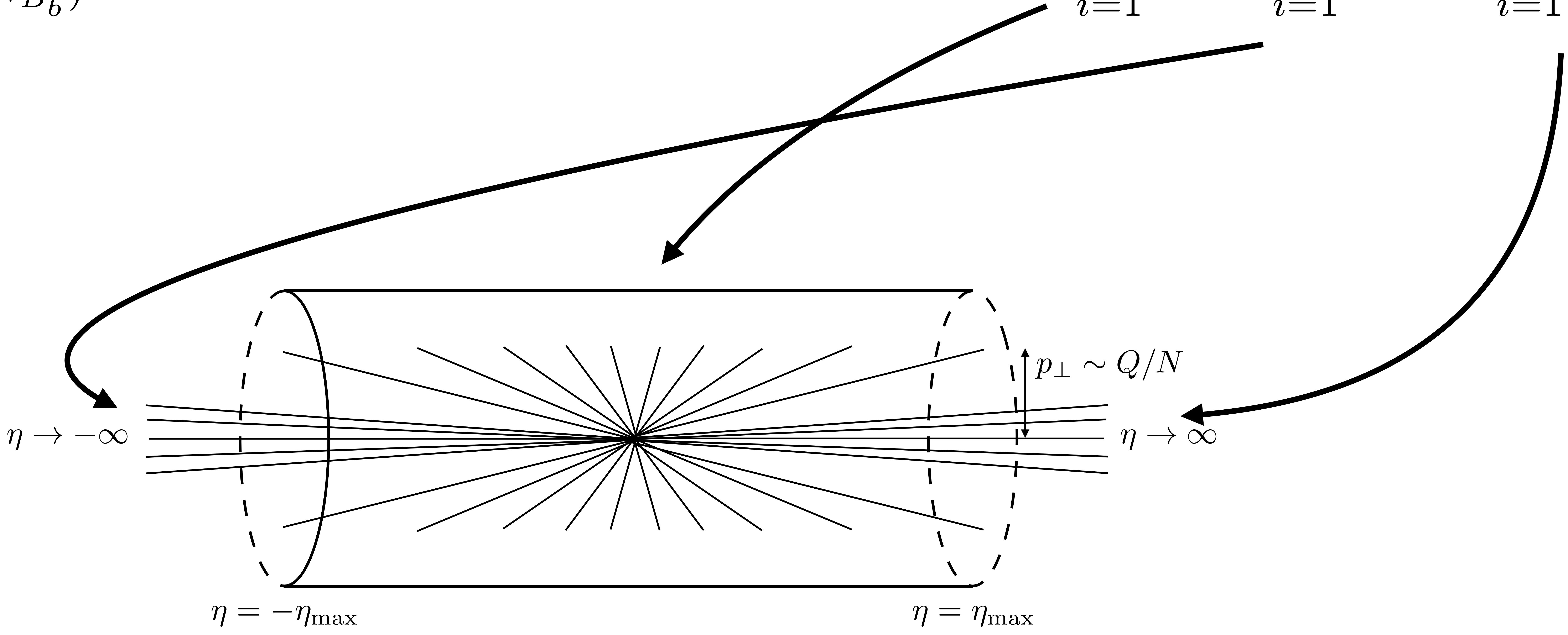
1. The beam is a small angular region outside the detection apparatus and we restrict our description of the event to far from the beam region, where detected particle pseudorapidity satisfies $|\eta| \sim 1 \ll \eta_{\rm max}$.
2. We assume that the mass of the particles is irrelevant and so detected particle transverse momentum $p_\perp$ is parametrically larger than the QCD scale or pion mass, $p_\perp \gg m_\pi$.
3. The momentum lost down the beam region is an order-1 fraction of the center-of-mass energy Q.
4. The number of detected particles N for which their pseudorapidity $|\eta| \ll \eta_{\rm max}$ is large: $N \gg 1$.
5. We assume that the mean transverse momentum of the detected particles is representative of all particles' momenta and so the mean and the root mean square momenta are comparable: $\langle p_\perp \rangle \sim \sqrt{\langle p_\perp^2 \rangle}$.

1. O(2) rotation and reflection symmetry about the beam,
2. reflection of the beam $\eta \to -\eta$
3. S_N permutation symmetry in all N detected particles,
4. translation symmetry in pseudorapidity, $\eta \to \eta + \Delta\eta$.

& finally, only measure momentum
(no species; but could be included)

Effective matrix element

$$\sigma = \int_{\mathrm{LIPS}(N+N_{B_a}+N_{B_b})} \sigma_N(p_1,\ldots,p_N,\{p_{a_i}\},\{p_{b_i}\}) \;\; \delta^{(4)}\Big(p_a + p_b - \sum_{i=1}^{N} p_i - \sum_{i=1}^{N_{B_a}} p_{a_i} - \sum_{i=1}^{N_{B_b}} p_{b_i}\Big)$$



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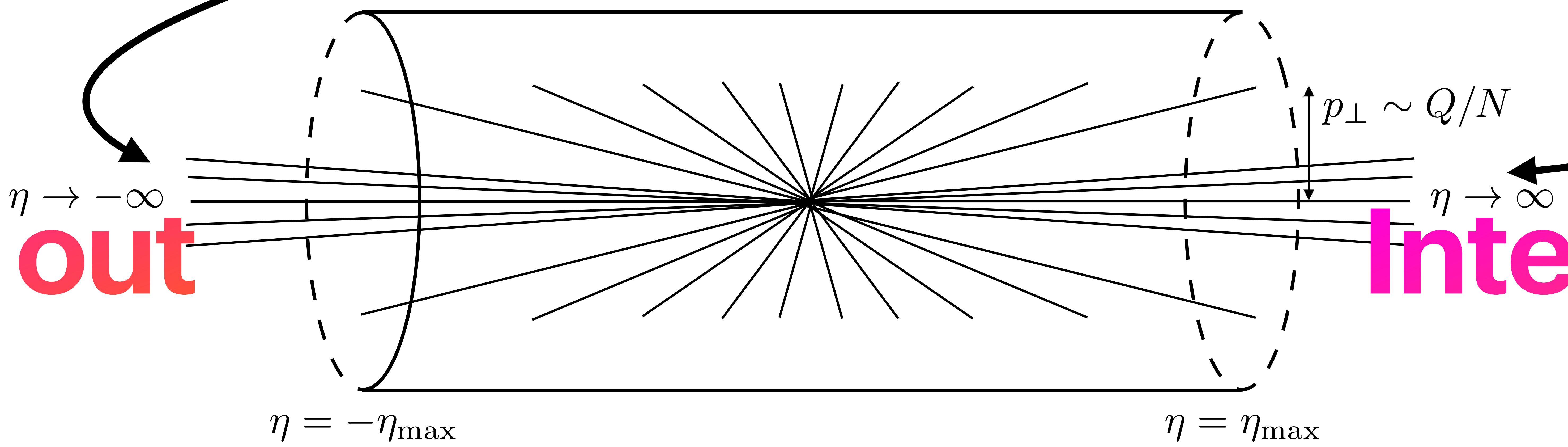

Integrate out

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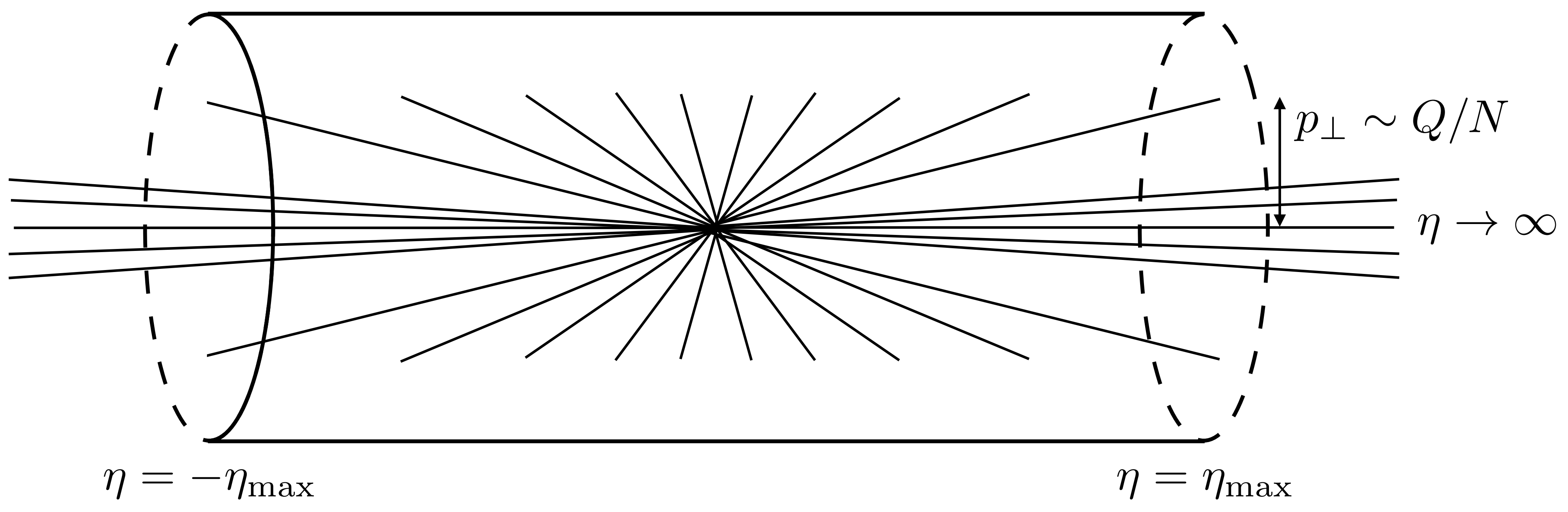
Effective matrix element

$$\sigma = \int_{\mathrm{LIPS}(N+N_{B_a}+N_{B_b})} \sigma_N(p_1,\ldots,p_N,\{p_{a_i}\},\{p_{b_i}\})\ \delta^{(4)}(p_a+p_b-\sum_{i=1}^{N}p_i-\sum_{i=1}^{N_{B_a}}p_{a_i}-\sum_{i=1}^{N_{B_b}}p_{b_i})$$

Light cone momentum down beam pipe



Light cone momentum down beam pipe

$$= \int_0^Q dk^+ \int_0^Q dk^- \int_{\mathrm{LIPS}(N)} f(k^+k^-)\ \tilde{\sigma}_N(p_1,\ldots,p_N)\ \delta(k^- - \sum_{i=1}^{N}p_{\perp i}e^{\eta_i})\,\delta(k^+ - \sum_{i=1}^{N}p_{\perp i}e^{-\eta_i})\,\delta^{(2)}(\sum_{i=1}^{N}\vec{p}_{\perp i})$$

Function (assumed analytic and finite) of the available energy for observed particles

Expansion of matrix element

$$\sigma = \int_0^Q dk^+ \int_0^Q dk^- \int_{\mathrm{LIPS}(N)} f(k^+k^-)\, \widetilde{\sigma}_N(p_1,\ldots,p_N)\ \delta(k^- - \sum_{i=1}^N p_{\perp i} e^{\eta_i})\, \delta(k^+ - \sum_{i=1}^N p_{\perp i} e^{-\eta_i})\, \delta^{(2)}(\sum_{i=1}^N \vec{p}_{\perp i})$$

$$= 1 + \frac{c_1^{(2)}}{Q^2} \sum_{i=1}^N p_{\perp i}^2 + \mathcal{O}(Q^{-4})$$

(After momentum conservation identities)

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In powers of 1/N

Ergodicity and power counting

$$p_\perp \sim Q/N$$

$$\Longrightarrow \frac{1}{Q^2} \sum_{i=1}^N p_{\perp i}^2 \sim \frac{1}{N}$$

N terms in the sum

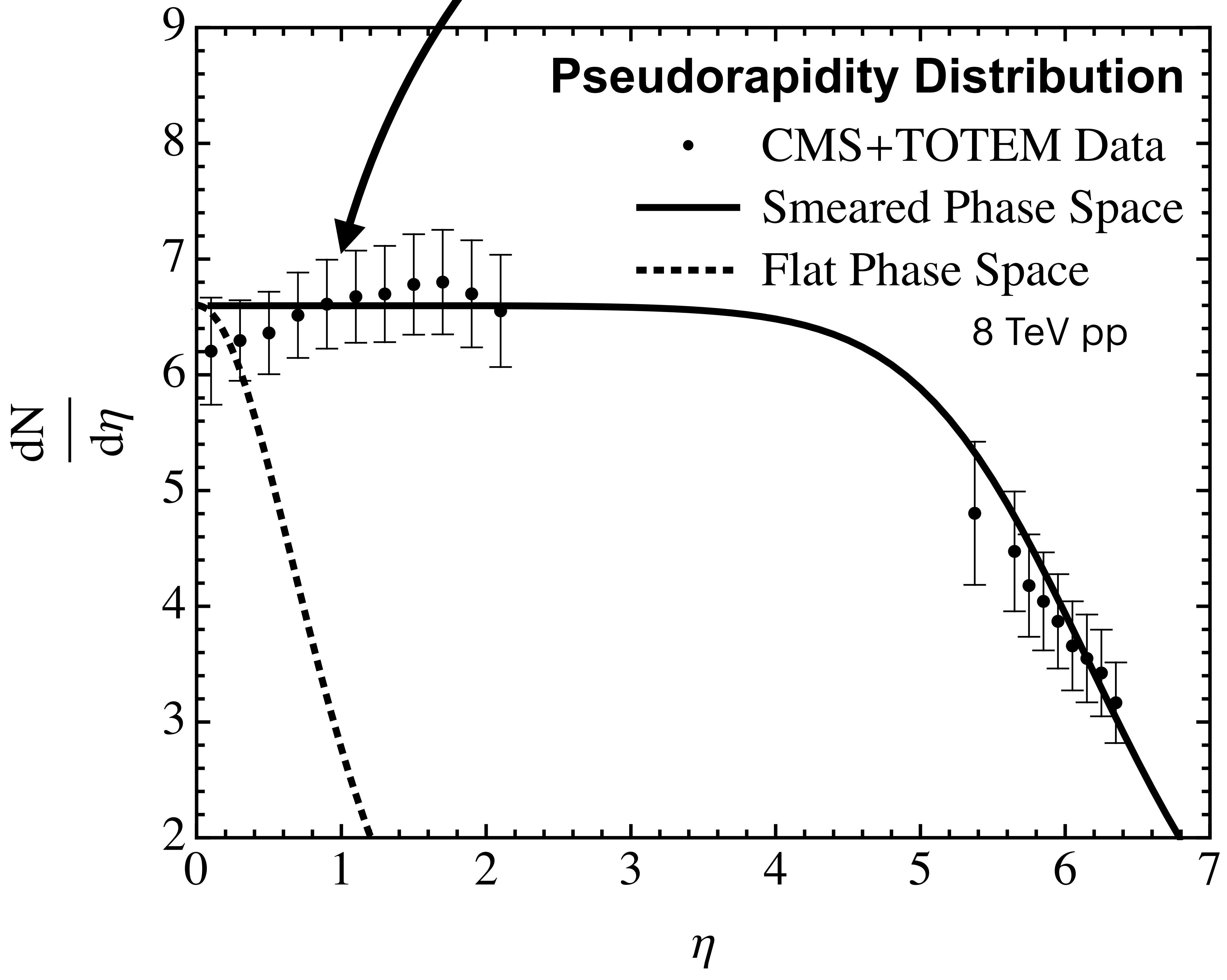
Expansion of matrix element

$$\sigma = \int_0^Q dk^+ \int_0^Q dk^- \int_{\mathrm{LIPS}(N)} f(k^+k^-)\, \widetilde{\sigma}_N(p_1,\ldots,p_N)\;\; \delta(k^- - \sum_{i=1}^N p_{\perp i}e^{\eta_i})\, \delta(k^+ - \sum_{i=1}^N p_{\perp i}e^{-\eta_i})\, \delta^{(2)}(\sum_{i=1}^N \vec{p}_{\perp i})$$

$$\begin{aligned}
1 &+ \frac{c_1^{(2)}}{Q^2}\sum_{i=1}^N p_{\perp i}^2 + \frac{c_1^{(4)}}{Q^4}k^+k^-\sum_{i=1}^N p_{\perp i}^2 + \frac{c_2^{(4)}}{Q^4}\sum_{i=1}^N p_{\perp i}^4 + \frac{c_3^{(4)}}{Q^4}\sum_{i\neq j}^N p_{\perp i}^2 p_{\perp j}^2 \\
&+ \frac{c_4^{(4)}}{Q^4}\sum_{i\neq j}^N p_{\perp i}^3 p_{\perp j}\cosh(\eta_i-\eta_j) + \frac{c_5^{(4)}}{Q^4}\sum_{i\neq j}^N p_{\perp i}^2 p_{\perp j}^2\cosh(2(\eta_i-\eta_j)) \\
&+ \frac{c_6^{(4)}}{Q^4}\sum_{i\neq j\neq k}^N p_{\perp i}^2 p_{\perp j} p_{\perp k}\cosh(\eta_i-\eta_j)\cosh(\eta_i-\eta_k) \\
&+ \frac{c_7^{(4)}}{Q^4}\sum_{i\neq j}^N p_{\perp i}^2 p_{\perp j}\cos(\phi_i-\phi_j) + \frac{c_8^{(4)}}{Q^4}\sum_{i\neq j}^N p_{\perp i}^2 p_{\perp j}^2\cos(2(\phi_i-\phi_j)) \\
&+ \frac{c_9^{(4)}}{Q^4}\sum_{i\neq j\neq k}^N p_{\perp i}^2 p_{\perp j} p_{\perp k}\cos(\phi_i-\phi_j)\cos(\phi_i-\phi_k) \\
&+ \frac{c_{10}^{(4)}}{Q^4}\sum_{i\neq j}^N p_{\perp i}^2 p_{\perp j}^2\cosh(\eta_i-\eta_j)\cos(\phi_i-\phi_j) \\
&+ \frac{c_{11}^{(4)}}{Q^4}\sum_{i\neq j\neq k}^N p_{\perp i}^2 p_{\perp j} p_{\perp k}\cosh(\eta_i-\eta_j)\cos(\phi_i-\phi_k) + \mathcal{O}(Q^{-6})\,.
\end{aligned}$$

Fixing the function f to give flat-in-rapidity

$$\sigma = \int_0^Q dk^+ \int_0^Q dk^- \int_{\mathrm{LIPS}(N)} f(k^+k^-)\ \widetilde{\sigma}_N(p_1,\ldots,p_N)\ \delta(k^- - \sum_{i=1}^N p_{\perp i}e^{\eta_i})\,\delta(k^+ - \sum_{i=1}^N p_{\perp i}e^{-\eta_i})\,\delta^{(2)}(\sum_{i=1}^N \vec{p}_{\perp i})$$



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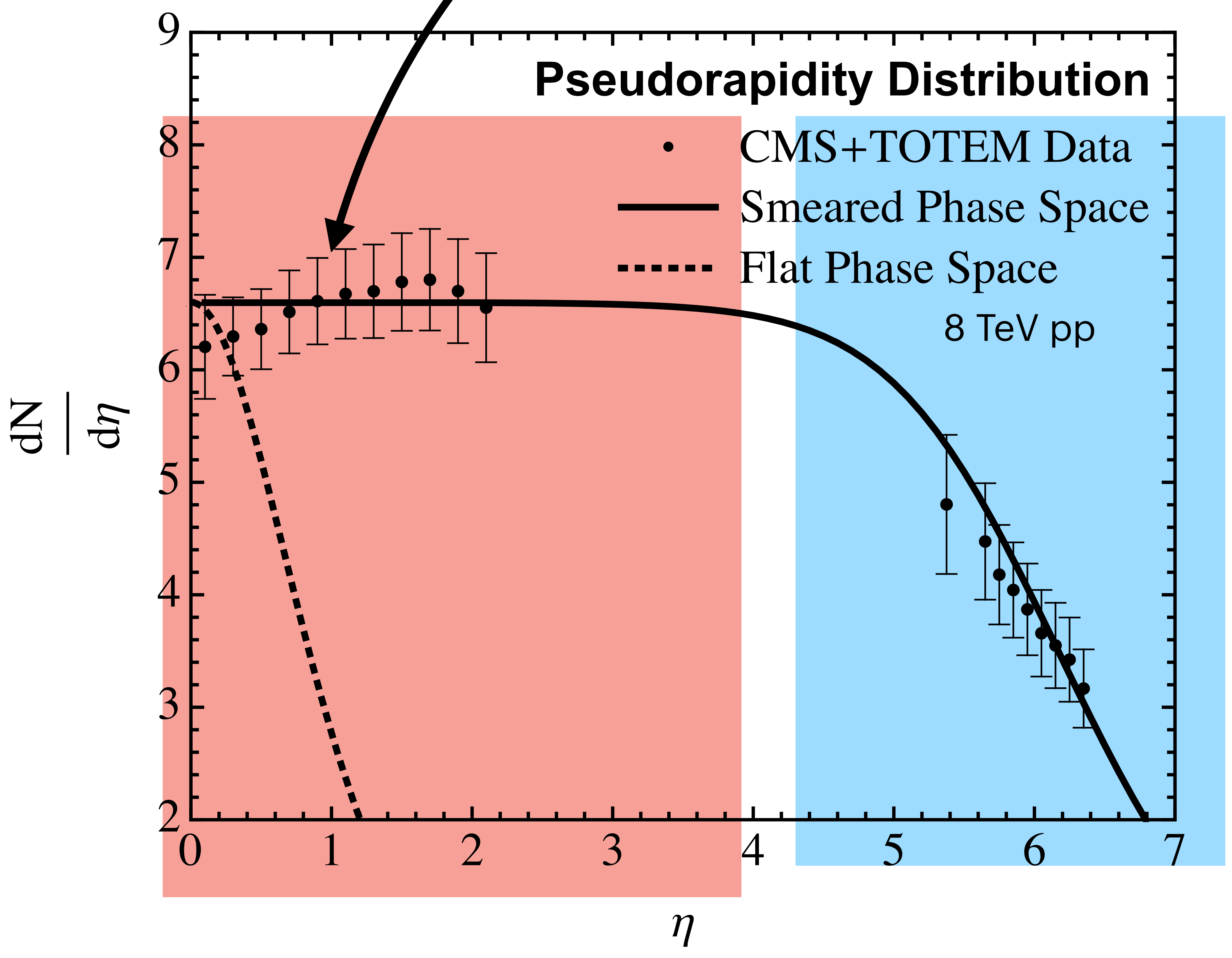

Any function f(x) that is analytic and highly peaked at x=0 produces the 'Feynman' plateau. Effective description is an Expansion around this

Fall-off can be fitted for useful self-consistency check, but it is <u>outside effective description</u>, so general results are agnostic to it

The predictions include (From power counting and symmetries)

- In the $N \to \infty$ limit, the symmetries of min bias events and central limit theorem require the matrix element is exclusively a function of the total energy of the observed final state particles

- The distribution of particle transverse momentum is universal, and depends on a single parameter, with fractional dispersion relation

- By a positivity condition, all azimuthal correlations vanish as $N \to \infty$ at fixed collision energy

- Scaling and factorisation of long-distance pairwise particle pseudo-rapidity correlations as $N \to \infty$ (relevant for the pp 'ridge')

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Transverse momentum distribution

The distribution on unsmeared phase space can be shown to be a Bessel function

$$p_{\text{flat}}(p_\perp) \propto p_\perp \int_{-\infty}^{\infty} d\eta\, e^{-\frac{k^+ e^{\eta} + k^- e^{-\eta}}{k^+ k^-} N p_\perp} = p_\perp\, K_0\left(\frac{2 N p_\perp}{\sqrt{k^+ k^-}}\right)$$

The function f is now fixed, no wiggle-room

$$p(p_\perp) = \frac{1}{Q^2} \int_0^Q dk^+ \int_0^Q dk^-\, f\left(k^+ k^-\right)\, p_{\text{flat}}(p_\perp)$$

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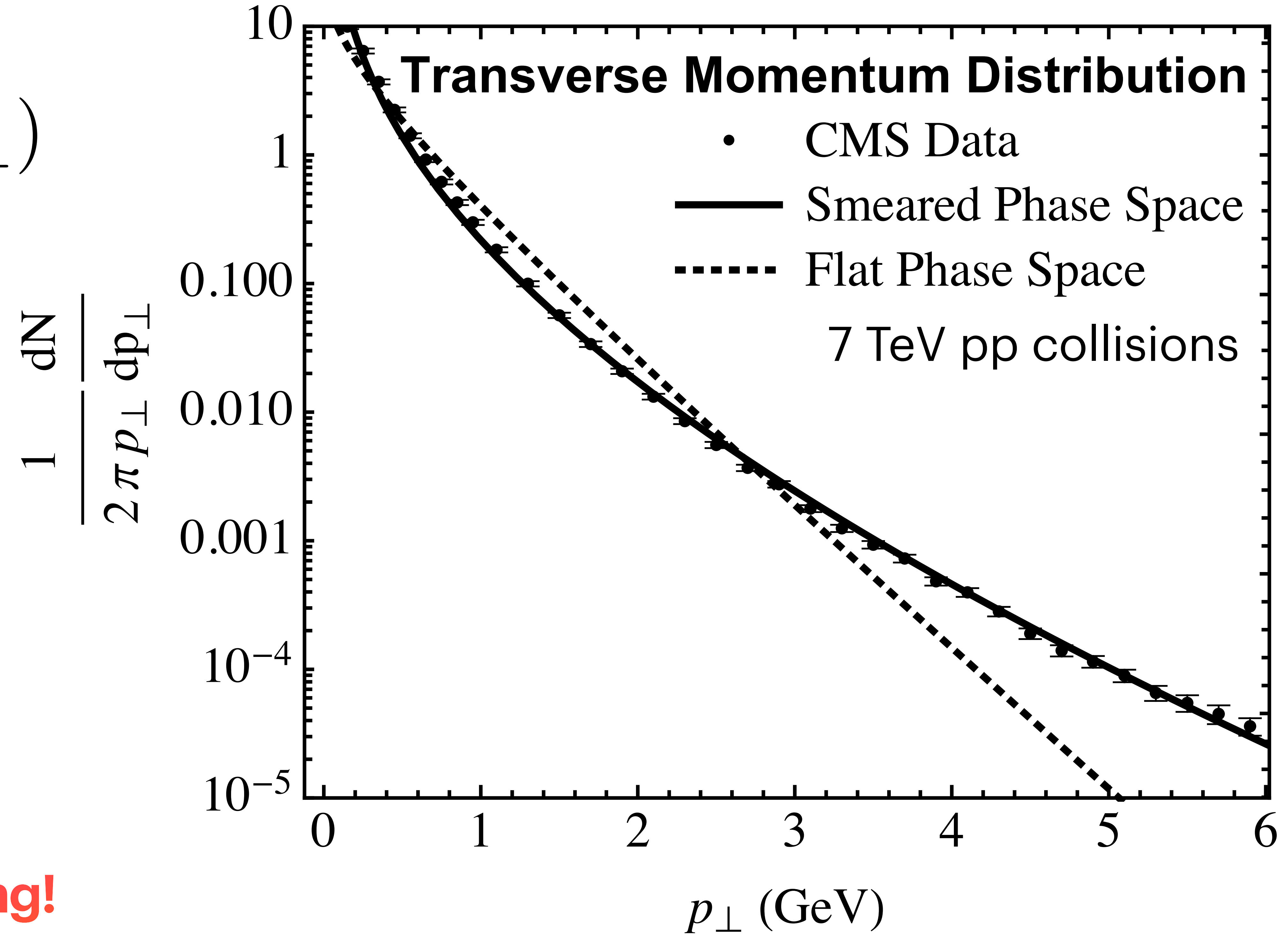
Expression for distribution parameter depends only on variable = average pT

$$p(p_\perp) \sim e^{-\frac{3\pi}{4}\frac{p_\perp^{2/3}}{\langle p_\perp\rangle^{2/3}}}$$

Fractional dispersion…interesting!



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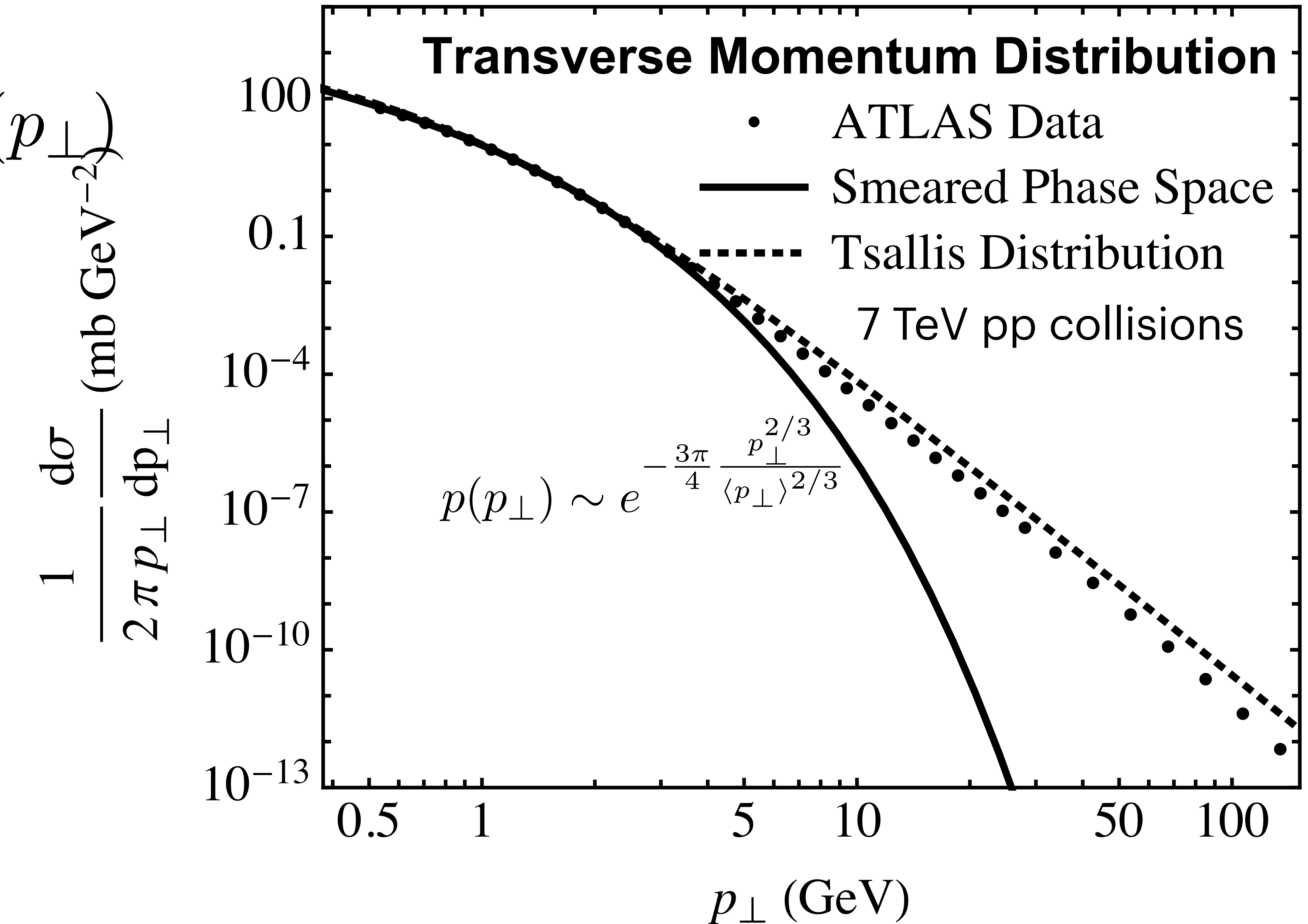
See edge of validity of the effective min bias description where we expect it, self-consistent



Conclusions

- Constructed an effective description of min bias, based on power counting and symmetry

- Framework predicts features, particularly in the $N \to \infty$ limit, that are borne out in collider data

- Future directions include detailed analysis, in particular away from $N \to \infty$, and comparison with data on small system collective behaviour, jet quenching

- Not presented here, but description works for other colliders: electron-hadron, and e+ e-, by accounting for different symmetries

- Underlying event, with different symmetries again, could be interesting to study by a similar approach

Backup slides

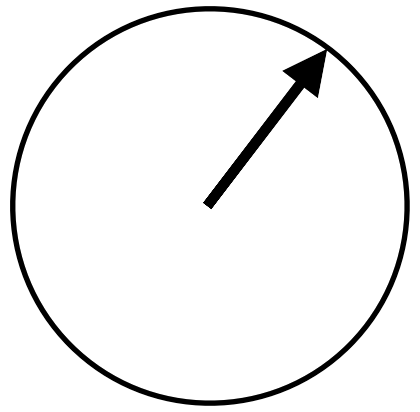
Lorentz invariant phase space is a Stiefel Manifold

Henning, TM arxiv:1902.06747

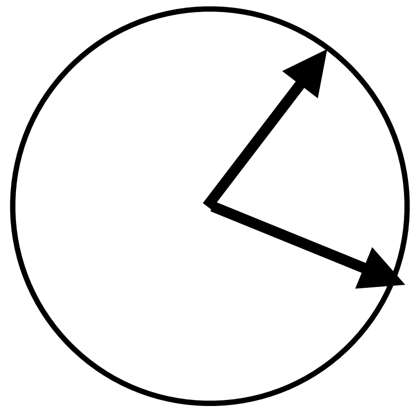
$$\delta^{(4)}(p_a + p_b - p_1 - \ldots - p_N) \prod_{i=1}^{N} \delta(p_i^2 - m_i^2)$$

Momentum conservation On-shell

Sphere $\dfrac{O(N)}{O(N-1)}$



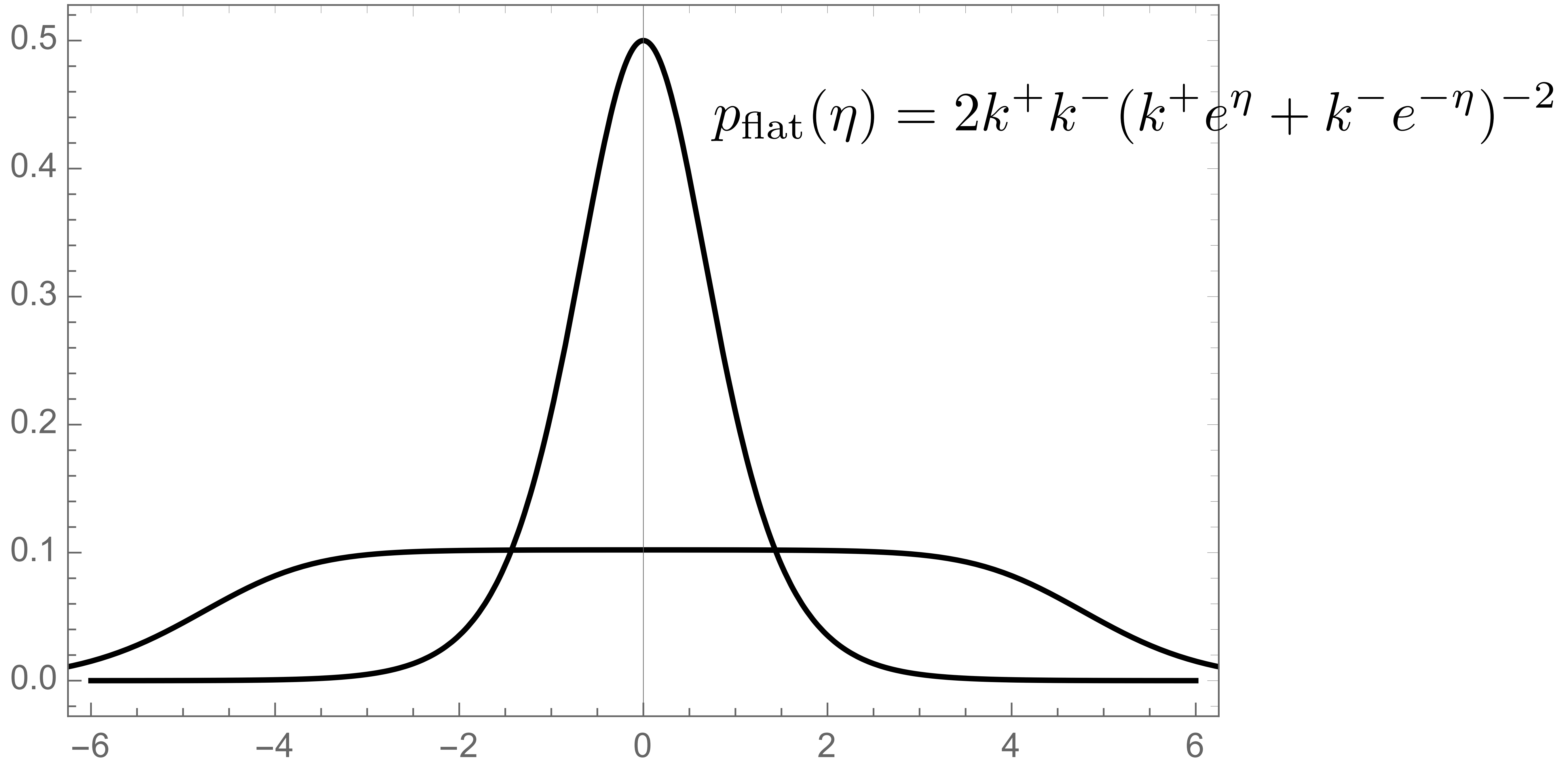
Stiefel $\dfrac{O(N)}{O(N-2)}$



(& then Complexified, O -> U)

Flat phase space to flat rapidity

$$
\begin{aligned}
p(\eta) &= \frac{1}{Q^2}\int_0^Q dk^+ \int_0^Q dk^- \, f\left(k^+k^-\right)\, p_{\text{flat}}(\eta) \\
&= \frac{1}{Q^2}\int_0^Q dk^+ \int_0^Q dk^- \, f\left(k^+k^-\right)\, 2k^+k^-\left(k^+e^{\eta} + k^-e^{-\eta}\right)^{-2} \\
&= \int_0^1 dx\, f(x)\, \frac{1-x^2}{1+x^2+2x\cosh(2\eta)}\,.
\end{aligned}
$$



Take e.g.

$$
f(k^+k^-) = \frac{n+1}{H_{n+1}}\left(1 - \frac{k^+k^-}{Q^2}\right)^n \simeq \frac{n}{\gamma_E + \log n}\, e^{-n\frac{k^+k^-}{Q^2}}
$$