Warm dense QCD matter in strong magnetic fields

Helena Kolešová

University of Stavanger

Joint work with Tomáš Brauner, Naoki Yamamoto and Georgios Filios

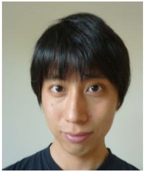
Outline

1. Motivation:
   - QCD phase diagram
   - Inhomogeneous phases of QCD matter
2. Chiral soliton lattice phase at LO
3. Chiral soliton lattice phase at finite temperature
4. Chiral soliton lattice phase seen in lattice simulations?

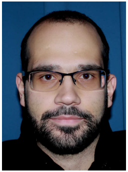
3 4

2 3 4

2 3

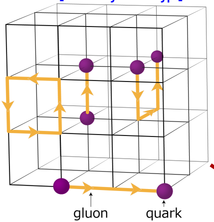


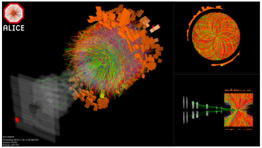
4



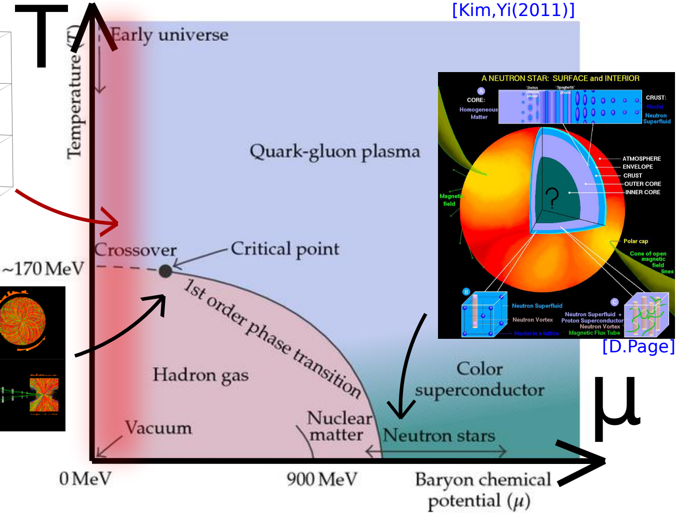
Motivation: QCD phase diagram

[www.jicfus.jp]





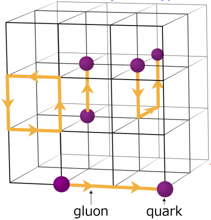
[Kim,Yi(2011)]

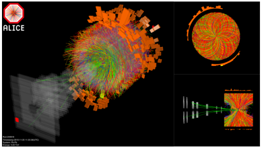


[D.Page]

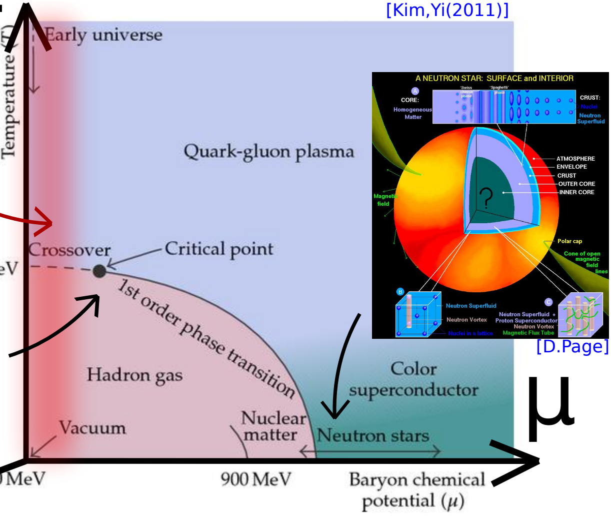
Motivation: QCD phase diagram

[www.jicfus.jp]





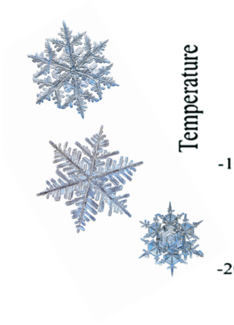
[Kim,Yi(2011)]

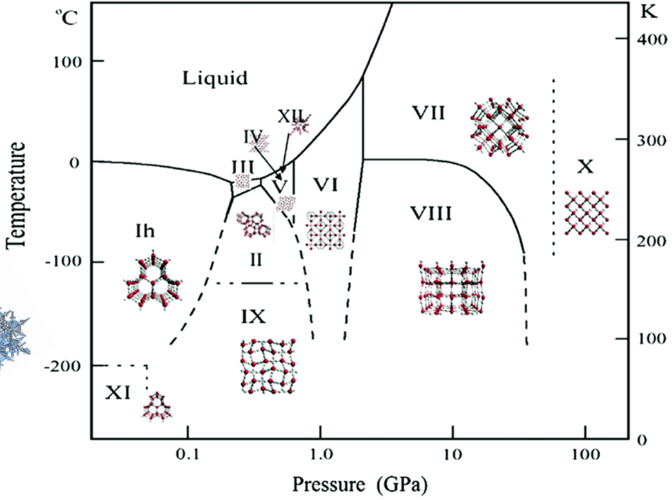


[D.Page]

B

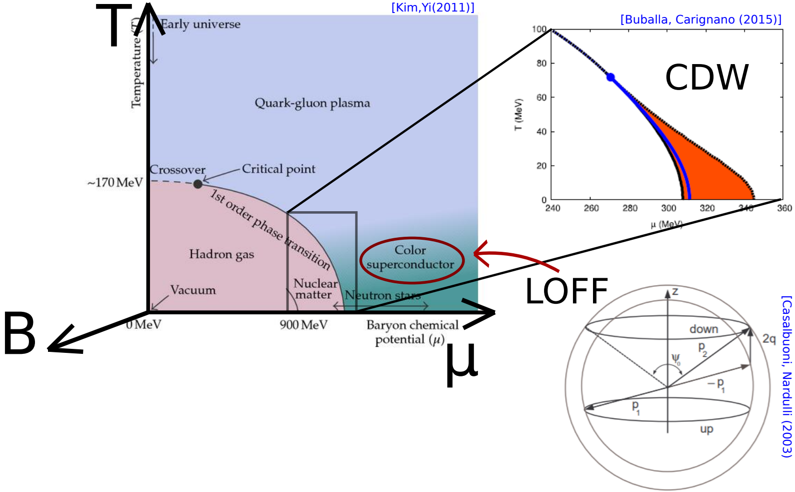
Motivation: Inhomogeneous phases of matter





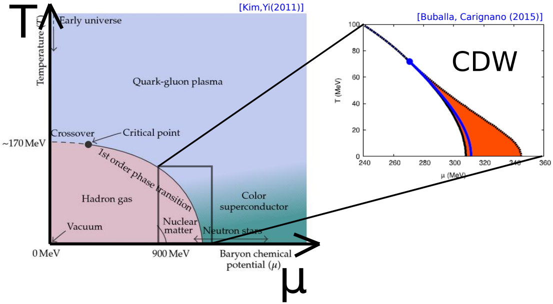
Inhomogeneous phases in the QCD phase diagram?

[Alford,Bowers,Rajagopal(2001)], review: [Buballa,Carignano(2015)]



Inhomogeneous phases in the QCD phase diagram?

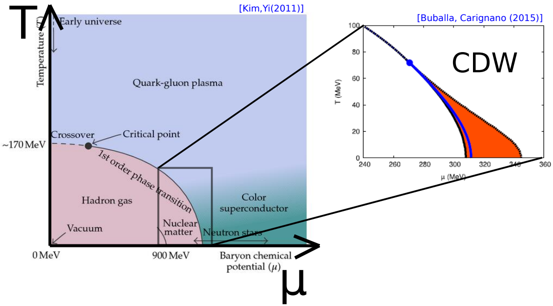
Review: [Buballa,Carignano(2015)]



- Chiral Density Wave: $\langle\overline{\psi}\psi\rangle = \Delta\cos(\vec{q}\cdot\vec{x})$, $\langle\overline{\psi}i\gamma_5\tau_3\psi\rangle = \Delta\sin(\vec{q}\cdot\vec{x})$

Inhomogeneous phases in the QCD phase diagram?

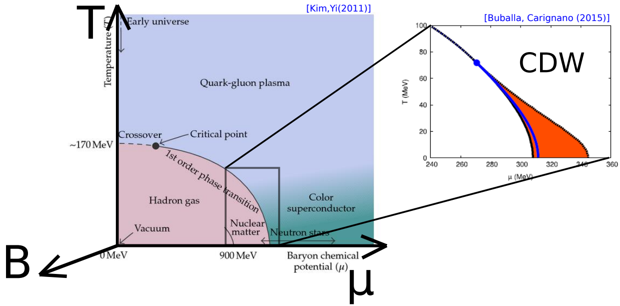
Review: [Buballa,Carignano(2015)]



- Chiral Density Wave: $\langle\bar{\psi}\psi\rangle = \Delta\cos(\vec{q}\cdot\vec{x})$, $\langle\bar{\psi}i\gamma_5\tau_3\psi\rangle = \Delta\sin(\vec{q}\cdot\vec{x})$
- 1D modulation in spherically symmetric 3D system unstable at any non-zero temperature! [Peierls (1934), Landau]
  (but quasi-long-range order may be sustained [Hidaka etal.(2015)][Pisarski etal.(2018)])

Inhomogeneous phases in the QCD phase diagram?

Review: [Buballa,Carignano(2015)]



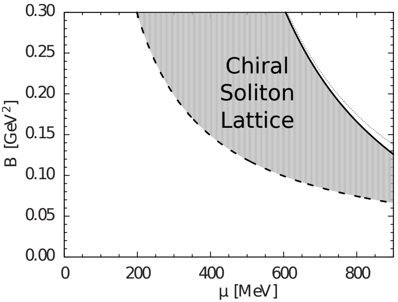
- Chiral Density Wave: $\langle\bar{\psi}\psi\rangle = \Delta\cos(\vec{q}\cdot\vec{x})$, $\langle\bar{\psi}i\gamma_5\tau_3\psi\rangle = \Delta\sin(\vec{q}\cdot\vec{x})$
- 1D modulation in spherically symmetric 3D system unstable at any non-zero temperature! [Peierls (1934), Landau]
  (but quasi-long-range order may be sustained [Hidaka etal.(2015)][Pisarski etal.(2018)])
- If the spherical symmetry broken by magnetic field, 1D modulations stable! [Tatsumi,Nishiyama,Karasawa(2015)] [Ferrer,Incera(2020)]

QCD matter in strong B: “new” inhomogeneous phase!

[Son,Stephanov(2008)][Brauner,Yamamoto(2017)]

Ground state of QCD matter for sufficiently strong magnetic field and large enough baryon chemical potential: inhomogeneous condensate of neutral pions

Shown using chiral perturbation theory!

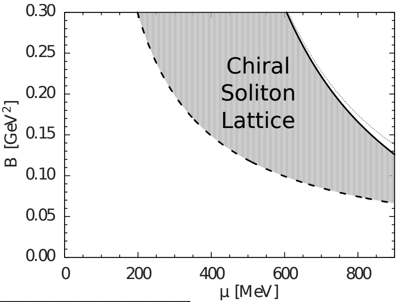


QCD matter in strong B: “new” inhomogeneous phase!

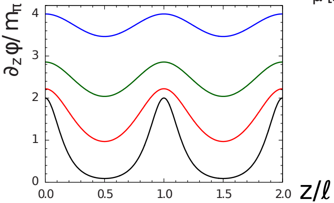
[Son,Stephanov(2008)][Brauner,Yamamoto(2017)]

Ground state of QCD matter for sufficiently strong magnetic field and large enough baryon chemical potential: inhomogeneous condensate of neutral pions

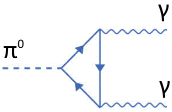
Shown using chiral perturbation theory!



Spatially varied neutral pion background field:



Neutral pions coupled to the magnetic field due to chiral anomaly!

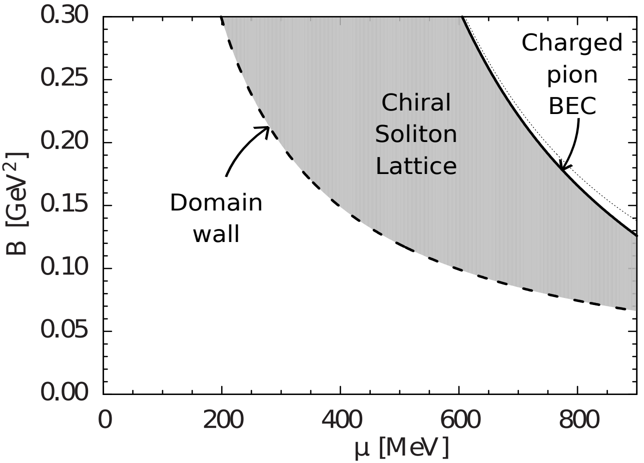


QCD matter in strong B: "new" inhomogeneous phase!

[Son,Stephanov(2008)][Brauner,Yamamoto(2017)]

Ground state of QCD matter for sufficiently strong magnetic field and large enough baryon chemical potential: inhomogeneous condensate of neutral pions

Shown using chiral perturbation theory!



QCD matter in strong B: “new” inhomogeneous phase!

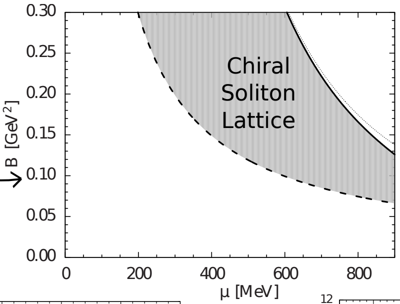
[Son,Stephanov(2008)][Brauner,Yamamoto(2017)]

Ground state of QCD matter for sufficiently strong magnetic field and large enough baryon chemical potential: inhomogeneous condensate of neutral pions

Shown using chiral perturbation theory!

?!

≈ 10^{19}G

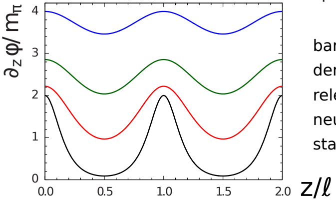


[Ferrer, Incera, Keith, Portillo, Springsteen(2010)]

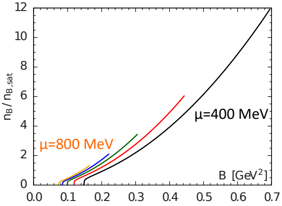
[Cardall, Prakash, Lattimer (2001)]

[Eto, Hashimoto, Hatsuda (2013)]

Spatially varied neutral pion background field:



baryon densities relevant for neutron stars:



Chiral soliton lattice phase @LO

[Brauner,Yamamoto(2017)]

- In strong magnetic fields charged pions get large effective masses $\Rightarrow$ only neutral pions remain relevant degrees of freedom!
- Ground state configuration of the neutral pion field? Minimize the energy functional (based on chiral perturbation theory):

$$\mathcal{H}_{\text{eff}} = \frac{f_\pi^2}{2}(\boldsymbol{\nabla}\phi)^2 + m_\pi^2 f_\pi^2(1-\cos\phi) - \frac{\mu}{4\pi^2}\boldsymbol{B}\cdot\boldsymbol{\nabla}\phi \qquad (\phi = \frac{\pi^0}{f_\pi})$$

Chiral soliton lattice phase @LO

[Brauner, Yamamoto(2017)]

- In strong magnetic fields charged pions get large effective masses $\Rightarrow$ only neutral pions remain relevant degrees of freedom!
- Ground state configuration of the neutral pion field? Minimize the energy functional (based on chiral perturbation theory):

$$\mathcal{H}_{\text{eff}} = \frac{f_\pi^2}{2}(\boldsymbol{\nabla}\phi)^2 + m_\pi^2 f_\pi^2(1-\cos\phi) \boxed{-\frac{\mu}{4\pi^2}\boldsymbol{B}\cdot\boldsymbol{\nabla}\phi} \leftarrow \text{Chiral anomaly!}$$

Chiral soliton lattice phase @LO

[Brauner,Yamamoto(2017)]

- In strong magnetic fields charged pions get large effective masses $\Rightarrow$ only neutral pions remain relevant degrees of freedom!
- Ground state configuration of the neutral pion field? Minimize the energy functional (based on chiral perturbation theory):

$$\mathcal{H}_{\text{eff}} = \frac{f_\pi^2}{2}(\boldsymbol{\nabla}\phi)^2 + m_\pi^2 f_\pi^2 (1 - \cos\phi) \boxed{-\frac{\mu}{4\pi^2}\boldsymbol{B}\cdot\boldsymbol{\nabla}\phi} \leftarrow \text{Chiral anomaly!}$$

$\Rightarrow$ $\partial_z^2\phi = m_\pi^2 \sin\phi$ (magnetic field in z direction)

Chiral soliton lattice phase @LO

[Brauner,Yamamoto(2017)]

- In strong magnetic fields charged pions get large effective masses $\Rightarrow$ only neutral pions remain relevant degrees of freedom!
- Ground state configuration of the neutral pion field? Minimize the energy functional (based on chiral perturbation theory):

$$\mathcal{H}_{\text{eff}} = \frac{f_\pi^2}{2}(\nabla\phi)^2 + m_\pi^2 f_\pi^2(1-\cos\phi) \boxed{-\frac{\mu}{4\pi^2}\boldsymbol{B}\cdot\nabla\phi} \leftarrow \text{Chiral anomaly!}$$

$\Rightarrow$ $\partial_z^2\phi = m_\pi^2 \sin\phi$ (magnetic field in z direction)

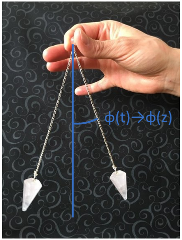
Chiral soliton lattice phase @LO

[Brauner,Yamamoto(2017)]

- In strong magnetic fields charged pions get large effective masses $\Rightarrow$ only neutral pions remain relevant degrees of freedom!
- Ground state configuration of the neutral pion field? Minimize the energy functional (based on chiral perturbation theory):

$$\mathcal{H}_{\text{eff}} = \frac{f_\pi^2}{2}(\nabla\phi)^2 + m_\pi^2 f_\pi^2(1-\cos\phi) \boxed{-\frac{\mu}{4\pi^2}\boldsymbol{B}\cdot\nabla\phi} \leftarrow \text{Chiral anomaly!}$$

$\Rightarrow \quad \partial_z^2\phi = m_\pi^2 \sin\phi \qquad$ (magnetic field in z direction)



Ground state for $\mu B \geq 16\pi m_\pi f_\pi^2$:

$$\cos\frac{\phi(z)}{2} = \text{sn}\left(\frac{m_\pi z}{k}, k\right),$$

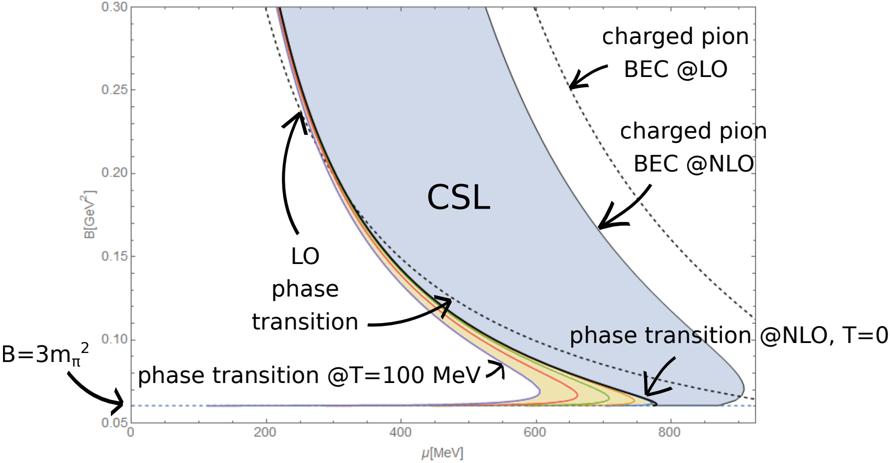
with elliptic modulus k fixed as

$$\frac{E(k)}{k} = \frac{\mu B}{16\pi m_\pi f_\pi^2}$$

Chiral soliton lattice phase at finite temperature

[Brauner,H.K.,Yamamoto (soon)]

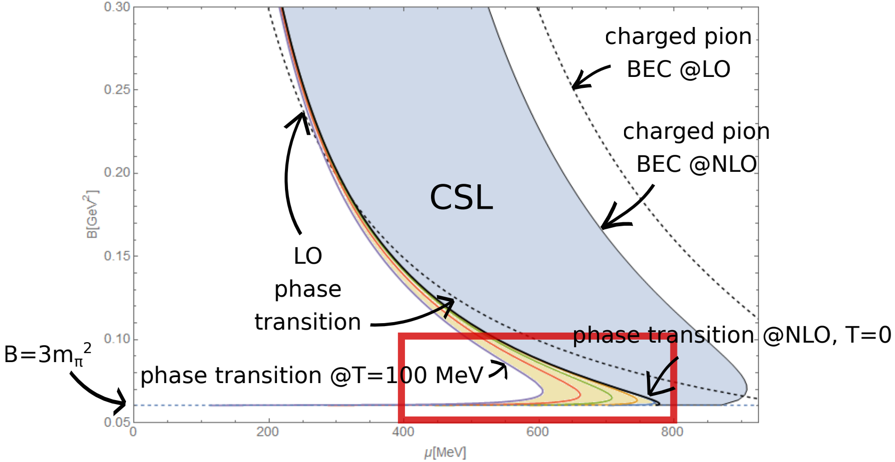
Chiral soliton lattice phase stabilized by finite temperature!



Chiral soliton lattice phase at finite temperature

[Brauner,H.K.,Yamamoto (soon)]

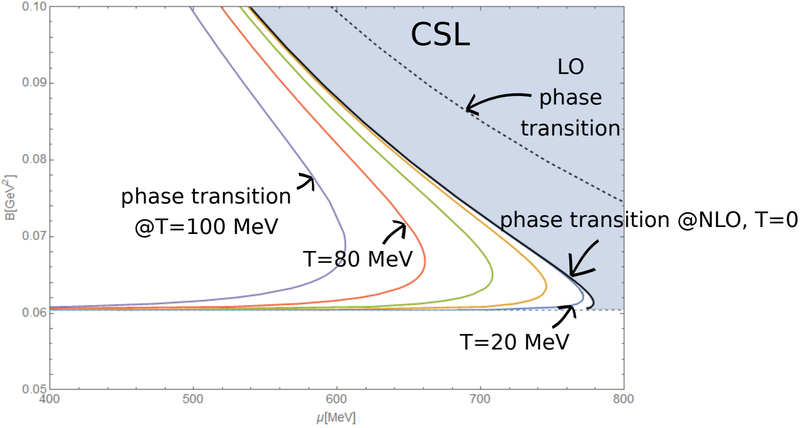
Chiral soliton lattice phase stabilized by finite temperature!



Chiral soliton lattice phase at finite temperature

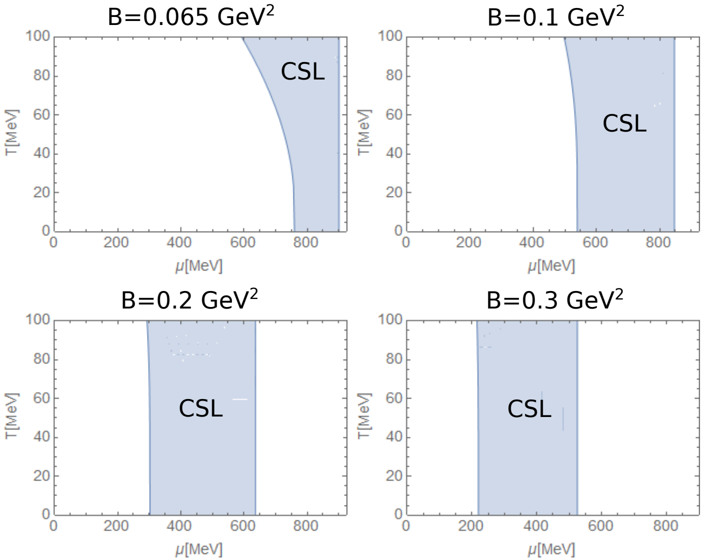
[Brauner,H.K.,Yamamoto (soon)]

Chiral soliton lattice phase stabilized by finite temperature!



Chiral soliton lattice phase at finite temperature

[Brauner,H.K.,Yamamoto, arXiv: 2108.10044]



Chiral soliton lattice phase at finite temperature

[Brauner,H.K.,Yamamoto, arXiv: 2108.10044]

- LO: domain wall formed at the phase transition $\Rightarrow$
  1. 1-loop free energy for domain wall background $\mathcal{F}^{NLO}(B,T)$ calculated
  2. phase transition occurs at such points in parameter space where $\mathcal{F}^{LO+NLO}(\mu,B,T)=0$

LO: $$\boxed{\frac{\mathscr{E}_{\text{wall}}}{S}=8m_\pi f_\pi^2-\frac{\mu B}{2\pi}.}$$

NLO, T=0

$$\frac{\mathscr{F}^{T=0}_{\text{wall,fin}}}{S}=\frac{B^{3/2}}{\sqrt{2}\pi}\Bigg\{\zeta\left(-\tfrac{1}{2},\tfrac{1}{2}-\tfrac{3m_\pi^2}{2B}\right)+\zeta\left(-\tfrac{1}{2},\tfrac{1}{2}\right)$$
$$-\int\frac{\mathrm{d}P}{2\pi}\frac{2}{1+P^2}\left[\zeta\left(-\tfrac{1}{2},\tfrac{1}{2}+\tfrac{m_\pi^2(1+P^2)}{2B}\right)+\frac{2}{3}\left(\frac{m_\pi^2}{2B}\right)^{3/2}(1+P^2)^{3/2}\right]$$
$$-\int\frac{\mathrm{d}P}{2\pi}\frac{4}{4+P^2}\left[\zeta\left(-\tfrac{1}{2},\tfrac{1}{2}+\tfrac{m_\pi^2(1+P^2)}{2B}\right)+\left(\frac{m_\pi^2}{2B}\right)^{3/2}\left(\frac{2}{3}(4+P^2)^{3/2}-3(4+P^2)^{1/2}\right)\right]\Bigg\}.$$

$$\frac{\mathscr{F}^{T=0}_{\text{wall,ren}}}{S}=\frac{m_\pi^3}{24\pi^2}\left(-17\log\frac{4\pi\Lambda^2_{\text{RG}}}{m_\pi^2}+40\log 2-\frac{121}{3}+17\gamma_{\text{E}}\right)+m_\pi^3\left(-\frac{64}{3}\bar{\ell}_1-\frac{8}{3}\bar{\ell}_2+\frac{8}{3}\bar{\ell}_3\right).$$

NLO, finite T:

$$\frac{\mathscr{F}^{T,(\pi^0)}_{\text{wall}}}{S}=-\frac{\zeta(3)T^3}{2\pi}-\frac{m_\pi^2 T}{\pi^2}\int_0^\infty Q\arctan Q\times\log\left(1-e^{-x\sqrt{1+Q^2}}\right)\mathrm{d}Q,$$

$$\frac{\mathscr{F}^{T,(\pi^\pm)}_{\text{wall}}}{S}=\frac{BT}{\pi}\sum_{m=0}^{\infty}\Bigg\{\log\left[1-e^{-\beta\sqrt{(2m+1)B-3m_\pi^2}}\right]+\log\left[1-e^{-\beta\sqrt{(2m+1)B}}\right]$$
$$-\frac{1}{\pi}\int_0^\infty\mathrm{d}P\left(\frac{2}{1+P^2}+\frac{4}{4+P^2}\right)\log\left[1-e^{-\beta\epsilon(m,P^2)}\right]\Bigg\}.$$

Chiral soliton lattice phase at finite temperature

[Brauner,H.K.,Yamamoto, arXiv: 2108.10044]

- LO: domain wall formed at the phase transition $\Rightarrow$
  1. 1-loop free energy for domain wall background $\mathcal{F}^{NLO}(B, T)$ calculated
  2. phase transition occurs at such points in parameter space where $\mathcal{F}^{LO+NLO}(\mu, B, T) = 0$

LO: $\boxed{\dfrac{\mathscr{E}_{\mathrm{wall}}}{S} = 8m_\pi f_\pi^2 - \dfrac{\mu B}{2\pi}.}$

NLO, T=0

$$\boxed{\begin{aligned}\frac{\mathscr{F}^{T=0}_{\mathrm{wall,fin}}}{S} = \frac{B^{3/2}}{\sqrt{2}\pi}\Big\{&\zeta(-\tfrac{1}{2}, \tfrac{1}{2} - \tfrac{3m_\pi^2}{2B}) + \zeta(-\tfrac{1}{2}, \tfrac{1}{2}) \\ &- \int \frac{\mathrm{d}P}{2\pi}\frac{2}{1+P^2}\left[\zeta(-\tfrac{1}{2}, \tfrac{1}{2} + \tfrac{m_\pi^2(1+P^2)}{2B}) + \frac{2}{3}\left(\frac{m_\pi^2}{2B}\right)^{3/2}(1+P^2)^{3/2}\right] \\ &- \int \frac{\mathrm{d}P}{2\pi}\frac{4}{4+P^2}\left[\zeta(-\tfrac{1}{2}, \tfrac{1}{2} + \tfrac{m_\pi^2(1+P^2)}{2B}) + \left(\frac{m_\pi^2}{2B}\right)^{3/2}\left(\frac{2}{3}(4+P^2)^{3/2} - 3(4+P^2)^{1/2}\right)\right]\Big\}.\end{aligned}}$$

$$\boxed{\frac{\mathscr{F}^{T=0}_{\mathrm{wall,ren}}}{S} = \frac{m_\pi^3}{24\pi^2}\left(-17\log\frac{4\pi\Lambda_{\mathrm{RG}}^2}{m_\pi^2} + 40\log 2 - \frac{121}{3} + 17\gamma_{\mathrm{E}}\right) + m_\pi^3\left(-\frac{64}{3}\bar{\ell}_1 - \frac{8}{3}\bar{\ell}_2 + \frac{8}{3}\bar{\ell}_3\right).}$$

NLO, finite T:

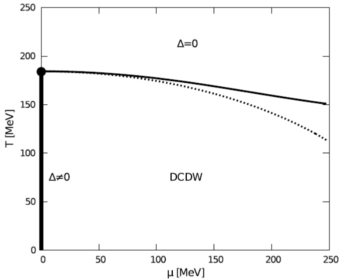
$$\boxed{\begin{aligned}\frac{\mathscr{F}^{T,(\pi^\pm)}_{\mathrm{wall}}}{S} = \frac{BT}{\pi}\sum_{m=0}^{\infty}\Big\{&\log\left[1 - e^{-\beta\sqrt{(2m+1)B - 3m_\pi^2}}\right] + \log\left[1 - e^{-\beta\sqrt{(2m+1)B}}\right] \\ &- \frac{1}{\pi}\int_0^\infty \mathrm{d}P\left(\frac{2}{1+P^2} + \frac{4}{4+P^2}\right)\log\left[1 - e^{-\beta\epsilon(m,P^2)}\right]\Big\}.\end{aligned}}$$

NB: Connection to Chiral Density Wave?

[Tatsumi,Nishiyama,Karasawa(2015)] [Ferrer,Incera(2020)]

- QCD at finite B, μ, T studied within NJL-like models, CDW found to be preferred in certain parameter range
- Important effect of chiral anomaly observed
- Only chiral limit considered
- CSL at chiral limit is equivalent to CDW! [Brauner,Yamamoto(2017)]

B=(300 MeV)^2:

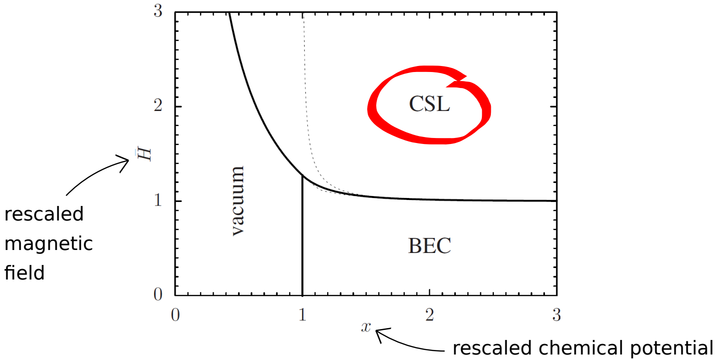


[Tatsumi,Nishiyama,Karasawa(2015)]

CSL phase in lattice simulations?

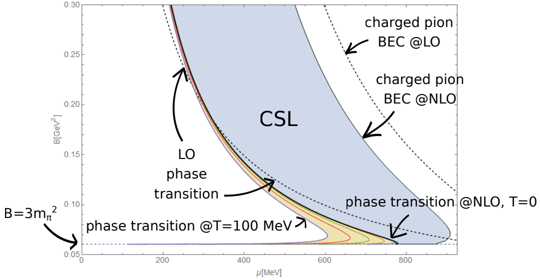
[Tomáš Brauner,Georgios Filios,H.K.;Phys.Rev.Lett.123(2019), JHEP 1912 (2019) 029]

- In certain QCD-like theories (e.g., two-color QCD) the sign problem is absent $\Rightarrow$ lattice simulations possible
- CSL-like phase present for sufficiently large magnetic fields! (Shown using chiral perturbation theory.)
- Conjecture of [Splittorff, Son, Stephanov (2001)] that the inhomogeneous phases exist only in theories with the sign problem disproved!



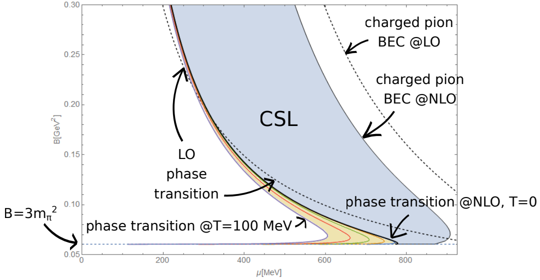
Conclusions

- "New" inhomogeneous phase of QCD matter appears for strong magnetic field and moderate baryon chemical potential
- Chiral soliton lattice is stable under thermal fluctuations! [arXiv: 2108.10044]
- Chiral soliton lattice phase in QCD-like theories may be in principle seen in lattice simulations!



Conclusions

- “New” inhomogeneous phase of QCD matter appears for strong magnetic field and moderate baryon chemical potential
- Chiral soliton lattice is stable under thermal fluctuations! [arXiv: 2108.10044]
- Chiral soliton lattice phase in QCD-like theories may be in principle seen in lattice simulations!

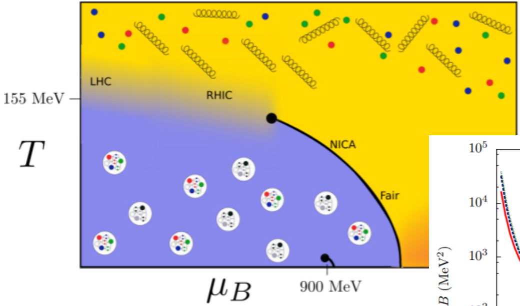


Thank you for your attention!

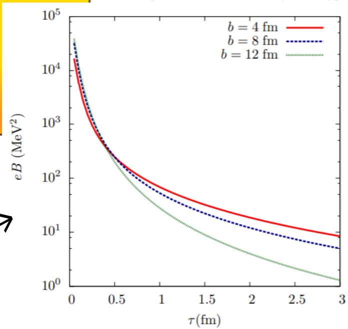
Backup Slides

Relevance of CSL for heavy ion collisions?

[Guenther(2020)]



[Kharzeev et al.(2007)]



Time evolution of the magnetic field created in heavy ion collisions with different impact parameters