Medium induced gluon spectrum in the Improved Opacity Expansion

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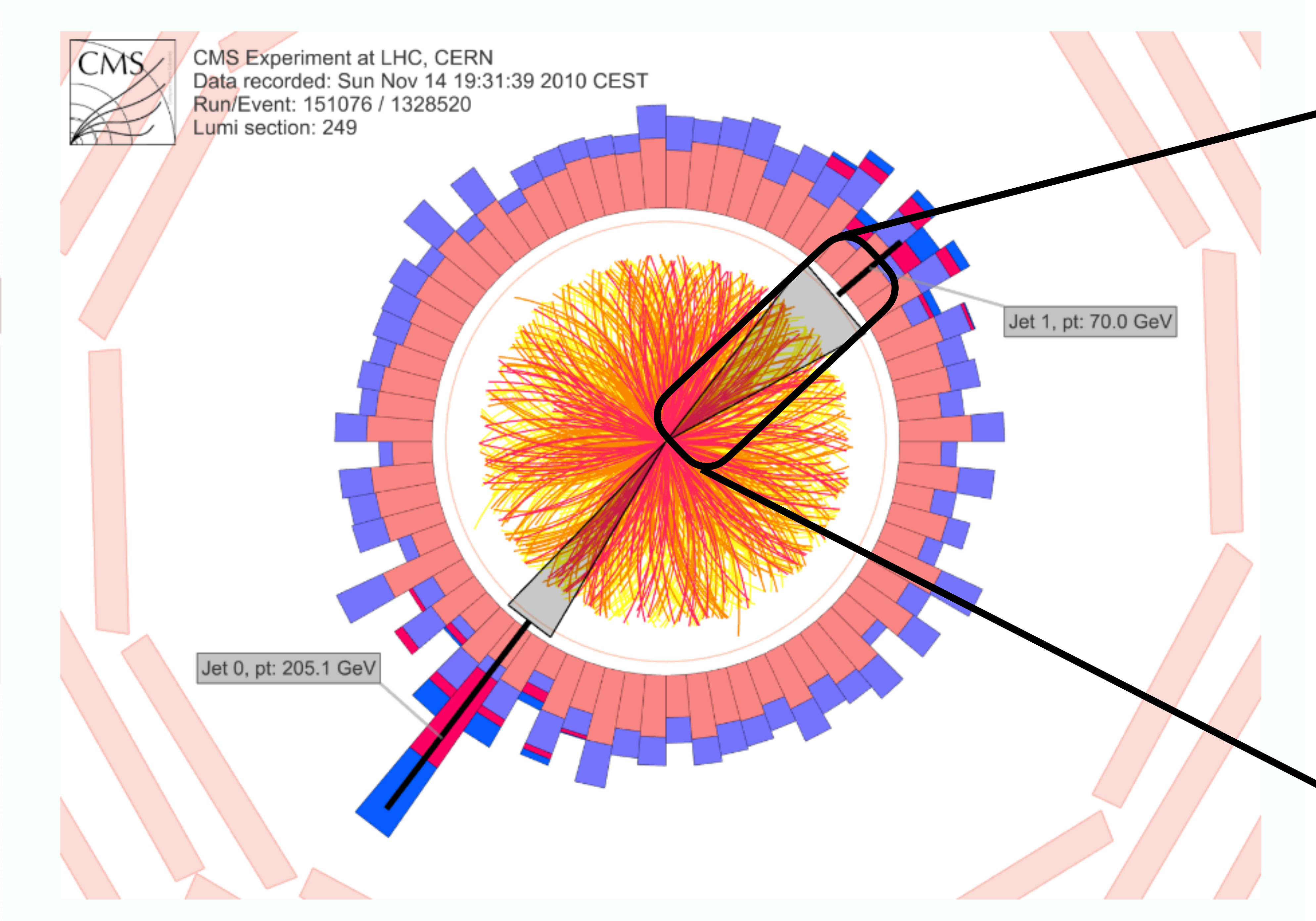
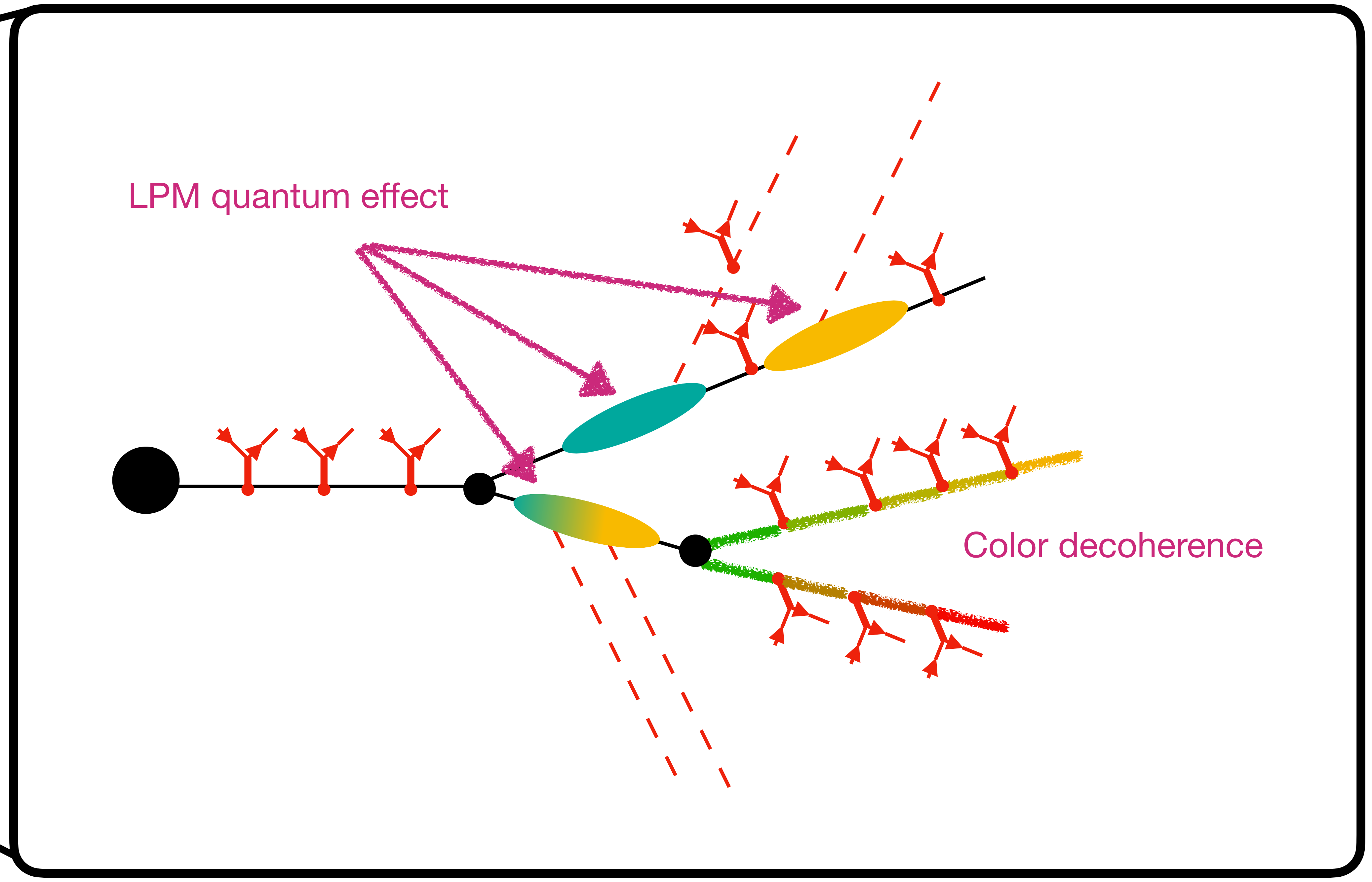
5th of September 2021, PANIC Lisbon

based on arXiv: 2104.04661 with Y. Mehtar-Tani, A. Soto-Ontoso, K. Tywoniuk

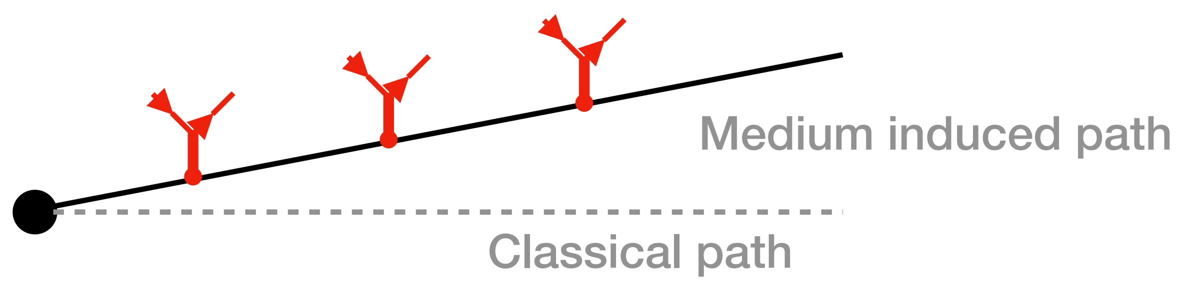
Jets in Heavy Ion Collisions





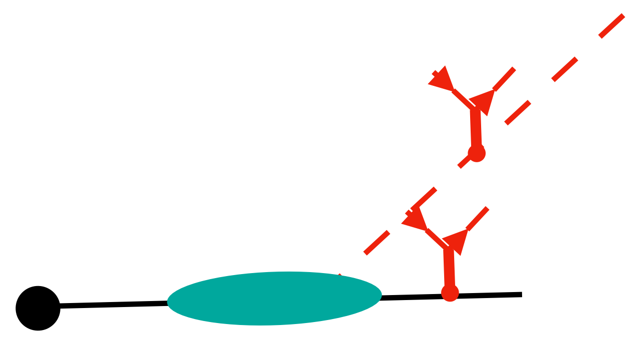
Basic building blocks for pheno:

Particle broadening $\mathcal{O}(\alpha_s^0)$



Easy to compute in the usual boosted kinematics

Induced radiation $\mathcal{O}(\alpha_s)$



Computable but far from being trivial

Complete numerical solutions are still actively studied

1006.2379 S. Caron-Huot, C. Gale

1810.08177 W. Ke, Y. Xu, S.A. Bass

1811.01591 X. Feal, R. Vazquez

2002.01517 C. Andrés, L. Apolinário, F. Dominguez

Jets in Heavy Ion Collisions

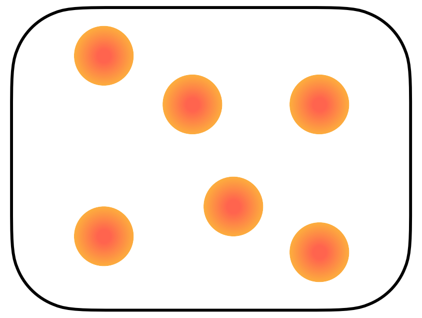
From an analytical perspective, one needs to make further approximations about the nature of the medium

Dense system

$L \gg \lambda$

Evolution dominated by multiple soft scatterings

Dilute system



$L \ll \lambda$

Evolution dominated by few hard scatterings

Can one unify both regimes?

Why is an analytical approach useful?

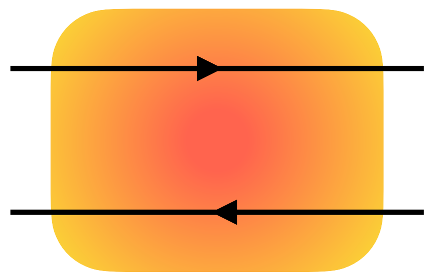
- Gain insight into the structure of the emission process
- Important to understand how the medium affects jet observables in detail, e.g. jet substructure
- Find simpler functional forms for the spectrum

The Improved Opacity Expansion: how it works

Consider the probability for a parton to acquire momentum $\mathbf{k}$ in-medium

$$\frac{\partial}{\partial L}\mathscr{P}(\mathbf{k},L) = C_R \int_{\mathbf{q}} \gamma(\mathbf{q}) \, \left[\mathscr{P}(\mathbf{k}-\mathbf{q},L) - \mathscr{P}(\mathbf{k},L)\right]$$

Collision rate



Typically, one goes to **x-space**

$$S(\mathbf{x},L) = e^{-\int_0^L ds\, v(\mathbf{x},s)} = e^{-v(\mathbf{x})\,L}$$

$$v(\mathbf{x}) \equiv C_R \int_{\mathbf{q}} \left(1 - e^{i\mathbf{q}\cdot\mathbf{x}}\right) \gamma(\mathbf{q})$$

dipole cross section

and then transforms back to **k-space**

$$\mathscr{P}(\mathbf{k},L) = \int_{\mathbf{x}} S(\mathbf{x},L)\, e^{-i\mathbf{k}\cdot\mathbf{x}}$$

When does $\mathscr{P}(\mathbf{k})$ admit a simple closed form?

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The Improved Opacity Expansion: how it works

Medium details are hidden in the in-medium scattering rate $v(\mathbf{x}) \equiv C_R \int_{\mathbf{q}} \left(1 - e^{i\mathbf{q}\cdot\mathbf{x}}\right) \gamma(\mathbf{q})$

Typically two models:

Gyulassy-Wang (GW) $\gamma^{\mathrm{GW}}(\mathbf{q}) = \dfrac{g^4 n}{(\mathbf{q}^2 + \mu^2)^2}$

PRL 68, 1480 X.-N. Wang, M. Gyulassy

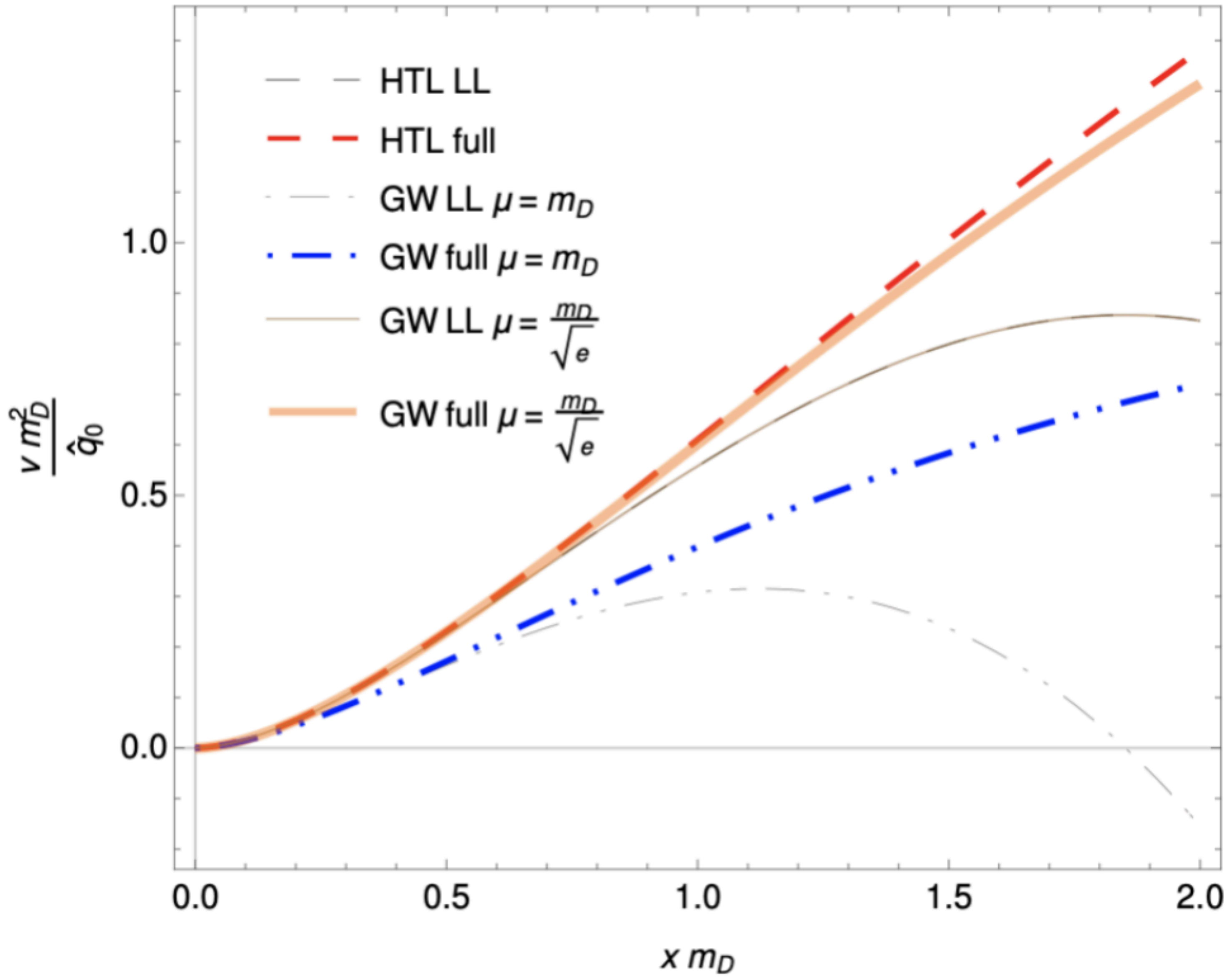
Hard Thermal-Loop (HTL) $\gamma^{\mathrm{HTL}}(\mathbf{q}) = \dfrac{g^2 m_D^2 T}{\mathbf{q}^2\,(\mathbf{q}^2 + m_D^2)}$

hep-ph/0204146 P. Aurenche, F. Gelis, H. Zaraket

In the ultraviolet, both models detail a Coulomb interaction, implying

$$v^{\mathrm{LP}}(\mathbf{x}) = \frac{\hat{q}_0}{4}\,\mathbf{x}^2 \log\left(\frac{1}{\mathbf{x}^2\mu_*^2}\right) + \mathcal{O}(\mathbf{x}^4\mu_*^2)$$

$$\mu_*^2 = \begin{cases} \dfrac{\mu^2}{4}\,\mathrm{e}^{-1+2\gamma_E} & \text{for the GW model} \\ \dfrac{m_D^2}{4}\,\mathrm{e}^{-2+2\gamma_E} & \text{for the HTL model} \end{cases}$$



In contrast with more standard prescription

$$\mu = m_D$$

0705.3439 M. Djordjevic, U. Heinz

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The Improved Opacity Expansion: how it works

When does $\mathscr{P}$ have a closed form?

Make further assumption on medium opacity $\chi \sim \frac{Q_{s0}^2 = \hat{q}_0 L}{\mu_*^2}$

$$S^{\rm LP}(\boldsymbol{x}) = \exp\left[-\frac{1}{4}Q_{s0}^2\,\boldsymbol{x}^2\,\log\frac{1}{\boldsymbol{x}^2\mu_*^2}\right] + \mathcal{O}(\boldsymbol{x}^2\mu_*^2)$$

$$\mathscr{P}(\mathbf{k},L) = \int_{\mathbf{x}} S(\mathbf{x},L)\,e^{-i\mathbf{k}\cdot\mathbf{x}}$$

In the dilute $\chi \ll 1$ approximation: $\mathbf{k}^2 \sim \frac{1}{\mathbf{x}^2} \gg Q_{s0}^2$

$$S^{\rm LP}(\boldsymbol{x})\Big|_{|\boldsymbol{x}|\ll 1/Q_{s0}} = 1 - \frac{1}{4}Q_{s0}^2\,\boldsymbol{x}^2\,\log\frac{1}{\boldsymbol{x}^2\mu_*^2} + \mathcal{O}\left(\boldsymbol{x}^4 Q_{s0}^4\right)$$

$$\mathscr{P}^{\rm SH}(\mathbf{k},L) = \frac{1}{4}Q_{s0}^2\,\vec{\nabla}_{\mathbf{k}}^2\frac{4\pi}{\mathbf{k}^2} = 4\pi\frac{Q_{s0}^2}{\mathbf{k}^4}$$

In the dense $\chi \gg 1$ approximation: $\mathbf{k}^2 \sim \frac{1}{\mathbf{x}^2} \leq Q_{s0}^2$

$$Q_s^2 = Q_{s0}^2 \log\frac{Q^2}{\mu_*^2}$$

$$S^{\rm LP}(\boldsymbol{x}) = \exp\left[-\frac{1}{4}Q_{s0}^2\,\boldsymbol{x}^2\,\log\frac{Q^2}{\cancel{\boldsymbol{x}^2}\mu_*^2}\right] + \mathcal{O}(\boldsymbol{x}^2\mu_*^2)$$

$$\mathscr{P}^{\rm MS}(\mathbf{k},L) = \frac{4\pi}{Q_s^2}\,e^{-\frac{\mathbf{k}^2}{Q_s^2}}$$

2009.13667 with Y. Mehtar-Tani, A. Soto-Ontoso, K. Tywoniuk

The Improved Opacity Expansion: how it works

When does $\mathscr{P}$ have a closed form?

So far:

$|\mathbf{k}| < Q_s$	$|\mathbf{k}| > Q_s$
Multiple Soft ✓	Multiple Soft ✗
Single Hard ✗	Single Hard ✓

Going beyond: Moliere's theory of scattering G. Moliere (1948)

1 $$v^{\rm LP}(\mathbf{x}) = \frac{\hat{q}_0}{4}\mathbf{x}^2 \log\left(\frac{1}{\mathbf{x}^2\mu_*^2}\right) \longrightarrow \underbrace{\frac{\hat{q}_0}{4}\mathbf{x}^2 \log\left(\frac{Q^2}{\mu_*^2}\right)}_{v^{\rm MS}\ \text{(fixed point)}} + \underbrace{\frac{\hat{q}_0}{4}\mathbf{x}^2 \log\left(\frac{1}{Q^2\mathbf{x}^2}\right)}_{\delta v\ \text{(perturbation)}}$$

2 $$Q^2 \gg \mu_*^2 \xrightarrow{Q^2 = Q_s^2 = Q_{s0}^2 \log \frac{Q_s^2}{\mu_*^2}}$$

$v^{\rm MS} \gg \delta v$ for $Q_s^2 \geq \mathbf{k}^2$

$v^{\rm MS} \ll \delta v$ for $Q_s^2 \leq \mathbf{k}^2$

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The Improved Opacity Expansion: how it works

When does $\mathscr{P}$ have a closed form?

In the **IOE** approximation:

$$\mathscr{P}^{\rm LP}(\mathbf{k},L) = \sum_n \int_{\mathbf{x}} e^{-i\mathbf{x}\cdot\mathbf{k}} \underbrace{e^{-\frac{1}{4}\mathbf{x}^2 Q_s^2}}_{\text{MS}} \underbrace{\frac{(-1)^n Q_{s0}^{2n}}{4^n n!}\mathbf{x}^{2n}\log^n\frac{1}{\mathbf{x}^2 Q^2}}_{\text{Hard pert. corrections}} \equiv \mathscr{P}^{(0)} + \overset{\delta v}{\mathscr{P}^{(1)}} + \overset{(\delta v)^2}{\mathscr{P}^{(2)}} + \cdots$$

Truncating **at first order** we find the simple result :

$$\lambda = \frac{\hat{q}_0}{\hat{q}} = \frac{1}{\log\frac{Q_s^2}{\mu_*^2}}$$

Expansion parameter in the soft sector

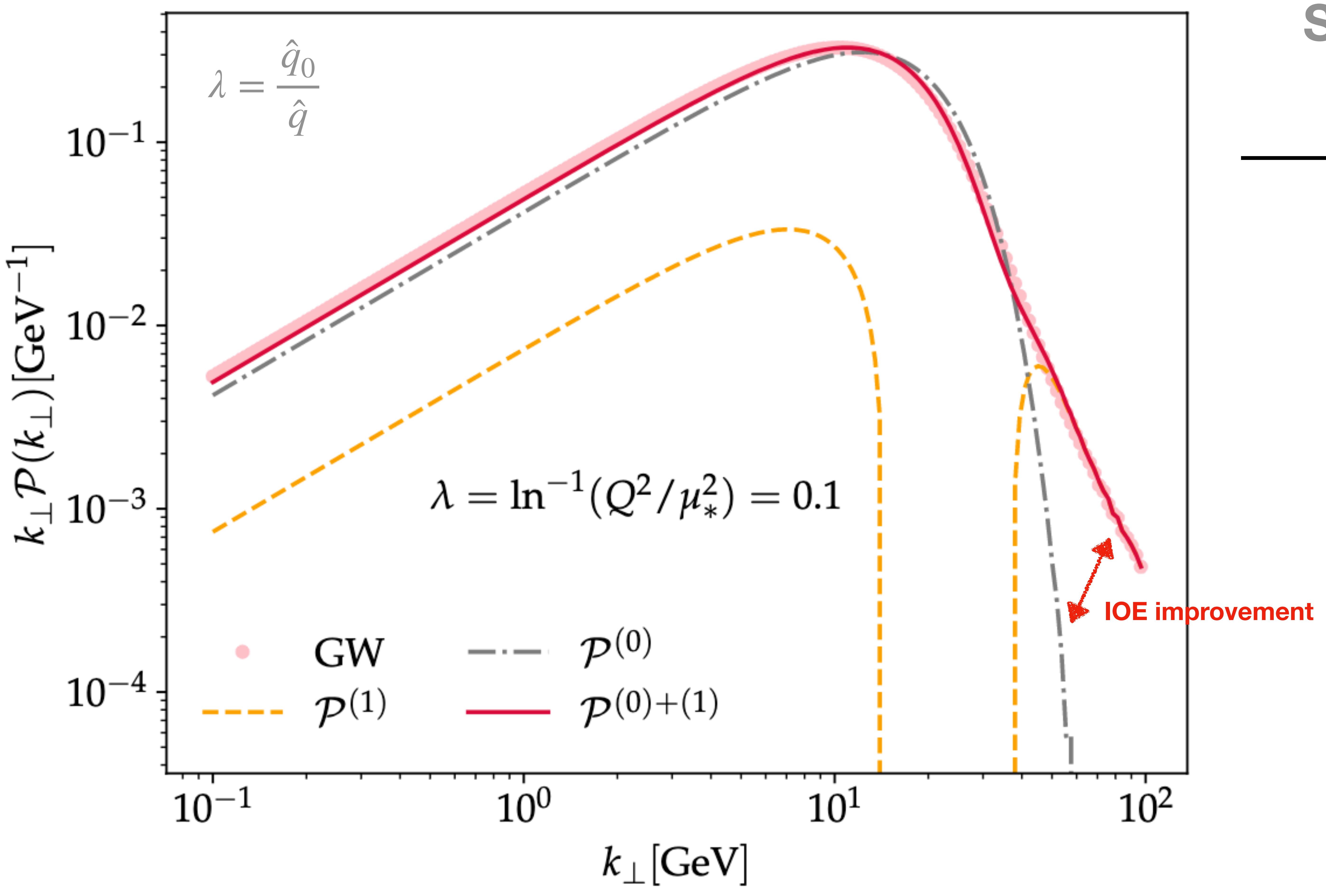
$$\boxed{\mathscr{P}^{(0)+(1)}(\mathbf{k},L) = \underbrace{\frac{4\pi}{Q_s^2}e^{-x}}_{\text{MS}}\left\{1-\lambda\left(e^x - 2 + (1-x)\left(\text{Ei}\,(x) - \log(4x)\right)\right)\right\}}$$

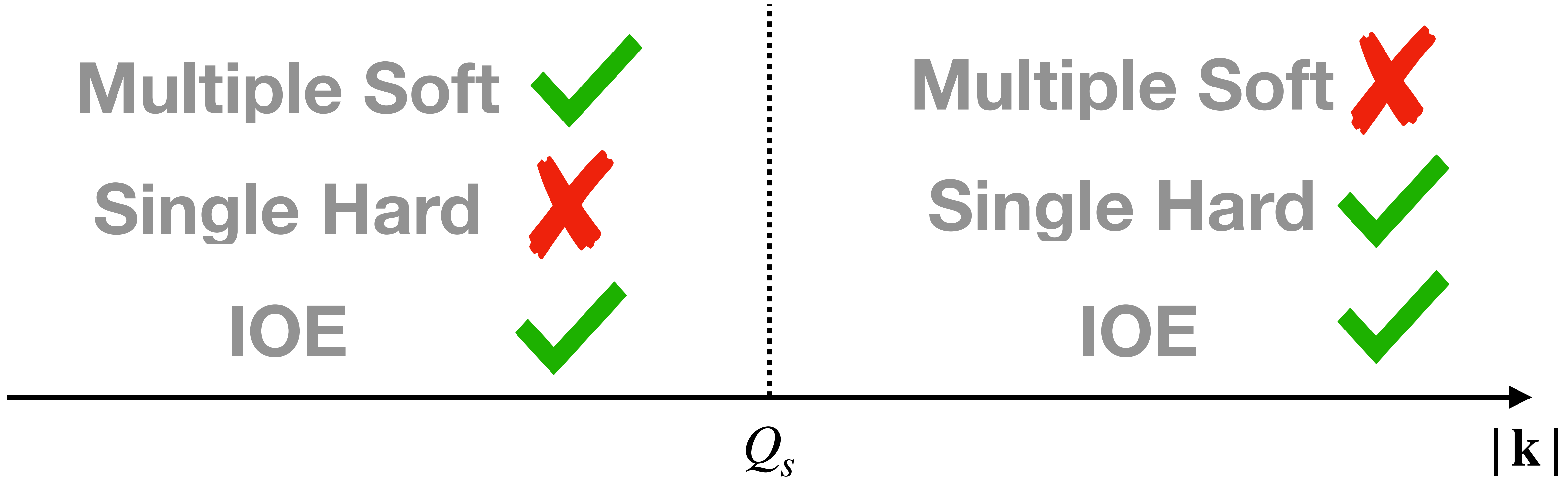
$$x = \frac{\mathbf{k}^2}{Q_s^2}$$

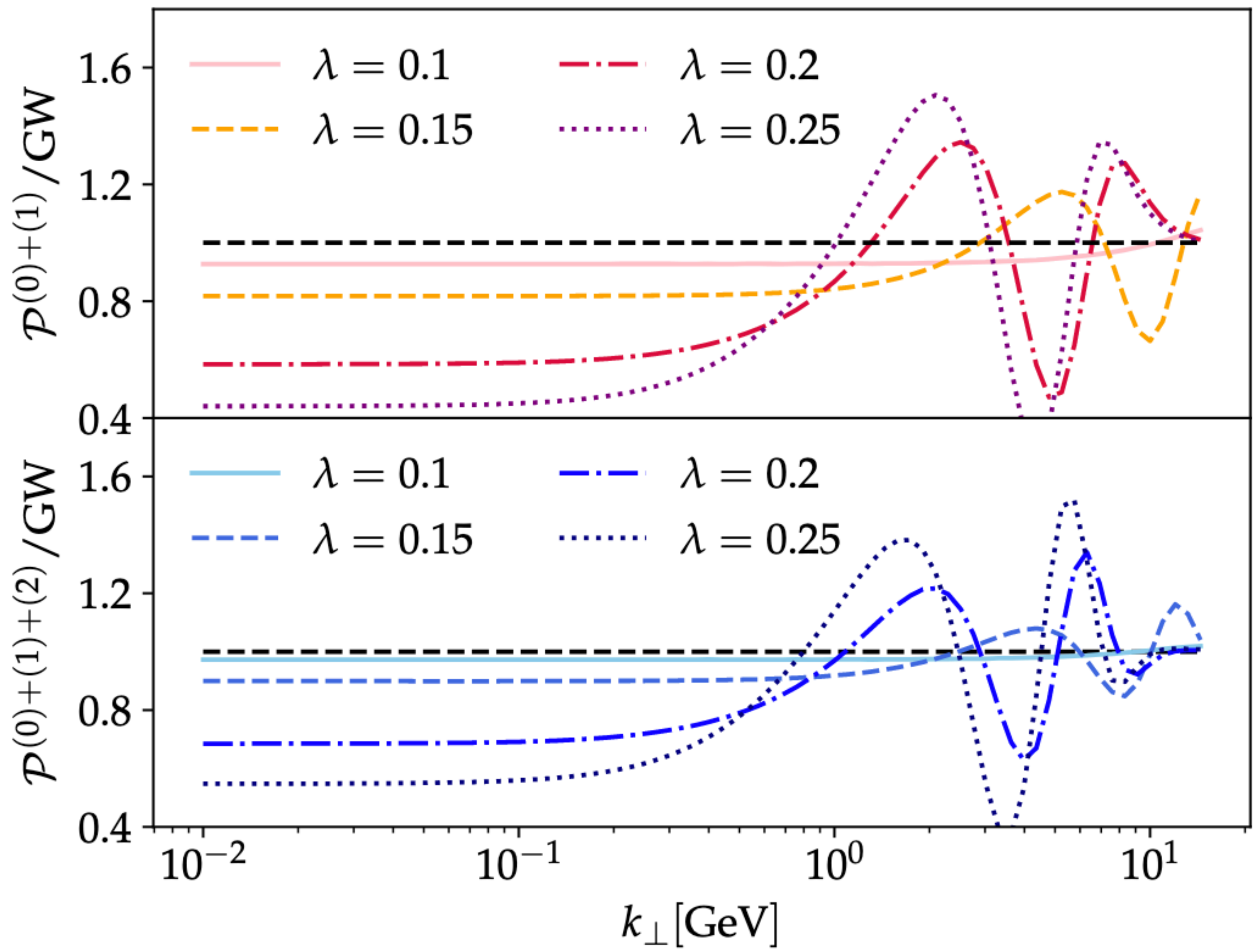
$$\mathscr{P}^{(1)}(\mathbf{k},L)\Big|_{\mathbf{k}^2\ll Q_s^2} = \underbrace{\frac{4\pi\lambda}{Q_s^2}\log 4e^{1-\gamma_E}}_{\text{MS (soft) } \times\lambda} \qquad ✓$$

$$\mathscr{P}^{(1)}(\mathbf{k},L)\Big|_{\mathbf{k}^2\gg Q_s^2} = \underbrace{4\pi\frac{Q_{s0}^2}{\mathbf{k}^4}}_{\text{SH}} \qquad ✓$$

The Improved Opacity Expansion: how it works







Medium induced spectrum in the IOE

The medium gluon spectrum can be written as:

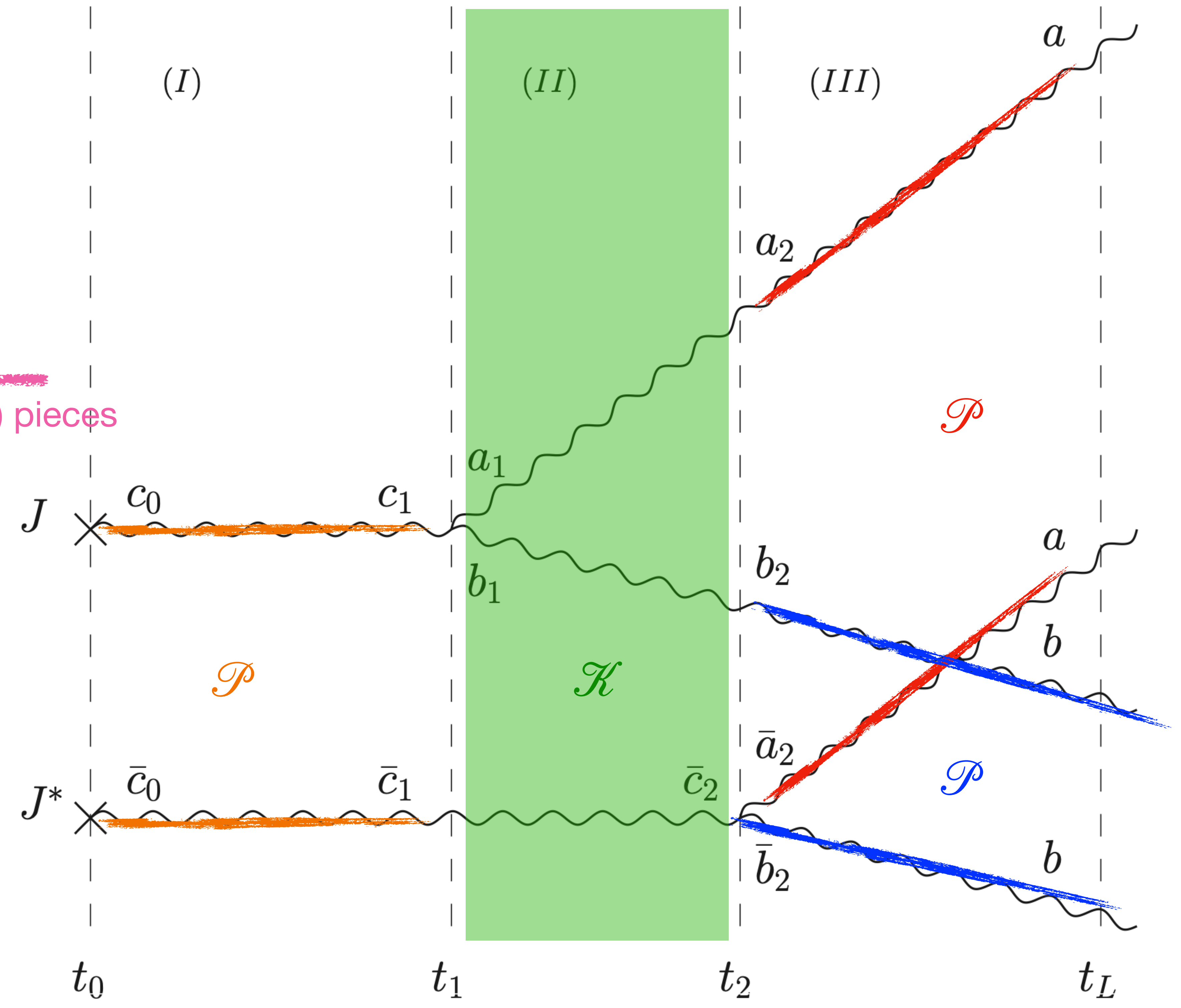
$$(2\pi)^2\omega\frac{\mathrm{d}I}{\mathrm{d}\omega\mathrm{d}^2\boldsymbol{k}} = \frac{2\bar{\alpha}\pi}{\omega^2}\mathrm{Re}\int_0^\infty \mathrm{d}t_2\,\mathrm{e}^{-\epsilon t_2}\int_0^{t_2}\mathrm{d}t_1\int_{\boldsymbol{x}}\mathrm{e}^{-i\boldsymbol{k}\cdot\boldsymbol{x}}\,\mathcal{P}(\boldsymbol{x},\infty;t_2)\,\boldsymbol{\partial_x}\cdot\boldsymbol{\partial_y}\mathcal{K}(\boldsymbol{x},t_2;\boldsymbol{y},t_1)_{\boldsymbol{y}=0} - \frac{8\bar{\alpha}\pi}{k_\perp^2}$$

Gluon energy and momentum

broadening

effective kernel

vacuum-(like) pieces



where the kernel satisfies

$$\left[i\partial_t + \frac{\partial^2}{2\omega^2} + iv(\mathbf{x})\right]\mathscr{K}(\mathbf{x},t_2|\mathbf{y},t_1) = i\delta(\mathbf{x}-\mathbf{y})\delta(t_2-t_1)$$

Schrodinger equation with imaginary (dissipative) potential

The IOE requires a series expansion of both $\mathscr{P}$ and $\mathscr{K}$!

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Medium induced spectrum in the traditional approaches

In the dilute $\chi \ll 1$ approximation: GLV/W

$$\mathcal{K}(\mathbf{x}, t_2 | \mathbf{y}, t_1) = \underbrace{\mathcal{K}_0(\mathbf{x}, t_2 | \mathbf{y}, t_1)}_{\text{known exactly}} - \int_{\mathbf{z}} \int_{t_1}^{t_2} ds\, \mathcal{K}_0(\mathbf{x}, t_2 | \mathbf{z}, s) v(\mathbf{z}, s) \mathcal{K}(\mathbf{z}, s | \mathbf{y}, t_1)$$

In the dense $\chi \gg 1$ approximation: BDMPS-Z/ASW

$$v(\mathbf{x}, s) \propto \hat{q}\mathbf{x}^2$$

$$\mathcal{K}_{\mathrm{HO}}(\mathbf{x}, t | \mathbf{y}, t_1) = \frac{\omega}{2\pi i S(t, t_1)} \exp\left[\frac{i\omega}{2S(t, t_1)} \left\{C(t_1, t)\, \mathbf{x}^2 + C(t, t_1)\, \mathbf{y}^2 - 2\mathbf{x} \cdot \mathbf{y}\right\}\right]$$

$$\left[\frac{d^2}{d^2t} + \Omega[t]\right] C(t, t_0) = 0\,, \quad C(t_0, t_0) = 1\,, \quad \partial_t C(t, t_0)_{t=t_0} = 0$$

$$\left[\frac{d^2}{d^2t} + \Omega[t]\right] S(t, t_0) = 0\,, \quad S(t_0, t_0) = 0\,, \quad \partial_t S(t, t_0)_{t=t_0} = 1$$

$$\Omega(t) = \frac{1-i}{2}\sqrt{\frac{\hat{q}(t)}{\omega}}$$

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Medium induced spectrum in the IOE

To first order in the IOE on can write the spectrum as

$$\frac{\mathrm{d}I}{\mathrm{d}\omega \mathrm{d}^2\boldsymbol{k}} = \frac{\mathrm{d}I^{\mathrm{LO}}}{\mathrm{d}\omega \mathrm{d}^2\boldsymbol{k}} + \frac{\mathrm{d}I^{\mathrm{NLO}}}{\mathrm{d}\omega \mathrm{d}^2\boldsymbol{k}} + \mathcal{O}(\delta v^2)\,.$$

The Leading Order term is simply the BDMPS-Z result

$\mathscr{K}_{\mathrm{HO}} = \mathscr{K}_{\mathrm{LO}}$ in what follows

$$(2\pi)^2\omega\frac{\mathrm{d}I^{\mathrm{LO}}}{\mathrm{d}\omega \mathrm{d}^2\boldsymbol{k}} = \frac{2\bar{\alpha}\pi}{\omega^2}\mathrm{Re}\int_0^\infty \mathrm{d}t_2\,\mathrm{e}^{-\epsilon t_2}\int_0^{t_2}\mathrm{d}t_1\int_{\boldsymbol{x}}\mathrm{e}^{-i\boldsymbol{k}\cdot\boldsymbol{x}} \times \mathcal{P}^{\mathrm{LO}}(\boldsymbol{x},\infty;t_2)\boldsymbol{\partial}_{\boldsymbol{x}}\cdot\boldsymbol{\partial}_{\boldsymbol{y}}\mathcal{K}^{\mathrm{LO}}(\boldsymbol{x},t_2;\boldsymbol{y},t_1)_{\boldsymbol{y}=0} - \frac{8\bar{\alpha}\pi}{k_\perp^2}$$

while the first correction reads

$$(2\pi)^2\omega\frac{\mathrm{d}I^{\mathrm{NLO}}}{\mathrm{d}\omega \mathrm{d}^2\boldsymbol{k}} = \frac{2\bar{\alpha}\pi}{\omega^2}\mathrm{Re}\int_0^\infty \mathrm{d}t_2\,\mathrm{e}^{-\epsilon t_2}\int_0^{t_2}\mathrm{d}t_1\int_{\boldsymbol{x}}\mathrm{e}^{-i\boldsymbol{k}\cdot\boldsymbol{x}} \times \Big[\mathcal{P}^{\mathrm{LO}}(\boldsymbol{x},\infty;t_2)\boldsymbol{\partial}_{\boldsymbol{x}}\cdot\boldsymbol{\partial}_{\boldsymbol{y}}\underline{\mathcal{K}^{\mathrm{NLO}}(\boldsymbol{x},t_2;\boldsymbol{y},t_1)}_{\boldsymbol{y}=0} + \underline{\mathcal{P}^{\mathrm{NLO}}(\boldsymbol{x},\infty;t_2)}\boldsymbol{\partial}_{\boldsymbol{x}}\cdot\boldsymbol{\partial}_{\boldsymbol{y}}\mathcal{K}^{\mathrm{LO}}(\boldsymbol{x},t_2;\boldsymbol{y},t_1)_{\boldsymbol{y}=0}\Big]$$

One can simplify these expressions to forms similar to the BDMPS-Z result !

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Medium induced spectrum in the IOE: asymptotics

The IOE allows for a simple treatment of the spectrum in the asymptotic regions

In the **Multiple Soft region** where $\omega \ll \omega_c$ and $t_f \ll L$

some relevant discussion in 1209.4585, J.-P. Blaizot, F. Dominguez, E. Iancu, Y. Mehtar-Tani

one can show that $$(2\pi)^2\omega\frac{\mathrm{d}I}{\mathrm{d}\omega\mathrm{d}^2\boldsymbol{k}} = \int_0^\infty \mathrm{d}t_2\,\mathcal{P}(\boldsymbol{k};L-t_2)\,\omega\frac{\mathrm{d}I}{\mathrm{d}\omega\mathrm{d}t_2}$$ $+$ $$\omega\frac{\mathrm{d}I^{\rm IOE}}{\mathrm{d}\omega} = \omega\frac{\mathrm{d}I^{\rm LO}}{\mathrm{d}\omega}(\hat{q}\to\hat{q}_{\rm eff})$$

A simple example :

$$(2\pi)^2\omega\frac{\mathrm{d}I^{\rm NLO}_{\text{in-out}}}{\mathrm{d}\omega\mathrm{d}^2\boldsymbol{k}} \approx \left((2\pi)^2\omega\frac{\mathrm{d}I^{\rm LO}_{\text{in-out}}}{\mathrm{d}\omega\mathrm{d}^2\boldsymbol{k}}\right)\frac{\hat{q}_0}{\hat{q}}\left(0.26(5)+\underbrace{\frac{1}{4}\log\frac{Q_r^2}{\sqrt{\hat{q}\omega}}}\right)$$

Enters redefinition of transport parameter

In the **Single Hard region** where $\omega_c \ll \omega$ and $k_\perp^2 \gg \hat{q}L$

IR dependence; must be absent from NLO term !

Exact / GLV :
$$(2\pi)^2\omega\frac{\mathrm{d}I^{\rm GLV}}{\mathrm{d}\omega\mathrm{d}^2\boldsymbol{k}} = \underbrace{\frac{8\bar{\alpha}\pi\hat{q}_0L}{k_\perp^4}}_{\text{Coulomb form}}\left(3\gamma_E-4+\log\left(\frac{k_\perp^2}{4\mu_*^2}\right)+\log\left(\frac{k_\perp^2L}{2\omega}\right)\right)$$

||

LO
$$\frac{8\bar{\alpha}\pi\hat{q}_0L}{k_\perp^4}\log\frac{\cancel{Q_b^2}}{\mu_*^2}$$

$+$

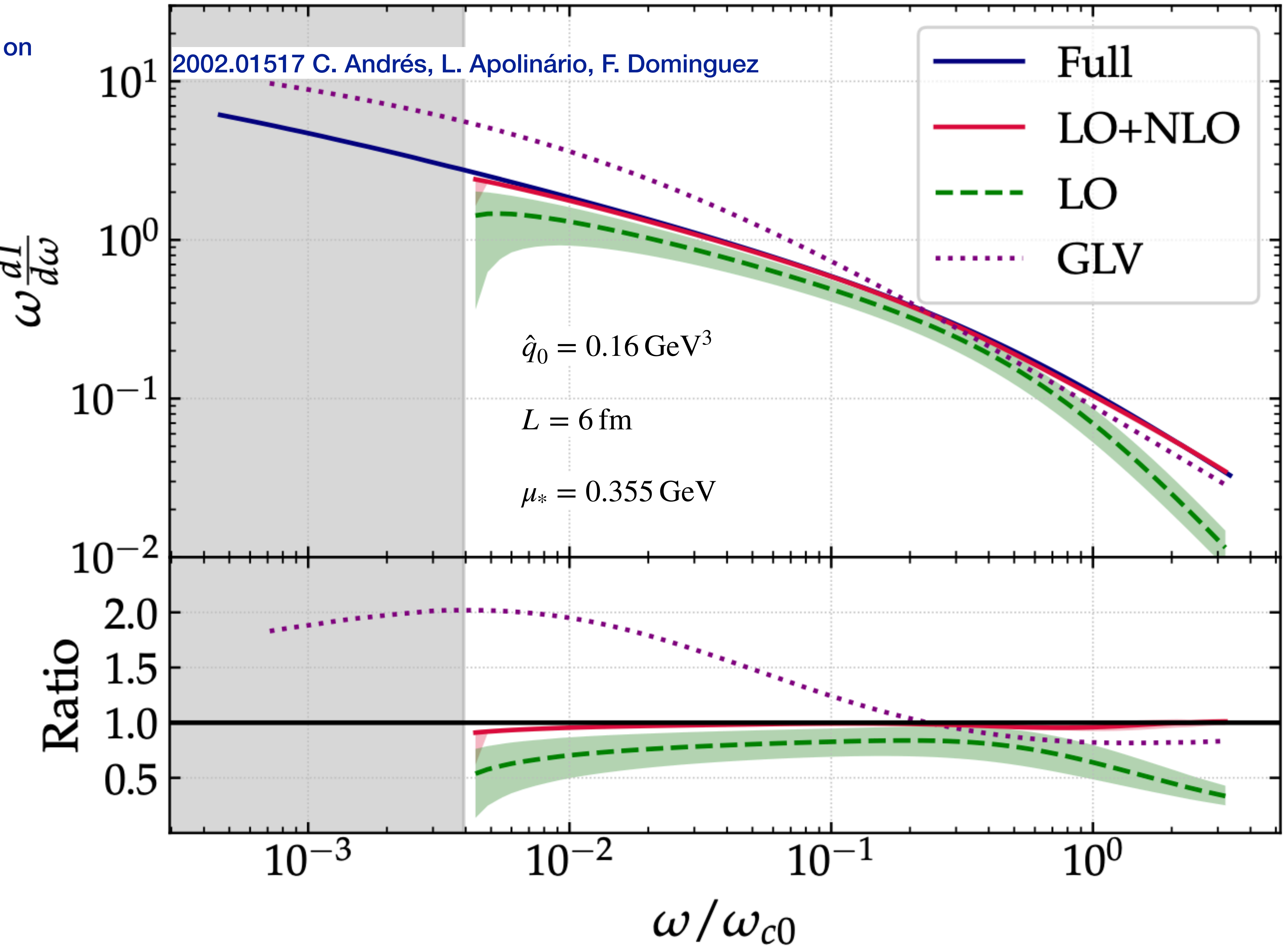
NLO
$$\frac{8\pi\bar{\alpha}\hat{q}_0L}{k_\perp^4}\left[3\gamma_E-4+\log\left(\frac{k_\perp^2}{4\cancel{Q_b^2}}\right)+\log\left(\frac{k_\perp^2L}{2\omega}\right)\right]$$

- ✓ Generic property of the IOE
- ✓ Cancellation with no constraint on Q

Medium induced spectrum in the IOE: numerical example

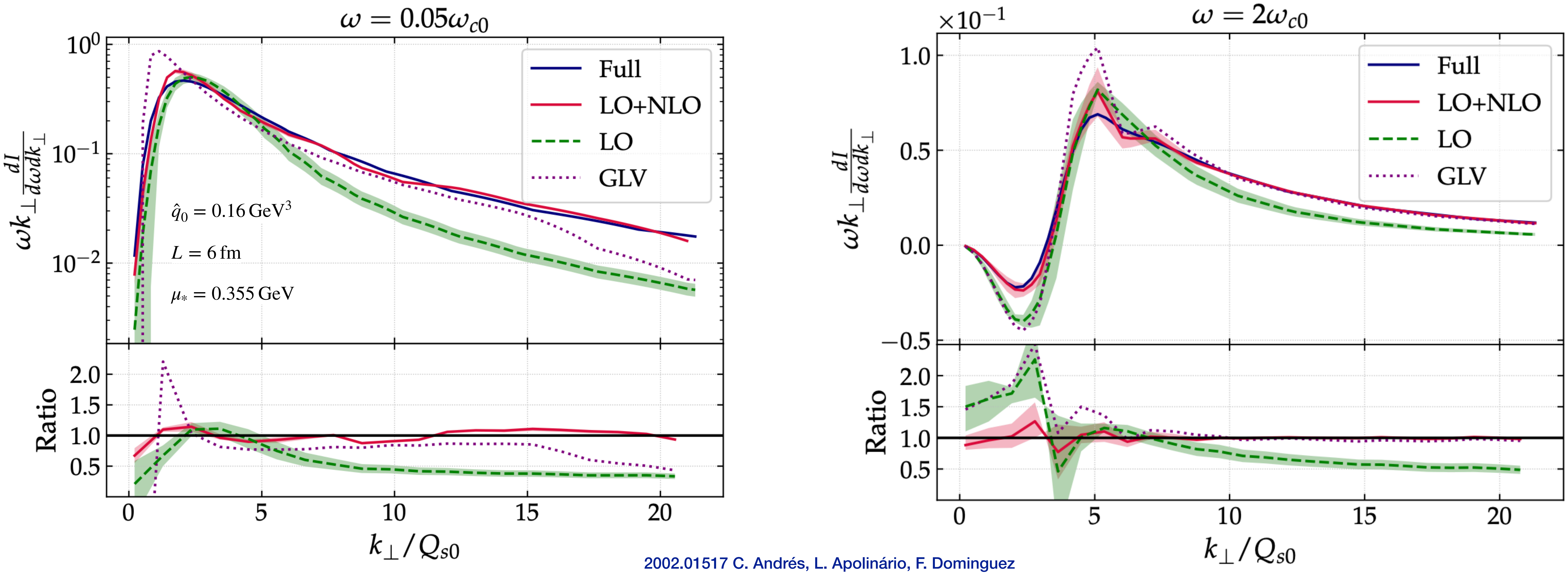
For the integrated spectrum, the expansion recovers the correct high energy tail

See Marcos G. Martinez talk later today on work related to this numerical approach



Medium induced spectrum in the IOE: numerical example

For the integrated spectrum, the expansion recovers the correct high energy tail



2002.01517 C. Andrés, L. Apolinário, F. Dominguez

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Conclusions and Outlook

- I have shown that the **IOE can be applied to compute the fully differential gluon spectrum** under the usual approximations

- The final expressions are of the **same "complexity" as the BDMPS-Z** result

- One can **analytically check that the IOE recovers the GLV** result in the appropriate kinematical region

- Immediate numerical application in LHC-inspire setting reveals the **IOE can be an interesting tool for pheno**; e.g. jet substructure