Studying chiral imbalance using Chiral Perturbation Theory.

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Abstract

We analize the most general low-energy effective lagrangian including local parity violating terms parametrized by an axial chemical potential μ_5. This result is obtained following the external source method, up to $\mathcal{O}(p^4)$ order in the chiral expansion for two light flavours. We show that the $\mathcal{O}(p^4)$ lagrangian includes new terms proportional to μ_5^2 and new low-energy constants. Finally, the μ_5 and temperature dependences of several observables related to the vacuum energy density are studied.

Chiral imbalance

A convenient way to parametrize a P-breaking source or chiral imbalance is by means of a constant axial chemical potential μ_5 to be added to the QCD action over a given finite space-time region.

The chiral charge may remain approximately conserved during the fireball evolution in a typical heavy-ion collision, giving rise in the light quark sector to a chemical potential term.

So we consider an axial source $a_\mu^0 = \mu_5 \delta_{\mu 0}$ in the QCD generating functional $Z_{QCD}(v, a, s, p, \theta)$. It is possible to perform a $U_1(A)$ rotation on the quark fields which allows to trade a and θ terms:

$$Z_{QCD}[0,0,\mathcal{M},0,\theta(x)] = Z_{QCD}\left[0, \frac{1}{2N_f}\partial_\mu\theta(x)\mathbb{1}, \mathcal{M}\cos\left(\theta(x)/N_f\right), \mathcal{M}\sin\left(\theta(x)/N_f\right), 0\right]. \quad (1)$$

Construction of the effective lagrangian

We consider the effective low-energy representation of $Z(v, a, s, p, \theta)$ in the case $v = p = 0$, $s = \mathcal{M}$ and the axial source given. The construction of the most general, model-independent, effective lagrangian can be carried out within the framework of the external source method.

To construct the lagrangian, the so called "spurion" fields $Q_{R,L}(x)$ are introduced:

$$\mathcal{L}_Q = A_\mu \bar{q}\gamma^\mu \left[Q_L(x)P_L + Q_R(x)P_R\right] q, \quad (2)$$

where in our case $Q_L = -Q_R = \mu_5/F$ and $A_\mu = F\delta_{\mu 0}$. There are additional terms in the effective lagrangian depending on $Q_{R,L}$.

The operator $\mathrm{tr}(U^\dagger d_\mu U)$ has to be considered, unlike in standard ChPT where that operator vanishes.

The leading order $\mathcal{O}(p^2)$ lagrangian

At $\mathcal{O}(p)$, the only nontrivial operator is $tr(Q_R + Q_L)$.

To $\mathcal{O}(p^2)$ the following operators are also allowed:

$$\mathrm{tr}\left(Q_R U Q_L U^\dagger\right), \quad \mathrm{tr}\left(U^\dagger d^\mu U\right)\mathrm{tr}\left(U^\dagger d^\mu U\right),$$

$$\mathrm{tr}\left(Q_L^2 + Q_R^2\right), \quad \mathrm{tr}\left(Q_L^2\right) + \mathrm{tr}\left(Q_R^2\right), \quad \mathrm{tr}\left(Q_R\right)\mathrm{tr}\left(Q_L\right).$$

Therefore, at this order, the only modification to the chiral lagrangian is a constant term:

$$\mathcal{L}_2 = \frac{F^2}{4}\mathrm{tr}\left[\partial_\mu U^\dagger \partial^\mu U + 2B_0\mathcal{M}\left(U + U^\dagger\right)\right] + 2\mu_5^2 F^2\left(1 - Z + \kappa_0\right). \quad (3)$$

Next to leading order $\mathcal{O}(p^4)$ lagrangian

At this order we have new terms constructed out of the Q operators and $\mathrm{tr}(U^\dagger d_\mu U)$. The possible contributions (including Q operators) are of the form: $ddQQ$, χQQ, $QQQQ$.

The form of the explicit μ_5 corrections are:

$$\mathcal{L}_4(\mu_5) = \mathcal{L}_4^0(\mu_5 = 0) + \kappa_1\mu_5^2\mathrm{tr}\left(\partial_\mu U^\dagger\partial^\mu U\right) + \kappa_2\mu_5^2\mathrm{tr}\left(\partial_0 U^\dagger \partial^0 U\right) + \kappa_3\mu_5^2\mathrm{tr}\left(\chi^\dagger U + \chi U^\dagger\right) + \kappa_4\mu_5^2. \quad (4)$$

Pion dispersion relation

The spatial and time components of the pion decay constant are different:

$$\left(F_\pi^t\right)^2(\mu_5) = F_\pi^2(0) + 4(\kappa_1 + \kappa_2)\mu_5^2 \quad (5)$$

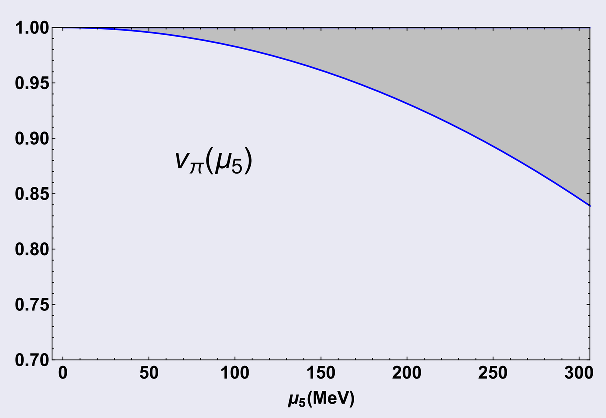
$$\left(F_\pi^s\right)^2(\mu_5) = F_\pi^2(0) + 4\kappa_1\mu_5^2 \quad (6)$$

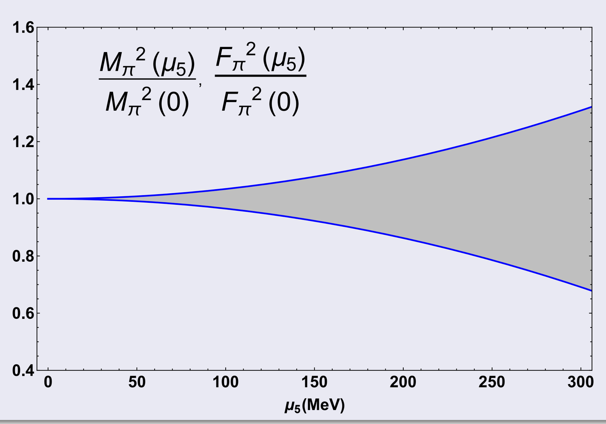
The two main physical consequences of that are the pion velocity and pion mass:

$$v_\pi(\mu_5) = 1 + 2\kappa_2\frac{\mu_5^2}{F^2} \quad (7)$$

$$\left[M_\pi^2\right]^{\mathrm{pole}}(\mu_5) = M_\pi^2(0) - 4\left(\kappa_1 + \kappa_2 - \kappa_3\right)\frac{\mu_5^2}{F^2}M^2 \quad (8)$$

Constrain for κ_2: $v_\pi(\mu_5) < 1 \to \kappa_2 < 0$.

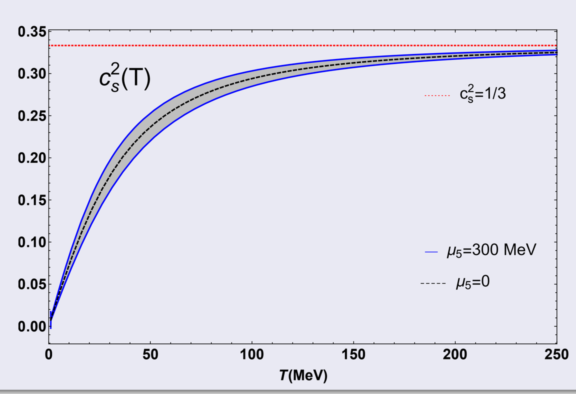




Pressure and speed of sound

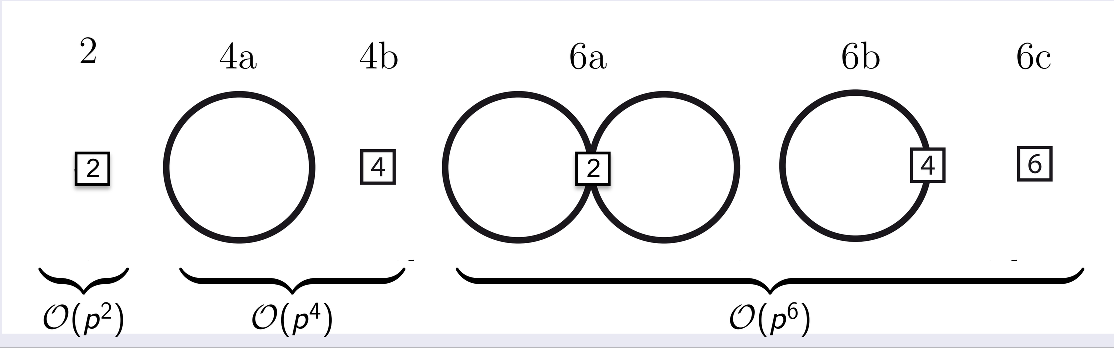
The μ_5 corrections to the pressure are parametrized by κ_2 and κ_b but in the chiral limit, which corresponds to the ultrarelativistic free pion gas, only the κ_2 term survives.

The speed of sound (c_s) depends only on κ_b and when we include the μ_5 corrections, c_s^2 remains below $1/3$. The uncertainty band for κ_b actually narrows as T increases.



The vacuum energy density

At $\mathcal{O}(p^2)$, it only contributes the constant part of the $\mathcal{L}_2$ lagrangian. The $\mathcal{O}(p^4)$ and $\mathcal{O}(p^6)$ include:



Quark condensate and scalar susceptibility

The first μ_5 correction to the quark condensate is temperature independent:

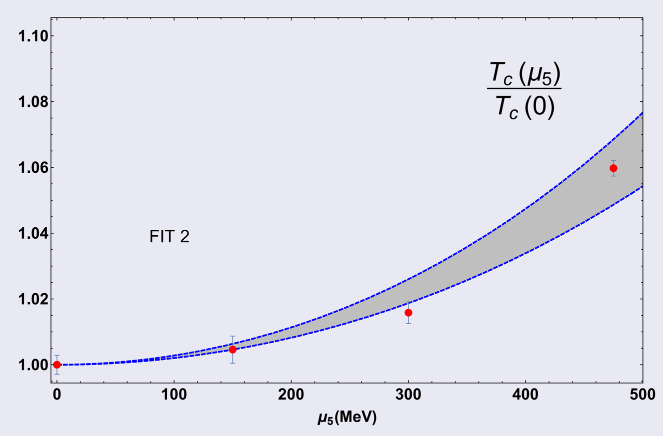
$$\langle\bar{q}q\rangle_l^{\mathrm{LO}} = -2B_0F^2, \quad \langle\bar{q}q\rangle_l^{\mathrm{NLO}}(T,\mu_5) = \langle\bar{q}q\rangle_l^{\mathrm{NLO}}(T,0) + 4\kappa_3\frac{\mu_5^2}{F^2}\langle\bar{q}q\rangle_l^{\mathrm{LO}}. \quad (9)$$

The coefficient that regulates the dependence of χ_S with μ_5 near the chiral limit is $\kappa_1 - \kappa_3$.

For the ratio $\langle\bar{q}q\rangle_l^{\mathrm{NNLO}}(T,\mu_5)/\langle\bar{q}q\rangle_l^{\mathrm{NNLO}}(0,\mu_5)$ the κ_i dependence reduces to the combinations κ_a and κ_b (although in the chiral limit it depends only on κ_a):

$$\kappa_a = 2\kappa_1 - \kappa_2, \quad \kappa_b = \kappa_1 + \kappa_2 - \kappa_3.$$

$T_c(\mu_5)$



The ChPT curve for the physical pion mass lies very close to the chiral limit one and the lattice points (Braguta et al) clearly fall into the uncertainty given by the natural values range of κ_a and κ_b. We compared a fit with two and three points in the chiral limit with a fit in the massive case fixing the κ_a parameter. That show that the chiral limit approach with just one parameter κ_a is a robust approximation.

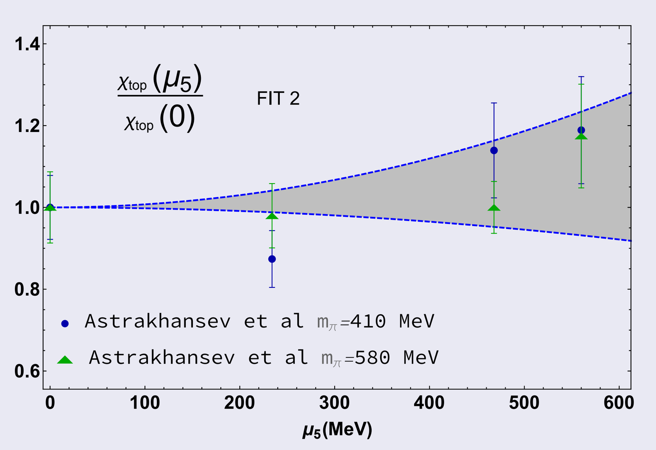
FIT	$\kappa_a \times 10^3$	$\kappa_b \times 10^3$	χ^2/dof	R^2	# points $\mu_5 \neq 0$
Fit 1 ($M = 0$)	1.7 ± 0.6	—	0.01	1.00	2
Fit 2 ($M = 0$)	2.3 ± 0.4	—	1.41	0.99	3
Fit 3	2.3 (fixed)	0 ± 1	1.36	0.99	3

The topological susceptibility

The dependence of χ_{top} with low and moderate μ_5 is controlled by the κ_3 constant:

$$\chi_{top}(\mu_5)/\chi_{top}(0) = 1 + 4\kappa_3\mu_5^2/F^2 + \mathcal{O}(1/F^4). \quad (10)$$

The fit of that observable to the lattice data shows that the results for κ_3 are compatible with zero and the error bands are much narrower than the natural values for this constant.



FIT	$\kappa_3 \times 10^3$	R^2	χ^2/dof	# points $\mu_5 \neq 0$
Fit 1	0.1 ± 1.4	0.99	1.20	2 ($m_\pi = 410$ MeV)+2 ($m_\pi = 580$ MeV)
Fit 2	0.5 ± 0.9	0.99	1.13	3 ($m_\pi = 410$ MeV)+3 ($m_\pi = 580$ MeV)

The chiral charge density

We perform a fit of $\rho_5(\mu_5)$

$$\rho_5(T = 0, \mu_5)|_{M=0} = 4F^2\mu_5^2(1 - Z + \kappa_0) + 4\kappa_4\mu_5^3 - 6\gamma_0\frac{\mu_5^2}{F^2} + \mathcal{O}(1/F^4) \quad (11)$$

The simple linear dependence setting $\kappa_4 = \gamma_0 = 0$ fits very well the lowest μ_5 lattice points. The prediction for κ_0 is consistent with the fit allowing the three parameters $\kappa_0, \kappa_4, \gamma_0$ to be free.

FIT	κ_0	$\kappa_4 \times 10^3$	$\gamma_0 \times 10^5$	R^2	# points $\mu_5 \neq 0$
Fit 1	3.2 ± 0.1	0 (fixed)	0 (fixed)	0.99	3
Fit 2	3.1 ± 0.1	7.1 ± 3.6	4.6 ± 2.4	0.99	4

This analysis does not favor a $\mu_5 \neq 0$ minimum for the free energy for moderate values of μ_5.

Conclusions

1. We have analyzed the effective chiral lagrangian for nonzero chiral imbalance for two light flavours, through its dependence with the axial chemical potential μ_5.
2. The pion dispersion relation is modified through a reduction in the pion velocity and the pion mass due to the Lorentz breaking effect.
3. We have calculated the μ_5 corrections to the vacuum energy density up to sixth order.
4. In the chiral limit T_c turns out to behave quite according to lattice analysis.
5. ρ_5 follows essentially a linear behaviour with μ_5, consistently with lattice data.
6. The lowest order corrections to χ_{top} are of order μ_5^2 controlled by the κ_3 constant.
7. The pressure at the order considered depends on κ_2 and κ_b.

References

- D. Espriu, A. Gómez Nicola and A. Vioque-Rodríguez, JHEP **06** (2020), 062 doi:10.1007/JHEP06(2020)062 [arXiv:2002.11696 [hep-ph]].
- V. V. Braguta, V. A. Goy, E. M. Ilgenfritz, A. Y. Kotov, A. V. Molochkov, M. Muller-Preussker and B. Petersson, JHEP **06** (2015), 094 doi:10.1007/JHEP06(2015)094 [arXiv:1503.06670 [hep-lat]].
- V. V. Braguta, E. M. Ilgenfritz, A. Y. Kotov, B. Petersson and S. A. Skinderev, Phys. Rev. D **93** (2016) no.3, 034509 doi:10.1103/PhysRevD.93.034509 [arXiv:1512.05873 [hep-lat]].
- N. Y. Astrakhantsev, V. V. Braguta, A. Y. Kotov, D. D. Kuznedelev and A. A. Nikolaev, Eur. Phys. J. A **57** (2021) no.1, 15 doi:10.1140/epja/s10050-020-00326-2 [arXiv:1902.09325 [hep-lat]].