Twist-3 gluon fragmentation contribution to transversely polarized hyperon production in semi-inclusive deep-inelastic scattering

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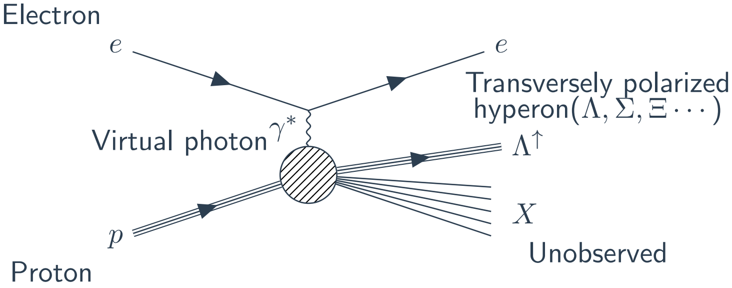
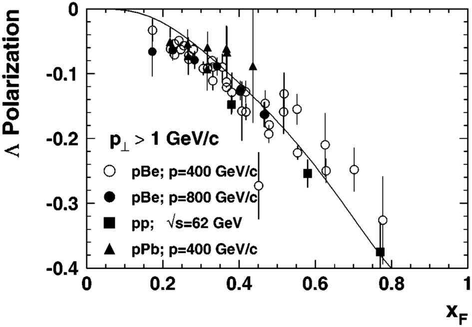
PURPOSE OF THIS TALK

We have calculated the twist-3 gluon FF contribution to
$ep \to e\Lambda^{\uparrow}X$,
which is relevant for the future EIC experiment.



INTRODUCTION

- Hyperon polarization in pp collision



Reprint from [L. Zuo-Tang, C. Boros, PRL79(1997)]
Data from [A. M. Smith et al, PLB185(1987)],
[B. Lundberg et al, PRD40(1989)] , [E. J. Ramberg et al, PLB338(1994)].

INTRODUCTION

- Collinear factorization framework of perturbative QCD.
- Twist-3 complete LO cross sectioncon for $pp \to \Lambda^{\uparrow} X$ has been already derived.

A **Twist-3 unpolarized PDF** in the initial proton

[Y. Kanazawa, Y. Koike, PRD64(2001)]
[J. Zhou, F. Yuan, Z. T. Liang, PRD78(2008)]

[Y. Koike, K. Yabe, S. Yoshida, PRD92(2015)]

B **Twist-3 quark FF** for the final hyperon

[Y. Koike, A. Metz, D. Pitonyak, K. Yabe, S. Yoshida, PRD95(2017)]

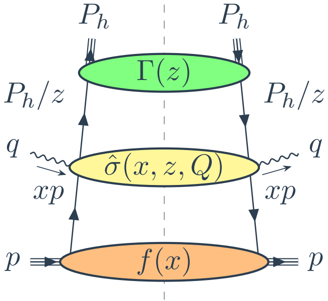
C **Twist-3 gluon FF** for the final hyperon

[Y. Koike, K. Yabe, S. Yoshida, [arXiv:2107.03113], PRD in press]
[RI, Y. Koike, K. Yabe, S. Yoshida, in preparation, (Concise derivation of the formalism and cancellation of ghost terms.)]

- We applied the formalism of C to $ep \to e\Lambda^{\uparrow} X$.
- Chiral-even twist-3 quark (B) and gluon (C) FFs mix under renormalization. Both need to be included.

INTRODUCTION

QCD factorization for SIDIS: $p(p) + \gamma^*(q) \to \Lambda^\uparrow(P_h) + X$



- **Distribution fn.** $f(x)$ with momentum fraction x
- **Hard parts** $\hat{\sigma}(x,z,Q)$ Perturbative parton interaction
- **Fragmentation fn.** $\Gamma(z)$ with momentum fraction z

Convolution with respect to momentum fractions x and z.

Cross section (hadronic tensor) $\sigma \sim f(x) \otimes \hat{\sigma}(x,z,Q) \otimes \Gamma(z)$

INTRODUCTION

3 types of the twist-3 contributions to $ep \to e\Lambda^{\uparrow}X$:

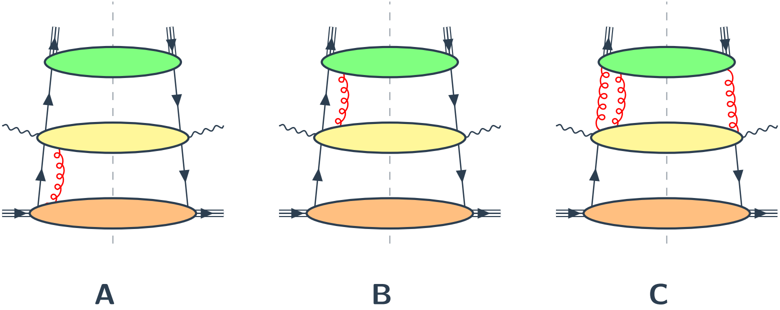
- A twist-3 unpolarized PDF $\otimes$ Twist-2 transversity FF [COMPLETED]

  [Y. Koike, S. Usui, K. Yabe, S. Yoshida, in preparation]
- B twist-2 unpolarized PDF $\otimes$ Twist-3 quark FF [COMPLETED]

  [Y. Koike, K. Takada, K. Yabe, S. Yoshida, in preparation]

  formalism by [K. Kanazawa, Y. Koike, PRD88(2013)] for $ep^{\uparrow} \to e\pi X$
- C twist-2 unpolarized PDF $\otimes$ Twist-3 gluon FF [THIS WORK]

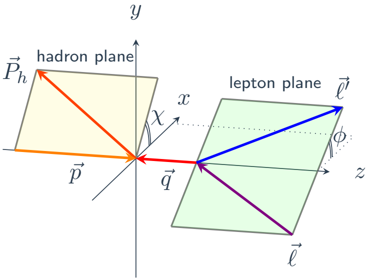
  [RI, Y. Koike, K. Yabe, S. Yoshida, in preparation]

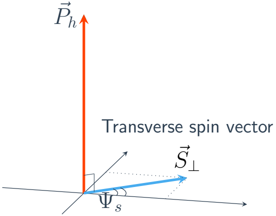


KINEMATICS

Hadron frame

$$e(\vec{\ell}) + p(\vec{p}) \to e'(\vec{\ell'}) + \Lambda^{\uparrow}(\vec{P}_h) + X$$





5 Lorentz invariants

1. $S_{ep} = (p+\ell)^2 \simeq 2p\cdot\ell$
2. $Q^2 = -q^2 = -(\ell-\ell')^2 > 0$
3. $x_{bj} = Q^2/2p\cdot q$
4. $z_f = p\cdot P_h/p\cdot q$
5. $q_{\mathrm{T}} = \sqrt{-q_t^2}, \quad q_t^\mu \equiv q^\mu - \frac{P_h\cdot q}{p\cdot P_h}p^\mu - \frac{p\cdot q}{p\cdot P_h}P_h^\mu$

KINEMATICS

- The cross section decribed by 5 Lorentz invarinats

$$\frac{\mathrm{d}^6\sigma}{\mathrm{d}x_{bj}\mathrm{d}Q^2\mathrm{d}z_f\mathrm{d}q_T^2\mathrm{d}\phi\mathrm{d}\chi}$$

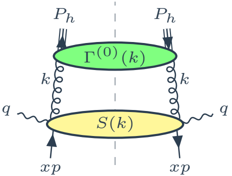
$$= \frac{\alpha_{em}^2 z_f}{128\pi^4 x_{bj}^2 S_{ep}^2 Q^2} \overset{\textbf{Leptonic tensor}}{\boxed{L^{\rho\sigma}(\ell,\ell')}} \overset{\textbf{Hadronic tensor}}{\boxed{\int \frac{\mathrm{d}x}{x} \underset{\textbf{Twist-2 PDF}}{f_1}(x) w_{\rho\sigma}(xp,q,P_h)}}$$

where $L^{\rho\sigma} = 2(\ell^\rho \ell'^\sigma + \ell^\sigma \ell'^\rho) - Q^2 g^{\mu\nu}$

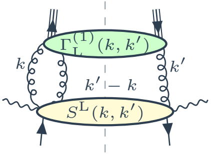
- $w_{\rho\sigma}$ consists of **FFs** and **hard parts**.
- Calculation of $w_{\rho\sigma}$ is essential.

HADRONIC TENSOR

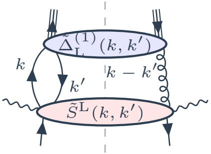
$$w_{\rho\sigma} \equiv w^{(a)}_{\rho\sigma} + w^{(b)}_{\rho\sigma} + w^{(c)}_{\rho\sigma} + w^{(d)}_{\rho\sigma} + w^{(e)}_{\rho\sigma}$$



$$w^{(a)} = \int \frac{\mathrm{d}^4k}{(2\pi)^4} \Gamma^{(0)\mu\nu}_{ab}(k) S^{ab}_{\mu\nu}(k)$$



$$w^{(b)} = \frac{1}{2} \iint \frac{\mathrm{d}^4k}{(2\pi)^4} \frac{\mathrm{d}^4k'}{(2\pi)^4} \Gamma^{(1)\mu\nu\lambda}_{\mathrm{L}abc}(k,k') S^{\mathrm{L}abc}_{\mu\nu\lambda}(k,k')$$



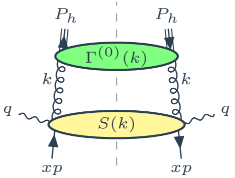
$$w^{(d)} = \mathrm{Tr} \iint \frac{\mathrm{d}^4k}{(2\pi)^4} \frac{\mathrm{d}^4k'}{(2\pi)^4} \tilde{\Delta}^{(1)\alpha}_{\mathrm{L}a}(k,k') \tilde{S}^{\mathrm{L}a}_{\alpha}(k,k')$$

$w^{(c)}$ and $w^{(e)}$ are of mirror of $w^{(b)}$ and $w^{(d)}$, respectively.

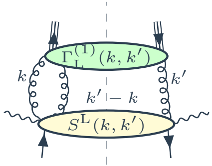
HADRONIC TENSOR

Definition of Fragmentation matrix elements

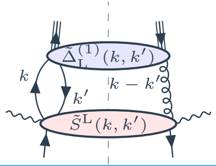
NOT color gauge invariant. (mirror diagrams also considered)



$$\Gamma^{(0)\mu\nu}_{ab}(k) \equiv \frac{1}{N}\sum_X \int \mathrm{d}^4\xi \mathrm{e}^{-\mathrm{i}k\cdot\xi}$$
$$\times \langle 0|A^\nu_b(0)|\Lambda^\uparrow X\rangle\langle\Lambda^\uparrow X|A^\mu_a(\xi)|0\rangle$$



$$\Gamma^{(1)\mu\nu\lambda}_{\mathrm{L}abc}(k,k') \equiv \frac{1}{N}\sum_X \iint \mathrm{d}^4\xi\mathrm{d}^4\eta \mathrm{e}^{-\mathrm{i}k\cdot\xi}\mathrm{e}^{-\mathrm{i}(k'-k)\cdot\eta}$$
$$\times \langle 0|A^\nu_b(0)|\Lambda^\uparrow X\rangle\langle\Lambda^\uparrow X|A^\mu_a(\xi)gA^\lambda_c(\eta)|0\rangle$$



$$\tilde{\Delta}^{(1)\alpha}_{\mathrm{L}a,ij}(k,k') \equiv \frac{1}{N}\sum_X \iint \mathrm{d}^4\xi\mathrm{d}^4\eta \mathrm{e}^{-\mathrm{i}k\cdot\xi}\mathrm{e}^{-\mathrm{i}(k'-k)\cdot\eta}$$
$$\times \langle 0|gA^\alpha_a(\eta)|\Lambda^\uparrow X\rangle\langle\Lambda^\uparrow X|\psi_i(0)\bar{\psi}_j(\xi)|0\rangle$$

HADRONIC TENSOR

- Collinear expansion with respect to k and k' around P_h:
  $k^\alpha = P_h^\alpha/z + \Omega^\alpha{}_\beta k^\beta \qquad k'^\alpha = P_h^\alpha/z' + \Omega^\alpha{}_\beta k^\beta$
  where $\Omega^\alpha{}_\beta = g^\alpha_\beta - P_h^\alpha w_\beta$
- Ward-Takahashi identity for the hard parts:

$$k^\mu S^{ab}_{\mu\nu}(k) = 0,$$

$$k^\mu S^{\mathrm{L}abc}_{\mu\nu\lambda}(k,k') = \frac{\mathrm{i} f^{abc}}{N^2-1}\delta^{a'b'} S^{a'b'}_{\lambda\nu}(k')$$

$$k'^\nu S^{\mathrm{L}abc}_{\mu\nu\lambda}(k,k') = 0$$

$$(k'-k)^\lambda S^{\mathrm{L}abc}_{\mu\nu\lambda}(k,k') = \frac{-\mathrm{i} f^{abc}}{N^2-1}\delta^{a'b'} S^{a'b'}_{\mu\nu}(k')$$

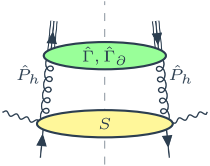
$$(k-k')^\alpha \tilde{S}^{\mathrm{L}a}_{\alpha}(k,k') = 0$$

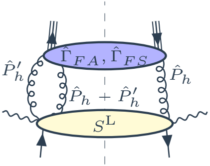
After very lengthy calculation ......

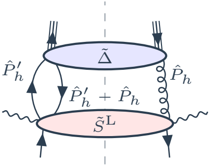
HADRONIC TENSOR

Color gauge invariant FFs

$$
\begin{aligned}
w_{\rho\sigma} = {} & \Omega^{\alpha}{}_{\mu}\Omega^{\beta}{}_{\nu}\int \mathrm{d}\left(\frac{1}{z}\right) z^2 \hat{\Gamma}^{\mu\nu}(z) S_{\alpha\beta,\rho\sigma}(1/z) \\
& - \mathrm{i}\Omega^{\alpha}{}_{\mu}\Omega^{\beta}{}_{\nu}\Omega^{\gamma}{}_{\lambda}\int \mathrm{d}\left(\frac{1}{z}\right) z^2 \hat{\Gamma}^{\mu\nu\lambda}_{\partial}(z) \left.\frac{\partial S_{\alpha\beta,\rho\sigma}(k)}{\partial k^{\gamma}}\right|_{k\to P_h/z(\equiv\hat{P}_h)} \\
& + \Re\Bigg[\mathrm{i}\Omega^{\alpha}{}_{\mu}\Omega^{\beta}{}_{\nu}\Omega^{\gamma}{}_{\lambda}\iint \mathrm{d}\left(\frac{1}{z}\right)\mathrm{d}\left(\frac{1}{z'}\right) zz'\frac{1}{1/z-1/z'} \\
& \times\left(-\frac{\mathrm{i}f^{abc}}{N}\hat{\Gamma}^{\mu\nu\lambda}_{FA}\left(\frac{1}{z'},\frac{1}{z}\right)+\frac{Nd^{abc}}{N^2-4}\hat{\Gamma}^{\mu\nu\lambda}_{FS}\left(\frac{1}{z'},\frac{1}{z}\right)\right) S^{\mathrm{L}abc}_{\alpha\beta\gamma,\rho\sigma}\left(\frac{1}{z'},\frac{1}{z}\right)\Bigg] \\
& - \Im\Omega^{\alpha}{}_{\mu}\iint \mathrm{d}\left(\frac{1}{z}\right)\mathrm{d}\left(\frac{1}{z'}\right) z\,\mathrm{Tr}\left[\tilde{\Delta}^{\mu}\left(\frac{1}{z'},\frac{1}{z'}-\frac{1}{z}\right)\tilde{S}^{\mathrm{L}}_{\alpha,\rho\sigma}\left(\frac{1}{z'},\frac{1}{z'}-\frac{1}{z}\right)\right]
\end{aligned}
$$







TWIST-3 GLUON FF

- *Intrinsic* FF

$$\hat{\Gamma}^{\alpha\beta}(z) = \frac{1}{N^2-1}\sum_X \int \frac{\mathrm{d}\lambda}{2\pi} \mathrm{e}^{-\mathrm{i}\lambda/z} \langle 0| \Big([\infty w, 0]\boldsymbol{F}^{w\beta}(0)\Big)_a |\Lambda^\uparrow X\rangle\langle\Lambda^\uparrow X| \big(\boldsymbol{F}^{w\alpha}(\lambda w)[\lambda w, \infty w]\big)_a |0\rangle$$

$$= M_h \epsilon^{P_h w S_\perp \{\alpha} w^{\beta\}} \underbrace{\Delta\hat{G}_{3\bar{T}}(z)} + \cdots$$

- *Kinematical* FF

$$\hat{\Gamma}^{\alpha\beta\gamma}_{\partial}(z) = \frac{1}{N^2-1}\sum_X \int \frac{\mathrm{d}\lambda}{2\pi} \mathrm{e}^{-\mathrm{i}\lambda/z} \langle 0| \Big([\infty w, 0]\boldsymbol{F}^{w\beta}(0)\Big)_a |\Lambda^\uparrow X\rangle\langle\Lambda^\uparrow X| \big(\boldsymbol{F}^{w\alpha}(\lambda w)[\lambda w, \infty w]\big)_a |0\rangle \overleftarrow{\partial}^{\gamma}$$

$$= -\mathrm{i}\frac{M_h}{2} g_\perp^{\alpha\beta} \epsilon^{P_h w S_\perp \gamma} \underbrace{\hat{G}_T^{(1)}(z)} - \mathrm{i}\frac{M_h}{8}\Big(\epsilon^{P_h w S_\perp \{\alpha} g_\perp^{\beta\}\gamma} + \epsilon^{P_h w \gamma\{\alpha} S_\perp^{\beta\}}\Big) \underbrace{\Delta\hat{H}_T^{(1)}(z)} + \cdots$$

where $[\lambda w, \infty w]$
gauge link in the adjoint representation connecting λw and ∞w.

- NOT gluon field A_a^μ, BUT strength tensor $F_a^{\mu\nu}$.
- $w_{\rho\sigma}$ is color gauge invariant in $\mathcal{O}(g)$ accuracy.

TWIST-3 GLUON FF

- *Dynamical* FF

$$\hat{\Gamma}^{\alpha\beta\gamma}_{FA}\left(\frac{1}{z},\frac{1}{z'}\right)$$

$$= \frac{-\mathrm{i}f_{abc}}{N^2-1}\sum_X\iint\frac{\mathrm{d}\lambda}{2\pi}\frac{\mathrm{d}\mu}{2\pi}\mathrm{e}^{-\mathrm{i}\lambda/z}\mathrm{e}^{-\mathrm{i}\mu(1/z'-1/z)}\langle 0|F_b^{w\beta}(0)|\Lambda^\uparrow X\rangle\langle\Lambda^\uparrow X|F_a^{w\alpha}(\lambda w)gF_c^{w\gamma}(\mu w)|0\rangle$$

$$= -M_h\left(\underline{\hat{N}_1\left(\frac{1}{z},\frac{1}{z'}\right)}g_\perp^{\alpha\gamma}\epsilon^{P_h w S_\perp \beta} + \underline{\hat{N}_2\left(\frac{1}{z},\frac{1}{z'}\right)}g_\perp^{\beta\gamma}\epsilon^{P_h w S_\perp \alpha} - \underline{\hat{N}_2\left(\frac{1}{z'}-\frac{1}{z},\frac{1}{z'}\right)}g_\perp^{\alpha\beta}\epsilon^{P_h w S_\perp \gamma}\right)$$

$$\hat{\Gamma}^{\alpha\beta\gamma}_{FS}\left(\frac{1}{z},\frac{1}{z'}\right)$$

$$= \frac{d_{abc}}{N^2-1}\sum_X\iint\frac{\mathrm{d}\lambda}{2\pi}\frac{\mathrm{d}\mu}{2\pi}\mathrm{e}^{-\mathrm{i}\lambda/z}\mathrm{e}^{-\mathrm{i}\mu(1/z'-1/z)}\langle 0|F_b^{w\beta}(0)|\Lambda^\uparrow X\rangle\langle\Lambda^\uparrow X|F_a^{w\alpha}(\lambda w)F_c^{w\gamma}(\mu w)|0\rangle$$

$$= -M_h\left(\underline{\hat{O}_1\left(\frac{1}{z},\frac{1}{z'}\right)}g_\perp^{\alpha\gamma}\epsilon^{P_h w S_\perp \beta} + \underline{\hat{O}_2\left(\frac{1}{z},\frac{1}{z'}\right)}g_\perp^{\beta\gamma}\epsilon^{P_h w S_\perp \alpha} + \underline{\hat{O}_2\left(\frac{1}{z'}-\frac{1}{z},\frac{1}{z'}\right)}g_\perp^{\alpha\beta}\epsilon^{P_h w S_\perp \gamma}\right)$$

$$\tilde{\Delta}^\alpha\left(\frac{1}{z},\frac{1}{z'}\right) = \frac{1}{N}\sum_X\iint\frac{\mathrm{d}\lambda}{2\pi}\frac{\mathrm{d}\mu}{2\pi}\mathrm{e}^{-\mathrm{i}\lambda/z}\mathrm{e}^{-\mathrm{i}\mu(1/z'-1/z)}\langle 0|F_a^{w\alpha}(\mu w)|\Lambda^\uparrow X\rangle\langle\Lambda^\uparrow X|\bar{\psi}_j(\lambda w)t^a\psi_i(0)|0\rangle$$

$$= M_h\left(\epsilon^{\alpha P_h w S_\perp}(\not{P}_h)_{ij}\underline{\tilde{D}_{FT}\left(\frac{1}{z},\frac{1}{z'}\right)} + \mathrm{i}S_\perp^\alpha(\gamma_5\not{P}_h)_{ij}\underline{\tilde{G}_{FT}\left(\frac{1}{z},\frac{1}{z'}\right)}\right)$$

TWIST-3 GLUON FF

- Twist-3 gluon FFs contributing to $ep \to e\Lambda^{\uparrow}X$

$$\Big\{\Delta\hat{G}_{3\bar{T}}(z),\ \hat{G}_{T}^{(1)}(z),\ \Delta\hat{H}_{T}^{(1)}(z),\ \Im\hat{N}_1\left(\frac{1}{z},\frac{1}{z'}\right),\ \Im\hat{N}_2\left(\frac{1}{z},\frac{1}{z'}\right),$$

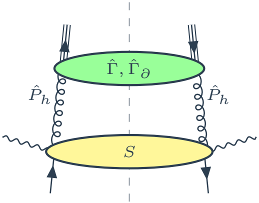
$$\Im\hat{O}_1\left(\frac{1}{z},\frac{1}{z'}\right),\ \Im\hat{O}_2\left(\frac{1}{z},\frac{1}{z'}\right),\ \Im\tilde{D}_{FT}\left(\frac{1}{z},\frac{1}{z'}\right),\ \Im\tilde{G}_{FT}\left(\frac{1}{z},\frac{1}{z'}\right)\Big\}$$

- Exact relations between FFs from QCD equation of motion and Lorentz invariance
  [Y. Koike, K. Yabe and S. Yoshida, PRD 101 (2020)]

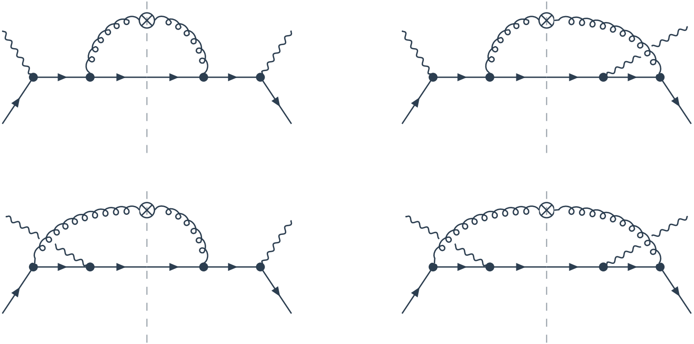
$$\Big\{\cancel{\Delta\hat{G}_{3\bar{T}}(z)},\ \hat{G}_{T}^{(1)}(z),\ \Delta\hat{H}_{T}^{(1)}(z),\ \Im\hat{N}_1\left(\frac{1}{z},\frac{1}{z'}\right),\ \Im\hat{N}_2\left(\frac{1}{z},\frac{1}{z'}\right),$$

$$\Im\hat{O}_1\left(\frac{1}{z},\frac{1}{z'}\right),\ \Im\hat{O}_2\left(\frac{1}{z},\frac{1}{z'}\right),\ \Im\tilde{D}_{FT}\left(\frac{1}{z},\frac{1}{z'}\right),\ \Im\tilde{G}_{FT}\left(\frac{1}{z},\frac{1}{z'}\right)\Big\}$$

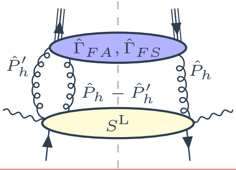
HARD PARTS

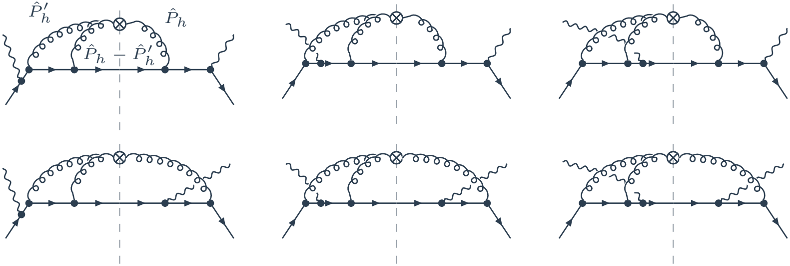


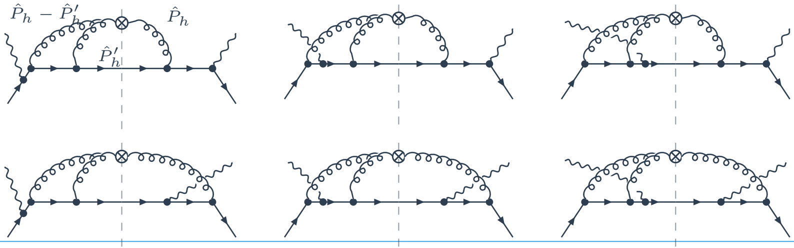
Regarding $\hat{\Gamma}^{\mu\nu}(z)$ and $\hat{\Gamma}_{\partial}^{\mu\nu\lambda}(z)$

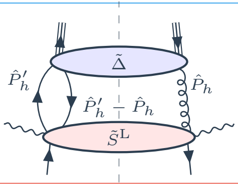


Regarding $\hat{\Gamma}^{\mu\nu\lambda}_{FA}\left(\frac{1}{z'},\frac{1}{z}\right)$ and $\hat{\Gamma}^{\mu\nu\lambda}_{FS}\left(\frac{1}{z'},\frac{1}{z}\right)$

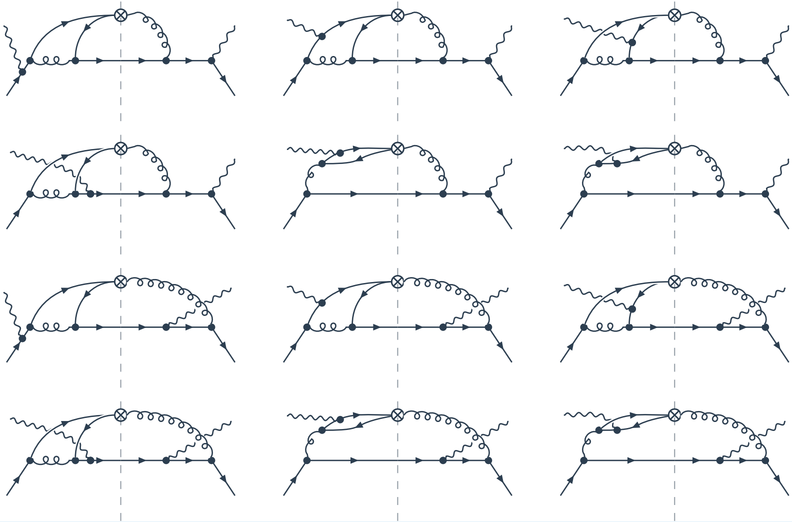








Regarding $\tilde{\Delta}^{\mu}\left(\frac{1}{z'}, \frac{1}{z'} - \frac{1}{z}\right)$



CROSS SECTION

$$\frac{\mathrm{d}^6\sigma}{\mathrm{d}x_{bj}\mathrm{d}Q^2\mathrm{d}z_f\mathrm{d}q_{\mathrm{T}}^2\mathrm{d}\phi\mathrm{d}\chi} \qquad \hat{x}\equiv x_{bj}/x,\ \hat{z}\equiv z_f/z$$

$$\begin{aligned}
&= \frac{\alpha_{em}^2\alpha_s M_h}{16\pi^2 x_{bj}^2 S_{ep}^2 Q^2}\sum_k \mathscr{A}_k(\varphi)\mathcal{S}_k(\Psi_s)\iint \mathrm{d}x\mathrm{d}\left(\frac{1}{z}\right)\frac{z^3}{x}f_1(x)\delta\left(\frac{q_T^2}{Q^2}-\left(1-\frac{1}{\hat{x}}\right)\left(1-\frac{1}{\hat{z}}\right)\right)\\
&\Big\{\hat{G}_T^{(1)}(z)\hat{\sigma}_G^k+\Delta\hat{H}_T^{(1)}(z)\hat{\sigma}_H^k\\
&+\int\mathrm{d}\left(\frac{1}{z'}\right)\Bigg[\frac{1}{1/z-1/z'}\Im\Bigg(\hat{N}_1\left(\frac{1}{z'},\frac{1}{z}\right)\hat{\sigma}_{N1}^k+\hat{N}_2\left(\frac{1}{z'},\frac{1}{z}\right)\hat{\sigma}_{N2}^k+\hat{N}_2\left(\frac{1}{z}-\frac{1}{z'},\frac{1}{z}\right)\hat{\sigma}_{N3}^k\Bigg)\\
&+\frac{1}{z}\left(\frac{1}{1/z-1/z'}\right)^2\Im\Bigg(\hat{N}_1\left(\frac{1}{z'},\frac{1}{z}\right)\hat{\sigma}_{DN1}^k+\hat{N}_2\left(\frac{1}{z'},\frac{1}{z}\right)\hat{\sigma}_{DN2}^k+\hat{N}_2\left(\frac{1}{z}-\frac{1}{z'},\frac{1}{z}\right)\hat{\sigma}_{DN3}^k\Bigg)\\
&+\{N\to O\}\Bigg]\\
&+\int\mathrm{d}\left(\frac{1}{z'}\right)\frac{2}{C_F}\Bigg[\Im\tilde{D}_{FT}\left(\frac{1}{z'},\frac{1}{z'}-\frac{1}{z}\right)\Bigg(\hat{\sigma}_{DF1'}^k+\frac{1}{z}\frac{1}{1/z-1/z'}\hat{\sigma}_{DF2}^k+\frac{z'}{z}\hat{\sigma}_{DF3}^k\\
&+\frac{1}{1-(1-q_T^2/Q^2)z_f/z'}\hat{\sigma}_{DF4}^k+\frac{1}{1-(1-q_T^2/Q^2)z_f(1/z-1/z')}\hat{\sigma}_{DF5}^k\Bigg)\\
&+\ \{D\to G\}\Bigg]\Big\}\,.
\end{aligned}$$

CROSS SECTION

$$\frac{\mathrm{d}^6\sigma}{\mathrm{d}x_{bj}\mathrm{d}Q^2\mathrm{d}z_f\mathrm{d}q_{\mathrm{T}}^2\mathrm{d}\phi\mathrm{d}\chi}$$

Twist-3 gluon FFs

$$\sim \sum_k \mathscr{A}_k(\varphi)\mathcal{S}_k(\Psi_s)\; f_1(x) \otimes \left\{\begin{matrix} \hat{G}_T^{(1)}, & \Delta\hat{H}_T^{(1)} \\ \Im\hat{N}_i, & \Im\hat{O}_i \\ \Im\tilde{D}_{FT}, & \Im\tilde{G}_{FT} \end{matrix}\right\} \otimes \left\{\hat{\sigma}^k\right\}$$

Twist-2 PDF

Hard parts

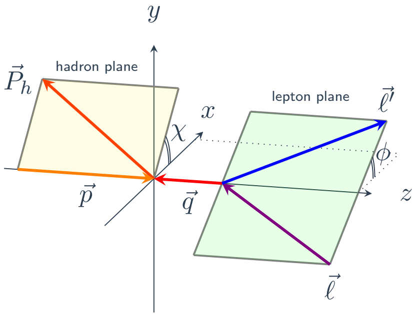
$\varphi \equiv \phi - \chi$

Ψ_s: azimuthal angle of $S_\perp$

$\hat{\sigma}^k$ depend on $\hat{x}, \hat{z}, Q$and q_T,

where $\hat{x} \equiv x_{bj}/x, \hat{z} \equiv z_f/z$.

$\mathscr{A}_k$ and $\mathcal{S}_k$ are defined
in the next slide



CROSS SECTION

Angle dependencies

$$\mathscr{A}_1(\varphi) = 1 + \cosh^2\psi, \qquad \mathscr{A}_2(\varphi) = -2$$
$$\mathscr{A}_3(\varphi) = -\cos\varphi\sinh 2\psi, \qquad \mathscr{A}_4(\varphi) = \cos 2\varphi\sinh^2\psi$$
$$\mathscr{A}_8(\varphi) = -\sin\varphi\sinh 2\psi, \qquad \mathscr{A}_9(\varphi) = \sin 2\varphi\sinh^2\psi$$

[R. Meng, F. I. Olness and D. E. Soper, NPB371 (1992)]

$$\cosh\psi \equiv 2x_{bj}S_{ep}/Q^2 - 1$$

$$\mathcal{S}_{1,2,3,4} \equiv \sin\Psi_s, \mathcal{S}_{8,9} \equiv \cos\Psi_s.$$

- 5 structure functions

$$\frac{\mathrm{d}^6\sigma}{\mathrm{d}x_{bj}\mathrm{d}Q^2\mathrm{d}z_f\mathrm{d}q_\mathrm{T}^2\mathrm{d}\phi\mathrm{d}\chi}$$
$$= F_0(\sin\Psi_s) + F_1(\sin\Psi_s\cos\varphi) + F_2(\sin\Psi_s\cos 2\varphi)$$
$$+ F_3(\cos\Psi_s\sin\varphi) + F_4(\cos\Psi_s\sin 2\varphi)$$

CONCLUSION

- We have derived the cross-section formula ($\mathcal{O}(\alpha_s)$) of $ep \to e\Lambda^{\uparrow}X$ process related to the twist-3 gluon fragmentation functions in the collinear framework.

A Twist-3 distribution effect [COMPLETED]

B Twist-3 quark fragmentation effect [COMPLETED]

C Twist-3 gluon fragmentation effect [THIS WORK]

- Twist-3 gluon FF contribution could play an important role, since gluons are ample in the collision environment.
- Satisfying EM and color gauge invariances.
- Classified into 5 structure functions.

Future outlook

- Numerical estimate of each effect.
- Inclusion of the NLO correction, $\mathcal{O}(\alpha_s^2)$.

*NLO contribution has large correction for other unpolarized cross section [Hinderer,Schlegel, Vogelsang PRD92(2015),erratum[PRD93(2016)]]